



**NANYANG
TECHNOLOGICAL
UNIVERSITY**
SINGAPORE

Entrance Examination Syllabus 2021

Maths (A)

updated as at 21 Oct 2020



CONTENT OUTLINE

Knowledge of the content of the O-Level Mathematics syllabus and of some of the content of the O-Level Additional Mathematics syllabuses are assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics. The assumed knowledge for O-Level Additional Mathematics is appended after this section.

	Topic/Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	Functions and graphs	
1.1	Functions	<p>Include:</p> <ul style="list-style-type: none"> concepts of function, domain and range use of notations such as $f(x) = x^2 + 5$, $f: x \mapsto x^2 + 5$, $f^{-1}(x)$, $fg(x)$ and $f^2(x)$ finding inverse functions and composite functions conditions for the existence of inverse functions and composite functions domain restriction to obtain an inverse function relationship between a function and its inverse <p>Exclude the use of the relation $(fg)^{-1} = g^{-1}f^{-1}$, and restriction of domain to obtain a composite function.</p>
1.2	Graphs and transformations	<p>Include:</p> <ul style="list-style-type: none"> use of a graphing calculator to graph a given function important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; \quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ $y = \frac{ax + b}{cx + d}$ $y = \frac{ax^2 + bx + c}{dx + e}$ determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of x and/or y effect of transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$, and combinations of these transformations relating the graphs of $y = f^{-1}(x)$, $y = f(x)$, $y = f(x)$, and $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$ simple parametric equations and their graphs

	Topic/Sub-topics	Content
1.3	Equations and inequalities	<p>Include:</p> <ul style="list-style-type: none"> • formulating an equation, a system of linear equations, or inequalities from a problem situation • solving an equation exactly or approximately using a graphing calculator • solving a system of linear equations using a graphing calculator • solving inequalities of the form $\frac{f(x)}{g(x)} > 0$ where $f(x)$ and $g(x)$ are linear expressions or quadratic expressions that are either factorisable or always positive • concept of x, and use of relations $x - a < b \Leftrightarrow a - b < x < a + b$ and $x - a > b \Leftrightarrow x < a - b$ or $x > a + b$, in the course of solving inequalities • solving inequalities by graphical methods
2	Sequences and series	
2.1	Sequences and series	<p>Include:</p> <ul style="list-style-type: none"> • concepts of sequence and series for finite and infinite cases • sequence as function $y = f(n)$ where n is a positive integer • relationship between u_n (the nth term) and S_n (the sum to n terms) • sequence given by a formula for the nth term • use of Σ notation • sum and difference of two series • summation of series by the method of differences • convergence of a series and the sum to infinity • formula for the nth term and the sum of a finite arithmetic series • formula for the nth term and the sum of a finite geometric series • condition for convergence of an infinite geometric series • formula for the sum to infinity of a convergent geometric series

	Topic/Sub-topics	Content
3	Vectors	
3.1	Basic properties of vectors in two and three dimensions	<p>Include:</p> <ul style="list-style-type: none"> addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations use of notations such as $\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}, xi + yj, xi + yj + zk, \overrightarrow{AB}, \mathbf{a}$ position vectors, displacement vectors and direction vectors magnitude of a vector unit vectors distance between two points collinearity use of the ratio theorem in geometrical applications
3.2	Scalar and vector products in vectors	<p>Include:</p> <ul style="list-style-type: none"> concepts of scalar product and vector product of vectors and their properties angle between two vectors geometrical meanings of $\mathbf{a} \cdot \hat{\mathbf{n}}$ and $\mathbf{a} \times \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector <p>Exclude triple products $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} \times \mathbf{c}$.</p>
3.3	Three-dimensional vector geometry	<p>Include:</p> <ul style="list-style-type: none"> vector and cartesian equations of lines and planes finding the foot of the perpendicular and distance from a point to a line or to a plane finding the angle between two lines, between a line and a plane, or between two planes relationships between <ul style="list-style-type: none"> (i) two lines (coplanar or skew) (ii) a line and a plane (iii) two planes <p>Exclude:</p> <ul style="list-style-type: none"> finding the shortest distance between two skew lines finding an equation for the common perpendicular to two skew lines

	Topic/Sub-topics	Content
4	Introduction to Complex numbers	
4.1	Complex numbers expressed in cartesian form	<p>Include:</p> <ul style="list-style-type: none"> extension of the number system from real numbers to complex numbers complex roots of quadratic equations conjugate of a complex number four operations of complex numbers equality of complex numbers conjugate roots of a polynomial equation with real coefficients
4.2	Complex numbers expressed in polar form	<p>Include:</p> <ul style="list-style-type: none"> representation of complex numbers in the Argand diagram complex numbers expressed in the form $r(\cos \theta + i \sin \theta)$, or $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$ calculation of modulus (r) and argument (θ) of a complex number multiplication and division of two complex numbers expressed in polar form
5	Calculus	
5.1	Differentiation	<p>Include:</p> <ul style="list-style-type: none"> graphical interpretation of <ul style="list-style-type: none"> (i) $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$ (ii) $f''(x) > 0$ and $f''(x) < 0$ relating the graph of $y = f'(x)$ to the graph of $y = f(x)$ differentiation of simple functions defined implicitly or parametrically determining the nature of the stationary points (local maximum and minimum points and points of inflexion) analytically, in simple cases, using the first derivative test or the second derivative test locating maximum and minimum points using a graphing calculator finding the approximate value of a derivative at a given point using a graphing calculator finding equations of tangents and normals to curves, including cases where the curve is defined implicitly or parametrically local maxima and minima problems connected rates of change problems <p>Exclude finding non-stationary points of inflexion and finding second derivatives of functions defined parametrically.</p>

	Topic/Sub-topics	Content
5.2	Maclaurin series	<p>Include:</p> <ul style="list-style-type: none"> standard series expansion of $(1 + x)^n$ for any rational n, e^x, $\sin x$, $\cos x$ and $\ln(1 + x)$ derivation of the first few terms of the Maclaurin series by <ul style="list-style-type: none"> repeated differentiation, e.g. $\sec x$ repeated implicit differentiation, e.g. $y^3 + y^2 + y = x^2 - 2x$ using standard series, e.g. $e^x \cos 2x$, $\ln\left(\frac{1+x}{1-x}\right)$ range of values of x for which a standard series converges concept of “approximation” small angle approximations: $\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$ <p>Exclude derivation of the general term of the series.</p>
5.3	Integration techniques	<p>Include:</p> <ul style="list-style-type: none"> integration of $f'(x)[f(x)]^n$ (including $n = -1$), $f'(x)e^{f(x)}$, $\sin^2 x$, $\cos^2 x$, $\tan^2 x$, $\sin mx \cos nx$, $\cos mx \cos nx$ and $\sin mx \sin nx$ $\frac{1}{a^2 + x^2}$, $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{a^2 - x^2}$ and $\frac{1}{x^2 - a^2}$ integration by a given substitution integration by parts
5.4	Definite integrals	<p>Include:</p> <ul style="list-style-type: none"> concept of definite integral as a limit of sum definite integral as the area under a curve evaluation of definite integrals finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves area below the x-axis finding the area under a curve defined parametrically finding the volume of revolution about the x- or y-axis finding the approximate value of a definite integral using a graphing calculator <p>Exclude finding the volume of revolution about the x-axis or y-axis where the curve is defined parametrically.</p>

	Topic/Sub-topics	Content
5.5	Differential equations	<p>Include:</p> <ul style="list-style-type: none"> solving for the general solutions and particular solutions of differential equations of the forms <ul style="list-style-type: none"> (i) $\frac{dy}{dx} = f(x)$ (ii) $\frac{dy}{dx} = f(y)$ (iii) $\frac{d^2y}{dx^2} = f(x)$ including those that can be reduced to (i) and (ii) by means of a given substitution formulating a differential equation from a problem situation interpreting a differential equation and its solution in terms of a problem situation

	Topic/Sub-topics	Content
SECTION B: PROBABILITY AND STATISTICS		
6	Probability and Statistics	
6.1	Probability	<p>Include:</p> <ul style="list-style-type: none"> • addition and multiplication principles for counting • concepts of permutation (nP_r) and combination (nC_r) • arrangements of objects in a line or in a circle, including cases involving repetition and restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $P(A') = 1 - P(A)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A B) = \frac{P(A \cap B)}{P(B)}$
6.2	Discrete random variables	<p>Include:</p> <ul style="list-style-type: none"> • concept of discrete random variables, probability distributions, expectations and variances • concept of binomial distribution $B(n, p)$ as an example of a discrete probability distribution and use of $B(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of binomial distribution (without proof) <p>Exclude finding cumulative distribution function of a discrete random variable.</p>

	Topic/Sub-topics	Content
6.3	Normal distribution	<p>Include:</p> <ul style="list-style-type: none"> concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $N(\mu, \sigma^2)$ as a probability model standard normal distribution finding the value of $P(X < x_1)$ or a related probability, given the values of x_1, μ, σ symmetry of the normal curve and its properties finding a relationship between x_1, μ, σ given the value of $P(X < x_1)$, or a related probability solving problems involving the use of $E(aX + b)$ and $\text{Var}(aX + b)$ solving problems involving the use of $E(aX + bY)$ and $\text{Var}(aX + bY)$, where X and Y are independent <p>Exclude normal approximation to binomial distribution.</p>
6.4	Sampling	<p>Include:</p> <ul style="list-style-type: none"> concepts of population and simple random sample concept of the sample mean \bar{X} as a random variable with $E(\bar{X}) = \mu$ and $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$ distribution of sample means from a normal population use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. $n \geq 30$) calculation and use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form Σx and Σx^2, or $\Sigma(x - a)$ and $\Sigma(x - a)^2$
6.5	Hypothesis testing	<p>Include:</p> <ul style="list-style-type: none"> concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance and p-value formulation of hypotheses and testing for a population mean based on: <ul style="list-style-type: none"> a sample from a normal population of known variance a large sample from any population 1-tail and 2-tail tests interpretation of the results of a hypothesis test in the context of the problem <p>Exclude the use of the term 'Type I' error, concept of Type II error and testing the difference between two population means.</p>

	Topic/Sub-topics	Content
6.6	Correlation and Linear regression	<p>Include:</p> <ul style="list-style-type: none"> • use of scatter diagram to determine if there is a plausible linear relationship between the two variables • correlation coefficient as a measure of the fit of a linear model to the scatter diagram • finding and interpreting the product moment correlation coefficient (in particular, values close to -1, 0 and 1) • concepts of linear regression and method of least squares to find the equation of the regression line • concepts of interpolation and extrapolation • use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model • use of a square, reciprocal or logarithmic transformation to achieve linearity <p>Exclude:</p> <ul style="list-style-type: none"> • derivation of formulae • relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients • hypothesis tests

ASSUMED KNOWLEDGE

Content from O-Level Additional Mathematics	
ALGEBRA	
A1	<p>Equations and inequalities</p> <ul style="list-style-type: none"> conditions for a quadratic equation to have: <ol style="list-style-type: none"> two real roots two equal roots no real roots conditions for $ax^2 + bx + c$ to be always positive (or always negative) solving simultaneous equations with at least one linear equation, by substitution
A2	<p>Indices and surds</p> <ul style="list-style-type: none"> four operations on indices and surds rationalising the denominator
A3	<p>Polynomials and partial fractions</p> <ul style="list-style-type: none"> multiplication and division of polynomials use of remainder and factor theorems partial fractions with cases where the denominator is not more complicated than: <ul style="list-style-type: none"> $(ax + b)(cx + d)$ $(ax + b)(cx + d)^2$ $(ax + b)(x^2 + c^2)$
A4	<p>Power, Exponential, Logarithmic, and Modulus functions</p> <ul style="list-style-type: none"> power functions $y = ax^n$, where n is a simple rational number, and their graphs functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs laws of logarithms equivalence of $y = a^x$ and $x = \log_a y$ change of base of logarithms function x and graph of $f(x)$, where $f(x)$ is linear, quadratic or trigonometric solving simple equations involving exponential and logarithmic functions
GEOMETRY AND TRIGONOMETRY	
B5	<p>Coordinate geometry in two dimensions</p> <ul style="list-style-type: none"> graphs of equations $y^2 = kx$ coordinate geometry of the circle with the equation in the form $(x - a)^2 + (y - b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$
B6	<p>Trigonometric functions, identities and equations</p> <ul style="list-style-type: none"> six trigonometric functions, and principal values of the inverses of sine, cosine and tangent trigonometric equations and identities (see List of Formulae) expression of $a \cos \theta + b \sin \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$

Content from O-Level Additional Mathematics	
CALCULUS	
C7	<p>Differentiation and integration</p> <ul style="list-style-type: none"> derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point derivative as rate of change derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences derivatives of composite functions derivatives of products and quotients of functions increasing and decreasing functions stationary points (maximum and minimum turning points and points of inflexion) use of second derivative test to discriminate between maxima and minima connected rates of change maxima and minima problems integration as the reverse of differentiation integration of x^n for any rational n, e^x, $\sin x$, $\cos x$, $\sec^2 x$ and their constant multiples, sums and differences integration of $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$ and $e^{ax + b}$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. Set Notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x: \dots\}$	the set of all x such that
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	universal set
A'	the complement of the set A
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Q}	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geq 0\}$
\mathbb{R}^n	the real n -tuples
\mathbb{C}	the set of complex numbers
\subseteq	is a subset of
\subset	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. Miscellaneous Symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\propto	is proportional to
$<$	is less than
$\leq; \nless$	is less than or equal to; is not greater than
$>$	is greater than
$\geq; \ngtr$	is greater than or equal to; is not less than
∞	infinity

3. Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
\sqrt{a}	the positive square root of the real number a
$ a $	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, ($0! = 1$)
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leq r \leq n$ $\frac{n(n-1)\dots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. Functions

f	the function f
$f(x)$	the value of the function f at x
$f: A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$g \circ f, gf$	the composite function of f and g which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x; \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... n th derivatives of $f(x)$ with respect to x
$\int y \, dx$	indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x for values of x between a and b
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to time

5. Exponential and Logarithmic Functions

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. Circular Functions and Relations

$\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$	$\left. \vphantom{\begin{matrix} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{matrix}} \right\}$ the circular functions
$\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	$\left. \vphantom{\begin{matrix} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \end{matrix}} \right\}$ the inverse circular functions

7. Complex Numbers

i	the square root of -1
z	a complex number, $z = x + iy$ $= r(\cos \theta + i \sin \theta), r \in \mathbb{R}_0^+$ $= re^{i\theta}, r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re}(x + iy) = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im}(x + iy) = y$
$ z $	the modulus of z , $ x + iy = \sqrt{x^2 + y^2}$, $ r(\cos \theta + i \sin \theta) = r$
$\arg z$	the argument of z , $\arg(r(\cos \theta + i \sin \theta)) = \theta, -\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $(x + iy)^* = x - iy$

8. Matrices

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the square matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$	the determinant of the square matrix \mathbf{M}

9. Vectors

\mathbf{a}	the vector \mathbf{a}
\overrightarrow{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} $	the magnitude of \mathbf{a}
$ \overrightarrow{AB} $	the magnitude of \overrightarrow{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10. Probability and Statistics

$A, B, C, \text{ etc.}$	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A , the event 'not A '
$P(A B)$	probability of the event A given the event B
$X, Y, R, \text{ etc.}$	random variables
$x, y, r, \text{ etc.}$	value of the random variables $X, Y, R, \text{ etc.}$
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations, x_1, x_2, \dots occur
$p(x)$	the value of the probability function $P(X = x)$ of the discrete random variable X
p_1, p_2, \dots	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x) \dots$	the value of the probability density function of the continuous random variable X
$F(x), G(x) \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$B(n, p)$	binomial distribution, parameters n and p
$\text{Po}(\mu)$	Poisson distribution, mean μ
$N(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}	sample mean
s^2	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample