Skill Distributions, Effective Endowment and Trade

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Skill Distributions, Effective Endowment and Trade*

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Abstract

This paper revisits the role of skill distributions in trade using a variant of the Heckscher-Ohlin model with multidimensional skill endowment and specialized production organized in teams. The equilibrium is characterized by the "effective endowment", the part of endowment that is actually utilized in production, which depends on the team matches and the task specialization within teams. It is shown that: (1) The endowment correlation between skill dimensions for each agent and the skill dispersion across agents, additional to the aggregate endowment, both matter for the pattern of trade; (2) There are new gains from trade, attributed to potential adjustments of the effective endowment upon trade integration; (3) Different endowment distributions can also generate different wage inequality levels across countries; An empirically found job polarization pattern can be generated in all developed countries in the global economy; (4) Additionally, it reveals a new channel through which the institutions may have effects on trade, by shaping the skill distributions. In particular, the potential effects of different educational policies and some labor market institutions on the skill distributions and trade are discussed.

Keywords: skill distribution, multidimensional endowment, team production, effective endowment, gains from trade, job polarization, wage inequality

JEL Classification Code: F11 F16 J31

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1 Introduction

The dispersion and correlation of individual skills in national populations affect a nation’s pattern of specialization and trade. This paper develops a model of multidimensional skill distributions and trade in the setting where production uses teams of agents who specialize in different tasks. Two novel features of specialization and trade emerge from considering this feature of reality. First, due to task specialization, population aggregate endowments of skills are replaced by endogenously generated effective endowments. Second, mobile individual agents become sector specific after forming teams in equilibrium.

Recent works on skill distributions in trade use the data from the International Adult Literacy Survey (IALS). It contains test scores of adults in different subjects. As approximations for skill levels, these scores are found to follow quite different distributions across countries. For instance, the U.S. has a relatively more diverse score distribution, and also the highest correlation between scores in different subjects for each individual. It has been theoretically shown by Grossman and Maggi (2000) and empirically supported by Bombardini et al. (2012) that skill dispersion matters in trade. This helps to explain the divergent trade patterns for countries with similar level of aggregate endowment, such as the United States and Japan, or Germany and Italy. However, the role of skill correlation is unclear.\(^1\) Presumably, when one can only utilize part of his skill endowment, the trade-off among his skills will be affected by the correlation between skill dimensions. And his optimal choice also depends on his colleagues’ skill endowments when production is organized in teams.\(^2\)

As highlighted by Grossman and Maggi (2000), there are systematic differences in trading patterns of countries with similar aggregate factor endowments. For instance, Germany exports more manufacturing goods requiring precision from a long sequence of production stages, such as automobiles. Whereas the United States exports more services such as software and financial services, whose value reflects disproportionately the input of a few very talented members in the team. In their paper, production is also organized in teams, with size equal two. However, the tasks for two members are symmetric. Thus they can not analyse the effects of skill distributions when endowment is multidimensional.

Also notice that the labor market outcomes across these countries with similar aggregate factor endowments differ.\(^3\) It is well known that the wage inequality is higher in the United

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\(^1\)Ohnsorge and Trefler (2007) assumes endowment of multiple skills for each worker, they focus on how each worker will optimally choose an industry that maximizes the total output of his whole skill set. There is no trade-off between skill dimensions in their paper.

\(^2\)The famous Roy (1950, 1951) model shows that with the choice from multiple skills, income distributions will be less unequal. This paper differs from Roy by introducing the team organization, hence one’s optimal choice depends on other team members’ endowments.

\(^3\)See Freeman and Katz (1995) among others for a review of these differences. This paper does not try
States than in Europe. Recent works find a common “job polarization” pattern of employment change in both regions. The current literature still lacks a unified framework that explains both the aforementioned differences in trade patterns and labor markets outcomes. This paper tries to bridge the gap.

I extends the standard Heckscher-Ohlin framework by adding multidimensional skill endowment and team-organized production. Individual agents are first matched into teams. Within teams, each agent utilizes certain dimension of his skill endowment to perform the corresponding task. The optimal labor allocation includes agents’ team matching, task assignments within teams and the industry choice for each team. Equilibrium determines the “effective endowment”, the part of population aggregate skill endowment that is actually utilized in production. Comparative static analysis for different skill distributions then show that both the endowment correlation between skill dimensions and the dispersion across labor force matters for the pattern of trade in each country. In particular, high correlation and high dispersion both generate comparative advantage in those industries with biased intensities in different dimensions of skill.

To see the intuition of the model, assume each agents’ skill has two dimensions: the entrepreneur’s managerial ability and the worker’s production skill. Correspondingly, production requires a manager task and a worker task to be performed, with different intensities across industries. In particular, some industries require one task much more intensitively than the other task, I call them biased-intensity industries; Other industries may require two tasks in similar, not necessary the same, intensities. I denote these industries as moderate-intensity industries. People are matched to form teams of size two and each team chooses its effective

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4. Job polarization is a pattern of employment change in which the top-wage jobs and bottom-wage jobs experience a higher increase than the middle-wage jobs. See Autor, Katz and Kearney (2006, 2008), Goos and Manning (2007) and Goos, Manning and Salomons (2009) for detail. It is also known as the “shrinking middle class” phenomenon in the news and media.

5. Within the trade literature, the effect of trade integration on inequality is still inconclusive; see Goldberg and Pavcnik (2007) and Harrison, McLaren and McMillan (2011) for reviews of related works. In the labor literature, the effect of trade and off-shoring is usually considered of small magnitude. For instance, when explaining the job polarization with the task-approach, Autor et al (2006, 2008) argues that the off-shoring ability of different tasks only has a quantitatively small role.

6. The matching process is not the focus of the current paper, thus only random matching and the social optimal matching are considered in this paper. A search process of team matching may be analyzed in future researches.


8. While one can think of other kinds of skill dimensions, it is not crucial what and how many the skill dimensions are. The key is that with completely specialized agents in team production, individuals face a trade-off between their skill attributes.
skill bundle, i.e. task assignment, and industry of production.\textsuperscript{9}

First, think of two countries with similar aggregate skill endowment along all dimensions. The skill correlation is the same, the only difference is the dispersion of skill levels. One country, the United States for example, has a higher dispersion than the other, for instance Japan.\textsuperscript{10} Given this difference, either with random matching or social optimal matching, there will be more teams with diverse skill levels along the two attributes in the United States than in Japan. As in Ohnsorge and Trefler (2007), more teams with diverse skill levels means comparative advantage in those biased-intensity industries. As a result, other things equal, a more diversified skill distribution generates comparative advantage in industries with biased task intensities.\textsuperscript{11}

Now think of two countries with similar skill distribution, except that one country has higher correlation between two skill dimensions, such as the United States; The other country has a lower skill correlation, Germany for example.\textsuperscript{12} Given the same average skill endowment, a higher correlation means more severe trade-off for each person between his endowment dimensions due to the team organization. Then effectively in production there will be a more diverse skill distribution along each dimension in the United States, which will generate more teams with diverse skill levels. As a result, a higher correlation has an effect on comparative advantage similar to a higher dispersion.

Consider the labor market outcomes. Biased-intensity industries tend have higher wage dispersion than moderate-intensity industries. Therefore, a country with comparative advantage in biased-intensity industries also has higher wage inequality. This says that the skill distributions in the United States and Europe may have also contributed to the wage inequality difference between these two regions over the past decades. When the North-South trade is analyzed in this model, under certain reasonable assumptions about skill distributions in the North and the South, a job polarization pattern of employment can be generated in all developed North economies.\textsuperscript{13}

\textsuperscript{9}In this paper, team size is fixed at two, equal to the skill dimensions. It is okay for the current paper’s goal. An extension of this paper with endogenous team size is to be done in the future.


\textsuperscript{11}This result is similar with Grossman and Maggi (2000), where diversity gives comparative advantage in industries with low task substitutability. Effectively both low substitutability and biased intensity demand team matches with diverse skill levels.

\textsuperscript{12}The high correlation between skill dimensions in the U.S. possibly results from its liberal arts education system, which emphasizes more general-skill training. Whereas in Germany the more subsidized vocational training, which is more skill-specific, may be the cause of the lower correlation. See Krueger and Kumar (2004a) for a comparison of educational systems in the US and European countries. See Ohnsorge and Trefler (2007) for evidences on skill correlation differences.

\textsuperscript{13}When combined with the trade between North economies, this model predicts that the job polarization pattern is more significant in the Unites States than in Europe. Current literature still lacks rigorous investigation on this comparison. However, the data and figures from Autor et al. (2006) and Goos and
A new source of gains from trade arises in this model. These new gains are generated from potential adjustments of the effective endowment through reshuffling the production teams or reassigning tasks within teams. Under the model setup, only part of the original skill endowment is utilized in production. Thus it is possible that when open to trade, each country may find itself better off by adjusting its effective endowment. For instance, if John has high level of managerial skill and also working skill, and Mike has low level of skills in both dimensions. They form a team and John performs the worker task if under autarky the worker-intensive industry is more demanding. When trade increases the manager-intensive good price, John and Mike’s team will move to the manger-intensive industry. This is the gain from trade identified in previous literature, i.e. factor reallocation. The new gain here is John and Mike can be better off by switching their tasks, i.e. changing their effective skills utilized in production.

Based on the previous results, this paper further reveals a new channel through which institutions can have effect on comparative advantage and trade. It is known that institutional quality does affect international trade, see Levchenko (2007) for example. But the channel found in this model is different. Institutions may not have direct effect on productivity or comparative advantage, instead it plays a role in shaping the skill distributions in each country. Through their effect on the skill distributions, some institutions can have indirect effect on the comparative advantage and trade. For instance, different education systems may affect the skill correlations in each country and then affect comparative advantage.\footnote{Chang and Huang (forthcoming) discussed the dynamic interaction between trade and education systems. They focus on how endogenous education policy are chosen to reinforce the initial comparative advantage.}

Some other labor market institutions, such as job protection, may affect worker’s choice of skill training strategy. For instance if current job is highly protected, he is more willing to take current-job-specific skill training, making his skill endowment more biased.

This paper is closely related to several strands of literature. Firstly, this paper adds to the line of works highlighting the role of higher moments of the endowment distribution in trade. Grossman and Maggi (2000) (henceforth GM) first highlight that the diversity of skill levels across labor force matters for the pattern of trade. Bombardini et al. (2012) find empirical support for this idea. Ohnsorge and Trefler (2007) (henceforth OT) further develop this idea with multidimensional skill endowment. This paper shares several insights with GM and OT. GM’s results about skill diversity can be replicated here. The key difference with OT is the team-production organization, which is crucial for my analysis on skill correlation and the new gains from trade. Bougheas and Riezman (2007) and Sly (2012) also extends the standard Heckscher-Ohlin framework to look at the effects of endowment distributions on

\footnote{Manning (2007) seems to support that the extent of job polarization is indeed greater in the US than in Europe.}
Secondly, this paper complements those works on the gains from trade by uncovering another potential source of gains. These new gains come from adjustments of the effective endowment upon integration.\(^{15}\) There is a large number of papers along this line, see Broda and Weinstein (2006), Feenstra (2004) for example. Moreover, this model may also generate gains from trade similar to the implications of scale economies under some conditions.

Thirdly, this paper also extends the Roy model by showing that different endowment distributions, higher correlation and dispersion in the US than Europe, contribute to the different inequality levels. The trade between these two regions may amplify this difference. Gould (2002) finds that the inequality increase in the US is increasingly characterized by the absolute advantage effect, indicating a high skill correlation. Blum (2008) finds that the increasing share of the service sector in the US explains almost 60% of the relative increase in wages of skilled workers between 1970 and 1996. On the other hand, this paper also indicates a role for globalization behind the job polarization pattern, complementing the existing explanations proposed by Autor et al. (2006, 2008), Autor and Dorn (2012) among others.\(^{16}\)

Finally, this paper links with papers on institutions and trade, see Levchenko (2007) for example. A new channel through which institutions may affect trade is revealed, i.e. by shaping the skill distributions. In particular, the potential role of education policies and some labor market institutions in trade is discussed. Tang (2012) shows that labor protection laws can be a source of comparative advantage.\(^{17}\)

The next section presents a simplified baseline model, illustrating the basic settings and main intuitions of this paper. The general model with continuous endowment levels and a continuum of industries is laid out in section 3. The generalized trade theories, the new gains from trade and the implications on wage inequality and job polarization are presented in the general model. Section 3 ends with some other model applications and discussions.

\(^{15}\)There are also papers emphasizing the change of utilized endowment after trade liberalization, due to the increased market competition for example. The under-utilization of initial endowment results from various distortions. However, in this paper the utilization of endowment is an optimal choice based on the trade-off between skill dimensions without any distortions. The new gains may still exist.

\(^{16}\)For other explanations for job polarization, see Costinot and Vogel (2010); Monte (2011); For more on the interdependence between trade and labor market, see Davidson and Sly (2012) for an example. The importance of team-production organization is also highlighted there.

\(^{17}\)This paper also relates with several other papers. Eeckhout and Jovanovic (2012) show that there are more managers in the developed countries, which is a natural result of this model when task off-shoring is allowed. Krueger and Kumar (2004a, 2004b) also highlight the effect of educational policies. Instead of focusing on the effects on technology adoption, hence growth rate, I focus on their comparative static effects on trade and wage structure. From a development point of view, this paper also shares with Buera and Kaboski (forthcoming) and Kaboski (2009) in explaining the increasing skill premium (or wage inequality) in the US with the structural changes towards service industries.
Section 4 concludes.

2 The Baseline Model

2.1 Setup

2.1.1 Endowment

In the baseline model I consider the two-dimensional skill endowment, think of them as the entrepreneurial ability and worker skill: \((E, W)\). Accordingly in production there are two tasks corresponding to each talent dimension.

In order to show the main intuitions in a simple way, without loss of generality, I assume that there are finite types of agents. Particularly, I assume two talent levels along each talent dimension: \(\{H, L\}\). Thus there are high-skill or low-skill entrepreneurs and high-skill or low-skill workers. Then the possible values for each agent’s talent endowment bundle \((E, W)\) in this economy are the following:

\[
(E, W) = \{(L, L), (H, H), (L, H), (H, L)\}
\]

Agents with endowment bundle \((L, L)\) have low level of skill in both attributes; Agents with bundle \((H, H)\) have high level of skill along both dimensions; \((H, L)\) agents have good entrepreneurial training but poor worker skill training; Whereas \((L, H)\) agents have good training as a worker but poor training as an entrepreneur.

Notice that the correlation of talent endowment along two dimensions for individual agent, \(E\) and \(W\), is one for the first two pairs of values, no matter how many there are each of them. And the correlation is negative one for the last two pairs. In general, the correlation can vary between these two extreme values. To simplify the analysis, I define two types of North economies. The type-I North economy has talent endowment bundles taking the first two pairs of values, with a perfect positive correlation between two dimensions skill endowment:

\[
(E, W) = \{(L, L), (H, H)\};
\]

The type-II North economy has endowment bundles taking values of the last two pairs, with a perfect negative correlation across dimensions:

\[
(E, W) = \{(L, H), (H, L)\}.
\]

When considering the case of North-South trade, I need to assume talent endowments for
the South economies. A consensus seems to be that the South has caught up fast along the worker-skill dimension over the past decades, and remains relatively poor in entrepreneurial capability either because of the biased education system, poor institutions, low level of financial development or lack of entrepreneurial culture.\footnote{There are many empirical papers documenting the catch-up in manufacturing in the developing economies. In contrast, in the service sector where there are more small business and entrepreneurial talent is more important, there is less catch-up in labor productivity. See Duarte and Restuccia (2010) for example.} Thus the South has a similar level of $W$ talent endowment as the North, but a relatively lower level of $E$ talent endowment. For simplicity, I assume that the South has the same level of talent endowment as the North along the $W$ dimension, and a lower level along the $E$ dimension. The potential values of skill endowment $(E, W)$ for agents in the South are then:

$$(E, W) = \{(L', L), (H', H), (L', H), (H', L)\}$$

where $H' = \theta H$ and $L' = \theta L$, $\theta < 1$. Two types of South economies with perfect positive and negative endowment correlations are then defined similarly as for the North.

### 2.1.2 Production

The technologies of production are the same in the North and South. There are two industries, service ($S$) and manufacturing ($M$), with a Cobb-Douglas production functions. I assume that in all industries managerial talent $E$ is always more intensively used than the $W$ talent.\footnote{In the real economy, this seems to be a reasonable assumption, given the extremely higher pay to those managerial jobs. The model does not require this assumption to generate implications on comparative advantage and gains from trade; It is only required in order to generate job polarization.} Moreover, in the service industry, entrepreneur’s personal success is even more important. Think of the software service as an example, the manager on the team performs a problem-solving task and the worker is writing codes with instructions from the manager. The output of this team largely depends on the manager’s problem-solving ability. Whereas for manufacturing, automobile for instance, it is important how the manager organizes the production process. It is also very important for the worker to do a good job in actually producing the car. The two production functions are given by:

$$F_s = E^\alpha W^{1-\alpha}, \quad F_m = E^\beta W^{1-\beta}, \quad \alpha > \beta \geq 0.5;$$

where $\alpha$ and $\beta$ are the entrepreneurial task intensities in two industries.
2.1.3 Preference

Agents' welfare comes from goods consumption. Utility function is given by:

\[ U = C_s^\mu C_m^{1-\mu}, \]

where \( C_s \) is the service consumption and \( C_m \) is the manufacturing good consumption.

The agent matching process can be modeled in many ways. For simplicity and focus on the analysis of skill distributions, I only consider the random matching and the social optimal matching. Two matching cases are presented in the following two subsections.

2.2 Random Matching

In the random matching case, agents meet and form teams with each other in a random fashion. Then each team chooses its task assignment, i.e. who will be the entrepreneur and who will be the worker, and which industry to enter. In this subsection, the role of skill correlation in trade is considered.\(^\text{20}\)

2.2.1 North-North Trade

Consider two North economies with the same aggregate skill endowment along both \( E \) and \( W \) dimensions. The only difference is the skill correlation between the two dimensions for each agent. In particular, Type-I North economy with a perfect positive correlation has endowment bundles:

\[ (E,W)_{N-I} = \begin{cases} 
(L,L) \text{ with measure } 1 \\
(H,H) \text{ with measure } 1
\end{cases} \]

Type-II North economy with a perfect negative correlation has skill endowments:

\[ (E,W)_{N-II} = \begin{cases} 
(L,H) \text{ with measure } 1 \\
(H,L) \text{ with measure } 1
\end{cases} \]

The measures of different bundles are chosen to ensure that the aggregate endowments along each dimension are the same for two economies.

Under the random matching process, I use the \( \otimes \) to denote matching, the matching

\(^{20}\)The role of skill dispersion in trade has been shown in many other models. GM for example. I will focus more on the skill correlation in the baseline model.
outcome for the Type-I North economy is:

\[
(E, W) \otimes (E, W) = \begin{cases} 
(L, L) \otimes (L, L) \text{ with measure } 0.25 \\
(L, L) \otimes (H, H) \text{ with measure } 0.5 \\
(H, H) \otimes (H, H) \text{ with measure } 0.25 
\end{cases}
\]

Each matched pair has to assign tasks among themselves and choose the industry (S or M) to enter based on their skill endowments. The effective skill bundles chosen by all matched teams in this type-I North economy will be:

\[
\text{Effective } (E, W)_{N-I} = \begin{cases} 
(L, L) \text{ with measure } 0.25 \\
(H, L) \text{ with measure } 0.5 \\
(H, H) \text{ with measure } 0.25 
\end{cases}
\]

Team \((L, L) \otimes (H, H)\) will assign the entrepreneurial task to the \((H, H)\) guy and the worker task to the \((L, L)\) agent due to the simplifying assumption that the \(E\) intensities \(\alpha\) and \(\beta\) for two industries are both greater than \(1/2\).

Analogously, in the Type-II North economy the matching outcome is the following:

\[
(E, W) \otimes (E, W) = \begin{cases} 
(L, H) \otimes (L, H) \text{ with measure } 0.25 \\
(L, H) \otimes (H, L) \text{ with measure } 0.5 \\
(H, L) \otimes (H, L) \text{ with measure } 0.25 
\end{cases}
\]

And the effective skill bundles in this type-II North economy will be:

\[
\text{Effective } (E, W)_{N-II} = \begin{cases} 
(L, H) \text{ with measure } 0.25 \\
(H, H) \text{ with measure } 0.5 \\
(H, L) \text{ with measure } 0.25 
\end{cases}
\]

Notice that for these two types of North economies, for any randomly matched team, the optimal choice of effective skill bundle is unique for all good prices. It is due to the simplifying assumption on the task intensities. This assumption will be relaxed later in the general model.

Now I can analyze the choice of industry for each team given different good prices, together with the consumption demand for both products, I can then pin down the equilibrium prices and output quantities. Without loss of generality, I let the manufacturing good be the numeraire and then \(P_m = 1, P_s = P\).

In the type-I North, teams with effective skill bundle \((L, L)\) or \((H, H)\) will choose to enter
the same industry for any given relative price $P$ since they have the same effective skill ratio. In particular, when $P = 1$ they are indifferent between producing $S$ and $M$; When $P < 1$, they choose to produce the $M$ good; When $P > 1$, they enter the $S$ industry. For teams with effective skill bundle $(H, L)$, the $S$ industry is chosen if and only if the price $P \geq \left( \frac{L}{H} \right)^{\alpha - \beta}$. Otherwise, they choose to produce the $M$ good.

In the type-II North, teams with effective skill bundles $(H, H)$ and $(H, L)$ have the same industry choice as teams in the type-I North with the same effective skill bundles. For teams with effective skill bundle $(L, H)$, they are more likely to choose the $M$ industry due to their high $W/E$ skill ratio which gives them comparative advantage in the more $W$-intensive $M$ industry. In particular, as long as the relative price $P < \left( \frac{H}{L} \right)^{\alpha - \beta}$, these teams will always stay in the $M$ industry.

The supply curves for both types of North economies are shown in Figure 1:

![Figure 1: North-North Trade](image)

where $A = \frac{2H^\alpha L^{1-\alpha}}{H+L}$, $B = \frac{H^\alpha L^{1-\alpha}}{H^\alpha L^{1-\beta} + 2H}$, and $C = \frac{H^\alpha L^{1-\alpha} + 2H}{H^\alpha L^{1-\beta}}$.

As we can see from Figure 1, when these two North economies open to trade, the type-I North has comparative advantage in the relatively more $E$-intensive industry $S$ and the type-II North has comparative advantage in the less $E$-intensive industry $M$. The intuition behind is straightforward. The high correlation between two dimensions of skill endowment in the type-I North means that agent with a high level of $E$ skill also has a high level of $W$ skill. However, each agent can only choose one skill dimension and perform the corresponding task, she has to waste her other skill. Given the same aggregate skill endowment along each dimension, a high correlation between two dimensions in the type-I North reduces its effective skill endowment that can be actually utilized. This further results in more teams with a high level skill along one dimension and a low level along the other in their effective skill bundles, generating comparative advantage in industries with more biased
task intensities. In contrast, in the type-II North with a low skill correlation, each agent has her own specialization along certain dimension. Hence when two agents with different skill endowments are matched together, they will assign the tasks according to their own comparative advantage. There is less waste of high skill levels. In the end, the type-II North has more teams with similar level of skill along two dimensions in their effective bundles, generating comparative advantage in industries with modest task intensities.\textsuperscript{21}

I apply these intuitions to analyze the divergent trade patterns between the United States and Germany. Given similar level of average skills in each dimension for individuals across two countries, the higher skill correlation in the United States generates comparative advantage in those more biased-intensity industries, such as those service industries. In contrast, low-correlation Germany has comparative advantage in those manufacturing industries with modest task intensities.

\subsection*{2.2.2 North-South Trade}

Now consider the case of North-South trade. Similarly two types of skill endowments for the South economies are defined. The Type-I South has a perfect positive correlation between the two dimensions of skill:

\[ (E, W)_{S-I} = \begin{cases} 
(L', L) \text{ with measure } 1 \\
(H', H) \text{ with measure } 1
\end{cases} \]

While the Type-II South economy has a perfect negative correlation:

\[ (E, W)_{S-II} = \begin{cases} 
(L', H) \text{ with measure } 1 \\
(H', L) \text{ with measure } 1
\end{cases} \]

Under random matching, the outcome of agent matching in the Type-I South is:

\[ (E, W) \otimes (E, W) = \begin{cases} 
(L', L) \otimes (L', L) \text{ with measure } 0.25 \\
(L', L) \otimes (H', H) \text{ with measure } 0.5 \\
(H', H) \otimes (H', H) \text{ with measure } 0.25
\end{cases} \]

\textsuperscript{21}If \( \alpha < \beta < 0.5 \), then type-I North will still have comparative advantage in the more biased-intensity industry, \( \alpha \) industry. A more general proposition will be high skill correlation generates comparative advantage in biased-intensity industries; In contrast, low skill correlation is associated with comparative advantage in those modest-intensity industries.
For the agent matches above, the effective skill bundles chosen by teams will be:

\[
\text{Effective } (E,W)_{S-I} = \begin{cases} 
(L', L) & \text{with measure } 0.25 \\
(H', L) & \text{with measure } 0.5 \\
(H', H) & \text{with measure } 0.25 
\end{cases}
\]

For the teams \((L', L) \otimes (H', H)\), as assumed \(H' = \theta H\) and \(L' = \theta L\), the skill bundle \((H', L)\) always outperforms the bundle \((L', H)\) due to the simplifying assumption that in both industries the \(E\) intensities \(\alpha\) and \(\beta\) are higher than 0.5. So the effective skill bundle will always be \((H', L)\) for these teams.

Similarly, in the Type-II South economy the effective talent bundles of all matched teams will be:

\[
\text{Effective } (E,W)_{S-II} = \begin{cases} 
(L', H) & \text{with measure } 0.25 \\
(H', H) & \text{with measure } 0.5 \\
(H', L) & \text{with measure } 0.25 
\end{cases}
\]

Since there are two types of economics in the North and two types in the South, there are four different cases for North-South trade. All four cases are analyzed below.

**Case 1:** Type-I North with Type-I South

![Figure 2: Type-I North with Type-I South](image)

where \(A' = A\theta^{\alpha - \beta}\).

As shown in Figure 2, the supply curve for the type-I South economy always locates above that for the type-I North economy without any intersection. Thus the price of \(S\) (relative to good \(M\)) is always higher in the South economy. The type-I North economy always has comparative advantage in the relatively more \(E\)-intensive \(S\) industry. In particular, it is those North teams with effective bundle \((E,W) = (H, L)\) that stay in the \(E\)-intensive \(S\)
industry and benefit the most from trade integration with the South.

It is useful to compare our results here with the case of a standard Heckscher-Ohlin (henceforth HO) model. The type-I North economy has exactly the same endowment of \( W \) talent as the type-I South economy. The South has a relatively lower endowment of \( E \) talent, \( \theta < 1 \). Skill correlation is the same. Under the HO framework, due to the relative higher endowment of \( E \) skill in the type-I North, the North economy has comparative advantage in the \( E \)-intensive industry \( S \). As shown above, the same result is obtained in this model with multidimensional skill endowment.

**Case 2**: Type-I North with Type-II South

\[
\begin{align*}
\left( \frac{H}{L} \right)^{\alpha - \beta} \\
\left( \frac{1}{\theta} \right)^{\alpha - \beta} \\
\left( \frac{L}{H} \right)^{\alpha - \beta} \\
\left( \frac{L}{H} \right)^{\alpha - \beta}
\end{align*}
\]

Figure 3: Type-I North with Type-II South

where \( B' = B \theta^{\alpha - \beta} \), and \( C' = C \theta^{\alpha - \beta} \).

As shown in Figure 3, the supply curve for the type-II South always locates strictly above the one for the type-I North. The same comparative advantage and trade pattern will be obtained as in Case 1. In this case, the North has a relatively higher endowment of talent \( E \), standard HO comparative advantage exist. In addition, the type-II South has a lower skill correlation. As shown in the case of North-North trade, *ceteris paribus*, a higher correlation generates comparative advantage in biased-intensity industries. In this Case 2, the type-I North economy has a relatively higher aggregate level endowment of skill \( E \), and also a higher correlation, both give it comparative advantage in the \( S \) industry. The traditional HO effect and the skill correlation effect work in the same direction here.

**Case 3**: Type-II North with Type-I South
As shown in Figure 4, where the relation between $B$ and $A'$ is not clear, depending on the values of $\theta$, $\alpha - \beta$, $H$ and $L$. Now the relative location of two supply curves is not clear. The trade pattern is also indeterminate. On one hand the type-II North economy has a relative higher level of aggregate endowment in talent $E$, which gives it comparative advantage in the $S$ industry; On the other hand, the type-II North economy also has a lower correlation compared with the type-I South economy, which gives the type-II North comparative advantage in the $M$ industry with modest task intensities. These two effects work in opposite directions. The equilibrium trade pattern depends on which effect dominates. In particular, when $\theta \leq \frac{L}{H}$, we have $(\frac{L}{H})^{\alpha - \beta} > 1$ and $(\frac{1}{\beta})^{\alpha - \beta} > (\frac{H}{L})^{\alpha - \beta}$. Then it is certain that the supply curve of the type-I South economy locates strictly above that of the type-II North economy, and the North will always have the comparative advantage in the relatively more $E$ intensive $S$ industry. In this case, the relative level of aggregate $E$ skill endowment is sufficiently high in the type-II North so that the standard HO effect dominates the effect of skill correlation in shaping the comparative advantages.

**Case 4**: Type-II North with Type-II South

![Figure 4: Type-II North with Type-I South](image-url)
As shown in Figure 5, where $B' < B$ and $C' < C$ always hold, the supply curve for the type-II South economy always locates strictly above the one for the type-II North economy. Thus it is always the case that the type-II North has comparative advantage in the $S$ industry and the type-II South has comparative advantage in the $M$ industry. This case is very much similar with case 1. The only endowment difference is that the type-II North has a higher aggregate endowment of $E$ skill. By standard HO theory, it generates comparative advantage in the more $E$-intensive $S$ industry for type-II North.

In all these cases, not only the relative aggregate endowment but also the correlation of endowment matters in trade. This leads to a key difference between this model and the standard HO model. What determines the comparative advantages and trade patterns here is the effective endowments instead of the original gross endowment as in the HO model. Effective endowment is the part of original endowment that is actually utilized in production. It differs from the original endowment because agents have to choose specialized task and production is organized in teams. As a result, part of each agent’s endowment is unused.

As we have shown in the North-North case, the skill correlation of endowment distributions plays an important role in shaping the effective endowment, hence comparative advantage, even with the same aggregate endowment across countries. In the cases of North-South trade, the comparative advantages are determined jointly by the relative aggregate endowment and endowment correlations. Sometimes the traditional HO results are amplified by the effect of skill correlation, as in case 2. While other times the traditional HO results are counteracted by the effect of skill correlation, as in case 3.
2.3 Optimal Matching

In this subsection I analyze the trade patterns under optimal agent matching. A benevolent social planner maximize the utility from goods consumption for the whole economy, given the skill endowment of each agent and the production technologies in two industries, by choosing the agent-matching arrangement, the task assignments and industry allocation for each team.

2.3.1 North-North Trade

First consider a type-I North economy. Since there are only two types of agents in this economy, I define two possible matching schemes. One is self-matching, the other is cross-matching. Under self-matching scheme, agents with the same skill endowment are matched together. Under cross-matching scheme, agents with different skill endowments are matched.

The social planner chooses the optimal agent matching arrangement from the self-matching scheme, cross-matching scheme, or a mix of two schemes. When self-matching scheme is chosen, the effective talent bundles in this economy will be half \( (H, H) \) and half \( (L, L) \). When cross-matching scheme is optimal, all teams have the same effective bundle \( (H, L) \), again due to the simplifying assumption that both industries are \( E \)-intensive.

Under the optimal arrangement, it is convenient to work with the production possibility frontier (henceforth PPF) for each economy. The possible PPFs for this type-I North economy under each matching scheme are shown below in Figure 6:
The line $ab$ is the PPF under the cross-matching scheme, line $cd$ is the one under self-matching scheme. Line $fg$ is parallel to $ab$ and $ef$ is parallel to $cd$. Thus $efg$ is the PPF under certain mix of the two schemes. In the case shown above in Figure 6, there is no dominant matching scheme for the social planner.\textsuperscript{22}

$$H^\alpha L^{1-\alpha} > \frac{1}{2}(H + L) > \frac{1}{2}(H + L) > H^\beta L^{1-\beta}$$

This condition ensures that there is no dominant matching scheme for this economy.

Even without any dominant matching scheme, the social planner can still achieve any possible production point on the line $ad$ by choosing a proper mix of the two schemes. As a result, the line $ad$ is the final PPF faced by the social planner in this economy.

In autarky, the social planner in this type-I North economy chooses a mix of two matching schemes to achieve a particular point on the line $ad$ to maximize the utility of consumption.

\textsuperscript{22}There may be dominant matching scheme in other cases. For instance, when $\alpha$ and $\beta$ are very close to 0.5, it is possible that $H^\alpha L^{1-\alpha}$ and $H^\beta L^{1-\beta}$ are both less than $\frac{1}{2}(H + L)$. In this case, the self-matching scheme dominates. This is similar with assortative matching in assignment models. It is also possible that cross-matching dominates.
Given the utility function, the equilibrium relative price of \( S \) will be determined by the reciprocal of (the absolute value of) the slope for line \( ad \), which is \( \frac{1}{\frac{(H+L)}{H^{\alpha}L^{1-\alpha}}} \).

In the case of no dominant matching scheme, \( H^{\alpha}L^{1-\alpha} > \frac{1}{2}(H + L) \), the relative price of \( S \) in this economy under autarky \( P = \frac{1}{\frac{(H+L)}{H^{\alpha}L^{1-\alpha}}} \) is smaller than one. The relative price of \( S \) will be \( P = \frac{H^{\beta}L^{1-\beta}}{H^{\alpha}L^{1-\alpha}} < 1 \) when cross-matching dominates; and 1 when self-matching dominates.

In a type-II North economy, the optimal matching scheme is always the cross-matching scheme. This is because two teams with effective talent endowment \((H, H)\) will always outperform two teams with effective endowments \((H, L)\) and \((L, H)\) in any industry. Hence the PPF faced by the social planner in a type-II North economy is always the one under complete cross-matching, a negative 45 degree line. Thus the relative price of \( S \) always equals one under autarky in the type-II North.

Notice that in all the cases, the relative good price of \( S \) in type-I North is always less or equal to one under autarky, with equality holds only when the self-matching scheme dominates, i.e. when \( H^{\alpha}L^{1-\alpha} < \frac{1}{2}(H + L) \). As a result, the type-I North economy always has a (weakly) lower \( S \) price than the type-II North. Thus the type-I North has comparative advantage in the \( S \) industry.

In general, when these two types of North economies open to trade, the type-I North economy has comparative advantage in the industry \( S \) with a more biased skill intensity. And the type-II North economy has comparative advantage in the \( M \) industry with a modest task intensity. Analogously, the type-I North economy will also has comparative advantage in industries with very high \( W \) intensity. A more general result about skill correlation in trade is presented in section 3 in a general model with a continuum of industries.

### 2.3.2 North-South Trade

For the North-South trade, I first look at a type-I South economy. Again the social planner chooses the agent matching scheme from self-matching, cross-matching, or a mix of the two schemes, to maximize welfare. Under the self-matching scheme, agents with the same talent endowment are matched together. Hence half of the teams have effective bundle \((H', H)\) and the other half have effective bundle \((L', L)\). Under cross-matching, agents with different talent endowment are matched and all teams choose the same effective bundle \((H', L)\), due to the assumption that the \( E \) intensities for both industries are greater than \( 1/2 \). The PPFs for all matching schemes are shown in the Figure 7 below.
The line \( ab \) is the PPF under complete cross-matching; The line \( cd \) is the PPF under self-matching; And the \( efg \) is the PPF under certain mix of the two matching schemes. In the case shown in Figure 7, \( H^\alpha L^{1-\alpha} > \frac{1}{2} \theta^\alpha (L + H) \) and \( \frac{1}{2} \theta^\beta (L + H) > H^\beta L^{1-\beta} \), there is no dominant agent matching scheme. Again, the social planner here can achieve any outcome along the line \( ad \) by choosing proper mixes of two matching schemes, hence the thick line \( ad \) is the PPF faced by the social planner.

Notice that the conditions for no dominant matching scheme is the same in the type-I South and type-I North economies:

\[
H^\alpha L^{1-\alpha} > \frac{1}{2} (L + H) \quad \text{and} \quad \frac{1}{2} (L + H) > H^\beta L^{1-\beta}.
\]

First consider trade between type-I North and type-I South. When there is no dominant agent matching scheme, the social planner chooses a best mix of two schemes to achieve the equilibrium production point at \( f \). The relative price for good \( S \) in type-I South is 

\[
P^* = \frac{\theta^\beta \frac{1}{2}(L+H)}{\theta^\alpha L^{1-\alpha}} = \frac{\theta^\beta}{\theta^\alpha} P.
\]

Since \( \theta^\beta > \theta^\alpha \), this price \( P^* \) is greater than \( P \). Thus type-I North has
comparative advantage in industry $S$.\footnote{I use asterisk to denote variables in the South whenever needed.} When $\frac{1}{2}(L+H) > H^\alpha L^{1-\alpha}$, the self-matching scheme dominates in both economies. The autarkic $S$ price $P = 1$ in the North, and $P^* = \frac{\theta^\beta}{\theta^\alpha} > 1$; When $H^\beta L^{1-\beta} > \frac{1}{2}(L + H)$, the cross-matching scheme dominates in both economies. The autarkic $S$ price $P = \frac{H^\beta L^{1-\beta}}{H^\alpha L^{1-\alpha}}$ in the type-I North, and $P^* = \frac{H'^\beta L^{1-\beta}}{H'^\alpha L^{1-\alpha}} = \frac{\theta^\beta}{\theta^\alpha} P > P$. In all these cases, the type-I North has a lower $S$ price than the type-I South.

Now consider the trade between the type-I South and type-II North economies, the relative price for good $S$ is always 1 in the type-II North economy. In cases when there is no dominant matching scheme or the cross-matching scheme dominates in the type-I South economy, the relative price for good $S$ in the South can be greater or less than one. Only when the self-matching scheme dominates in the South, the relative price of good $S$ is certain to be greater than one. Thus only in this case, the type-I South is sure to have comparative advantage in the $M$ industry. This result is very similar as that we get under the random matching process. The trade pattern between a type-I South and type-II North is not determined. Intuition is that relative aggregate endowment and skill correlation have counteracting effects on comparative advantage.

Finally, I consider the trade between a type-II South economy and the North economies. In a type-II South economy, the optimal matching scheme is always cross-matching, as two teams with effective skill $(H', H)$ always outperform two teams with $(L', H)$ and $(H', L)$ no matter which industry they choose. Hence under autarky, the relative good price for good $S$ is always equal to $\frac{\theta^\beta}{\theta^\alpha}$, which is always greater than one. When integrated with North economies, either type-I or type-II North, this type-II South economy always has comparative advantage in the modest-intensity $M$ industry.

From the analysis in the previous and current subsection, we can see that very similar results are generated under the random agent matching process and the optimal agent matching chosen by a benevolent social planner. These results have universal applications.

### 2.4 Dispersion and Trade

The previous subsections have shown how the skill correlation will affect trade. The important role of skill dispersion in trade has been highlighted in many other papers. I will skip this part in the baseline model.

It is well shown in GM that dispersion matters, when tasks in production have different elasticity of substitution across industries. In particular, higher dispersion generates comparative advantage in those industries with high elasticity of substitution between tasks. Bombardini et al. (2012) uses industry wage dispersion to approximate this elasticity and
found empirical support for GM’s model.

Under the settings in my model, industries differ in their task intensities. And higher dispersion will naturally generate more teams with biased effective skill ratios, hence comparative advantage in industries with biased task intensities. Notice that biased task intensity in an industry also means high wage dispersion in that industry. So the empirical evidence found by Bombardini et al. (2012) also supports the predictions of the current model.

2.5 Gains From Trade in the Baseline Model

In this subsection, I analyse the gains from trade in this baseline model. The gains under random matching is first presented. Then I compare the new source of gains from trade with the conventional gains that identified in the literature. An example is presented in the end.

2.5.1 Gains from Trade in Random Matching

Under random matching, the PPF of each economy is fixed given all the assumptions in the baseline model. This is because the share of teams are fixed, and for each team the choice of effective endowment bundle is also fixed. Thus in this baseline model, the gains from trade is the same as those already identified in the literature.

There are overall gains from trade integration in this baseline open economy model under random matching. However, the distribution of these gains across countries might be uneven. In this simple baseline model, with all the assumptions on endowment and industry intensities, we have the following lemma on gains from trade.

**Lemma 1.** With random matching, the economies whose relative good price changes from autarky to integration gain from trade. The economies with the same price after integration stay the same in welfare level.

**Proof.** Without change of production technology, the production possibility frontier of each economy is not changed due to the same matching outcome and same effective talent bundle choices under random matching. This is because given the assumptions on production functions and endowment levels, the effective bundle chosen by each team is already a dominant choice for every team and for all relative good prices. An economy will gain if the new price differs from its autarky price by the revealed preference. And due to the same reason, welfare stays the same if price does not change from autarky to integration for some other economies.

When there is a continuum of talent levels and industries, this lemma may be gone. Then there might be changes of the effective endowment, i.e. the production possibility frontier,
due to the possible change of effective endowment bundle choice for each team. In that case, there exists another possible source of gains from trade generated by the option to switch effective talent bundle within each team. This new source of gains from trade differs from the conventional gains in the sense that the effective endowment changes.

2.5.2 The Conventional Gains and New Gains

In the standard trade models, such as Ricardian model and HO model, countries trade due to different technology and relative aggregate endowment. In each economy, all its endowment is utilized. Countries will gain from the trade integration as they can consume goods at a lower price and total output increases because more resources are allocated to its comparative-advantage industries. This form of conventional gains from trade still exists in this paper. Moreover, there is a new source of gains from trade, originating from the potential adjustment of the effective endowment in each country.\textsuperscript{24}

As previously shown in this paper, the effective endowment is determined by the initial endowment, the agent-matching scheme and the task-assignment on each team. When open to trade, because of the price changes, the social planner may choose to change the agent matching outcome or switch task assignments within teams to improve the overall utilization of initial endowment. For example, think of a small type-I North economy with no dominating matching scheme under autarky, the relative price of $S$ is $P = \frac{\frac{1}{2}(H+L)}{\frac{1}{2}(H+L)}$, which is less than one in this case. When it opens up to trade with the rest of the world, suppose the world price for good $S$ is greater than $P$, then this small type-I North economy will completely specialize in producing good $S$. It will change the mixed matching scheme into complete cross-matching. In this way, this economy improves its utilization of its initial endowment. Notice that, when there is dominant agent matching scheme under autarky, then effective endowment cannot be improved by changing agent matching outcome upon integration.

To show how the effective endowment can be improved by changing within-team task assignment, let $\alpha > \frac{1}{2} > \beta$, and suppose in the type-I North economy the cross-matching scheme is dominant. Then the PPF for this economy is shown in Figure 8. Thus the teams in this economy have the same effective skill bundle choices, $(H, L)$ or $(L, H)$. For those teams entering the $S$ industry the bundle $(H, L)$ is preferred, and for those producing $M$ the bundle $(L, H)$ is chosen. The equilibrium allocation of teams determines the equilibrium effective endowment in this economy.

\textsuperscript{24}The adjustment of effective endowment is different from conventional reallocation of resources across industries. It is based on the trade-off for each agent among their multidimensional skill endowment. In reality, it is associated with inter-occupation movement.
As shown in Figure 8, $E$ is the equilibrium under autarky. When open to trade, suppose that the world price of good $M$ is higher ($P'_S < P_S$), then this economy will completely specialize in the $M$ industry. If the teams can not switch the effective skill bundles by changing the task assignments within teams, then the equilibrium in the open economy will be $E''$ and consumption is $C''$. However, if the teams moving from $S$ industry to the $M$ industry are able to switch their effective bundles by re-assigning the tasks within teams, the open economy equilibrium will be $E'$ and consumption $C'$. The consumption at $C'$ is strictly better than that at $C''$. Thus this economy gains from the switching of effective talent bundles within teams.

In the optimal matching process for a type-I North economy, when there is no dominant matching scheme, the social planner chooses a proper mix of the two matching schemes to maximize the welfare of the economy overall. The PPF for the social planner is the line $ad$. In contrast, if the social planner is not able to change the agent matching outcome after opening to trade, the PPF faced by her will be an inferior set below $ad$. Hence, allowing possible agent re-matching introduces new potential gains from trade.

Preceding are examples of gains from trade through agent re-matching and within team task re-assignment. I now present an example of another possible gains from trade in my framework, which is very similar with those in scale economies.

Consider again a type-I North economy, under the optimal matching, the PPF facing the social planner is $ad$ as shown in Figure 9 for the case when there is no dominant matching scheme. And $f$ is the optimal production point chosen by the planner under autarky.

When open to trade, $efg$ will be the PPF in the open economy for this country if the cost of breaking up existing teams and forming new teams is infinite. Suppose now the planner is able to create a market platform for those broken up agents to form new teams with a fixed
cost. Then the cost of breaking up existing teams and forming new teams features decreasing return to scale. The more teams a planner chooses to break up, the less the average cost will be. Then the PPF facing this social planner consists of two curves starting from the autarky point $f$, which is $e'fg'$ in Figure 9:

![Figure 9: From Autarky to Trade: Agent Re-Matching with Cost](image)

Notice that, in this case, the production possibility set for this country after opening to trade is a non-convex set.

The non-convexity of the production possibility set is similar with the one in scale economy models. Thus this potential new source of gains from trade also resembles that in scale economy models. However, the mechanism behind the gains here is totally different from that in scale economy models. Again this extra source of gains from trade comes from the potential changes of the utilized effective endowments in this economy. In contrast, the gains in scale economies come from the increasing return to scale in production technology.
2.6 Wages and Job Polarization

In this subsection I turn to the effects of trade on the wages and employment. First I need to determine the output division within teams.

2.6.1 Wage Determination

In the random matching case without re-matching after separation, each matched team will enter into production and divide the output evenly within teams. This is because when they bargain on their shares of surplus, they both have the same zero payoff in case of separation as the threat point. When there is search, then separated agents can enter the matching market and form new teams again. In this case, the threat points of each agent depends on the expected value of her endowment bundle.

In the type-I North economy, agent with endowment \((H, H)\) has probability 0.5 to be matched with a \((H, H)\) agent and 0.5 with a \((L, L)\) agent. Suppose that she has a bargaining power of \(\gamma\) when she is matched with \((L, L)\), thus her wage is \(\gamma H^\alpha L^{1-\alpha}\) and agent \((L, L)\)'s wage is \((1 - \gamma)H^\alpha L^{1-\alpha}\). When \((H, H)\) is matched with a same type agent, her payoff is simply \(0.5H\); when \((L, L)\) is matched with a same type agent, his payoff is \(0.5L\).\(^{25}\)

Assume a extreme case that search cost is zero, then the expected value of separation is \(0.5\gamma H^\alpha L^{1-\alpha} + 0.25H\) for agent \((H, H)\) and \(0.5(1 - \gamma)H^\alpha L^{1-\alpha} + 0.25L\) for agent \((L, L)\). The efficient bargaining solves the following problem by choosing \(\gamma\):

\[
\text{argmax } (\gamma H^\alpha L^{1-\alpha} - (0.5\gamma H^\alpha L^{1-\alpha} + 0.25H)) \left( (1 - \gamma)H^\alpha L^{1-\alpha} - (0.5(1 - \gamma)H^\alpha L^{1-\alpha} + 0.25L) \right)
\]

Solving this problem gives the efficient bargaining outcome for agent \((H, H)\):

\[
\gamma_1 = \frac{1}{2} + \frac{H - L}{4H^\alpha L^{1-\alpha}}
\]

Similarly, in the type-II North economy, when agent \((H, L)\) is matched with a \((L, H)\) agent, she gets \(\gamma H\) and her partner gets \((1 - \gamma)H\). When she is matched with a same type agent, they both get \(0.5H^\alpha L^{1-\alpha}\); when two \((L, H)\) agents are matched, they each gets \(0.5L^\beta H^{1-\beta}\). The efficient bargaining solves the following problem by choosing \(\gamma\):

\[
\text{argmax } (\gamma H - (0.5\gamma H + 0.25H^\alpha L^{1-\alpha})) \left( (1 - \gamma)H - (0.5(1 - \gamma)H + 0.25L^\beta H^{1-\beta}) \right)
\]

\(^{25}\)For simplicity, I neglect the good prices in the autarky analysis, assuming relative price equals one. This is a convenient short-cut in comparing the autarkic wage inequality levels for the two types of North economies.
Solving this problem gives the efficient bargaining outcome for agent \((H, L)\):

\[
\gamma_2 = \frac{1}{2} + \frac{H^\alpha L^{1-\alpha} - L^\beta H^{1-\beta}}{4H}
\]

Following the assumptions in the baseline model, it is obvious that:

\[
H > H^\alpha L^{1-\alpha} > L^\beta H^{1-\beta} > L.
\]

As we can see, \(\gamma_1 > \gamma_2 > 0.5\), thus the wage inequality between two types of agents is higher in the type-I North economy than in the type-II North economy. The intuition is that in the type-II North economy, low correlation means for every agent has his own specialization in skills, no one is significantly dominated by the other in all dimensions. Thus the bargaining power allocation within teams is more equal because of the lower endowment correlation. In contrast, in the type-I North economy, agent with endowment \((H, H)\) outperforms agent \((L, L)\) in both entrepreneurial ability and working skills dimensions. As a result, there is higher inequality level in the type-I North economy than in the type-II North economy in equilibrium due to different correlations.

In a competitive environment under optimal matching, since there are many teams of the same type in a particular industry, each agent on the team gets the share that she contributes to the total team surplus. This is because there many teams with the same agent endowments. There is no searching cost, each team can find many alternatives for any one of its member. In this case each agent’s wage is determined by the industry intensity and the total team surplus.

For the type-I North economy, I consider different optimal matching arrangements for the social planner. If pure self-matching dominates, then relative price of \(S\) relative to \(M\) is 1. Agent \((H, H)\) gets a wage \(\frac{1}{2}H\), and agent \((L, L)\) gets \(\frac{1}{2}L\).

If pure cross-matching dominates, relative price for \(S\) will be \(P_s = \frac{H^\beta L^{1-\beta}}{H^\alpha L^{1-\alpha}}\). In the \(S\) industry, \((H, H)\) type agents get a wage \(\alpha H^\beta L^{1-\beta}\), and \((L, L)\) agents get \((1 - \alpha)H^\beta L^{1-\beta}\). In the \(M\) industry, \(\beta H^\beta L^{1-\beta}\) is the wage for \((H, H)\) agents and \((1 - \beta)H^\beta L^{1-\beta}\) for \((L, L)\) agents.

If the optimal matching is a mix of the two, the relative good price is \(P_s = \frac{\frac{1}{2}(H + L)}{H^\alpha L^{1-\alpha}}\). In the \(S\) industry, \((H, H)\) agents get \(\alpha \frac{1}{2}(H + L)\) and \((L, L)\) agents get \((1 - \alpha)\frac{1}{2}(H + L)\). In the \(M\) industry, \((H, H)\) agents get \(\frac{1}{2}H\) and \((L, L)\) agents get \(\frac{1}{2}L\).

For the type-II North economy, cross-matching always dominates. In the \(S\) industry, wage for \((H, L)\) agents is \(\alpha H\), \((1 - \alpha)H\) for \((L, H)\) agents; In the \(M\) industry, \((H, L)\) agents get \(\beta H\) and \((L, H)\) agents get wage \((1 - \beta)H\).

From these wages, we see that the wage inequality is higher in the \(S\) industry. The
high-correlation type-I North thus also have higher wage inequality than the type-II North under optimal matching.

As we can see from the above analysis, the random matching case and the competitive environment yield very similar results on wage distributions. For simplicity, when I analyse the wage and job evolutions I assume the wage determination in the competitive environment.

In order to analyse the job polarization, we need to first identify what jobs are top-wage jobs, middle-wage jobs and bottom-wage jobs. In the high-correlation type-I North economy, in most cases, we have that entrepreneurs in $S$ industry earn the highest wage and workers in $S$ industry earn the lowest wage. On the other hand, entrepreneurs and workers in the $M$ industry earn middle-level wages. For the low-correlation type-II North economy, we always have that entrepreneurs in $S$ industry earn the highest wage and workers in $S$ industry earn the lowest wage; Entrepreneurs and workers in the $M$ industry earn middle-level wages.

2.6.2 Trade and Wage Inequality

In the case of North-North trade, the type-I North has comparative advantage in the $S$ industry, those teams with endowment $(H,H) \otimes (L,L)$ benefit the most from this integration. They are the top-wage entrepreneurs and bottom-wage workers in this industry. The inequality in type-I North will then increase. The opposite is true for the type-II North economy. This is true when we apply this into the comparison between the US and the EU. The US has a higher inequality level than the EU because the endowment correlation shaped by the general education system in the US is higher than that in the EU shaped by the skill-specialized vocational education system in those European countries.

The more interesting case to consider is the North-South trade. From the previous analysis we know that the South economy has comparative advantage in the relatively more $W$-intensive $M$ industry. Two types of North economies both have comparative advantage in the $E$-intensive $S$ industry. More teams in the North will choose to move into the $S$ industry. Since in either type of North economy, $S$ industry has the higher wage inequality than the other industry, inequality levels increase in both economies when open to trade with the South.

2.6.3 Globalization and Job Polarization

Over the last three decades or so, most of the North economies has experienced a pattern of job evolution, the “job polarization”. It denotes the fact that the employment growth for low-wage and high-wage jobs has increased more than those middle-wage jobs. This

$^{26}$In all cases if $\frac{H}{L} < \frac{\alpha}{1-\alpha}$.
phenomenon has been addressed pretty well in the closed economy with the task approach by David Autor and other labor economists. Here I show that this simple baseline model may also generate such a job change pattern in the open economy, specifically in the North-South integration.

As shown in the wage determination subsection, in both types of North economies, entrepreneurs in the $S$ industry earn the highest wages in the economy and workers in the $S$ industry earn the lowest wages. Those entrepreneurs and workers in the $M$ industry earn middle-level wages.

After integration with the South in the globalization era, those agents working in the $S$ industry in the North benefit the most due to the relative price increase for $S$. They are the top-wage entrepreneurs and bottom-wage workers in the North. After integration there will be more such matches formed and entering the $S$ industry in the North. As a result, the employment of these top-wage jobs and bottom-wage jobs increases relative to middle-wage jobs in the $M$ industry. Thus a pattern of “job polarization” exist in all the North economies.

Notice that even though this “job polarization” exists in all North economies. The extent of this pattern may differ in different types of North economies. Additionally, the effects on wage evolution will definitely be different for different types of North economies. In fact, there are indeed evidences for divergent wage changes in different North economies. Autor et al (2006) documents both “job polarization” and “wage polarization” in the U.S. However, the second polarization pattern is not observed in the European countries.

All in all, this baseline model provide an alternative explanation of the job polarization using North-South trade, highlighting the potential role for globalization behind these observed labor market outcomes in the North economies.

3 The Continuous Model

In this section I present a more general model with a continuum of industries and continuous multidimensional talent endowment.

There are a continuum of industries with $E$ intensities range from $\alpha$ up to $\bar{\alpha}$. For any particular industry with intensity $\alpha_n$, $n \in [0, 1]$, the production function is given by:

$$Y_n = E^{\alpha_n}W^{1-\alpha_n}.$$ (3)

The $n$ industries are ranked such that for $n > n'$, $\alpha_n > \alpha_{n'}$.\(^{27}\)

\(^{27}\)A more general form of constant return to scale production function with asymmetric tasks is $Y = \left[\alpha E^{x-1} + (1 - \alpha)W^{x-1}\right]^{\frac{1}{x-1}}$. I use the Cobb-Douglas form for simplicity, all the results are the same.
The talent endowments of agents in this economy has two dimensions: \((E,W)\). Each follows a distribution with probability density function \(g(E)\) and \(h(W)\) respectively, \(E \in [\omega, \bar{\omega}], W \in [\omega, \bar{\omega}]\). The correlation between the two endowment dimensions is \(\rho, \rho \in [-1, 1]\).

### 3.1 Agent Matching

I consider two different agent matching assumptions, the random matching and the social optimal matching. Under random matching, agents have incomplete information about others’ endowment before matching. The matched agents observe each other’s endowment bundle and then choose task assignment, i.e. the effective bundle, and industry to maximize team output. The wages within each team are determined by solving the bargaining problem. Each agent’s bargaining power depends on his alternative option value, which is the expected value of his endowment bundle.

Under the optimal matching mechanism, the benevolent social planner has complete information about agents’ endowments. He chooses the agent matching arrangement, the task assignments and industry choices for each team in order to maximize the aggregate social welfare, given the available industries. In this competitive environment, each agent’s wage equals her share of contribution to the final output. Particularly, in the Cobb-Douglas production case these shares of contribution to the final output equal the talent intensities.

### 3.2 Generalized Trade Theories

Under this new framework I reinvestigate several well-known trade theories. Generalizations of some established trade theories are found, while some other trade theories no longer exist under new settings. Also new implications on trade and wages are drawn from this new model.

The framework in this paper shares many common features with the OT paper. They investigate how agents with different talent ratios choose their industries. This model mimic their logic after teams are formed and effective bundles are chosen. When teams with effective talent bundle of different ratios choose their industries, it follows exactly the same rationale as the agents in OT model. So the key of my proof in this paper lies in the determination of effective talent bundles.

#### 3.2.1 The Generalized Heckscher-Ohlin Theorem

Compare two economies, North and South, with different relative talent endowment across two dimensions. Assume the two economies have the same endowment distribution along the \(W\) dimension. The talent endowment along the \(E\) dimension is lower in the South.
Other aspects of endowment distributions are all the same, such as the correlations, available industries and technologies. Particularly, for any agent in the North with endowment \((E,W)_N = (E_i,W_i)\); in the South there is an agent with endowment \((E,W)_S = (\theta E_i,W_i)\), where \(\theta < 1\).

**Proposition 1.** Given that South has relative less endowment along the \(E\) skill dimension, *ceteris paribus*, the North always has comparative advantage in those industries with high \(E\) intensities.

*Proof.* Since the only difference between two economies is that agents in the South have lower levels of \(E\) endowment. With random matching process, the agent matching outcome in two economies will be the same in the sense that for each matched team in the North with endowments \((E_i,W_i) \otimes (E_j,W_j)\), there is a counterpart team in the South with endowments \((\theta E_i,W_i) \otimes (\theta E_j,W_j)\). When producing a particular good with \(E\) intensity \(\alpha\), these two teams will choose effective talent bundle in the same way because

\[
(\theta E_i)^\alpha W_j^{1-\alpha} \geq (\theta E_j)^\alpha W_i^{1-\alpha} \iff E_i^\alpha W_j^{1-\alpha} \geq E_j^\alpha W_i^{1-\alpha}.
\]

Thus in the production possibility set along the PPF, the maximum output potential of any good in the North will be higher than that in the South by \(\left(\frac{1}{\theta}\right)^\alpha\) times. Since \(\frac{1}{\theta} > 1\), the higher \(E\) intensity \(\alpha\) is, the more increase in output potential in the North than that in the South. As a result, the North has comparative advantage in those industries with high \(E\) intensities.

Under the optimal matching, consider any pair of goods with \(E\) intensities \(\alpha_i\) and \(\alpha_j\), \(\alpha_i > \alpha_j\). Given the optimal output of a good in the South, the North can always increase the output potential of that good by at least \(\left(\frac{1}{\theta}\right)^{\alpha_i}\) times through the increase of the talent \(E\) level, without changing the agent matching outcome or task assignments. Since North has a higher level endowment of \(E\) talent, the optimal matching and task assignment there allow it to utilize more of its \(E\) talent. More high level \(E\) talent endowment will be actually utilized. In this case, the higher the \(E\) intensity of an industry, the more output of it can be improved in the North from the optimal matching in the South. Hence again the relative output of the two industries \(\alpha_i\) and \(\alpha_j\) is always higher in the North than the South. This ratio is even higher under the optimal matching than that under the random matching process. Thus the North economy always has a PPF more in favor of those more \(E\) intensive goods than the South. 

In the standard H-O theorem, a country will export goods that use its abundant factors intensively, and import goods that use its scarce factors intensively. As shown by Proposition
Proposition 2. Assume the same $W$ endowment in both economies and less $E$ in the South, ceteris paribus, the effective endowment along the $W$ dimension in the South is always higher or equivalent to that in the North.

Proof. This lemma directly follows from proposition 1. Under random matching, the two economies have the same effective endowment along the $W$ dimension, since for each pair of randomly matched teams in two economies their choice of effective bundles are the same, as shown in the proof of proposition 1. Under optimal matching, the North economy is more willing to sacrifice some $W$ talent to utilize the higher level $E$ talent than the South. As the North and South have the same endowment distribution along the $W$ dimension, in the optimal matching equilibrium the South ends up with a relatively higher effective endowment of $W$ talent than the North.

This proposition is to emphasize the importance of effective endowment. In this new framework with multidimensional endowment, it is the effective endowment that determines the trade patterns. And for two economies with same aggregate original endowment, their effective endowments that are actually utilized in production may differ. Thus it is important to investigate how the effective endowment is generated in different economies. This paper takes a first step in this direction.

3.2.2 Factor Prices

In the H-O model, the relative prices for two identical factors of production in the same market will eventually equal each other because of competition.

However in this model, the factor prices are usually not equalized, since the return to a particular factor also depends on the factor matched with it. In the multi-dimensional endowment case, the return to a particular dimension talent also depends on the endowments along other dimensions, since they will affect the final choice of effective talent bundle. One particular dimension of talent may end up idle without any price.

Two agents from one country with the same talent endowment along all dimensions tend to have the same (at least expected) return. However, two agents with the same talent endowment in countries with different endowments may differ in their expected returns, due to the different pools of potential team mates they are able to find.

3.2.3 The Generalized Stolper-Samuelson Theorem

In the H-O model, a rise in the relative price of a good will lead to a rise in the return to that factor which is used most intensively in the production of the good, and conversely, to
a fall in the return to the other factor.

Similarly to OT, in this model the industry choice is governed by the relative talent ratio in each team’s effective talent bundle. Each matched team chooses her industry according to her effective talent ratio. When the price of a particular good $\alpha_i$ increases, the returns to those effective bundles with this talent ratio $\frac{\alpha_i}{1-\alpha_i}$ are increased. Since factor prices are not equalized in this framework, we can not tell as in the standard H-O model whether the return to certain dimension of talent increases or not.

3.2.4 The Generalized Rybczynski Theorem

In standard H-O model, when only one of two factors of production is increased there is a relative increase in the production of the good using more of that factor. This leads to a corresponding decline in the relative price of that good as well as a decline in the production of the good that uses the other factor more intensively.

Within the current framework, a similar result holds. A universal increase across all agents in the endowment level of certain dimension increases the production of goods with high intensities in that talent. This is because after the increase there are more effective bundles with high ratios of that talent. Similar with OT, this means there are more teams choosing to produce goods with high intensities in that talent. On the other hand, the production of goods with small intensities in that talent shrinks.

**Theorem 1.** In the framework with multidimensional endowment and team production, an increase in one dimension of talent endowment increases the relative production of goods using that dimension more intensively and decreases the relative production of goods using the other dimensions more intensively.

**Proof.** I illustrate this theorem by doing comparative statics analysis between two equilibriums before and after the increase of talent endowment. Given equilibrium of the economy, each team has an effective talent bundle $(E,W)$ utilized in production of certain industry $\alpha$. Now consider an universal increase in the endowment levels of one particular talent dimension, without loss of generality, assume that the $E$ talent levels are all increased by $\lambda$ times. And $\gamma$ is only slightly greater than 1, in order to keep all the other characteristics of endowment distribution unchanged, such as the variance, correlation etc.

Under random matching, after the talent increase the agent-matching outcome is not changed, but for each team the two choices of effective bundle both become more $E$-intensive. If the effective bundle choices, hence industry choices, of all teams are still the same as before the increase, then the output of industry $\alpha$ increases by $\lambda^\alpha$ times, where $\alpha$ is the $E$ intensity of this industry. In this case, the output of industries with high $\alpha$ increases more than that
of industries with low $\alpha$ values. On the other hand, if some teams choose to switch their effective bundles and hence industry choices, it can only be the case that they are moving from less $E$-intensive industries into more $E$-intensive industries. This argument can be easily shown by contradiction. If a team moves from a more $E$-intensive industry to a less $E$-intensive industry after the increase of talent $E$, then they will choose that less $E$-intensive industry before the increase. The reason is similar with the proof of proposition 1. Hence under random matching, the Rybczynski theorem extends into this new framework.

Under optimal matching, the proof of this theorem works with the new PPF for this economy after the increase of talent $E$. For any pair of two goods, the increase of talent $E$ to $\lambda E$ increases the output possibility of all goods. And the more $E$-intensive a good is, the higher the increase of its output potential will be. In the end, the new PPF is more biased to those more $E$-intensive goods. Then the new optimal choice of output composition consists of more $E$-intensive goods and less $W$-intensive goods relative to the optimal choice before the increase. Thus the Rybczynski theorem still holds under optimal matching in the new framework.

OT also proved a generalized version of Rybczynski theorem, which is very similar with the one shown above. There the increase of the correlation between $s$, the talent ratio, and $l$, the absolute level of the ratio, increases the relative level of high-$s$ talent bundles, hence the relative production of those goods with high-ratio intensities. Here in my framework, the increase in the level of one talent increases the ratios of that talent for all endowment bundles in this economy, resulting in more high-ratio effective bundles in that talent and thus more output of goods with high-intensity in that talent. In fact, the Theorem 4 in OT on the effect of an increase in the distribution mean of $s$, is very similar in effect as the mechanism presented here in Theorem 1.

### 3.2.5 Correlation and Trade

There is not many measures in the literature for the correlation between endowment dimensions. But this difference do exist due to different education systems in different countries. For instance, the most recognized vocational education system in EU provide the economy with many specialized workers with some specific skills. They are proficient in performing certain tasks but poor in others. While in US education system where the general purpose technology is more emphasized provide students with general trainings in all dimensions. The difference in skills between agents in US lies more in absolute levels of skills instead of comparative differences.

The role of correlation between endowment dimensions in shaping the wage inequality
has been emphasized by Gould (2002). He shows that workers in the US are increasingly finding that they are either good in all sectors or bad in all sectors. The negative effect of comparative advantage on inequality from the Roy model is decreasing in the US economy. The level of inequality is rising in US as the economy is increasingly characterized by the pursuit of absolute advantage rather than comparative advantage.

The International Adult Literacy Survey (IALS) has shown that people in different countries do have different mix of different talents/skills. The correlations do differ across countries with different education systems.

In this part, I will investigate the role of this endowment correlation in shaping the effective endowment, hence the production, comparative advantage, and trade patterns.\textsuperscript{28}

\textbf{Theorem 2.} Consider a world with two economies that are identical except for the correlation between endowment dimensions $E$ and $W$. (1) There exists an equilibrium. (2) In equilibrium there are two cut-off industries $\alpha_{\frac{1}{2}}$ and $\alpha_{i}$ such that the high-$\rho$ economy imports all goods with middle $E/W$ intensities ($\alpha_{\frac{1}{2}} < \alpha < \alpha_{i}$) and exports all goods with extreme $E/W$ intensities ($\alpha < \alpha_{\frac{1}{2}}$ or $\alpha > \alpha_{i}$).

\textit{Proof.} Given the Theorem 3 in OT, I only need to prove that in this model the effective bundles in the high-$\rho$ country are more unequally distributed.

I show this by assuming a specific distribution for both dimensions of the talent endowment, which is the normal distribution. Particularly the talent endowment of $E$ and $W$ follow the following bivariate normal distribution:

$$\begin{bmatrix} E \\ W \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma_{E}^{2} & \rho \sigma_{E} \sigma_{W} \\ \rho \sigma_{E} \sigma_{W} & \sigma_{W}^{2} \end{bmatrix} \right),$$

(4)

where $\rho$ is the correlation between $E$ and $W$. When considering the role of correlation, I set $\sigma_{E} = \sigma_{W}$ to restrict out the role of relative diversity, which will be analysed later.

Under normality the expectation of $E$ given $W$ is

$$\xi(E|W) = \mu + \frac{\sigma_{E}}{\sigma_{W}}(W - \mu) = \mu + \rho(W - \mu), \text{ when } \sigma_{E} = \sigma_{W}. \quad (5)$$

Similarly we also have the expectation of $W$ given the level of $E$.

The two countries have the same endowment distribution along each talent dimension, the only difference is the correlation. For any two agent $i, j$, with talent levels $E_{i}$ for $i$ and

\textsuperscript{28}Notice the difference between the correlation here and the one in OT. The correlation here is between different endowment dimensions for each agent. In OT instead, each agent’s endowment is characterized by his talent ratio and the talent level. The correlation in OT is the correlation between this ratio and level for each agent.
W_j for j, they show up with the same probability in two countries. In the high-correlation country A, the corresponding talent levels along the other dimension for these two agents i, j are expected to be: \( \xi(W_i|E_i) = \rho_A(E_i - \mu) + \mu \) and \( \xi(E_j|W_j) = \rho_A(W_j - \mu) + \mu \); In the low-correlation country B, the corresponding talent levels for these two agents are expected to be: \( \xi(W_i|E_i) = \rho_B(E_i - \mu) + \mu \) and \( \xi(E_j|W_j) = \rho_B(W_j - \mu) + \mu \), where \( \rho_A > \rho_B \).

Given \( E_i \) and \( W_j \), the (expected) talent ratio for the other potential talent bundle for this team is

\[
R \equiv \frac{E_j}{W_i} = \frac{\rho(W_j - \mu) + \mu}{\rho(E_i - \mu) + \mu}.
\]

We are interested in the role of \( \rho \) in this ratio, taking derivative with respect of \( \rho \), we have

\[
R' = \frac{\partial E_j}{\partial \rho} = \frac{(W_j - E_i)\mu}{\rho(E_i - \mu) + \mu}.
\]

When \( E_i > W_j \), we have \( \frac{E_i}{W_j} > R \) for any value of \( \rho \leq 1 \); \(^{29}\) and \( R' < 0 \), thus the high-correlation country A has another bundle of smaller \( E/W \) ratio than country B. As a result, the range of talent ratios is larger in country A than in country B.

When \( E_i < W_j \), we have \( \frac{E_i}{W_j} < R \) for any value of \( \rho \); and \( R' > 0 \), thus the high-correlation country A has another bundle of higher \( E/W \) ratio than country B. As a result, the range of talent ratios is again larger in country A than in country B. The rest of the proof mimics the Theorem 3 in OT.

At the end, with identical talent distribution along each single dimension, the high-correlation country has comparative advantage in industries with extreme talent intensities while the low-correlation country has comparative advantage in middle \( E/W \) intensities. \( \Box \)

Thus even with different definition of correlations, both my model and OT find an important role for the higher moment of endowment distributions in international trade. This point will be further emphasized in the next part about the effect of diversity on trade.

### 3.2.6 Diversity and Trade

The role of endowment diversity has been shown by GM by assuming two different production technologies: the super-modular and sub-modular technology. This part investigate the role of endowment diversity in this new framework.

**Theorem 3.** In an open economy of two countries, suppose they have the same aggregate endowment along every talent dimension. The only difference is the diversity (variance) of the endowment distribution along certain dimension. Then the high-diversity country

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\(^{29}\)To see this point, \( \frac{E_i}{W_j} - R = \frac{\rho(E_i^2 - W_j^2)(\mu - \rho \mu)(E_i - W_j)}{W_j(\rho(E_i - \mu) + \mu)} \), thus \( \frac{E_i}{W_j} - R > (<) \Rightarrow E_i > (<) W_j \).
has comparative advantage in industries with extreme (high or low) intensities of that skill dimension, while the other has comparative advantage in industries with middle intensities of that dimension.

Proof. Consider the case of two countries with the same aggregate endowment along all dimensions, same diversity along the W dimension, and different diversity along the E dimension. Compare the change of equilibrium from the low-diversity country (A) to the high-diversity country (B).

Under the random matching mechanism, agent matching outcome will be the same along the W dimension. For any team in country A with agents endowment \((E_i, W_i)\) and \((E_j, W_j)\), there is a team in country B with agents endowment \((E_i', W_i')\) and \((E_j', W_j')\), where \(E_i + E_j = E_i' + E_j'\), and \((E_i' - E_j')^2 > (E_i - E_j)^2\). Then the effective talent bundles in country A are \((E_i, W_j)\) or \((E_j, W_i)\), and \((E_i', W_j')\) or \((E_j', W_i')\) for country B. Without loss of generality, assume \(E_i > E_j\) and \(W_i > W_j\)\(^{30}\), then \(E_i' > E_i\), \(E_j' < E_j\) the effective talent ratios has the following relationship:

\[
\frac{E_i'}{W_i} < \frac{E_j'}{W_j} \leq \frac{E_i}{W_i} < \frac{E_j}{W_j};
\]

Thus in country B, the effective bundles have more extreme ratios than those in country A. As a result, the high-diversity country B has comparative advantage in those industries with extreme talent intensities.

Under the optimal matching mechanism, again the proof is dealing with the PPFs in these two countries. For any pair of agents in country A with endowments \((E_i, W_i)\) and \((E_j, W_j)\), if there is a dominating choice of effective bundle, when \(E_i > E_j\) and \(W_i < W_j\) \((E_i < E_j\) and \(W_i > W_j)\), then effective bundle will be \((E_i, W_j)\) \(((E_j, W_i))\) in country A and \((E_i', W_j)\) \(((E_j', W_i))\) in country B. Country B has comparative advantage in industries with high (low) \(E/W\) intensities. When \(E_i > E_j\) and \(W_i > W_j\), there is no dominating choice of effective bundle. Consider the output possibility of two goods, one with a high \(E/W\) intensity \(\alpha\) and the other with modest \(E/W\) intensity \(\beta\). Country A chooses \((E_i, W_j)\) to produce the high \(\alpha\) good, chooses \((E_i, W_j)\) when \(E_i^\beta W_j^{1-\beta} > E_j^\beta W_i^{1-\beta}\) or \((E_j, W_i)\) when \(E_i^\beta W_j^{1-\beta} < E_j^\beta W_i^{1-\beta}\) to produce the \(\beta\) good. For country B, \((E_i', W_j)\) is always chosen to produce the \(\alpha\) good, it chooses \((E_i', W_j)\) when \(E_i'^\beta W_j^{1-\beta} > E_j'^\beta W_i^{1-\beta}\) or \((E_j', W_i)\) when \(E_i'^\beta W_j^{1-\beta} < E_j'^\beta W_i^{1-\beta}\). Notice that when country A chooses \((E_i, W_j)\) to produce the \(\beta\) good, B will choose \((E_i', W_j)\), in this case, country B has comparative advantage in the high \(E\) intensity \(\alpha\) good; When country B chooses \((E_i', W_j)\) and country A chooses \((E_j, W_i)\) to produce the \(\beta\) good, this comparative advantage is amplified; When country B chooses \((E_i', W_i)\) to produce the \(\beta\) good, country...

\(^{30}\)If instead \(E_i > E_j\) and \(W_i < W_j\), then \((E_i, W_j)\) is the dominant choice for any industry. In that case, we still have \(\frac{E_i}{W_i} < \frac{E_j}{W_j}\).
A will choose \((E_j, W_i)\), country A has comparative advantage in this \(\beta\) good. Analogously, in case of two goods with one modest intensity and one high \(W/E\) intensity, similar results will be obtained, the high diversity country will have comparative advantage in the industry with high \(W/E\) intensity and the low-diversity country has comparative advantage in the industry with modest intensity.

This theorem can be easily extended in the following way.

**Proposition 3.** In an open economy of two countries, assume same aggregate endowment along every talent dimension. One country has more diverse endowment distributions along one or more dimensions than the other does. Then the high-diversity country has comparative advantage in industries with extreme (high or low) intensities of that or those dimensions, while the other has comparative advantage in industries with middle intensities of that or those dimensions.

**Proof.** This proposition directly follows from Theorem 3. When there is a second dimension with higher diversity, it generates comparative advantage in industries with extreme intensities in this second dimension. This goes the same way with the first high-diversity dimension, since the extremely high relative intensity of one dimension corresponds to the extremely low relative intensity of the other dimension.

Compared to GM, similar results on the role of endowment diversity are generated here. In GM, the high-diversity economy has comparative advantage in industries featuring submodular production technology where the complementarity between tasks is lower, while the low-diversity economy has comparative advantage in industries featuring super-modular technology where the complementarity between tasks is higher. On the other hand, in my framework, industries differ in their task/talent intensity instead of task complementarity. The high-diversity economy has comparative advantage in producing industries with extreme talent intensities, while the low-diversity economy has comparative advantage in producing goods with middle talent intensities. The U.S. has a more diverse skill (talent) distribution compared with Japan. Thus U.S. has comparative advantage in those industries requiring very intensive usage of certain talent, acting like super stars on a team. While Japan’s comparative advantage lies in those industries require similar levels of various skills.

Again when compared with the model in OT, the endowment inequality comes from the increase of variances of \(s\) and \(l\). Increase of \(s\) variance means more bundles with extreme talent ratios, while the increase of \(l\) variance with correlation held constant results in more relative production of high-\(s\) goods. The diversity here denotes variances of different dimensions of talent endowment. It works similarly with the variance of \(s\) in OT.
3.3 The New Gains From Trade

There are new gains from trade attributed to the changes of the effective endowments in each country through two channels. First, when there is no dominant agent matching scheme, then optimal agent matching contains a proper mix of different agent matching schemes. The effective endowment can be changed by using a different mix of schemes. Moreover, assuming decreasing cost to scale for agent re-matching, the production possibility set is non-convex when there is no dominating agent matching scheme as discussed in the baseline model. This leads to another type of gains similar with the gains from trade in scale economy models.

Second, the effective endowment can be changed if for some teams there is no dominant choice of effective bundle for all industries. Then these teams may switch their effective bundles and change industries as good prices change when opening to trade.

In the general model with continuous endowment levels and a continuum of industries, there are several conditions required for these new gains to exist. I formally state them below.

**Assumption 1.** Agents have multidimensional endowment of talents; The production is performed by teams of completely specialized workers; There is a continuum of industries with different task intensities.

**Assumption 2.** There is no dominating matching scheme, thus the PPFs under different matching schemes have intersections.

**Assumption 3.** There is no dominating choice of effective talent bundle across industries for all teams along the PPF.

**Proposition 4.** Given assumptions 1, with either or both assumptions 2 and 3 hold, the new source of gains from trade will exist.

**Proof.** This proposition directly follows the intuitions shown in the baseline model. When assumptions 1, 2, 3 hold, there will always be potential to change the effective endowment of the economy to achieve a better utilization and allocation of the initial multi-dimensional endowments.

**Proposition 5.** Given proposition 1, the countries with higher endowment correlations have higher probability to obtain these new gains from trade.

**Proof.** With higher endowment correlation, the trade-off when choosing effective endowment bundles is relatively tied. Thus when outside conditions change, such as good price changes, there are more teams who want to change their effective bundles. Hence it is easier for this economy to gain from trade by changing the effective endowment.
The intuition of this proposition can be connected with mobility. High correlation endowment results in close trade-off between effective bundles, which differ in talent ratios. It is also saying these teams are more flexible in choosing effective bundles and industries. When outside conditions change, they are more mobile, thus also easier to gain from integration.

**Proposition 6.** As the production specialization increases, the number of asymmetric tasks, i.e. skill dimensions, increases. Given assumption 1, ceteris paribus, there is increasing probability for the extra source of gains to exist.

**Proof.** Given the assumption of multi-dimensional endowment, as the specialization of production increases, the dimension of talents increases, there are more potential choices of effective bundles. Hence there will be more potential to improve when outside conditions change upon trade integration.

### 3.4 Wage Inequality and Job Polarization

As shown in the baseline model, this new framework with multidimensional endowment and team production seems promising in explaining both the job polarization in all North economies and the different wage evolutions across the developed world.

Ever since the Roy model, multidimensional endowment has drawn attention from economists. Particularly, the pursuit of comparative advantage is shown to reduce the level of inequality from what would occur in a random assignment of workers into occupations. And this comparative advantage effect depends on the correlation between endowment dimensions. Gould (2002) finds evidence that the increasing correlation in US indeed contributed to the rising wage inequality.

In this new framework, endowment correlation has richer implications not only on the trade patterns as stated in the Theorem 2 but also on wage inequality and employment changes.

Multidimensional endowment has two effects on wage inequality. The selection effect decreases wage inequality when agents can choose their tasks according to their comparative advantages. The endowment also provide the option value when teams bargain on the shares of final output.

In the competitive case, assume that there are many potential firms that can freely enter the market and employ each team. Thus the two agents on each team get the total surplus of that team. In a competitive environment, there are also many teams with the same effective bundle in each industry, thus each agent on the team gets the share that he contributes to

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31 Each dimension could be a particular subset of different talents. The increase of dimensions gives increasing number of ways to utilize the multidimensional talents.
the total surplus. Particularly, in the Cobb-Douglas production case, each gets a share that equals his talent intensity.

**Proposition 7.** Given assumption 1, in the competitive open economy, countries with comparative advantage in those extreme-intensity industries experience a higher wage inequality level; In contrast, the wage inequality level decreases in those countries with comparative advantage in middle-intensity industries.

*Proof.* In the competitive environment, agents’ share on team surplus depends on their talent intensities. With more teams enter the extreme-intensity industries, the surplus division between team members becomes more uneven. Thus the wage inequality increases. The contrary applies to the economies with comparative advantage in middle-intensity industries.

Blum (2008) finds that changes in the sectoral composition of the economy were the most important force behind the widening of the wage gap, accounting for about 60% of the relative increase in wages of skilled workers between 1970 and 1996. This fact seems to be in line with the above proposition. Service sector is usually recognized as a sector where personal success is more important in final output. Assuming team production, in service sector there are more teams with one superman and one sidekick. The crucial skill is more intensively used than the other. And wage gap between the superman and the sidekick is large. Thus an expansion of the service sector may results in an increase in wage inequality.

**Proposition 8.** In an open economy with multidimensional endowment, assume that the industry intensities are biased toward certain dimension. The developed North has higher endowment along that dimension relative to the South. Then a “job polarization” employment pattern will exist in the North after integration.

*Proof.* When the industry intensities are truncated, i.e. biased toward certain dimension. For instance, in all industries the entrepreneurial talent is equally or more intensively used than the worker skill. And the South has relatively poor endowment in the entrepreneurial talent, thus relatively rich endowment in the worker skill. In the open economy, the North has comparative advantage in those high $E$-intensity industries. In the competitive environment, these industries contain agents who earns the highest and lowest wages in the North. After trade integration, these industries expand the most. Hence employment in these industries increases relative to that in industries with middle intensities.

In most of the developed economies, a relatively higher increase in the employment of low-wage and high-wage jobs, namely the “job polarization”, has been found over the last
three decades (Goos et al., 2007, 2009). Most of the researches on this pattern has been focused on the closed economy (Autor et al., 2006, 2008). Only a few tries to consider the role of trade integration (Costinot and Vogel, 2010 and Monte, 2011).\textsuperscript{32} The task approach used by Autor et al. is the most recognized framework to investigate the job polarization pattern. They assume that workers on those middle wage jobs mainly perform some routine tasks, which are substitutable by machines and computers. In fact, when Dustmann, Ludstech and Schönberg (2009) dissect the German wage structure, they find it may not be true that those middle-wage workers are the ones who mainly perform routine tasks.

Even though this employment evolution pattern, job polarization, has been found in most North economies. There is however not a common pattern of wage and inequality evolution. The stylized fact in the literature says that US has a higher inequality level than the EU. Also Autor et al (2006) also finds a “wage polarization” in the US labor market besides the job polarization. The wage increase for low-wage workers and high-wage workers is higher than that for middle-wage workers. They propose the task approach and predict wage polarization should accompany job polarization. However, there is no strong evidence showing that wage polarization also exist in other developed North economies. The upper-tail inequality does not increase in France or Japan. In Germany, below the median, the correlation between employment and wage changes is negative rather than positive in the US.

This new framework with multidimensional endowment seems to be able to reconcile these differences. Based on Proposition 8 I argue that the common pattern of job polarization in the North may be a result of the North-South trade integration. To explain the different wage evolutions across North countries, this model embeds the difference in endowment correlations. European countries and also Japan have very specialized workers due to their education system and also labor market institutions. This low endowment correlation generates comparative advantage in middle-intensity industries, thus lower wage inequality and small or no wage polarization.

\section*{3.5 Discussion}

In this section, I will roughly discuss several interesting implications of this model.

\subsection*{3.5.1 Educational Policy}

As shown in the above analysis, multidimensional endowment plays an important role in trade and labor market. The endowment correlation and dispersion both matters. In each

\footnote{\textsuperscript{32}Costinot and Vogel (2010) needs to assume a extreme-biased technology change to generate job polarization in all the North economies. Monte (2011) shows the skill-biased technology and trade integration between identical countries can produce the same pattern.}
country, agents’ talent endowments are largely shaped by its education system, thus different educational policies may have profound influence on the economy.

For example, the Germany has a very skill-specialized vocational educational system as compared with a more general-skill education system in the US. Youths in Germany are divided into schools with very different missions at a very early age (around 10), some of them are then going to secondary schools to prepare for college, while others are going to schools prepared for various vocational trainings. This early division increases the specialization of the German labor force, and decreases the correlation of their skill endowments. In the Unites States however, general purpose technology and general training are more emphasized in most liberal arts schools. Thus the correlation of talent endowment across dimensions is very high in the US. Workers with more years of education outperform those with less education in almost every dimension. As shown in my model, this difference in correlations generates comparative advantage in the service industries with extreme skill/task intensities for the US, and in manufacturing with middle skill intensities for the Germany.

Given the set of available industries, educational policies should be set in particular ways to accompany one’s comparative advantage. Particularly, for countries with comparative advantage in middle-intensity industries, such as manufacturing, more diversified skill endowments are preferred. Hence education policies should encourage specialization. Government subsidies for vocational training, unemployment insurance and directed agent match are optimal choices. The reverse is true for other economies with comparative advantage in extreme-intensity industries.

As specialization increases, the dimension of talent/skills increases, an efficient education system should allocate more resources to diversify the talent endowments, in order to limit the waste of talents and increase the equilibrium effective endowments that are actually utilized in production.

3.5.2 Task Off-shoring

The baseline model discussed in section 2 can be easily extended to allow international teams, or off-shoring certain tasks. It is interesting to consider the changes of occupational structure in each economy and its effects on the wage structure.

Consider the integration of a type-I North economy and a type-I South economy, if under autarky the self-matching scheme dominates. Then the effective bundles are: half \((L, L)\) and half \((H, H)\) in the North economy, half \((L', L)\) and half \((H', H)\) in the South economy. When open to trade, the North specialize in the \(E\) intensity good and South specialize in the \(W\) intensive good.

\[^{33}\text{This case exists when } H/L \text{ is not too big and talent intensities are not too extreme.}\]
When agents in these two economies are allowed to form international teams, the optimal matching includes task off-shoring. North teams with effective bundle \((L, L)\) and South teams with effective bundle \((L', L)\) will break up. Each North agent with endowment \((E, W) = (L, L)\) is matched with a South agent with endowment \((E, W) = (L', L)\), each new team has effective bundle \((L, L)\), all North agents are doing the entrepreneurial task and South agents are doing the worker task. In the world economy, the effective bundles are half \((H, H)\), half \((H', H)\) and a unit of \((L, L)\). It is a welfare improvement allowing task off-shoring. In the end, there are more entrepreneurs in the North and more workers in the South.

Eeckhout and Jovanovic (2012) builds a model to investigate the occupational choice and development. Their predictions on occupational choice are similar with the model in this paper. And they do find support from the cross-country data that after integration, the high-skill countries see a disproportionate increase in managerial occupations and low-skill countries see an increase in wage work occupations.

### 3.5.3 Trade and Inequality

The effect of trade integration on income inequality, particularly in developing countries, still puzzles economists. Standard theories predict that inequality increases in the North developed economies, but decrease in the South developing economics. However, empirical evidences for this effect of trade on income distribution are mixed.

This paper emphasizes the role of industrial structure in shaping the income distribution. As economies develop, they experience different stages. Industrial specialization moves from manufacturing to service. As Blum (2008) has pointed out the rise of service sector is the most important contributor to the increase of US wage inequality between 1970 and 1996. When open to trade, the structural change is accelerated in the North. Thus inequality increases. The South also specialize in some industries with extreme high intensities in its abundant talent, the inequality level may not decrease as the standard trade model predicts.

### 4 Conclusions

This paper extends the standard Heckscher-Ohlin model with multidimensional skill endowment and team production to revisit the role of skill distributions in the open economy. Under the model settings, part of the initial endowment is utilized in production, which is defined as “effective endowment”. Specialization and trade pattern in equilibrium is governed by this effective endowment, which in turn is determined by the optimal choice of team-matching and task assignment within teams. It is shown that different skill distributions will generate
divergent effective endowment even with same aggregate factor endowment across countries. In particular, more diverse skill distribution across the labor force and more correlated endowment between skill dimensions of each individual both gives a country comparative advantage in those biased-intensity industries. This further generates higher wage inequality for these countries. These implications are broadly in line with the empirical facts. Thus this model is able to explain the puzzling trade patterns between similar countries, along with some labor market outcomes. Moreover, this model generates a job polarization pattern in the North economies with North-South trade under reasonable assumptions. This is in line with the fact that the job polarization pattern took place roughly in the second era of globalization, when the South economies open up to trade with the North. Hence this model suggests a role for globalization behind the job polarization pattern, complementing the existing explanation based on routine-task substitute technology changes in the labor literature.

Most importantly, this paper finds another source of gains from trade. These gains come from the potential adjustments of the effective endowment when a country moves from autarky to free trade.

Lastly, a new channel through which institutions in a country can affect its specialization and trade is revealed in this paper. It is by shaping the skill distributions. As previously mentioned, more skill-specific vocational training usually results in a low skill correlation compared to the liberal arts education system where general skill training is more emphasized. In this way, education polices in a country may have effect on its comparative advantages.

This framework can be adopted to investigate many interesting questions. Several applications of the model have been considered in this paper, such as educational policy, task off-shoring and the effect of trade on wage inequality. There are also interesting empirical implications that might lead to promising empirical exercises when proper data sets are available. For example, in the case of North-South trade, the best managers and worst workers in the North benefit the most. Thus within the manager occupation, inequality increases. On the other hand, the inequality within the worker occupation decreases. The reverse is true for the South economy. These qualitative implications calls for more rigorous empirical investigations in the future.

The current model is significantly simplified in several ways and can be further extended. The agent matching process in this model is not specified in details given the focus of current paper. In reality agents are able to direct their searches towards particular jobs based on their endowments, and the matching outcome may differ. The search model is promising because it links to possible labor and trade policy interventions that may improve efficiency. Unlike
the adjustment costs discussions of the earlier literature (Leamer (1980), Feenstra and Lewis (1994), etc.), inefficiencies related to team production and matching appear to be long lasting and thus especially suitable targets for policy. Moreover, the team production organization is also extremely simplified. The team size is fixed. In contrast, many papers have pointed out that the size of the production unit is usually bigger in manufacturing than in services.\(^{34}\) The size of teams may also be endogenously chosen by different firms. For countries with high-correlation endowment distributions, smaller team size may be chosen so that less talents are left idle. This might introduce another dimension of effect that endowment correlation may have on comparative advantages across countries. A more detailed and micro-founded characterization of the team production seems promising and will be pursued in my future research.

References


\(^{34}\)Buera and Kaboski (2012) for example


