Diamonds are a Government's Best Friend: Burden-Free Taxes on Goods Valued for their Values

By YEW-KWANG NG*

To most economists, it seems almost axiomatic that taxes (except corrective taxes) impose not just a burden equal to the amount of the tax collected, but also an excess burden by distorting individual choices, not to mention administrative, compliance, and policing costs (loosely called transaction costs below). Lump sum taxes with no excess burden exist only in theory. Yet there exists at least one class of goods that can be taxed and the tax will not only not produce an excess burden, but it will not be a burden at all (ignoring transaction costs)! This sounds like a miracle but it is really quite simple once it is recognized that some goods are valued for their values, not for their intrinsic consumption effects. Taxes on these goods increase their prices. But consumers can reduce quantities consumed without changing the values of these goods, suffering no loss in utility. Thus, no burden is imposed, not to mention excess burden. For example, after a doubling in the price of diamond, a $1000 gift of diamond is still valued at $1000, though the size of the stone is smaller.

A carat of diamond can be worth thousands of dollars, but costume jewelry that looks similar may cost only a few dollars. Imitation diamonds look virtually the same as real diamonds and it takes experts with fine instruments to tell the difference. Surely, diamonds are valued not for their intrinsic consumption effects but because they are costly. Consumers of diamonds either derive utility by showing off their wealth (Veblen's conspicuous consumption), by using it as a store of value, or by giving it as a gift of value. In virtually all cases, it is the value, not the diamond itself, that counts. This applies to most other precious stones and metals, including gold. In various lesser degrees, this "diamond effect" also applies to other items of conspicuous consumption such as expensive fur coats and luxurious cars. With increasing affluence, "diamond goods" will become more important.

The diamond effect (valuing something for its value rather than the consumption effect) must be distinguished from other similar or related but different phenomena. First, "the habit of judging quality by price" (Tibor Scitovsky, 1945) is the belief that higher-priced brands give higher intrinsic consumption utilities. Second, while Thorstein Veblen's (1899) conspicuous consumption may partly contribute to the diamond effect, they are conceptually distinct. A person's desire to go on a world trip may be partly to show off to his (her) admiring friends who cannot afford to go. But as long as he can and they cannot, it has value for conspicuous consumption. Increases in the price of the trip may add little to the value and half a world trip is not as good, even if the price of the world trip has doubled. On the other hand, a man's gift of a $1000 diamond ring to his wife is worth that much irrespective of the size of the stone, and they may never show the ring to anybody. Also, some people show off their heavy gold bracelets (conspicuous consumption with a diamond effect) while others hide their gold bullion (a diamond effect without conspicuous consumption). Third, there are goods whose intrinsic consumption effects depend on whether other people also consume them (for example, telephones, unusual clothing, fashions). To some extent, this consideration may also affect the diamond effect. However, to concentrate on the pure diamond effect, I will abstract these complications away.

As in the distinction between private and public goods where different degrees of pub-

*Department of Economics, Monash University, Clayton, Victoria 3168 Australia, and University of Maryland-College Park.

licness are involved, most goods are valued partly for their intrinsic consumption effects (approaching 100 percent for ordinary items like bread) and their values (approaching 100 percent for precious jewels). However, for analytical simplicity, I consider only two polar cases and adopt an atemporal model commonly used in welfare analysis. The complicated questions of the dynamics of transition and some other practical complications are touched on in the concluding section, but are not formally analyzed.

The basic result is that a change in the price of a diamond good leaves its own value unchanged and the amount of all other goods consumed, and hence the utility levels of consumers unchanged, and that it is optimal to place arbitrarily high taxes on diamond goods which impose no burden and no excess burden. A corollary is that the demand curve for a diamond good is a rectangular hyperbola with unit price elasticity throughout.

Such an obvious phenomenon as the diamond effect has not of course completely escaped the economist's attention. For example, Pigou touched on the "desire to possess what other people do not possess" (1932, p. 226) and used diamonds as an example. But there is a curious lack of formal analysis,[1] and almost complete disregard in the public finance texts (for example, Richard Musgrave and Peggy Musgrave, 1980) and actual policy debate on taxation issues (for example, the great Australian tax reform in 1985). This paper provides a modest attempt at a formal analysis which may attract, hopefully, more attention both by theorists and those concerned with policy decision.

I. A Simple Analysis

For simplicity, consider the case with only one diamond good (say, the first) and ignore all other complications (externalities, etc.). Generalization to impure diamond goods is straightforward, but impure or mixed goods bring some complications not considered in this paper. The utility function of an individual may thus be written as

$$U\left(p^1x^1/p^n, x^2, \dots, x^n\right), \tag{1}$$

where x^i is the amount of the ith good consumed, p^i its price, and the last good is being used as a numeraire. In the long run, the consumer is unlikely to suffer from significant money illusion. Thus, instead of the money value of the diamond good p^1x^1, we should replace the money price p^1 by real or relative price p^1/p^n.[2] For impure diamond goods, both p^1x^1/p^n and x^1 enter the utility function.

Taking prices as given, the consumer maximizes (1) subject to

$$\sum p^ix^i = M, \tag{2}$$

where the summation is over all the n goods and M is the given amount of income. This problem is homogeneous of degree zero in all prices and money income; no money illusion is involved.

Assuming an interior solution for notational simplicity, the first-order conditions for optimality are

$$U_1 = \lambda p^n \tag{3a}$$

$$U_i = \lambda p^i \qquad (i = 2,\dots,n) \tag{3b}$$

where U_i is the partial derivative (marginal utility) of the ith element in the utility function (1), and λ is the Lagrangian multiplier

[1] Peter Kalman (1968) provides a rigorous analysis of consumer behavior when prices enter the utility function. This is a very general analysis which may be said to include "judging quality by price," conspicuous consumption, and the diamond effect. However, partly because it is too general and partly because of its exclusive concern with the positive theory of consumer behavior, it reaches none of the results of this paper.

[2] Alternatively, we may replace p^n in (1) by P, a price index of all nondiamond goods, with the same result except that p^n below is replaced by P.

associated with (2) or the marginal utility of income.

The price of a diamond, p^1, does not appear in the system of equations (3) describing the optimal solution. It is tempting but wrong to infer from this that the optimal x's are independent of p^1. This is wrong because p^1 appears in (2) which is included in the set of equations, together with (3), defining the optimal solution. However, it is true that p^1x^1/p^n and $x^2,\dots,x^n$ and hence U are independent of p^1, as shown below.

The maximization problem above may be written, with no change of any substance, as the maximization of

$$U(y^1, y^2,\dots, y^n) \tag{4}$$

subject to

$$\sum q^i y^i = M, \tag{5}$$

where

$$y^1 \equiv p^1x^1/p^n, \quad y^i \equiv x^i \qquad (i=2,\dots,n),$$

$$q^1 \equiv p^n, \qquad q^i \equiv p^i \qquad (i=2,\dots,n).$$

This rewritten problem is identical in its mathematical form to the traditional consumer optimization problem with no diamond effect, and with the following familiar first-order conditions for an interior solution,

$$U_i = \lambda q^i \qquad (i=1,\dots,n), \tag{6}$$

which, with constraint equation (5), define the optimal y's.

With the rewritten problem, if we work in terms of y's instead of x's (the only difference is to take p^1x^1/p^n as an integral variable instead of breaking it up into its constituent parts), it is clear that p^1 appears neither in the constraint (5) nor in the first-order condition (6). The optimal set of y's and hence the maximized utility level are thus independent of p^1. This result may be expressed as

PROPOSITION 1: *A change in the price of a diamond good leaves its value and the amounts of all other goods consumed, and hence the utility level of the consumer unaffected.*

COROLLARY 1: *The demand curve for a diamond good is a rectangular hyperbola with unit elasticity throughout the whole range where it remains a pure diamond good.*

This is true not only for an individual demand curve, but also for a market demand curve as long as the good is viewed by all consumers as a diamond good (assumed here for simplicity) because the horizontal summation of rectangular hyperbolas is also a rectangular hyperbola.

For simplicity, assume a horizontal supply curve. A 100 percent tax on the diamond good then doubles its price and a 200 percent tax triples it, etc. The higher the tax rate, the larger the tax revenue, while consumers remain no worse off. The tax revenue collected thus represents pure gain, imposing not only no excess burden, but also no burden at all!

There is an upper limit beyond which the tax revenue cannot exceed. This supremum (the maximum does not exist) is the pre-tax (= post-tax) value of the good. The amount of tax revenue that can be raised without burden is limited by the amount of expenditure on diamond goods (which may be expected to increase relatively and absolutely with increasing affluence). The net gain is the amount of resources saved due to a smaller output after the imposition of the diamond tax.

II. A Model of Optimal Taxation

The above analysis may be regarded as somewhat partial and/or intuitive. Here, I present a standard model of optimal taxation, except that I allow for pure diamond goods. Since no changes in the relative price between private goods is considered, I lump them into a composite good y. Similarly, I lump all diamond goods into another composite good d. As in the standard optimal taxation literature, I concentrate on the tax side by assuming a constant government revenue requirement and assume that the consumer side of the economy can be represented by one consumer or a community utility function,

$$(7) \qquad U(D, y),$$

where $D \equiv (q+t)d/(Q+T)$ is the (relative) value of the diamond good, q and Q are the fixed producer prices, and t and T the per unit taxes on diamond and the private good, respectively. The assumption of a representative consumer does assume away distributional issues, but may be justified by the predominant concern on efficiency issues and the argument on separating equity and efficiency issues even in the presence of second-best factors and other complications (see my 1984 paper).

The consumer maximizes (7) with respect to d and y, subject to

$$(8) \qquad (q+t)d+(Q+T)y = M,$$

where money income M is taken as given. While this may seem to abstract away work-leisure choice, we may alternatively interpret M as full income and include leisure in the composite good y, with the result that leisure is regarded as taxable. If I can establish the result on the optimality of imposing a high tax on diamond even in a model where leisure is taxable, the desirability of doing so where leisure is not taxable seems to apply a fortiori.

The first-order condition for the consumer maximization is

$$(9) \qquad U_D/U_y = 1,$$

where U_D is the marginal utility of the value of diamond (relative to the price of private good) consumer and U_y the marginal utility of the private good. In other words,

PROPOSITION 2: *In equilibrium, the marginal rate of substitution between the (relative) value of diamond and the private good equals unity.*

This may appear too simple to be true. But if I write the budget constraint (8) as

$$(8') \qquad (Q+T)D+(Q+T)y = M,$$

it can immediately be seen that the consumer price $(Q+T)$ of the private good serves as the price for both the private good y and for the relative value of diamond D, and equation (9) is thus obvious. The consumer allocates his (her) income M between two goods D and y that have the same price, so $MRS = 1$ for an interior maximum.

As discussed in Section I, the consumer's optimal choice between D and y is independent of the consumer diamond price, $q+t$, which entered neither (8′) nor (9). We thus have

$$(10) \qquad \partial D/\partial t = 0 = \partial y/\partial t.$$

From the first inequality in (10) and the definition of D, we have

$$(11) \qquad \eta^{dt} = -t/(q+t),$$

where $\eta^{dt} \equiv (\partial d/\partial t)d/t$ is the elasticity of d with respect to t.

The government maximizes (7) with respect to t and T, subject to the consumer's choice described above and to the fixed revenue constraint

$$(12) \qquad dt + yT = \bar{R}.$$

The first-order conditions for an interior solution are

$$(13) \qquad \left(\frac{d}{Q+T}+\frac{q+t}{Q+T}\frac{\partial d}{\partial t}\right)U_D+\frac{\partial y}{\partial t}U_y = \theta\left(d+t\frac{\partial d}{\partial t}+T\frac{\partial y}{\partial t}\right)$$

$$(14) \qquad \left\{\frac{q+t}{Q+T}\frac{\partial d}{\partial T}-\frac{(q+t)d}{(Q+T)^2}\right\}U_D + \frac{\partial y}{\partial T}U_y = \theta\left(y+T\frac{\partial y}{\partial T}+t\frac{\partial d}{\partial T}\right),$$

where θ is the Lagrangian multiplier associated with (12). Eliminate θ between (13) and (14) and rewrite expressions in elasticity form, that is, $\eta^{xy} = (\partial x/\partial y)y/x$, we have

(15)

$$\frac{\frac{d}{t}\left(\frac{t}{Q+T}+\frac{q+t}{Q+T}\eta^{dt}\right)U_D+\frac{y}{t}\eta^{yt}U_y}{\frac{d}{T}\left\{\frac{q+t}{Q+T}\eta^{dT}-\frac{(q+t)T}{(Q+T)^2}\right\}U_D+\frac{y}{T}\eta^{yt}U_y}$$

$$=\frac{d(1+\eta^{dt})+\frac{yT}{t}\eta^{yt}}{y(1+\eta^{yT})+\frac{dt}{T}\eta^{dT}}.$$

Substitute η^{dt} from (11) and $\eta^{yt}=0$ (from the second equality in equation (10)) into (15), the numerator of the left-hand side of (15) becomes zero. The denominator of the right-hand side does not equal infinity unless t itself is infinite, since an infinitesimal change in T (equivalent to a reverse change in M) does not cause a jump in either y or d under traditional assumptions about the consumer. Therefore, the numerator of the right-hand side must equal zero. Since $\eta^{yt}=0$ (from equation (10)), $d \neq 0$ for an interior solution, we have $1+\eta^{dt} = 1 - t/(q+t)$ (from equation (11)) $= 0$), or

$$(16) \qquad t/(q+t) = 1.$$

Since q is nonzero, (16) can hold if and only if t is infinite. This gives us

PROPOSITION 3: *A pure diamond good has an infinite tax in an optimal tax system.*

This result confirms the analysis of Section I. Of course, in practice, as the tax on diamonds gets to be very high, the physical amount of diamonds of a given value becomes very small. This will eventually affect the intrinsic consumption value of diamonds, or at least increase the cost of handling tiny quantities. Thus my model of pure diamond goods ceases to be an accurate approximation when t becomes very high. Taking account of this, a very high tax rather than an infinite tax is optimal.

III. Concluding Remarks

Other practical considerations also suggest a reasonably high, instead of an arbitrarily huge, tax on diamond goods. Too high a tax induces tax evasion (including smuggling), especially if only a few countries are imposing high taxes. This suggests that international cooperation to raise taxes on diamond goods may be desirable.

My analysis, being conducted in an atemporal framework, also ignores the complication of dynamic transition. Ideally, when taxes on diamond goods are introduced, the preexisting possessors of diamond goods should also pay the taxes. This enlarges the tax base (and hence the tax revenue) and also avoids the distributional problem if only new diamond goods are taxed. This problem arises because existing possessors of diamond goods are actually made better off by the taxes, while new consumers of diamond goods are not. However, it may not be administratively and politically feasible to tax existing diamond goods. The distributional problem may then suggest that taxes on diamond goods should be lower. The following example will make clear this distributional-dynamic transition issue. Assume that a 100 percent tax on all future production (assumed competitive) or consumption of a (set of) diamond good is imposed. A distributionally neutral policy is to impose the same 100 percent tax on existing stocks in the form of conscripting 50 percent of all holdings which are then destroyed. The gain in revenue consists only in taxes on future production. No one is worse off as all diamond goods double in prices. If the conscripted 50 percent are not destroyed but put back into the market (as equivalent, ignoring transaction costs, to a 100 percent monetary tax on existing stocks), this will depress the prices of diamond goods (from their doubled values) and make existing holders worse off. However, reasonably assuming continuity, there exists a tax rate t $(0 < t < 100\%)$ on existing holdings that will leave them indifferent. The government will then immediately gain the revenue from taxes on existing holdings, at a cost (in comparison to the alternative of destroying half of the existing holdings) of foregoing revenue in the immediate future when no diamond goods are produced, at least not from existing marginal producers, since prices are below marginal production costs plus taxes. The transition raises some interesting dynamic problems (including the relative desirability of alternative policies). However, a detailed analysis requires an explicitly dynamic model beyond the scope of this paper.

In any case, the argument for treating a dollar as a dollar to whomsoever it goes (see my earlier paper) suggests that it is better to impose the full burden-free taxes on efficiency considerations with possible adjustments in income and wealth taxes to achieve the objective of equality.

REFERENCES

Kalman, Peter J., "Theory of Consumer Behavior when Prices Enter the Utility Function," *Econometrica*, July-October 1968, *36*, 497–510.

Musgrave, Richard A. and Musgrave, Peggy B., *Public Finance in Theory and Practice*, 3rd ed., New York: McGraw-Hill, 1980.

Ng, Yew-Kwang, "Quasi-Pareto Social Improvements," *American Economic Review*, December 1984, *74*, 1033–50.

Pigou, Arthur C., *The Economics of Welfare*, 4th ed., London: Macmillan, 1932.

Scitovsky, Tibor, "Some Consequences of the Habit of Judging Quality by Price," *Review of Economic Studies*, No. 2, 1945, *12*, 100–05.

Veblen, Thorstein, *The Theory of the Leisure Class*, New York: Macmillan, 1899.