INDIRECT PRICING THEORY OF THE FIRM:
A GENERAL-EQUILIBRIUM ANALYSIS
INVOLVING PRODUCTION TECHNOLOGY AND
MANAGEMENT SERVICE

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Abstract. This paper develops a general-equilibrium model that involves production technology and management service to study the emergence of firms from the perspective of saving transaction costs. Inframarginal analysis is used to formalize Coase and Cheung’s theory of the firm and to generalize Yang and Ng’s indirect pricing theory of the firm. Not only the emergence of firms, their growth and contraction, but also the relevant conditions for the existence of technology entrepreneurs and professional management are also investigated.

1. INTRODUCTION

This paper employs Yang and Ng’s analytical framework (Yang and Ng, 1993; 1995) to investigate the existence of firms in the line of Coase and Cheung’s theory of the firm (Coase, 1937; Cheung, 1983). (See also related contributions by Borland and Yang, 1995; Liu and Yang, 2000; Yang, 2001.)

There are two alternative ways to coordinate production: one is using the market, which is decentralized and achieves resources allocation through the price mechanism; the other is through the internal organization of the firm, which suppresses the price mechanism with central planning and direct control. According to Coase (1937, p. 392), ‘The operation of a market costs something and by forming an organization and allowing some authority (an “entrepreneur”) to direct the resources, certain marketing costs are saved.’ The costs to use the market are: ‘the brokerage cost of finding a correct price, the cost of defining the obligations of parties in a contract, the risk of scheduling and related input costs; and the taxes paid on exchange transactions in a market.’ (Rugman, 1980, p. 369)

Cheung (1983) interprets Coase’s theory of the firm in the light of contractual arrangements. Cheung argues that ‘we do not exactly know what the firm is – nor is it vital to know. The word “firm” is simply a shorthand description of a way to organize activities under contractual arrangements that differ from

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1 Li and Ng (2004) also use the Yang-Ng framework to analyse the emergence of multinational enterprises. Both being based on the same framework, the two papers contain some similar equations and graphs. However, while the previous paper focuses on multinational enterprises, the present paper focuses on the indirect pricing theory of the firm.
those of ordinary product markets.’ He claims ‘The growth of a firm may then be viewed as the replacement of a product market by a factor market, resulting in a saving in transaction costs.’

Yang and Ng (1993; 1995) and Yang (2001) refine Coase and Cheung’s theory of the firm and put forward an indirect pricing theory of the firm. They claim that ‘The institution of the firm can get the activity with the lowest transaction efficiency involved in the division of labor, while avoiding the direct pricing and marketing of the outputs and inputs of that activity. The residual returns claimed by the employer are the indirect price of her efforts’. In their model, a single intermediate good is needed in the production of the final good. The intermediate good can be produced inside a firm, or it can be purchased from the market. Yang and Ng interpreted Cheung’s argument of the replacement of the product market with the labour market as a necessary condition for the emergence of the firm.

The model in this paper is different from that of Yang and Ng (1993; 1995). First, two intermediate goods are introduced into our analytical framework. They can be interpreted as production technology and management service. We also assume that there are two methods to produce a final good. There is the non-firm mode of production, where every individual can transform the production technology self-produced into the final good, or some individuals in the economy fully specialize in the production of production technology and all others fully specialize in transforming the production technology purchased into the final good. The other method is production within a firm, where all employees are assigned to specialize in the production of different goods and their production activities are well organized and carefully monitored. Without management service a firm cannot exist.

The other difference between our model and Yang and Ng’s is that management service is considered a necessary input in the firm’s mode of production. Not only can the model with two intermediate goods formalize Coase and Cheung’s theory of the firm, it can also test whether or not our model supports Yang and Ng’s indirect pricing theory of the firm. Since our model involves production technology and management service, we hope that this model can further refine Coase and Cheung’s theory of the firm by explaining not only the emergence of the technology entrepreneur but also the emergence of professional management.

The rest of the paper is organized as follows. In Section 2, the model with two kinds of intermediate goods is introduced and two types of the production of the final good are explained. In Section 3, all possible market structures are analysed and their corner-equilibrium solutions are derived. In Section 4, the emergence of the firm, its growth and contraction and the conditions for the existence of technology entrepreneurs and professional management are presented. Finally, a brief conclusion is provided in Section 5.

2. A MODEL WITH TWO INTERMEDIATE GOODS

Let us consider an economy with $M_1$ ex ante identical consumer-producers. There are one consumer or final good and two intermediate goods. The amounts
self-provided of the consumer good and the two intermediate goods are \( y, x, \) and \( z \) respectively. The amounts of the three goods sold to markets are \( y^s, x^s \) and \( z^s \) respectively. The amounts of the three goods purchased from the markets are \( y^d, x^d \) and \( z^d \) respectively.

We assume that there are two methods to produce the final good: the non-firm mode and the firm mode. Intermediate good \( x \) can be interpreted as the production technology needed in the production of \( y \). Intermediate good \( z \) is the management service. If the final good is produced by the non-firm mode, production technology \( x \) is the only intermediate good needed in the production of the final good. In addition to production technology, management service is also a necessary input in the firm mode of production of \( y \).

2.1. The non-firm mode of production

First, let us look at the non-firm mode of production of the final good \( y \).

An individual’s production function for the final good is

\[
y + y^s = [(x + t_x x^d) l_y]^a, \quad 0.5 < a < 1, \tag{1}
\]

where \( x^d \) is the amount of the intermediate good purchased from the market and \( t_x \) is its transaction efficiency coefficient. Obviously, \( 1 - t_x \) is the transaction cost coefficient. \( t_x x^d \) is the net amount an individual receives from the purchase of this intermediate good. \( x + t_x x^d \) is the total amount of the intermediate good employed in the production of final good \( y \). \( l_y \) is the labour share in producing the final good and \( y + y^s \) is the output of the final good. A person’s labour share in producing a good is taken as her level of specialization in producing this good.

A production function for a good is said to exhibit economies of specialization if the total factor productivity of the good is an increasing function of the level of specialization in producing this good. Total factor productivity of the final good \( y \), \( (y + y^s) / [(x + t_x x^d) l_y] \), increases with the level of specialization \( l_y \) if \( a > 0.5 \). Parameter \( a \) represents the degree of economies of specialization in producing the final good.

The production function of the intermediate good is

\[
x + x^s = (l_x)^b \tag{2}
\]

where \( x + x^s \) is the output of the intermediate good and \( l_x \) is the amount of time spent producing \( x \) (it also measures the level of specialization in producing the intermediate good \( x \)). If \( b > 1 \), the production function displays economies of specialization. Similarly, parameter \( b \) represents the degree of economies of specialization in producing the intermediate good.

We assume that each individual is endowed with one unit of labour, so we have

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2 Please see footnote 4 for explanation.

3 We do not explicitly model the source of the transaction costs. Here, iceberg type transaction costs are assumed. Yang and Ng (1995) pointed out that the exogeneity of transaction costs allows us to capture in a simple way the main ideas which seem to have emerged from the transaction-cost literature, namely that transaction costs exist and have implications for production and exchange.
\[ l_x + l_y = 1, \quad 0 \leq l_i \leq 1, \quad i = x, y. \] (3)

The utility function of each individual is represented by

\[ U = y + ky^d \] (4)

where the amount self-provided of the final good is \( y \), \( y^d \) is the amount of \( y \) purchased from the market and \( k \) is the transaction efficiency coefficient of the final good market. \( ky^d \) is the net amount an individual receives from the purchase of this final good.

Finally, the budget constraint for each individual is

\[ P_x x^d + y^d = P_x x^s + y^s \]

where we assume that the price of final good is 1 and \( P_x \) is the price of the intermediate good in terms of the final good.

2.2. The firm mode of production

Yang and Ng (1993; 1995) and Yang (2001) define the institution of the firm as a structure of transactions based on the division of labour that satisfies the following conditions: (i) there are two types of partners who are associated with a firm, employer and employees; (ii) in relevant contracts, only the payment to employees is specified, the employer claims residual returns; and (iii) a firm must involve a process that transforms the labour of employees into something that is sold in the market by the owner of the firm. Here, another condition is added: (iv) management service is a necessary input in the firm mode of production. Management service includes monitoring and management. Without monitoring, production cannot be guaranteed due to shirking. More importantly, the firm mode of production involves many parties (or employees) and they are assigned to specialize in the production of different goods. Hence, the production function with the help of management service in the firm mode of production is different from that in the non-firm mode of production; Alchian and Demsetz (1972) claim that firm production is team production.

The production function of an individual in a firm is

\[ y_p + y_p^s = \{(x + t_xx^d)l_y^p(z + t_zz^d))^a, \quad 0.5 < a < 1 \} \] (5)

where \( [(x + t_xx^d)l_y^p(z + t_zz^d)]^a \) is the amount of the final good contributed by each individual in the process of production inside the firm. As the production of each individual has to be monitored, management service is necessary. \( z^d \) is the amount of the management service purchased from the market and \( t_z \) is its transaction efficiency coefficient. \( t_z z^d \) is the net amount an individual receives from the purchase of this intermediate good. \( y_p + y_p^s \) is the ultimate amount

\[ 4 \text{ If } a > 1, \text{ intuitively, the employer will hire all the individuals in the economy. This is not realistic. From equation (5), we obtain the output of each individual } y_p + y_p^s = [y^d(z + t_z z^d)]^a. \text{ The total factor input is } (y^d)^{0.5}(z + t_z z^d)^{0.5}. \text{ The total factor productivity is the ratio of the output of the final good to total factor input, given by } (y^d)^{0.5}(z + t_z z^d)^{-0.5}. \text{ The total factor productivity increases with the input, such as management service } (z + t_z z^d) \text{ if } a - 0.5 > 0. \text{ Hence we obtain } a > 0.5. \]
of the final good produced by each individual inside the firm with the assistance of management service.

\[ x + x' = (l_x)^b, \quad b > 1 \]  

Equation (6) is the same as (2).

\[ z + z^s = (l_z)^b, \quad b > 1 \]  

where \( l_x \) is a person’s level of specialization in producing management service. Parameter \( b \) represents the degree of economies of specialization in producing management service.

Because each individual is endowed with one unit of labour, we have

\[ l_x + l_y + l_z = 1, \quad 0 \leq l_i \leq 1, \quad i = x, y, z. \]  

The utility function of each individual is given by

\[ U = y_p + ky^d. \]  

In addition, everyone faces a budget constraint,

\[ P_x x^d + y^d_p + P_z z^d = P_x x^s + y^s_p + P_z z^s, \]

where \( P_z \) is the price of management service in terms of the final good and \( P_x \) is the price of production technology in terms of the final good.

Finally, free entry for all individuals into any sector and a large \( M_1 \) are assumed. Free entry implies that every individual can choose to be a production technology specialist, a management service specialist, or a worker who transforms production technology into a final good.

3. MARKET STRUCTURE AND CORNER EQUILIBRIUM

3.1. All the possible market structures

Each individual makes a decision about which goods to produce and on her demand for and supply of any traded good to maximize her utility. A given structure of production and trade activities for any individual is defined as a configuration. There are \( 2^6 = 64 \) combinations of zero and non-zero values of \( x, x', x^d, y, y', y^d \) and therefore 64 possible configurations for the non-firm mode of production.

Using the Kuhn-Tucker sufficiency theorem, we can rule out interior solutions and many corner solutions from the list of candidates for an optimal decision. Yang and Ng (1993) and Wen (1998) used Kuhn-Tucker conditions to establish the following lemma (see also Yao, 2002 and Diamantaras and Gilles, 2004 for the results under more general conditions).

**Lemma 1:** An individual sells at most one good and does not buy and self-provide the same good. She self-provides the consumer good if she sells it. If \( b > 1 \) and \( a \in (0.5, 1) \), she does not self-provide the intermediate good unless she produces the final good.
The combination of the configurations of the $M_1$ individuals in the economy is defined as a market structure. A feasible market structure is composed of a set of choices of configurations by individuals such that market-clearing conditions can be maintained. Each market structure has a corner equilibrium solution. A corner equilibrium is defined as a set of relative numbers of individuals choosing different configurations such that: (i) market clearing conditions are satisfied; and (ii) each individual maximizes her utility at a given set of prices for a given market structure.\(^5\)

After having considered Lemma 1 above, we can identify two feasible structures: A and D for the non-firm mode of production, as shown in Figures 1 and 2 respectively. Structure A is autarky.\(^6\) Each individual spends some time to self-provide intermediate good $x$, then she uses her remaining time and $x$ produced to produce the final good. Structure D is called the structure of the division of labour with no firms. In structure D, the markets for intermediate good $x$ and final good $y$ emerge. Some individuals in the economy specialize in producing $x$ and all others specialize in producing the final good. In structure D, let $(x/y)$ denote a configuration in which an individual sells $x$ and buys $y$, and let $(y/x)$ denote a configuration in which an individual sells $y$ and buys $x$.

There are $2^9 = 512$ combinations of zero and non-zero values of $x$, $x'$, $x''$, $y$, $y'$, $y''$, $z$, $z'$, $z''$ and therefore 512 possible configurations for the firm mode of production. Using Lemma 1 and other conditions such as the four conditions to form a firm, we can identify all the possible market structures of firms as shown in the following figures.

In structure F1 (shown in Figure 3), the owner of a firm is not fully specialized. She spends some time producing production technology, and her remaining time producing management service. The owner hires workers to transform her production technology into a final good. In structure F2 (shown in Figure 4), the owner is not fully specialized either. She spends some time producing management service, and her remaining time transforming the production technology produced by hired production technology specialists into the final good.

\(^5\) Please see Yang and Ng (1995) and Yang (2001) for the detailed explanation of relevant concepts such as configuration, market structure and corner equilibrium.

\(^6\) A criticism by Auerbach (1988) focuses on the disjointed way in which Coase and his followers see firms and markets, in particular the presupposition of the existence of markets and failure to see the role of firms in the making of markets. So, this paper begins with the study of market structures from autarky.

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In structures F3 (shown in Figure 5) and F4 (shown in Figure 6), a management service specialist is the owner of a firm and she is fully specialized in producing management service. In Structure F3, she hires production technology specialists to produce production technology, and workers to transform the production technology produced into a final good. In Structure F4, she directly purchases production technology from the market instead of hiring production technology specialists to produce it in her firm.

In structures F5 (shown in Figure 7) and F6 (shown in Figure 8), a production technology specialist is the owner of a firm and she is fully specialized in producing production technology. In Structure F5, she hires workers to transform her production technology into a final good and hires management service specialists to organize and monitor the production of the final good. In Structure F6, the owner directly purchases the management service rather than hires management service specialists to produce it in her firm.
In structures F7 (shown in Figure 9) and F8 (shown in Figure 10), a worker is the owner of a firm and she is fully specialized in producing the final good. In Structure F7, the owner hires production technology specialists to produce production technology, and hires management service specialists to organize and monitor the production of the final good. In Structure F8, the owner purchases the management service from the market instead of hiring management service specialists to produce it in her firm.

3.2. Corner equilibrium solutions

Since $l_x$, $l_y$, and $l_z$ are not independent of other decision variables, each of the nine decision variables $x$, $x'$, $x''$, $y$, $y'$, $y''$, $z$, $z'$, $z''$ can take 0 and positive values. When a decision variable takes a value of 0, a corner solution is chosen. Let us examine the 10 market structures one by one and obtain their solutions.

3.2.1. Structure A (autarky)

In autarky, there is no need to produce intermediate good $z$ (management service) and each individual self-provides intermediate good $x$ and then the final good...
After a simple mathematical manipulation, we can solve for the corner solution for Structure A,

\[ l_x = b(1 + b)^{-1}, \quad l_y = (1 + b)^{-1}, \quad x = b^b(1 + b)^{-b}, \quad U_A = b^{ab}(1 + b)^{-a(1 + b)} \]

where \( U_A \) is individual's per capita real income as well as the maximum per capita output level of the final good in autarky.

### 3.2.2. Structure D

In structure D, there is no need for each individual to produce intermediate good \( z \) (management service) and individuals specialize in either intermediate good \( x \) or final good \( y \). Structure D consists of configurations of \((x/y)\) and \((y/x)\). Authority and residual rights are equally distributed between the two parties to a contract.

In configuration \((x/y)\), the utility of the intermediate good’s producer can be represented by

\[
\text{Max} \quad U_x = ky^d \\
\text{s.t.} \quad x' = (l_y^b), \quad l_x = 1 \quad \text{(production function)} \\
P_x x' = y^d \quad \text{(budget constraint)}
\]

where \( k \) is the transaction efficiency coefficient of the final good’s market. We can obtain the indirect utility function for configuration \((x/y)\): \( U_x = kP_x \).

In configuration \((y/x)\), the final good’s producer’s utility can be represented by

\[
\text{Max} \quad U_y = y \\
\text{s.t.} \quad y + y' = (t_x x' l_y^a), \quad l_y = 1 \quad \text{(production function)} \\
P_x x' = y^s \quad \text{(budget constraint)}
\]

where \( t_x \) is the transaction efficiency coefficient of the market of intermediate good \( x \). We can obtain the indirect utility function for configuration \((y/x)\): \( U_y = (1 - a)(a t_x P_x^{-1})^a(x^a) \).

After we have obtained individual optimal decisions for these two configurations, market clearing conditions and utility equalization condition are used to solve for corner equilibrium,

\[ U_D = a^a(1 - a)^{1-a} t_x^a k^a \]

where \( U_D \) is the corner equilibrium solution to structure D and individual’s per capita real income.

From the market clearing condition, we have

\[ M_x x' = M_y x'. \]

In the mathematical manipulation above, we have obtained the values of \( x' \) and \( x' \). So in the corner equilibrium, the relative number of individuals selling intermediate good and final good is \( M_x / M_y = a(1 - a)^{-1}k \).

### 3.2.3. Structure F3

Let \((z/l_x, l_y)\) denote a configuration in which an individual is the owner of a firm. She is fully specialized in producing management service. She hires production
technology specialists and workers from relevant labour markets and commands the former to produce production technology and the latter to transform the production technology into the final good. Let \((l_x/y)\) denote a configuration in which an individual sells her labour, buys the final good and becomes a production technology specialist. Let \((l_y/y)\) denote a configuration in which an individual sells her labour, buys the final good and becomes a worker.

The individual decision problem for configuration \((z/l_x, l_y)\) is

\[
\text{Max } U_z = y
\]

s.t. \(y + y^* = (Ny^p z)^a \) (production function)

\[
z = (l_y)^b, l_x = 1 \quad \text{(output of management service)}
\]

\[
y^p = (x^d r_w l_x)^a, l_y = 1 \quad \text{(production function of each worker)}
\]

\[
x^d = x/N_1 \quad \text{(consumption of production technology by each worker)}
\]

\[
x^* = (r_x l_x)^b, l_x = 1 \quad \text{(production function of production technology specialist)}
\]

\[
y^s = w_2 N + w_1 (N/N_1) \quad \text{(budget constraint)}
\]

where \(y + y^*\) is the total output of the firm. \(y\) is the residual returns of the owner and \(y^*\) is the amount of the final good sold to market. \(N\) is the number of workers hired. \(y^p\) is the output of each worker. \(w_1\) is the wage rate of production technology specialists in terms of the final good and \(w_2\) is the wage rate of the workers in terms of the final good. One production technology specialist can provide the necessary production technology for \(N_1\) workers. Hence, the number of production technology specialists hired is \(N/N_1\), and the total number of the employees in the firm is \(N + N/N_1\).

The individual decision problem for configuration \((l_x/y)\) is

\[
\text{Max } U_x = k y_d^d
\]

s.t. \(y_d^d = w_1 l_x, l_x = 1 \) (budget constraint)

where \(y_d^d\) is the amount of the final good purchased by one production technology specialist hired by the firm.

The individual decision problem for configuration \((l_y/y)\) is

\[
\text{Max } U_y = k y_d^y
\]

s.t. \(y_d^y = w_2 l_y, l_y = 1 \) (budget constraint)

where \(y_d^y\) is the amount of the final good purchased by one worker hired by the firm.

Manipulating the market-clearing condition and utility equalization condition yields the corner equilibrium in structure F3, given by

\[
N = ak, \quad N_1 = a^{-1}(1 - a), \quad U_{F3} = a^{a^2 - 1} r_w a^2 (r_x^y a^2) k^a k^a
\]

where \(U_{F3}\) is the per capita real income in structure F3. The ratio of employer to production technology specialists to workers is \(1:(N/N_1):N = 1:a^2(1-a)^{-1}k:k\).

Following the same procedure, we can solve the corner equilibria in other market structures of firms. Let us put all the per capita real incomes of 10 market structures together into Table 1.
Table 1. Per capita real income for each market structure

<table>
<thead>
<tr>
<th>Market structure</th>
<th>Per capita real income for each structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure A</td>
<td>$U_A = b^{ab}(1 + b)^{-a^{(1+b)}}$</td>
</tr>
<tr>
<td>Structure D</td>
<td>$U_D = a^{ab}(1 - a)^{-a^{ab}}$</td>
</tr>
<tr>
<td>Structure F1</td>
<td>$U_{F1} = a^{a\bar{b} - b^{ab}}(1 - a)^{-a^{ab}}(1 + a)^{-a^{ab}}(1 - a + a^{2})^{-a^{ab}}k^{2-a^{ab}}$</td>
</tr>
<tr>
<td>Structure F2</td>
<td>$U_{F2} = b^{ab}(1 - a)^{-a^{ab}}(a + b)^{-a^{ab}}k^{a^{ab}}$</td>
</tr>
<tr>
<td>Structure F3</td>
<td>$U_{F3} = a^{a^{ab}}(1 - a)^{-a^{ab}}k^{a^{ab}}$</td>
</tr>
<tr>
<td>Structure F4</td>
<td>$U_{F4} = a^{a^{ab}}(1 - a)^{-a^{ab}}k^{a^{ab}}$</td>
</tr>
<tr>
<td>Structure F5</td>
<td>$U_{F5} = a^{2a^{ab}}(1 - a)^{-a^{ab}}k^{2a^{ab}}$</td>
</tr>
<tr>
<td>Structure F6</td>
<td>$U_{F6} = a^{2a^{ab}}(1 - a)^{-a^{ab}}k^{2a^{ab}}$</td>
</tr>
<tr>
<td>Structure F7</td>
<td>$U_{F7} = a^{2a^{ab}}(1 - a)^{-a^{ab}}k^{2a^{ab}}$</td>
</tr>
<tr>
<td>Structure F8</td>
<td>$U_{F8} = a^{2a^{ab}}(1 - a)^{-a^{ab}}k^{2a^{ab}}$</td>
</tr>
</tbody>
</table>

Table 2. The meaning of transaction efficiency coefficients

<table>
<thead>
<tr>
<th>Category</th>
<th>Coefficient</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Final good market</td>
<td>$k$</td>
<td>The transaction efficiency of the final good market</td>
</tr>
<tr>
<td>2. Intermediate good market</td>
<td>$t_x$</td>
<td>The transaction efficiency of the Intermediate good market for production technology ($x$)</td>
</tr>
<tr>
<td></td>
<td>$t_z$</td>
<td>The transaction efficiency of the Intermediate good market for management service ($z$)</td>
</tr>
<tr>
<td>3. Labour market</td>
<td>$r_w$</td>
<td>The transaction efficiency of the labour market for workers</td>
</tr>
<tr>
<td></td>
<td>$r_t$</td>
<td>The transaction efficiency of the labour market for production technology specialists</td>
</tr>
<tr>
<td></td>
<td>$r_m$</td>
<td>The transaction efficiency of the labour market for management service specialists</td>
</tr>
</tbody>
</table>

The meanings of all the transaction efficiency coefficients are summarized in Table 2.

4. FORMALIZING AND REFINING COASE AND CHEUNG’S THEORY OF THE FIRM

From the analysis above, we can see that the standard marginal analysis of the interior solution is insufficient and corner solutions are allowed in our model. Therefore, we need a three-step inframarginal analysis. In the first instance, a list of candidates for an individual’s optimal decision is identified by excluding all inefficient solutions. In the second step all the solutions for possible corner structures are derived. The third step is total cost–benefit analysis. We have already completed the first two steps. Now we move on to the third step. After we have obtained the corner equilibrium solutions for 10 market structures, we can find the relevant conditions under which one market structure will emerge.

4.1. The evolution of economic organization

Normally the total cost–benefit analysis is very tedious and cumbersome. The following result helps to simplify the analysis. Sun et al. (2003) show that, in
an economy with a continuum of ex ante identical consumer-producers and with production functions displaying individual specific economies of specialization, a Walrasian general equilibrium exists and it is the Pareto optimal corner equilibrium. (For the existence of equilibrium with non-identical individuals, see Sun et al., 2004.) Hence, the Pareto optimal corner equilibrium is one that generates the highest per capita real income.

Per capita real incomes in 10 structures are dependent on the transaction efficiency parameters $t_x$, $t_z$, $k$, $r_w$, $r_m$, and $r_t$, the degree of economies of specialization $b$, and the degree of economies of roundabout production $a$. If we study the per capita real incomes in different structures, we can find all the independent variables summarized in Table 3.

If the transaction efficiency coefficients of both goods markets and labour markets are not sufficiently high, the transaction costs will overweigh the benefit from the division of labour and specialization, and autarky will be the optimal structure.

If $t_x k > a^{-1}(1 - a)^{-a^{-1}} b^b (1 + b)^{-d+b}$, we have $U_D > U_A$. This inequality indicates the conditions under which the economy will evolve from autarky to the structure of the division of labour with no firms. In addition, if $U_D > U_i = F \_1, F \_2, \ldots, F \_8$, the market structures of firms are not optimal. Obviously, then structure D is the general-equilibrium structure, which can generate the highest per capita real incomes for all the consumer-producers in the economy.

If $U_i > U_D$ and $U_i > U_A$, $i = F \_1, F \_2, \ldots, F \_8$, one of the market structures of firms is the optimal structure. The economic organization of the firm emerges. The emergence of firm is a discontinuous jump from autarky to a market structure of firms or from division of labour with no firms to a market structure of firms. The general-equilibrium analysis of the emergence of firm can be summarized in Table 4.

In order to make the results more concrete, we explicitly solve for general equilibrium and its inframarginal comparative statics for some specific ranges of parameter values. We first assume that the structures of $A$, $D$, $F \_1$, and $F \_2$ are excluded and only the structures of $F \_3$, $F \_4$, $F \_5$, $F \_6$, $F \_7$, and $F \_8$ will be taken into consideration. We can divide the parameter space into several subspaces. All the subspaces for relevant corner equilibria are summarized in Table 5 and Table 6.

4.2. The growth and contraction of the firm

For $U_{F \_3} > U_{F \_4}$, we have the following inequality of structure $F \_3$ to structure $F \_4$,

$$r_i^b > t_x.$$
This means that if the above inequality holds, the firm will hire production technology specialists to produce production technology rather than directly purchase it from the market. Similarly, for $U_{F5} > U_{F6}$ and $U_{F7} > U_{F8}$, we have the inequality of structure $F5$ to structure $F6$ and that of structure $F7$ to structure $F8$. These two inequalities are the same as

$$ r_m^b > t_z. $$

In structure $F3$ and structure $F4$, a management service specialist is the employer of a firm and the intermediate good is production technology $x$. In structures $F5$ and $F6$, the production technology specialist is the employer and the intermediate good is management service $z$. In structures $F7$ and $F8$, the worker is the employer and the intermediate good is management service $z$.

These two inequalities show that firms will hire labour to produce the intermediate good rather than directly purchase it from the market if the transaction efficiency for labour employed to produce the intermediate good is greater than that for the intermediate good. The replacement of the intermediate good by the market for labour hired to produce the intermediate good implies that more employees will be hired by the firm. This is exactly the growth of a firm. Hence, our analysis supports Cheung’s claim that ‘The growth of a firm may then be viewed as the replacement of a product market by a factor market, resulting in a saving in transaction costs.’ (Cheung, 1983).

Conversely, if the transaction efficiency for an intermediate good is greater than that for labour employed to produce the intermediate good, firms will directly purchase the intermediate good from the market rather than hire labour to produce it in the firm, and some employees will be fired. When the transaction efficiency of the intermediate good market is improved, it is possible for us to witness the contraction of firms even without economic recession.\(^7\)

---

7 Liu and Yang (2000) construct a general-equilibrium model to explain the evolution of firms and changes in firm size by the trade-off between economies of specialization and transaction costs. Lambing and Kuehl (1997) point out that ‘A study done by the company found corporate profits rose 11 percent in 1994, after increasing 13 percent in 1993. During 1994, however, corporate American cut 516,069 jobs. This was nearly as many as were laid in 1991, the depth of our last recession.’ These figures suggest that efficiency-improving restructure similar to the contraction mentioned in the text here might be taking place.
Table 5. Equilibrium structures if \( t_x < r_t^b \)

<table>
<thead>
<tr>
<th></th>
<th>( r_m &lt; \beta_1 r_t^a )</th>
<th>( r_m &gt; \beta_1 r_t^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_x &lt; \beta_2 t_z^a )</td>
<td>( t_x &gt; \beta_2 t_z^a )</td>
<td>( t_x &gt; \beta_2 t_z^a )</td>
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<td>( r_m &lt; \beta_2 r_m^a )</td>
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<td>( t_z &lt; \beta_2 t_m^a )</td>
<td>( t_z &gt; \beta_2 t_m^a )</td>
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<td>( r_m &lt; \beta_2 r_m^b )</td>
<td>( r_m &gt; \beta_2 r_m^b )</td>
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<td>( t_z &lt; \beta_2 t_m^b )</td>
<td>( t_z &gt; \beta_2 t_m^b )</td>
<td>( t_z &gt; \beta_2 t_m^b )</td>
</tr>
</tbody>
</table>

Where \( \beta_1 = a^{2(2a+1)b^2} (1-a)^{-2(2a+1)^2b^2} k^{a-1}j^a b^{-2a} \), \( \beta_2 = a^{2(2a+1)} (1-a)^{-2(2a+1)^2j} k^{a+1}b^a \), \( \epsilon_1 = l_{m}^{b-1} l_{m}^{b+a+1} t_{z}^{-a} b^{a} \), and \( \epsilon_2 = r_{m}^{b-1} l_{m}^{b+a+1} t_{z}^{-a} b^{a} \).
Table 6. Equilibrium structures if $t_x > r_i^b$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_m &lt; \beta r_x^{ab}$</td>
<td>$t_z &lt; \beta r_i^{ab}$</td>
<td>$t_z &gt; \beta r_i^{ab}$</td>
</tr>
<tr>
<td>$t_z &lt; \beta r_i^{ab}$</td>
<td>$r_i &lt; \beta_3$</td>
<td>$r_i &gt; \beta_3$</td>
</tr>
<tr>
<td>$t_z &gt; r_m^b$</td>
<td>$t_z &lt; r_m^b$</td>
<td>$t_z &gt; r_m^b$</td>
</tr>
<tr>
<td>$r_w &lt; r_i$</td>
<td>$r_w &gt; r_i$</td>
<td></td>
</tr>
<tr>
<td>$t_z &lt; t_m^b$</td>
<td>$t_z &gt; t_m^b$</td>
<td></td>
</tr>
<tr>
<td>$r_i &lt; r_w$</td>
<td>$r_i &gt; r_w$</td>
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</tr>
<tr>
<td>$r_i &lt; r_i$</td>
<td>$r_i &gt; r_i$</td>
<td></td>
</tr>
</tbody>
</table>

F4 F7 F7 F8 F6 F7 F7 F8 F7 F5 F7 F7 F6 F7 F8 F8 F8

Where $\beta_3 = a^{(2\alpha-1)\beta^{-1}}(1-a)^{(-2\alpha+3-a^{-1})\beta^{-1}}k_p t_x^{b-1}t_m^{b-1}$, $\beta_4 = a^{(2\alpha-1)\beta^{-1}}(1-a)^{(-2\alpha+3-a^{-1})\beta^{-1}}k_p t_x^{b-1}t_m^{b-1}$.
4.3. Who is the owner of a firm?

If $U_{F3} > U_{F5}$, $U_{F3} > U_{F6}$, $U_{F3} > U_{F7}$, and $U_{F3} > U_{F8}$, we obtain the following inequalities of $F3$ to $F5$, $F3$ to $F6$, $F3$ to $F7$, and $F3$ to $F8$:

$$
\begin{align*}
    r_h^b &> a^{a-1-2}(1 - a)^{2-3a^{-1}+a^{-2}}(l_m^b)^{a-1}k^{-1}a^{-1} \\
r_h^l &> a^{a-1-2}(1 - a)^{2-3a^{-1}+a^{-2}}(l_z^l)^{a-1}k^{-1}a^{-1} \\
r_w^a &> a^{a-1-2}(1 - a)^{2-3a^{-1}+a^{-2}}(l_m^a)^{a-1}k^{-1}a^{-1} \\
r_w^b &> a^{a-1-2}(1 - a)^{2-3a^{-1}+a^{-2}}(l_z^l)^{a-1}k^{-1}a^{-1}.
\end{align*}
$$

The first two inequalities show that a management service specialist will be the boss of a firm if the transaction efficiency of the labour market for production technology specialists is sufficiently high. The last two inequalities show that a management service specialist will be the boss of a firm too if the transaction efficiency of the labour market for workers is sufficiently high. Similarly, we can analyse the conditions for a production technology specialist as the boss of a firm, or the conditions for a worker as the boss of a firm.

Let us give an example to explain the analysis above. If I can produce some kind of management know-how, I may want to transfer it to you for S$US$2m. You think that it is too expensive. In order to convince you, I have to give you some information about my know-how. However, if you have obtained sufficient information, you have actually acquired my know-how without any cost. It is obvious that the transaction efficiency of management know-how $z$ is very low. Hence, structures $F6$ and $F8$ are not our favourite choices because both involve the trading of management know-how. If you also doubt the efficiency of the know-how, you will be reluctant to hire me to produce the know-how in your firm. Hence, structures $F5$ and $F7$ are not our favourite choices either because both involve the pricing of the effort of a management service specialist. In this case, I will not sell my management know-how, and I try to avoid the pricing and trading of my know-how. I can hire employees, and ask them to produce a final good in my firm. I claim the residual return after all other factors have been paid. The residual return is my real income, and it is the indirect price of my know-how, or my efforts. So, the structures of $F3$ and $F4$ are our best choices, which can generate higher per capita real incomes. This is the main idea of Yang and Ng’s indirect pricing theory of the firm (Yang and Ng, 1993, 1995; Yang, 2001). Without doubt, the model with two kinds of intermediate goods in this paper supports and generalizes their theory.

4.4. The emergence of professional management

If $U_{F3} > U_{F1}$, which means that structure $F3$ can generate higher per capita real income than structure $F1$, we have

$$
r_r > a^{1-2b^{-1}}(1 - a)^{a^{-1}b^{-1}-b^2}(1 + a)^{1-a^{-1}a^{-2}b^{-1}+b^2}(1 - a + a^2)^{a^{-1}b^{-1}+b^2-a^{-2}b^{-1}}k^{-b^{-1}}.
$$

Therefore, if the transaction efficiency coefficient of the labour market for production technology specialists is sufficiently high, as the owner of a firm, the
management specialist will hire production technology specialists to produce intermediate good $x$, and she is fully specialized in producing management service. The discontinuous jump from structure $F_1$ to structure $F_3$ indicates the emergence of professional management.

Similarly, if $U_{F_4} > U_{F_1}$, which means that structure $F_4$ can generate higher per capita real income than structure $F_1$, we have

$$t_x > a^{b-2}(1 - a)^{a-1}(1 + a)^{-b-a^{-1}-a^{-2}}(1 - a + a^2)^{-a-1+a^{-1}a^{-2}} k^{-1}. \tag{11}$$

The discontinuous jump from structure $F_1$ to structure $F_4$ shows that, as the owner of a firm, a management service specialist will not spend any time producing intermediate good $x$ and will not hire production technology specialists to produce intermediate good $x$, she will directly purchase intermediate good $x$ from the market. In this case, the emergence of professional management requires that the transaction efficiency of the market for intermediate good $x$ should be sufficiently high, given by the inequality in equation (11).

4.5. The emergence of technology entrepreneurs

Of the 10 corner structures, structures $F_5$ and $F_6$ are the structures of technology entrepreneurs. If structure $F_5$ (or structure $F_6$) is the general-equilibrium structure, we can say that technology entrepreneurs emerge. Now, let us look at the conditions for the emergence of technology entrepreneurs.

The reason for structure $F_5$ to be the general-equilibrium structure is that it can generate the highest per capita real income for all the individuals in the economy. We have $U_{F_5} > U_i$, $i = A, D, F_1, F_2, F_3, F_4, F_6, F_7, F_8, F_9, F_9$.

$U_{F_5} > U_{F_1}$ and $U_{F_5} > U_{F_2}$. From Table 3, we can find that the independent variables affecting the per capita real income of structure $F_1$ are $k$ and $r^n$, and that the independent variables in structure $F_5$ are $k$, $r^n$, and $r_m$. Therefore, $r_m$, the transaction efficiency coefficient of the labour market for management service specialists should be high enough. Otherwise, it is not a good idea to hire management service specialists to produce management service due to high transaction costs. As the owner of a firm, the technology entrepreneur will have to produce both production technology and management service by herself. The independent variables affecting per capita real income of structure $F_2$ are $k$ and $r_t$. Hence, compared with $r_n$, $r^n$, and $r_m$ should be sufficiently high. It is such a sufficiently low $r_t$ that any efficient structure must avoid the direct pricing and trading of the efforts of the production technology specialists. The structure of technology entrepreneurs is then the ideal structure, which not only can avoid the direct pricing and trading of the efforts of production technology specialists, but also can make full use of more efficient labour markets for management service specialists and workers.

$U_{F_5} > U_{F_3}$ and $U_{F_5} > U_{F_4}$. In structures $F_3$ and $F_4$, management specialists are the owners of firms. From Table 3, we can find that the independent variables in structure $F_3$ are $k$, $r^n$, and $r_t$, and that the independent variables in structure $F_5$ are $k$, $r^n$, and $r_m$. Therefore, compared with $r_t$, $r_m$ should be
sufficiently high. Obviously, the structure of technology entrepreneurs is then the ideal structure, which not only can avoid the direct pricing and trading of the efforts of production technology specialists, but also can make full use of the more efficient labour market for management service specialists. The independent variables in structure $F_4$ are $k$, $r_w$, and $t_x$. If $U_{F_5} > U_{F_4}$, compared with $r_n$, $r_m$ should be sufficiently high. In this case, the transaction efficiency of production technology is very low. So the ideal structure could avoid the trading of production technology.

$U_{F_5} > U_{F_7}$ and $U_{F_5} > U_{F_8}$. In structures $F_7$ and $F_8$, workers are the owners of firms. The independent variables in structure $F_7$ are $k$, $r_m$, and $r_r$, and the independent variables in structure $F_5$ are $k$, $r_m$, and $r_w$. Therefore, compared with $r_f$, $r_w$ should be sufficiently high. Without doubt, the ideal structure must make full use of the more efficient labour market for workers. Naturally, the structures with the workers as the owners of firms are not then ideal. The independent variables in structure $F_8$ are $k$, $r_r$, and $t_z$. If $U_{F_5} > U_{F_8}$, compared with $r_i$ and $t_z$, $r_w$ and $r_m$ should be sufficiently high. In this case, the structure of technology entrepreneurs can avoid the direct trading of management service as well as the using of a low efficient labour market for production technology specialists.

In summary, there are two basic conditions for the emergence of technology entrepreneurs. First, the transaction efficiency of the good’s market for production technology and the transaction efficiency of the labour market for production technology specialists are not sufficiently high, so that ideal structures should avoid the direct pricing and trading of production technology, and avoid the use of low efficient labour market for production technology specialists. This condition implies that the production technology specialist should be the owner of a firm. Secondly, the transaction efficiency of other intermediate goods except for production technology and/or the transaction efficiency of the labour markets for workers and management service specialists except for production technology specialists should be sufficiently high, so that the ideal market structures of technology entrepreneurs save transaction costs. Otherwise, the structure of autarky or the structure of the division of labour with no firms may be ideal market structures. The evolution of economic organization from autarky to the division of labour with no firms, and to the structures of firms implies that the economy grows discontinuously.

4.6. The case with no transaction costs

If we assume that there are no transaction costs in the economy, we could find that

$$U_i \neq U_D \neq U_{F_1} \neq U_{F_2} \neq U_{F_3}(= U_{F_4}) \neq U_{F_5}(= U_{F_6} = U_{F_7} = U_{F_8}),$$

which shows that the structure of ownership makes a difference to the efficiency of the market. Thus our analysis does not support Coase’s Theorem (Coase, 1960). Certainly, we obtain this conclusion from our special model and relevant assumptions.
In this paper, a model with two kinds of intermediate goods within the analytical framework of Yang and Ng has been constructed to investigate the emergence of firms, the growth and contraction of firms, and the conditions for the emergence of technology entrepreneurs and professional management. The role of a structure of residual rights associated with the institution of firms has also been explored.

On top of the three conditions contained in the definition of firms given by Yang and Ng, one more condition is added: management service is a necessary input in a firm’s mode of production. It is clearly indicated that without management service, firms cannot exist because of shirking and other problems. By introducing production technology and management service into our analytical model, we not only have formalized Coase and Cheung’s theory of the firm, we also have refined Coase’s ideas about transaction costs. Coase claims that the institution of the firm can be used to reduce transaction costs. Compared with the structure of autarky and the structure of the division of labour with no firms, the structures with firms have more market transactions and entail more transaction costs. The institution of the firm will emerge as long as increased economies of the division of labour outweigh the increased transaction costs. When transaction costs are zero, we have found the structure of ownership makes a difference to the efficiency of the market. This result does not support the Coase Theorem.

REFERENCES


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