

Throughput-Lifetime Tradeoffs in Multihop Wireless Networks under an SINR-based Interference Model

Jun Luo*, Aravind Iyer[†] and Catherine Rosenberg[‡]

Abstract—High throughput and lifetime are both crucial design objectives for a number of multihop wireless network applications. As these two objectives are often in conflict with each other, it naturally becomes important to identify the tradeoffs between them. Several works in the literature have focused on improving one or the other, but investigating the tradeoff between throughput and lifetime has received relatively less attention. We study this tradeoff between the network throughput and lifetime, for the case of fixed wireless networks where link transmissions are coordinated to be conflict-free. We employ a realistic interference model based on the Signal-to-Interference-and-Noise Ratio (SINR) which is usually considered statistically sufficient to infer success or failure of wireless transmissions. Our analytical and numerical results provide several insights into the interplay between throughput, lifetime, and transmit power. Specifically, we find that with a fixed throughput requirement, lifetime is not monotonic with power – neither very low power nor very high power result in the best lifetime. We also find that, for a fixed transmit power, relaxing the throughput requirement may result in a more than proportional improvement in the lifetime, for small enough relaxation factors. Taken together, our insights call for a careful balancing of objectives when designing a wireless network for high throughput and lifetime.

Index Terms—Multihop wireless network, throughput, lifetime, tradeoff, interference model.

I. INTRODUCTION

Improving the network *throughput* (how fast the network may deliver data), and improving the network *lifetime* (how long the network may last) are two important design objectives for multihop wireless networks. These two objectives appear to be in conflict with each other – intuitively, higher throughput means greater energy consumption, and hence reduced network lifetime. In order to be able to balance these two design objectives, it becomes important to identify the tradeoffs between throughput and lifetime. In particular, it is not clear if it is possible to set the transmit power in a network, so as to improve both throughput and lifetime. Further, how much throughput needs to be traded off to achieve a certain improvement in lifetime? Our aim is to investigate

the tradeoff between throughput and lifetime in multi-hop wireless networks, to address questions of the above nature. We consider scenarios where it is required to achieve an adequate network throughput (not necessarily the maximum) as well as a sufficiently long lifetime. This would be the case for wireless sensor networks which need to collect a large amount of data, such as multimedia sensor networks [1]. Wireless mesh networks in developing countries whose nodes have occasional access to the power grid serve as another example.

We focus on the case of fixed scheduled wireless networks. More precisely, we consider wireless networks which are operated by scheduling link transmissions to be conflict-free, as opposed to a random access MAC (medium access control) protocol. There are several reasons for focusing on such networks. Firstly, upcoming standards such as IEEE 802.16 and LTE (Long-Term Evolution) for fourth generation (4G) cellular networking support completely scheduled modes of operation, whereby link transmissions in the network can be precisely controlled and scheduled to be conflict-free. Secondly, our centralized and scheduled solutions bound the performance of distributed solutions, and serve as benchmarks for designing wireless networks. We also focus on the case where the traffic requirements are static. Both IEEE 802.16 and LTE are envisaged for static deployment of wireless networks, which would carry aggregated traffic from several users, thereby motivating aggregated (and relatively static) traffic requirements. Hence, we use a fluid model of data, i.e., we offer long-term guarantees on network throughput.

We assume that the network is specified in terms of a set of nodes and a set of flows described in terms of their origin and destination. We use a realistic interference model based on the Signal-to-Interference-and-Noise Ratio (SINR) for modeling the conflicts to avoid when scheduling the wireless links. This *conflict set model* (also used in [12], [14]) captures the fact that the interference to a certain link is the cumulative interference from the multiple links that are activated during the same period of time. Our notion of network throughput is the *max-min flow rate* [14] and our notion of network lifetime is the *max-min node lifetime* [8].

We formulate the following three optimization problems:

- P1** To maximize the network lifetime while achieving the max-min network throughput;
- P2** To maximize the network throughput while achieving a pre-specified network lifetime; and
- P3** To maximize the network lifetime while achieving a

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fraction of the max-min throughput.

A solution to any of the problems above is a network configuration which achieves the corresponding objective. By network configuration, we mean the complete choice of parameters for operating the network including the set of links, the link transmission schedules and the routes for the flows. In other words, we jointly select flow routes and link schedules, to achieve the desired objective.

In order to solve problems **P1**, **P2** and **P3**, and study the trend of their solutions as a function of transmit power, we adopt the following approach. We assume a network-wide reference power level. All the nodes may use the reference power, or a finite number of power levels which have a fixed offset from the reference power. Nodes may also use a finite set of modulation and coding schemes. We solve the problems **P1**, **P2** and **P3** for a fixed setting of the network-wide reference power (and possibly, a fixed set of offsets), and study the trend of the solution by varying the reference power. We consider this model a realistic representation of the capabilities of modern wireless radios, as compared to using the Shannon capacity formula to model wireless link rates. For ease of exposition, we assume first that all the nodes use a single power level (the reference power) and a single modulation and coding scheme. We will later show how to extend this case to multiple power offsets and modulation and coding schemes.

Even in the case of a single power and modulation, solving these problems requires searching among combinatorially many configurations due to the intricate conflict structure. In [17], we have developed computational tools based on column generation to circumvent this issue which we use to obtain all the numerical examples in this work. It must be emphasized that all our solutions are *exact*. Whereas our formulation makes no assumptions on the traffic flows, for our numerical results we consider traffic patterns that converge to a gateway/sink. Examples of such traffic patterns can be found in wireless mesh and sensor networks, where only a few gateways or sinks attract or initiate traffic. We have also studied cases where all flows are initiated by the gateway/sink. We have not observed any major differences in trends, and hence we do not present them for lack of space. Note that our analytical results are very general, and would apply regardless of the traffic pattern.

Both our analytical and numerical results show that the optimal tradeoffs between throughput and lifetime are usually not obtained at the minimum power that enables network connectivity. In addition, our results show that, by fixing the throughput requirement, the lifetime is not a monotonically decreasing function of the transmit power. Finally, for a given power level, our results indicate the existence of a throughput threshold, below which a small sacrifice of throughput leads to a large (more than proportional) improvement of lifetime and beyond which a reduction in throughput only leads to a proportional improvement in lifetime. We provide both theoretical and intuitive explanations for these phenomena in the paper. In addition to the above, we highlight the importance of the interference model, and point out why results based on the *interference range model* (to be defined later) may be misleading. For further discussions, please see [12].

The rest of the paper is organized as follows. We first

discuss related work in Section II. We then describe our network model and study the problem **P1** in Section III. We then focus on **P2** and **P3**, and we address them in Sections IV and V, respectively. Section VI extends our approach to multiple power and modulation levels. In Section VII, we report the numerical results. Finally, we conclude our paper in Section VIII.

To facilitate readability, we present all the proofs in the appendix. In order to facilitate understanding, we will use a simple network (shown later in Fig. 1) to explain notations, propositions and other remarks, throughout the paper. The use of the simple example should not be seen as limiting our results, but rather as a tool to illustrate them.

II. RELATED WORK AND DISCUSSIONS

In the research community, maximizing the *throughput* (or a *network utility* in general) [13], [21], [6], [7], [14] and maximizing the *lifetime* [8], [20], [16], [18] of multihop wireless networks, have long been treated as two separate problems.

Approaches to throughput maximization have (roughly) been of the following three kinds: (i) offline design with exact solution (e.g., [13], [14]), (ii) offline design with approximate solution (e.g., [21], [6]), and (iii) online dynamic control (e.g., [10] and the references therein). We would like to point out that the throughput maximization approaches we discuss here lead to explicit solutions. This differs from the approach that aims at characterizing the *capacity* of a network in an asymptotic sense (e.g., [11]).

The first approach explicitly formulates an optimization problem with the throughput as the objective. Typically, such problems turn out to be NP-hard. In [13], the authors derive bounds rather than exact solutions. In [14], more realistic SINR-based interference models are considered, and exact solutions are derived for medium size networks, by intelligently enumerating independent sets of link. Since solving the offline design problem exactly could be problematic for large networks, the second approach resorts to heuristics: either randomized algorithms [21] or deterministic algorithms with provable performance [6]. Whereas the first two approaches assume a quasi-static environment, the third approach considers the cases where no information about the environment is available *à priori*. However, the price paid for the lack of *à priori* information, is the increased algorithmic complexity: NP-complete problems (e.g., finding maximum weight independent set [10]) need to be solved online, repeatedly.

Approaches to lifetime maximization include [8], [9], [20], [16], [18], which aim at identifying the network configuration that gives the longest lifetime. Now, if the required throughput is considered to be extremely low as in [8], [20], [16], [15], interference (or collisions) happens with very low probability and can hence be neglected. As a result, the conflict structure (and therefore, the issue of scheduling) does not come into the picture, and finding the optimal configuration only involves the routing strategy. However, the required throughput cannot always be assumed to be very low: even wireless sensor networks may demand a sustained throughput [1]. Other

approaches [9], [18] take into account the issues of interference mitigation and frequency reuse to reduce energy consumption under high throughput requirements. In the latter set of works, the link rate is derived from Shannon capacity formula, which neglects the limitation imposed by the availability of only a finite set of modulation schemes in practice.

It has been only very recently that the tradeoff between network utility and lifetime has been investigated [19], [22]. In these recent works, the tradeoff is identified by means of scalarizing the two conflicting objectives. However, scalarization (i.e., a linear combination through a *weight vector*) [5] yields results that are not always easy to interpret: what does the weighted sum of throughput (in, e.g., bit per second) and lifetime (in, e.g., second) mean? These papers aim at devising distributed algorithms for solving the optimization problem online, and they apply either a predefined scheduling [19] or a randomized collision avoidance mechanism [22]. These are costs that have to be paid to make the problem tractable to be solved online. Since we want to address the offline design problem for dimensioning a network, we are able to build a more general analytical framework and therefore able to characterize the optimal solution.

III. NETWORK LIFETIME WITH MAXIMUM THROUGHPUT

In this section, we introduce our network model, and precisely formulate problem **P1**, namely that of maximizing network lifetime while achieving the max-min throughput, for a given assignment of physical layer parameters. Please use Table I as a handy reference for notation.

A. Network Model

We model the network as a set \mathcal{N} of nodes and a set \mathcal{F} of flows, with $|\mathcal{N}| = N$. Each node $n \in \mathcal{N}$ is associated with a geographical location. We assume that time is slotted and all the nodes are synchronized.

Physical Layer Model: We assume that all the nodes use the same network-wide *reference power* P_{tx} and the same modulation and coding scheme z with a data-rate of unity. Note that, as we remarked in Section I, this is only for ease of exposition. We will show in Section VI how our approach can accommodate multiple power levels and modulation and coding schemes. We assume that the channel gain from a node i to a node j is quasi-static, since we are looking at fixed wireless networks. For simplicity, we model the channel gain as isotropic path-loss given by $(\frac{d_{ij}}{d_0})^{-\eta}$ where d_{ij} denotes the distance from node i to node j , d_0 is the near-field crossover distance and η is the path-loss exponent. Incorporating non-isotropic attenuation, and in general, shadowing in our framework, is straightforward. The feasibility of a wireless link is based on whether a bit-error-rate (BER) less than a tolerable maximum can be achieved on the link. We assume that this BER requirement translates into a minimum SINR requirement corresponding to an SINR threshold $\beta(z)$, depending on z . The set \mathcal{L} is defined as the set of all feasible links. Specifically, a link $l = (i, j)$ exists (or $l \in \mathcal{L}$) if $\frac{P_{\text{tx}}}{N_0} (\frac{d_{ij}}{d_0})^{-\eta} \geq \beta(z)$ where N_0 is the thermal noise power in the frequency band of operation. Let $|\mathcal{L}| = L$, and let l_O and l_D respectively denote the origin and destination of link l .

TABLE I
NOTATION USED FOR PROBLEM FORMULATIONS

\mathcal{N}, \mathcal{F} and \mathcal{L}	Respectively the set of Nodes, Flows and Links
P_{tx}	Transmit Power
P_{rx}	Power Consumption during packet reception
$\beta(z)$	SINR threshold for unit rate modulation scheme z
$(\frac{d}{d_0})^{-\eta}$	Channel Gain as a function of distance d
N_0	Thermal Noise
E_i	Initial Battery Energy of node i
ζ	A set of links
$\gamma_l(\zeta)$	SINR of link l when ζ is active
\mathcal{D}_l	Collection of Conflict Sets of l
\mathcal{I} or $\mathcal{I}(P_{\text{tx}})$	Collection of all Independent Sets
\mathcal{I}_l or $\mathcal{I}_l(P_{\text{tx}})$	Collection of all Independent Sets containing l
\mathcal{S}	The power set of \mathcal{L}
α_ζ	Time Fraction for which ζ is active
\mathcal{R}_f	Set of all routes of flow f
\mathcal{R}_f^l	Set of all routes of flow f which pass through l
r	A route (a sequence of links)
ϕ_f^r	Fraction of traffic of flow f on route r
λ_f	Throughput of Flow f
P_i^c	Power consumption of Node i

Link Conflict Model: In order to characterize the simultaneous conflict-free operation of sets of wireless links, we use the following link conflict model derived from the SINR-based model. This link conflict model was introduced in our previous work [14]. Let $\zeta \subset \mathcal{L}$ denote a set of links. When all the links in ζ are simultaneously active, the SINR perceived by link $l \in \zeta$ is given by

$$\gamma_l(\zeta) = \frac{P_{\text{tx}} (\frac{d_{l_O l_D}}{d_0})^{-\eta}}{N_0 + \sum_{k \in \zeta \setminus \{l\}} P_{\text{tx}} (\frac{d_{k_O l_D}}{d_0})^{-\eta}} \quad (1)$$

For data transmissions on link l to be successful under activation of the set ζ , we require $\gamma_l(\zeta) \geq \beta(z)$. Now, given a link l and a set of links ζ such that $l \notin \zeta$, we define ζ to be a *conflict set* of link l , if $\gamma_l(\zeta \cup \{l\}) < \beta(z)$. In other words, if l is concurrently active with all the links in ζ , then l would be infeasible (i.e., would not meet the SINR threshold). We define \mathcal{D}_l to be the collection of conflict sets of link l :

$$\mathcal{D}_l = \{\zeta | l \notin \zeta, \zeta \subset \mathcal{L} \text{ and } \gamma_l(\zeta \cup \{l\}) < \beta(z)\} \quad (2)$$

A dual concept of the conflict set is the *independent set* (not in the conventional graph-theoretical sense). We define a set of links ζ to be an independent set if for every link $l \in \zeta$, we have $\gamma_l(\zeta) \geq \beta(z)$. In other words, links belonging to an independent set ζ can operate concurrently without conflicts. We define \mathcal{I} to be the collection of all independent sets:

$$\mathcal{I} = \{\zeta | \gamma_l(\zeta) \geq \beta(z) \forall l \in \zeta\} \quad (3)$$

Let \mathcal{I}_l denote the set of independent sets that contain link l . We will also use the notation $\mathcal{I}(P_{\text{tx}})$ and $\mathcal{I}_l(P_{\text{tx}})$ in order to emphasize their dependence on P_{tx} .

This link conflict structure will be compared later with the *interference range* model [12] that we describe now. Denote by $R(P_{\text{tx}}, z)$ the maximum communication range of a node given the reference transmit power P_{tx} and modulation scheme z . Specifically, $R(P_{\text{tx}}, z) = d_0 (\frac{P_{\text{tx}}}{N_0 \beta(z)})^{1/\eta}$. A transmission

on link l is successful under the interference range model, characterized by the constant parameter $\sigma \geq 1$:

$$d_{l_O l_D} \leq R(P_{\text{tx}}, z) \text{ and} \\ d_{k_O l_D} > \sigma R(P_{\text{tx}}, z) \quad \forall \text{ links } k \text{ concurrently active} \quad (4)$$

The *independent set* for this model is the set of links that do not interfere directly with each other; it is the graph-theoretical independent set in the *conflict graph* [13] induced by (4). It may also be noted that the SINR-based model and the interference range model are respectively analogous to the physical and protocol models described in [11].

Routing: A flow $f \in \mathcal{F}$ is identified by its source-destination pair and is assumed to have a rate λ_f . Let \mathcal{R}_f be the set of all routes used by f and \mathcal{R}_f^l be the set of routes of f going through link l . The fraction of f routed on $r \in \mathcal{R}_f$ is denoted by ϕ_f^r . Thus, $\sum_{r \in \mathcal{R}_f} \phi_f^r = 1$. Let $\phi = [\phi_f^r]_{r \in \mathcal{R}_f, f \in \mathcal{F}}$.

Scheduling: The network is assumed to use conflict-free scheduling as opposed to a random access protocol. Let \mathcal{S} denote the power set of \mathcal{L} . A transmission schedule is an $|\mathcal{S}|$ -dimensional vector $\alpha = [\alpha_\zeta]_{\zeta \in \mathcal{S}}$ such that $\alpha_\zeta > 0$ only if the set ζ is an independent set (otherwise $\alpha_\zeta = 0$) and $\sum_{\zeta \in \mathcal{S}} \alpha_\zeta = 1$. We can interpret α_ζ as the fraction of time allocated to the set of links ζ .

Energy Model: We assume a simple model: the power P_{tx} is expended by the source node l_O of a link l during transmission, and a fixed power P_{rx} is consumed by a node during reception.¹ We assume that nodes have an initial energy denoted by $\vec{E} = [E_i]_{i \in \mathcal{N}}$. We also allow for the possibility of $E_i = \infty$ for certain nodes (see Section III-C for details). Now, the energy consumed per unit time, in operating a link l , depends on the time fraction l is active. We compute this fraction as the ratio of the amount of flow rate carried by a link to the link data-rate (which is the same as the amount of flow if the link data-rate is unity). Consequently, the power consumption P_i^c of a node i is the sum of the power consumptions on all the links (i, j) and $(j, i) \in \mathcal{L}$ where $j \neq i$.

$$P_i^c = \sum_{\substack{(i, j) \in \mathcal{L} \\ (j, i) \in \mathcal{L} \\ f \in \mathcal{F}}} \lambda_f \left(P_{\text{tx}} \sum_{r \in \mathcal{R}_f^{(i, j)}} \phi_f^r + P_{\text{rx}} \sum_{r \in \mathcal{R}_f^{(j, i)}} \phi_f^r \right) \quad (5)$$

The lifetime of a node i then, is simply E_i/P_i^c . We define the network lifetime as the time when the first node runs out of its initial energy, or $\min_{i \in \mathcal{N}} \left(\frac{E_i}{P_i^c} \right)$. In other words, the network lifetime is the *max-min node lifetime*.

B. Constraints on Flows

Given the network model described above, the set of flows \mathcal{F} is constrained by the following.

Flow Conservation: For each flow $f \in \mathcal{F}$, we have

$$\sum_{r \in \mathcal{R}_f} \phi_f^r = 1 \quad (6)$$

This is actually the path formulation of the flow conservation law: at the source node, the flow out of the source balances the

¹This is the power consumption of a transceiver in the receiving mode. It is **not** the signal power at the receiver antenna.

flow injected to the source; the conservations at other nodes are implied by the path formulation.

Link Capacity Bound: For each link $l \in \mathcal{L}$, we have

$$\sum_{f \in \mathcal{F}} \lambda_f \left(\sum_{r \in \mathcal{R}_f^l} \phi_f^r \right) \leq \sum_{\zeta \in \mathcal{I}_l} \alpha_\zeta \quad (7)$$

It means that the amount of flow going through a link is bounded from above by the link capacity (represented by the product of the link data-rate (unity) and the scheduled time).

Scheduling Constraint: As explained in Section III-A, any feasible scheduling has to abide by the following equality

$$\sum_{\zeta \in \mathcal{I}} \alpha_\zeta = 1 \quad (8)$$

We have replaced \mathcal{S} with \mathcal{I} , because $\alpha_\zeta > 0$ only if $\zeta \in \mathcal{I}$.

Energy Constraint: Let T denote the minimum node lifetime in the network. Thus, for every node $i \in \mathcal{N}$, we need $E_i/P_i^c \geq T$. In other words,

$$TP_i^c \leq E_i \quad (9)$$

where P_i^c is given by (5).

C. P1: Max-min Lifetime with Max-min Throughput

The problem of maximizing the throughput in a network given in terms of the set of nodes and flows, can be posed as follows:

$$\begin{aligned} \text{Maximize } \lambda \quad & \text{Subject to: (6)–(8) and} \\ & \lambda \leq \lambda_f \quad f \in \mathcal{F} \end{aligned} \quad (10)$$

where λ is the minimum flow throughput. Here the notion of maximum throughput is basically the max-min flow rate, and the optimization is over all possible routes ϕ and all possible schedules α . This problem was introduced and studied in [14] and [17]. We refer to this problem as the *throughput optimization (TO)* problem.

The solution of TO (10) is in general not unique: there may exist different configurations that achieve the same optimal throughput. Among all these configurations, we are interested in one (not necessarily unique) that results in the longest lifetime. Therefore, we formulate the following *lifetime optimization problem (P1)*:

$$\begin{aligned} \text{Maximize } T \quad & \text{Subject to: (6)–(9) and} \\ & \lambda_f \geq \lambda^* \quad f \in \mathcal{F} \end{aligned} \quad (11)$$

where λ^* is the optimal solution of TO (10).

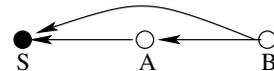


Fig. 1. A Simple Example Network

To understand the problems TO and P1 and their solutions, let us first work them out for the simple network shown in Fig 1. The network consists of 3 nodes S, A and B with two flows, one from A destined to S, and another from B

destined to S. The nodes are equally spaced. Let $P_{\text{tx}}^{(1)}$ be the minimum transmit power level such that the links (A, S) and (B, A) are feasible, but not the link (B, S), and let $P_{\text{tx}}^{(2)}$ be the minimum transmit power level such that the link (B, S) is feasible. Clearly, $P_{\text{tx}}^{(1)} \leq P_{\text{tx}}^{(2)}$. Also, for $P_{\text{tx}} < P_{\text{tx}}^{(1)}$ the network is disconnected.

For $P_{\text{tx}}^{(1)} \leq P_{\text{tx}} < P_{\text{tx}}^{(2)}$, the collection of independent sets is just the collection of singleton sets of the following links: (A, B), (B, A), (A, S) and (S, A). The best throughput is achieved by scheduling the independent set {(B, A)} for 1/3rd of the time, and the set {(A, S)} for 2/3rd of the time. This solution is also unique so $\mathbf{P1}$ is trivial in this case. The maximum throughput (i.e., the solution of TO) is 1/3. The power consumption of node A is $\frac{1}{3}P_{\text{rx}} + \frac{2}{3}P_{\text{tx}}$, that of node B is $\frac{1}{3}P_{\text{tx}}$, and that of node S is $\frac{2}{3}P_{\text{rx}}$. Let us assume that the initial battery energies are $E_A = E_B = E$ and $E_S = \infty$. Clearly, the power consumption of node A is greater than that of node B, and the power consumption of node S is irrelevant, since it has no energy constraint. Therefore, the lifetime (i.e., the solution of $\mathbf{P1}$) is nothing but the lifetime of node A, $\frac{E}{\frac{1}{3}P_{\text{rx}} + \frac{2}{3}P_{\text{tx}}}$.

For $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$, two new independent sets become feasible, viz., the singletons {(B, S)} and {(S, B)}. Hence, the best throughput is achieved by scheduling the set {(B, S)} for 1/2 the time, and the set {(A, S)} for 1/2 the time. Again, the solution is unique so $\mathbf{P1}$ is trivial. The solution of TO is 1/2, and the solution of $\mathbf{P1}$ is $\frac{E}{\frac{1}{2}P_{\text{tx}}}$ since both nodes A and B are transmitting for 1/2 the time. So the overall trend in the solutions of TO and $\mathbf{P1}$ is the following. The solution of TO is non-decreasing and shows non-negative jumps whenever new independent sets are created. The solution of $\mathbf{P1}$ is decreasing between the points of creation of independent sets, and the jump is positive in this case. Of course, note that there are exactly two points at which new independent sets are created, viz., $P_{\text{tx}}^{(1)}$ and $P_{\text{tx}}^{(2)}$.

To further illustrate the solutions of TO (10) and $\mathbf{P1}$ (11), we solve them for the two networks shown in Fig. 2. The set

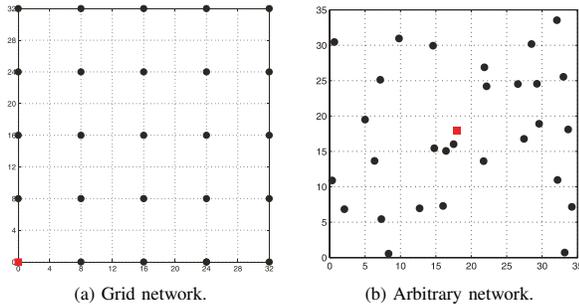


Fig. 2. Two networks with 25 nodes (a) and 30 nodes (b).

of nodes in each case is apparent from the figures. The set of flows for both networks consists of a flow to the gateway node (marked as a red square) from every other node (marked as a black dot). We assign $E = 1\text{mJ}$ to all nodes (except to the gateway whose energy is infinite). We take $\beta(z) = 6.4\text{ dB}$. For radio propagation, we assume $d_0 = 0.1\text{m}$ and $\eta = 3$. In Fig. 3, we plot the solution of both problems (10) and (11), viz., $\lambda^*(P_{\text{tx}})$ and $T^*(P_{\text{tx}})$, respectively, as functions of the

reference power P_{tx} . Note that, unless otherwise specified, the results are obtained for the SINR-based interference model.

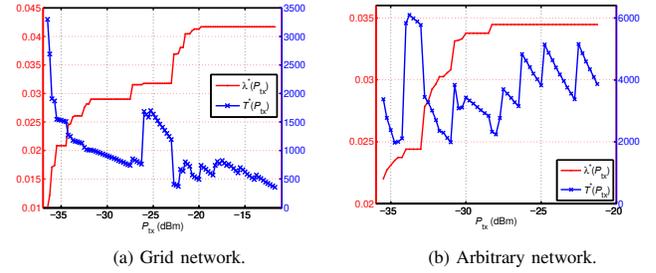


Fig. 3. Throughput and lifetime versus P_{tx} for (a) the grid network from Figure 2 ($P_{\text{rx}} = -26\text{ dBm}$) and (b) the arbitrary network from Figure 2 ($P_{\text{rx}} = -29\text{ dBm}$).

It can be seen that the maximum throughput is an increasing function of P_{tx} , which was proved in [2] and [14]. Interestingly, the maximum lifetime is **not** strictly decreasing in P_{tx} . The longest lifetime is obtained for the grid network at the lowest power level enabling connectivity, but it is not the case for the arbitrary network. These results seem to indicate that the interplay between throughput, lifetime and transmit power are far from trivial. Given the differences between the results for the grid and the arbitrary networks, it may be also noted that the results obtained for grid networks should not be overinterpreted. In order to investigate the interplay between throughput, lifetime and transmit power, we look at the problems $\mathbf{P2}$ and $\mathbf{P3}$ next. In particular, studying $\mathbf{P3}$ will allow us to explain the behavior of the lifetime in Fig. 3.

IV. MAXIMIZING THROUGHPUT UNDER A CONSTRAINT ON LIFETIME ($\mathbf{P2}$)

In this section, we consider the case where we try to maximize the minimum flow throughput given a lower bound for the lifetime. This is the problem $\mathbf{P2}$ introduced earlier. Specifically, the problem we consider in this section is:

$$\text{Maximize } \lambda \quad \text{Subject to: (6)—(8), (10) and} \quad (12)$$

$$\phi, \alpha$$

$$P_i^c \leq \hat{E}_i \quad \forall i \in \mathcal{N} \quad (13)$$

where $\hat{E}_i = E_i/T_{\text{req}}$ is the bound on the energy consumed per unit time for node i . Note that we directly take $T = T_{\text{req}}$ in the formulation, because the maximum λ obtained for $T = T_{\text{req}}$ is no less than that obtained for $T > T_{\text{req}}$. In what follows, we first study the optimal configuration for a given reference transmit power. Then, we characterize the evolution of the throughput as a function of the reference power. Finally, we comment on the unsuitability of the interference range model in investigating $\mathbf{P2}$.

A. The Throughput Optimal Configuration

Let the optimal solution of $\mathbf{P2}$ be $\lambda^*(P_{\text{tx}}, T_{\text{req}})$. In order to study the structure of the optimal configuration, we express the optimal throughput in terms of the routing variables ϕ .

Our approach is to use results from graph theory. In order to treat the problem under a graph theoretical framework,

we need to embed the link conflict structure (as defined in Section III-A) into a graph the way it is done in [14]. Such a graph is termed an *extended conflict graph* (ECG) [14]. The idea behind the transformation is the following. Each set of links $\zeta \in \mathcal{D}_l$ (refer to (2)) conflicts with the link l . Say, $\zeta = \{m_1, m_2, m_3\}$. Then, scheduling l, m_1, m_2 and m_3 simultaneously is infeasible. This constraint can be represented as “scheduling l means m_1 should not be scheduled OR m_2 should not be scheduled OR m_3 should not be scheduled”. To represent these constraints, we replicate each physical link into multiple copies called “virtual” links, with each copy realizing one of the OR-clauses derived from all $\zeta \in \mathcal{D}_l$. Then, in the ECG, a vertex represents a virtual link and an edge exists between two vertices if and only if the corresponding virtual links are involved in a scheduling constraint.

This extension allows us to apply graph theoretical results. For example, a graph theoretic *independent set* in an ECG is equivalent to the definition in (3). Also, a *clique* q in an ECG represents a set of virtual links such that for every pair of them (say) l and m , the real link l belongs to some conflict set of the real link m , and vice versa. We say a node $i \in \mathcal{N}$ is “involved” in clique q (or $i \vdash q$) if at least one “virtual” copy of (i, j) or (j, i) belongs to q for some node $j \in \mathcal{N}$.

Now recall that in any graph, the size of the largest clique is a lower bound on its chromatic number². A perfect graph is one in which the chromatic number is equal to the size of the largest clique, for every induced subgraph. Thus, for a perfect graph, the lower bound is tight. Vertex coloring of the ECG is analogous to creating a feasible link schedule. A clique in an ECG is basically a set of virtual links all of which contend with one another. Clearly, in a feasible schedule, each of these links would have to be scheduled at different times (analogous to being assigned a different color). In other words, the sum of the time-fractions for which virtual links are active should be less than or equal to unity, in every clique in the ECG.

Now, this is necessary but not sufficient to guarantee a feasible schedule. If the ECG is a perfect graph, it would also be sufficient. An alternate way to derive a sufficient condition for a feasible schedule is to restrict the sum of the time-fractions for which virtual links are active to be less than a factor $\kappa \in (0, 1]$ within every clique in the ECG. Clearly, κ depends on the network topology and the transmit power P_{tx} . It can now be noted that the ECG being a perfect graph is equivalent to κ being equal to unity.

Proposition 1: The optimal throughput $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ is bounded above and below as follows:

$$\lambda^*(P_{\text{tx}}, T_{\text{req}}) \leq \max_{\phi} \left\{ \min_{q, i \vdash q} \left[\frac{1}{w_q(\phi)}, \frac{1}{w_i(\phi)} \right] \right\} \quad (14)$$

$$\lambda^*(P_{\text{tx}}, T_{\text{req}}) \geq \max_{\phi} \left\{ \min_{q, i \vdash q} \left[\frac{\kappa}{w_q(\phi)}, \frac{1}{w_i(\phi)} \right] \right\} \quad (15)$$

²The chromatic number of a graph is the minimum number of colors required for vertex coloring. Vertex coloring is the assignment of colors to vertices of a graph in such a way that no two vertices which are connected by an edge share the same color.

where $w_q(\phi)$ and $w_i(\phi)$ are given by:

$$w_q(\phi) = \sum_{l \in q} \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f^l} \phi_f^r$$

$$w_i(\phi) = \hat{E}_i^{-1} \sum_{\substack{(i, j) \in \mathcal{L} \\ (j, i) \in \mathcal{L} \\ j \in \mathcal{F}}} \left(P_{\text{tx}} \sum_{r \in \mathcal{R}_f^{(i, j)}} \phi_f^r + P_{\text{rx}} \sum_{r \in \mathcal{R}_f^{(j, i)}} \phi_f^r \right)$$

Remark: The upper bound on the optimal throughput comes from the necessary condition for a feasible schedule: that the time-fractions for which virtual links are active should add up to less than or equal to unity in every clique in the ECG. The lower bound comes from the sufficient condition: that such time-fractions need to add up to less than or equal to κ in every clique. In addition to the scheduling constraints, the energy constraint due to the lifetime requirement T_{req} also figures in the upper and lower bounds on the optimal throughput. The quantities w_q and w_i can be thought of as the “cost” of using the clique q for scheduling time, and the node i for its energy, respectively.

Now, let us define $\hat{\phi}$ as a routing scheme which maximizes the right-hand side term within braces in (14), i.e., $\hat{\phi} = \arg \max_{\phi} \min_{q, i \vdash q} \left[\frac{1}{w_q(\phi)}, \frac{1}{w_i(\phi)} \right]$. Also, given a routing ϕ , let us define $\lambda^*(\phi)$ as the solution of the problem:

$$\begin{aligned} & \text{Maximize } \lambda && \text{Subject to: (6)—(8), (10) and (13)} && (16) \\ & \alpha && && \\ & && \text{Given } \phi && \end{aligned}$$

In other words, $\lambda^*(\phi)$ is the maximum achievable throughput under the routing ϕ , while ensuring a minimum lifetime of T_{req} . Now, *Proposition 1* can be equivalently stated as follows, to capture the performance of the routing $\hat{\phi}$ defined above. Note that the constant κ is the same as that in Proposition 1.

Proposition 2: For some $\kappa \in (0, 1]$ which depends on the conflict structure and a given reference transmit power P_{tx} ,

$$\kappa \lambda^*(P_{\text{tx}}, T_{\text{req}}) \leq \lambda^*(\hat{\phi}) \leq \lambda^*(P_{\text{tx}}, T_{\text{req}})$$

In particular, we have $\lambda^*(\hat{\phi}) = \lambda^*(P_{\text{tx}}, T_{\text{req}})$ if the ECG induced by the link conflict structure is perfect.

Remark: The above proposition states that the routing scheme $\hat{\phi}$ achieves a throughput which is within a factor κ of the optimal throughput, provided the optimal schedule is used once the routing is fixed. This can be viewed as separating the optimal routing problem from the optimal scheduling problem. Such an approach would achieve optimal performance only if the ECG is a perfect graph.

B. Optimal Throughput vs. Transmit Power

We now look at how changing the reference power P_{tx} affects the optimal throughput $\lambda^*(P_{\text{tx}}, T_{\text{req}})$. We first consider TO (**P2** with $T_{\text{req}} = 0$) and then **P2** in general. We will use \bar{P} to denote the maximum allowed value of P_{tx} . Please refer to Table II for a summary of additional notation pertaining to the Propositions to follow.

Definition 1: Let N_p for $p \in [0, \bar{P}]$ be the counting process which counts the number of independent sets in a network when all the network nodes use a transmit power $P_{\text{tx}} = p$. Let

TABLE II
NOTATION USED FOR PROPOSITIONS

N_p	Process counting independent sets vs. transmit power
$\{P_n\}$	Point Sequence of N_p
$\lambda^*(P_{tx}, 0)$	Solution of TO
$\lambda^*(P_{tx}, T_{req})$	Solution of P2
$T^*(P_{tx}, \rho)$	Solution of P3
$\Delta\lambda(p)$	Variation of solution of TO at p
$\Delta\lambda(p, T_{req})$	Variation of solution of P2 at p
$\Delta T(p, \rho)$	Variation of solution of P3 at p
$g(p, T_{req})$	Derivative of continuous part of solution of P2 at p
$h(p, \rho)$	Derivative of continuous part of solution of P3 at p
$\mathcal{C}(P_{tx}, \rho)$	Configuration Ensemble

$\{P_n\}$ denote the sequence of transmit power levels at which new independent sets are created.

Remark: For a fixed set of nodes \mathcal{N} , N_p would actually be a deterministic process, and the points $\{P_n\}$ would be determined only by the locations of the nodes.

We first show that $\lambda^*(P_{tx}, 0)$ is a non-decreasing function of P_{tx} . Let us define $\Delta\lambda(p) = \lim_{p' \uparrow p} (\lambda^*(p, 0) - \lambda^*(p', 0))$.

Proposition 3: Let $P_{tx} \in [0, \bar{P}]$. Then we have that:

$$\lambda^*(P_{tx}, 0) = \int_0^{P_{tx}} \Delta\lambda(p) N_p(dp) \text{ and } \Delta\lambda(p) \geq 0$$

Remark: In other words, the optimal throughput is constant in $[P_n, P_{n+1})$ and has a non-negative jump at each P_n . We have already seen exactly this trend for the case of the simple network of Fig. 1. The proposition is actually just a special case of the general result that high transmit power increases throughput [2], [14]. However, it is instructive to state and prove it using the notation of the point process N_p , since its point sequence actually determines the evolution of the solution of problems **P2** and **P3**, and not just of TO.

Now let us look at the case where $T_{req} > 0$. Again, let us define $\Delta\lambda(p, T_{req}) = \lim_{p' \uparrow p} (\lambda^*(p, T_{req}) - \lambda^*(p', T_{req}))$ and $g(p, T_{req}) = \lim_{p' \rightarrow p} \frac{\lambda^*(p, T_{req}) - \lambda^*(p', T_{req})}{p - p'}$. In other words, $g(p, T_{req})$ is the derivative of the continuous part of $\lambda^*(P_{tx}, T_{req})$ at p and $\Delta\lambda(p, T_{req})$ is the throughput variation at power level p .

Proposition 4: Let $P_{tx} \in [0, \bar{P}]$. Then we have that:

$$\lambda^*(P_{tx}, T_{req}) = \int_0^{P_{tx}} \Delta\lambda(p, T_{req}) N_P(dp) + \int_0^{P_{tx}} g(p, T_{req}) dp$$

We also have that $\Delta\lambda(p, T_{req}) \geq 0$ and $g(p, T_{req}) \leq 0$.

Remark: In other words, the optimal throughput $\lambda^*(P_{tx}, T_{req})$ as a function of P_{tx} has non-negative variations at $\{P_n\}$ but is non-increasing between any two successive points P_n and P_{n+1} .

Remark: Let us work out the trend of the solution of **P2** vs transmit power for the simple example of Fig. 1. As we saw earlier, for $P_{tx}^{(1)} \leq P_{tx} \leq P_{tx}^{(2)}$, scheduling the set $\{(B, A)\}$ for 1/3rd the time, and the set $\{(A, S)\}$ for 2/3rd the time, provided the solution for TO. Now, due to the minimum lifetime requirement, T_{req} , this schedule may not be feasible. So, we schedule $\{(B, A)\}$ for a time fraction $\frac{\rho}{3}$ and $\{(A, S)\}$ for a time fraction $\frac{2\rho}{3}$, where $0 \leq \rho \leq 1$ is a factor which we will use for meeting the lifetime requirement. This yields

a throughput of $\frac{\rho}{3}$. Clearly, we require the lifetime of node A to be greater than T_{req} , i.e., $\frac{E}{\frac{\rho}{3} P_{tx} + \frac{2\rho}{3} P_{tx}} \geq T_{req}$. In other words, $\rho = \min(\frac{3E}{T_{req}(P_{tx} + 2P_{tx})}, 1)$. Therefore, the max-min throughput (i.e., the solution of **P2**) is $\min(\frac{E}{T_{req}(P_{tx} + 2P_{tx})}, \frac{1}{3})$. Similarly, for $P_{tx} \geq P_{tx}^{(2)}$, the solution of **P2** can be worked out to be $\min(\frac{E}{T_{req} P_{tx}}, \frac{1}{2})$. Thus, as per *Proposition 4*, $\lambda^*(P_{tx}, T_{req})$ is non-increasing between jumps, and has non-negative jumps.

In Fig. 4 we illustrate *Propositions 3 and 4*, using the network and parameters of Fig. 2 (a). The curves labeled SINR-based Interference Model in Fig. 4 show that $\lambda^*(P_{tx}, 0)$ is indeed non-decreasing, and that $\lambda^*(P_{tx}, T_{req})$ shows non-negative ‘‘jumps’’ and is non-increasing between jumps. Note that we use the term ‘‘jumps’’ to indicate points on the curves where there is a step-change or a discontinuity. The jumps may be difficult to discern if they are closely spaced, since our plots are numerical, rather than based on an explicit formula.

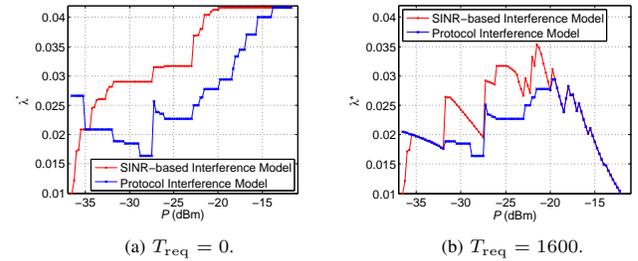


Fig. 4. Optimal throughput for different lifetime requirements under different interference models.

C. The Case of the Interference Range Model

If we consider the interference range model (instead of the SINR-based model used above), the throughput varies in a totally different way as we increase the transmit power. The following proposition serves as a counterpart to *Propositions 3 and 4*. It shows that, under the interference range model, $\lambda^*(P_{tx}, T_{req})$ may decrease at points $\{P_n\}$.

Proposition 5: Let $P_{tx} \in [0, \bar{P}]$. Then we have that:

$$\lambda^*(P_{tx}, T_{req}) = \int_0^{P_{tx}} \Delta\lambda(p, T_{req}) N_P(dp) + \int_0^{P_{tx}} g(p, T_{req}) dp$$

We also have that $g(p, T_{req}) \leq 0$. However, $\Delta\lambda(p, T_{req})$ can be of either sign.

We illustrate $\lambda^*(P_{tx}, 0)$ and $\lambda^*(P_{tx}, T_{req})$ under the interference range model with $\sigma = 2$ also in Fig. 4. The differences with the SINR-based model are very remarkable, and the trend described by *Proposition 5* is clearly demonstrated.

Remark: As per the interference range model, a link is feasible if the source and destination nodes are within $R(P_{tx}, z)$ of each other and every other transmitter is at least $\sigma R(P_{tx}, z)$ from the destination node. Thus, as the transmit power increases, a link would eventually become feasible. Subsequently, the link would remain feasible, but its exclusion circle of $\sigma R(P_{tx}, z)$ around the destination node would keep growing, causing more interference, and possible decreasing the number of independent sets. This is in contrast with the

SINR-based model in which a link actually becomes more immune to interference as the power increases, and hence more independent sets are created. Thus, the interference range model is inaccurate: the throughput is overestimated at low powers but underestimated at intermediate power levels.

V. MAXIMIZING LIFETIME UNDER A CONSTRAINT ON THROUGHPUT (P3)

In this section, we consider the case where the data rate of each flow $f \in \mathcal{F}$ is required to be greater than λ_{req} . The problem aims at maximizing the network lifetime T under the constraint that $\lambda_f \geq \lambda_{\text{req}}, \forall f \in \mathcal{F}$. We refer to this problem as **P3**. Let us define the cumulative data from flow f transported over link (i, j) over the entire lifetime T as $x_{(i,j)}^f = \lambda_f T \left(\sum_{r \in \mathcal{R}_f^{(i,j)}} \phi_r^f \right)$ and the total time for which the independent set ζ is scheduled over the lifetime T as $\hat{\alpha}_\zeta = T \alpha_\zeta$. Now **P3** can be formulated as:

$$\text{Maximize}_{\mathbf{x}, \hat{\alpha}} T \quad (17)$$

$$\sum_{j:(i,j) \in \mathcal{L}} x_{(i,j)}^f - \sum_{j:(j,i) \in \mathcal{L}} x_{(j,i)}^f \geq \lambda_{\text{req}} T 1_{\{i=f_s\}} \quad \forall f \in \mathcal{F} \quad (18)$$

$$\sum_{j:(i,j) \in \mathcal{L}} x_{(i,j)}^f - \sum_{j:(j,i) \in \mathcal{L}} x_{(j,i)}^f \leq -\lambda_{\text{req}} T 1_{\{i=f_d\}} \quad \forall i \in \mathcal{N}$$

$$\sum_{f \in \mathcal{F}} x_l^f - \sum_{\zeta \in \mathcal{I}_l} \hat{\alpha}_\zeta \leq 0 \quad \forall l \in \mathcal{L} \quad (19)$$

$$\sum_{\zeta \in \mathcal{I}} \hat{\alpha}_\zeta = T \quad (20)$$

$$\sum_{\substack{j \in \mathcal{N} \\ f \in \mathcal{F}}} P_{\text{tx}} x_{(i,j)}^f + P_{\text{rx}} x_{(j,i)}^f \leq E_i \quad \forall i \in \mathcal{N} \quad (21)$$

where $\mathbf{x} = [x_{(i,j)}^f]_{(i,j) \in \mathcal{L}, f \in \mathcal{F}}$ and $\hat{\alpha} = [\hat{\alpha}_\zeta]_{\zeta \in \mathcal{S}}$. The constraint (18) comes from flow conservation: the incoming flow should balance the outgoing flow for a given node. Since we require $\lambda_f \geq \lambda_{\text{req}}$, this constraint is an inequality rather than an equality as suggested by the conservation law. The objective T has been absorbed into the cumulative data flow variables x_l^f and the aggregated scheduling variables $\hat{\alpha}_\zeta$ in order to formulate **P3** as a linear program.

Note that, given an arbitrary λ_{req} , the problem might not be feasible. However, we can guarantee feasibility (and thus avoiding the complexity of feasibility verification) by requiring $\lambda_{\text{req}} = \rho \lambda^*(P_{\text{tx}}, 0)$ where $\rho \leq 1$ and $\lambda^*(P_{\text{tx}}, 0)$ is the optimal solution of TO (**P2** with $T_{\text{req}} = 0$).

A. Optimal Lifetime vs. Transmit Power

We investigate the trend of the optimal lifetime T^* , as a function of the reference transmit power. We want to characterize the trend of $T^*(P_{\text{tx}}, \rho)$ as P_{tx} varies in $[0, \bar{P}]$. Note that the lower bound $\lambda_{\text{req}}(P_{\text{tx}}) = \rho \lambda^*(P_{\text{tx}}, 0)$ is also a function of P_{tx} .³ The following proposition shows that $T^*(P_{\text{tx}}, \rho)$ has a similar trend (i.e., is piecewise continuous) as $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ given by *Proposition 4*. Let us define $\Delta T(p, \rho) = \lim_{p' \uparrow p} (T^*(p, \rho) - T^*(p', \rho))$ and $h(p, \rho) = \lim_{p' \rightarrow p} \frac{T^*(p, \rho) - T^*(p', \rho)}{p - p'}$.

³This differs from **P2**, where the lower bound of the lifetime is a constant for all $P_{\text{tx}} \in [0, \bar{P}]$. The reason is that λ_{req} cannot be taken arbitrarily; it has to be defined with respect to the maximum achievable throughput for the given P_{tx} in order to preserve the feasibility of the problem **P3**.

Proposition 6: Let $P_{\text{tx}} \in [0, \bar{P}]$. Then we have that:

$$T^*(P_{\text{tx}}, \rho) = \int_0^{P_{\text{tx}}} \Delta T(p, \rho) N_p(dp) + \int_0^{P_{\text{tx}}} h(p, \rho) dp$$

We also have that $h(p, \rho) \leq 0$. However, $\Delta T(P_n, \rho)$ can be of either sign.

Remark: In other words, the optimal lifetime $T^*(P_{\text{tx}}, \rho)$ as a function of P_{tx} is non-increasing between any two successive points P_n and P_{n+1} , but could show positive or negative variations at $\{P_n\}$. Thus $T^*(P_{\text{tx}}, \rho)$ need not be a decreasing function of P_{tx} as a whole.

Remark: Again, let us try to work out the trend of the solution of **P3** vs. transmit power, for the simple network of Fig. 1 to understand *Proposition 6*. For $P_{\text{tx}}^{(1)} \leq P_{\text{tx}} \leq P_{\text{tx}}^{(2)}$, the max-min throughput is $1/3$. So, to meet a throughput requirement of $\frac{\rho}{3}$, we need to schedule $\{(B, A)\}$ for a time fraction $\frac{\rho}{3}$ and $\{(A, S)\}$ for a time fraction $\frac{2\rho}{3}$. Since this is the only possible configuration, the solution of **P3** is trivial. The lifetime of the network is nothing but $\frac{3E}{\rho(P_{\text{tx}} + 2P_{\text{tx}})}$. Similarly, for $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$, the maximum network lifetime can be worked out to be $\frac{2E}{\rho P_{\text{tx}}}$. This trend is accordance with *Proposition 6*.

We illustrate *Proposition 6* by plotting $T^*(P_{\text{tx}}, 1)$ and $T^*(P_{\text{tx}}, 0.75)$ in Fig. 5 (labeled SINR-based interference model). The network and parameters are taken from Fig. 2 (a).

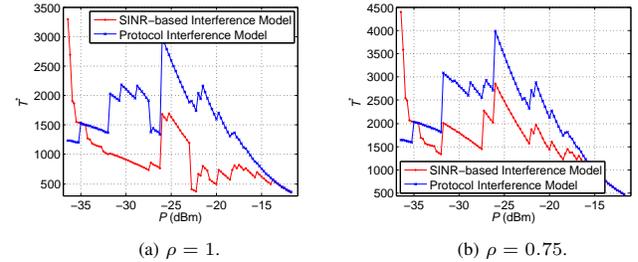


Fig. 5. Optimal lifetime under different interference models.

Remark: As $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ is non-decreasing at points of N_p (*Proposition 4*) and $T^*(P_{\text{tx}}, \rho)$ may increase or decrease at the same points (*Proposition 6*), it is straightforward to see that very low power levels may benefit lifetime but lead to very low throughput; the reversed case happens for very high power levels. Consequently, our results imply that the optimal tradeoffs between λ and T are usually obtained at a moderately high transmit power. Our numerical results in Section VII give a strong evidence of this point.

B. The Case of the Interference Range Model

If we use the interference range model, the maximum lifetime $T^*(P_{\text{tx}}, \rho)$, as a function of P_{tx} would vary in a similar way as with the SINR-based interference model. In Fig. 5, we compare $T^*(P_{\text{tx}}, 1)$ and $T^*(P_{\text{tx}}, 0.75)$ for both the interference range and SINR-based interference models. However, remember that **P3** is defined with respect to the optimal value of the TO problem. Since the interference range model, as shown in Section IV-C, is inadequate for investigating the TO problem, the same model should not be applied to **P3** either.

C. The Tradeoff between Throughput and Lifetime

The solution of **P3** clearly depends on the choice of ρ , since $\lambda_{\text{req}} = \rho\lambda^*(P_{\text{tx}}, 0)$. Here, we investigate the trend of the solution of **P3** as a function of ρ . As ρ decreases, we are essentially relaxing the throughput requirement, and expecting to improve the lifetime $T^*(P_{\text{tx}}, \rho)$. This is the tradeoff we investigate. Now, unlike **P2** (whose parameter T_{req} is unbounded), the parameter $\lambda_{\text{req}}(P_{\text{tx}})$ of **P3** is bounded within $[0, \lambda^*(P_{\text{tx}}, 0)]$ for all $P_{\text{tx}} \in [0, \bar{P}]$. This simplifies our investigation. We first give a definition to facilitate further discussions.

Definition 2: Duty cycle scaling refers to a specific strategy of trading throughput for lifetime. Given a schedule α and a routing ϕ which achieve a flow throughput of λ_f for flow $f \in \mathcal{F}$, duty cycle scaling by a factor $\rho < 1$ refers to scaling the operation time of each link l , $\sum_{f \in \mathcal{F}} \lambda_f \left(\sum_{r \in \mathcal{R}_l^r} \phi_f^r \right)$, by ρ which is feasible since the operating time is less than or equal to the time for which the link is scheduled, $\sum_{\zeta \in \mathcal{I}_l(P_{\text{tx}})} \alpha_\zeta$.

When the operation time of a link is strictly shorter than its scheduled time, we assume that both the transmitter and the receiver will switch to sleep mode during the spare time to conserve energy. The duty cycle scaling utilizes this mechanism to trade throughput for lifetime. The next proposition is a direct consequence of this definition. It shows that, if we accept a decrease in throughput, duty cycle scaling leads to a lower bound on the optimal lifetime. In other words, the tradeoff is biased towards the lifetime in proportional sense: sacrificing a certain fraction of throughput gains a larger or equal fraction of improvement in lifetime.

Proposition 7: Given a fixed P_{tx} and two required source rates $\lambda_{\text{req}}^{(1)}(P_{\text{tx}}) = \rho_1\lambda^*(P_{\text{tx}}, 0)$ and $\lambda_{\text{req}}^{(2)}(P_{\text{tx}}) = \rho_2\lambda^*(P_{\text{tx}}, 0)$ with $\rho_1 > \rho_2$, we have

$$T^*(P_{\text{tx}}, \rho_2) \geq \frac{\rho_1}{\rho_2} T^*(P_{\text{tx}}, \rho_1)$$

An interesting question that we turn to now, is whether the inequality of *Proposition 7* is strict, and if so, under what conditions. In order to answer this question, let us consider the feasible network configurations that achieve a throughput of $\rho\lambda^*(P_{\text{tx}}, 0)$. A feasible network configuration is an assignment of the flow variables x_l^f and the scheduling variables $\hat{\alpha}_\zeta$ such that (18) holds with $\lambda_{\text{req}} = \rho\lambda^*(P_{\text{tx}}, 0)$. For each such network configuration, we consider the collection of independent sets ζ for which $\hat{\alpha}_\zeta > 0$. We denote this collection by $\mathcal{I}_i^*(P_{\text{tx}}, \rho)$, where i is a generic index. We now define a *configuration ensemble* $\mathcal{C}(P_{\text{tx}}, \rho)$ as the set

$$\mathcal{C}(P_{\text{tx}}, \rho) = \{\mathcal{I}_i^*(P_{\text{tx}}, \rho) | i = 1, 2, \dots, |\mathcal{C}(P_{\text{tx}}, \rho)|\} \quad (22)$$

The problem **P3** can be solved by finding among all the configurations in the configuration ensemble $\mathcal{C}(P_{\text{tx}}, \rho)$, one that achieves the best lifetime.

Now, for $\rho_1 > \rho_2$, we know that any configuration achieving a throughput $\rho_1\lambda^*(P_{\text{tx}}, 0)$ can be *duty cycle scaled* to achieve $\rho_2\lambda^*(P_{\text{tx}}, 0)$. Therefore, we have $\mathcal{C}(P_{\text{tx}}, \rho_1) \subseteq \mathcal{C}(P_{\text{tx}}, \rho_2)$. Therefore, $\mathcal{C}(P_{\text{tx}}, \rho)$ grows with decreasing ρ . In other words, more collections of independent sets are included in the configuration ensemble as ρ is decreased. This accounts for the inequality in the *Proposition 7*. However, the size of $\mathcal{C}(P_{\text{tx}}, \rho)$

is bounded from above by the size of the power set of $\mathcal{I}(P_{\text{tx}})$. As a consequence, $\mathcal{C}(P_{\text{tx}}, \rho)$ will reach its maximum size for some small enough $\rho_0 \in [0, 1]$. We formally define this in the following:

Definition 3: We say $\mathcal{C}(P_{\text{tx}}, \rho)$ is *complete* if $\mathcal{C}(P_{\text{tx}}, \rho)$ includes all the elements of the power set of $\mathcal{I}(P_{\text{tx}})$ that yield a connected routing topology for $f \in \mathcal{F}$. Let ρ_0 be the largest ρ for which $\mathcal{C}(P_{\text{tx}}, \rho)$ is complete.

Remark: In order to understand the notion of configuration ensembles, and the *Definition 3*, let us use the example given in Fig. 1. Let $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$. As we have discussed earlier, the maximum throughput achievable $\lambda^*(P_{\text{tx}}, 0) = \frac{1}{2}$. Now, observe that by scheduling the independent sets $\{(B, S)\}$ and $\{(A, S)\}$ for a time-fraction $\frac{\rho}{2}$ each, any throughput between 0 and $\frac{1}{2}$ can be achieved. However, by scheduling the independent set $\{(B, A)\}$ for a time-fraction $\frac{1}{3}$ and the independent set $\{(A, S)\}$ for a time-fraction $\frac{2}{3}$, the maximum achievable throughput is only $\frac{1}{3}$. Thus, for a max-min throughput requirement between $\frac{1}{3}$ and $\frac{1}{2}$ (i.e., for $\frac{2}{3} < \rho \leq 1$), only the configuration consisting of independent sets $\{(B, S)\}$ and $\{(A, S)\}$ can be used. For $\rho \leq \frac{2}{3}$, the other configuration consisting of independent sets $\{(B, A)\}$ and $\{(A, S)\}$ can also be used. Thus, we have for $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$:

$$\begin{aligned} \mathcal{C}(P_{\text{tx}}, \rho) &= \left\{ \left\{ \{(B, S)\}, \{(A, S)\} \right\} \right\} \text{ for } \frac{2}{3} < \rho \leq 1 \quad (23) \\ &= \left\{ \left\{ \{(B, S)\}, \{(A, S)\} \right\}, \left\{ \{(B, A)\}, \{(A, S)\} \right\} \right\} \\ &\quad \text{for } \rho \leq \frac{2}{3} \quad (24) \end{aligned}$$

Thus, we see that $\mathcal{C}(P_{\text{tx}}, \rho)$ is increasing with decreasing ρ . Also, it is easy to see that $\mathcal{C}(P_{\text{tx}}, \rho)$ cannot grow any larger than the right-hand side of (24). Thus, $\rho_0 = \frac{2}{3}$ for the simple network in Fig. 1, for $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$.

Based on *Definition 3*, we can state the sufficient condition for the equality in *Proposition 7* to hold:

Proposition 8: If $\rho_0 \geq \rho_1 > \rho_2$, then

$$T^*(P_{\text{tx}}, \rho_2) = \frac{\rho_1}{\rho_2} T^*(P_{\text{tx}}, \rho_1)$$

Essentially, if the configuration ensemble is sufficiently large (or in other words, the throughput requirement $\rho\lambda^*(P_{\text{tx}}, 0)$ is sufficiently low), the tradeoff between throughput and lifetime is strictly proportional.

Remark: Note that the condition of ρ_1 and ρ_2 being less than or equal to ρ_0 in *Proposition 8* is not a necessary condition, but a sufficient one. In particular, for the simple example of Fig. 1, we know that *Proposition 8* would apply for $\rho \leq \frac{2}{3}$. However, as we saw earlier in the remarks following *Proposition 6*, the solution of **P3** for $P_{\text{tx}} \geq P_{\text{tx}}^{(2)}$ is obtained by duty-cycle scaling for any ρ , by scheduling the independent sets $\{(B, S)\}$ and $\{(A, S)\}$ for a time-fraction $\frac{\rho}{2}$ each. Thus, the equality in *Proposition 8* applies for any $\rho \leq 1$, although $\rho_0 = \frac{2}{3}$.

The previous results describe the tradeoffs at a fixed transmit power. Based on *Propositions 4* and *6*, the following property for the optimal solutions of both **P2** and **P3** within the full range of variation of the reference power $[0, \bar{P}]$ is immediate:

Proposition 9: The optimal solutions of both **P2** and **P3** are achieved at a point of N_p .

This proposition reduces our search scope for the optimal tradeoffs from a continuous spectrum of $[0, \bar{P}]$ to a few discrete points given by N_p .

Remark: In fact, *Proposition 9* applies to any trade-off utility function $\Gamma(\lambda, T)$ which is a monotonically non-decreasing function of both the throughput and lifetime when the other is fixed. We actually prove this general case in the appendix.

D. Identifying the Optimal Lifetime Given a Feasible Throughput Requirement λ_{req}

As we saw earlier, a throughput requirement λ_{req} is feasible at P_{tx} if $\lambda_{\text{req}} = \rho \lambda^*(P_{\text{tx}}, 0)$ for some $\rho \leq 1$. Clearly, λ_{req} is feasible for some $P_{\text{tx}} \in [0, \bar{P}]$, if $\lambda_{\text{req}} = \theta \lambda^*(\bar{P}, 0)$ for some $\theta \leq 1$. Now, given a feasible λ_{req} , among all combinations of P_{tx} and ρ which satisfy $\lambda_{\text{req}} = \rho \lambda^*(P_{\text{tx}}, 0)$, we would like to identify that combination for which $T^*(P_{\text{tx}}, \rho)$ is the highest. This corresponds to the maximum lifetime that the network can provide while delivering a minimum throughput of λ_{req} .

If we were to directly use **P3** to identify this optimal lifetime, we would have to solve a tremendous number of instances of **P3** (each with a different P_{tx} and ρ). Fortunately, *Proposition 9* allows us to significantly reduce the computational complexity. In summary, we take the following steps to identify the optimal lifetime and its corresponding network configurations.

- 1) Given a known network topology, compute the points $\{P_n\}$ of N_p , where optimal tradeoff points can be found (by *Proposition 9*), as well as $\{\lambda^*(P_n, 0)\}$. The computation is done by solving **P3** instances at P_n for which $\lambda^*(P_n, 0) \geq \lambda_{\text{req}}$.
- 2) At each power $P \in \{P_n\}$, solve a **P3** instance with a ρ such that $\lambda_{\text{req}} = \rho \lambda^*(P_n, 0)$.
- 3) An optimal tradeoff point with respect to a certain λ_{req} is obtained by maximizing over $T^*\left(P, \frac{\lambda_{\text{req}}}{\lambda^*(P, 0)}\right)$ for all $P \in \{P_n\}$.

Similarly, we can compute $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ for $P_{\text{tx}} \in [0, \bar{P}]$, by solving a sequence of **P2** over $\{P_n\}$. In general, the complexity of this algorithm is mainly determined by the complexity of solving a **P3** (or **P2**) instance and the number of jumps in $\{P_n\}$. For the complexity of solving a **P3** (or **P2**) instance, we refer to [17] for details.

VI. EXTENDING OUR APPROACH TO MULTIPLE POWERS AND MODULATIONS

The results presented so far are based on the assumption of a single power level and modulation scheme. In practice, a node may have a choice of using a power level and a modulation scheme from a finite set of power levels, and a finite set of modulation schemes, respectively. To incorporate such cases, we assume a network-wide reference power level. We assume that each node can transmit at a finite number of power levels each obtained via a fixed offset with respect to the network-wide reference power. Then, with multiple power

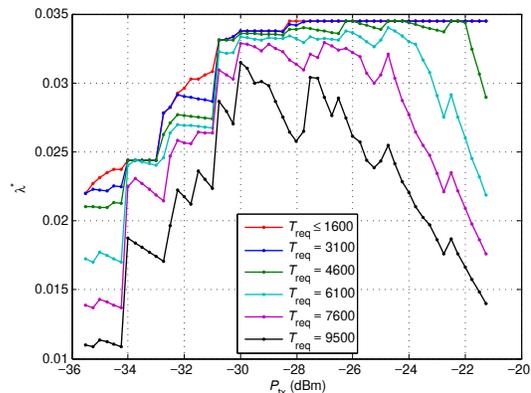


Fig. 6. Throughput (with lifetime lower bound) vs. transmit power for the 30-node arbitrary network.

and modulation levels, we replicate each link between two nodes into multiple “logical links” – each representing a certain feasible power and modulation level (see also [17]). For example, a link (i, j) where nodes can use two power level offsets from the reference power (P_1 and P_2) and two modulation schemes (z_1 and z_2), would be replicated as (i, j, P_1, z_1) , (i, j, P_2, z_1) , (i, j, P_1, z_2) and (i, j, P_2, z_2) . An independent set is now a collection of “logical links” each of which can operate simultaneously without packet decoding failures. The point process N_p counts such independent sets and is defined with respect to the network-wide reference power $p \in [0, \bar{P}]$. All the propositions that we have presented are based only on N_p and would still hold. We provide examples in Section VII-C to illustrate this scenario.

VII. NUMERICAL RESULTS

We report our numerical results in this section. Since we have focused on the grid topology of Fig. 2 (a) so far, we now focus on the arbitrary network of Fig. 2 (b).

A. Optimal Throughput vs. Lifetime Requirement and Transmit Power

As proved in Section IV, the optimal throughput is a piecewise continuous function of the transmit power. In case there is no lower bound on lifetime, then the optimal throughput is non-decreasing with increasing transmit power. But with a non-zero lower bound on lifetime, the throughput shows non-negative “jumps” at power levels where new independent sets get created, and is non-increasing between the “jumps”. This behavior is illustrated in Fig. 6 which plots solutions of **P2** for different values of T_{req} . In the figure, the highest value of T_{req} (resp. the lowest value) is chosen such that the constraint in equation (13) is active for all $P \in [0, \bar{P}]$ (resp. not active for any $P \in [0, \bar{P}]$). Consequently, the curve with $T_{\text{req}} = 1600$ in the figure is exactly the same as the optimal throughput with $T_{\text{req}} = 0$. For higher values of T_{req} , the throughput eventually starts decreasing as P_{tx} increases. This behavior is more prominent the higher the value of T_{req} . Overall, the curves seem to suggest that intermediate power levels maybe where good tradeoffs between throughput and lifetime performance can be found.

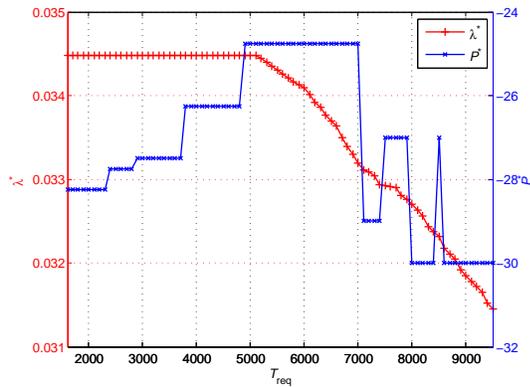


Fig. 7. Maximum Achievable Throughput and transmit power curves as function of T_{req} for the 30-node arbitrary network.

It is also instructive to look at the maximum achievable throughput for a fixed lifetime requirement, by optimizing over all possible reference transmit power levels. These results have been depicted in Fig. 7. The curve marked λ^* represents the maximum achievable throughput as a function of the lifetime requirement T_{req} . It is obtained by maximizing the solution of **P2** over $P_{\text{tx}} \in [0, \bar{P}]$. The curve marked P^* represents the minimum transmit power level at which the solution of **P2** achieves the corresponding throughput. There are a number of points to note. The curve marked λ^* is smooth, remains flat for low values of T_{req} and starts decreasing for $T_{\text{req}} \geq 5000$ time units. In other words, throughput has to be sacrificed to improve lifetime. The value of the plateau is given by the (unconstrained) maximum achievable throughput, i.e., for a 30-node network with unit link rate, $1/30$. At lower values of T_{req} , this throughput can be reached for lower value of the reference power, but when T_{req} increases while remaining below 5000 time units, the reference power at which the maximum throughput is achieved *increases*. This is again in support of our intuition that very low powers do not provide sufficient flexibility (in terms of independent sets) to achieve a high lifetime and throughput. Note that the λ^* curve in Fig. 7 does not describe a Pareto frontier [5] as a whole since all T_{req} s less than 5000 correspond to the same λ^* . But the portion of the curve for $T_{\text{req}} \geq 5000$ does. This is because, for a vector $\mathbf{v} \succeq [T_{\text{req}}, \lambda^*]$ on the curve, one can easily show that any vector $\mathbf{v} \succeq [T_{\text{req}}, \lambda^*]$, except $[T_{\text{req}}, \lambda^*]$ itself, is not feasible.

B. Optimal Lifetime vs. Throughput Requirement and Transmit Power

As proved in Section V, the optimal lifetime is also a piecewise continuous function of the transmit power which shows non-negative “jumps” at power levels where new independent sets get formed, and is non-increasing between the “jumps”. This behavior is illustrated in Fig. 8 which plots solutions of **P3**, for various values of ρ . The “jumps” provide improvements to the lifetime performance for low to intermediate power levels, and then the non-increasing trend between the “jumps” starts dominating. Again, overall the curves suggest that for good tradeoffs between lifetime and

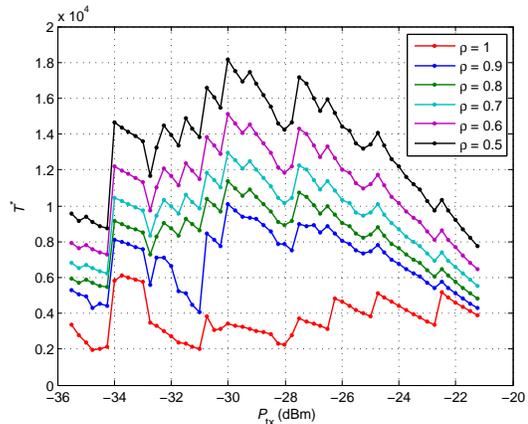


Fig. 8. Lifetime (with throughput lower bound) vs. transmit power for the 30-node arbitrary network.

throughput, intermediate power levels are better. Also, based on Fig. 6 and Fig. 8, it seems that neither throughput nor lifetime is better off at a very low power level.

Next, we investigate the tradeoff between optimal lifetime and throughput requirement, at a fixed transmit power. In order to represent the results in the context of all available power levels in $[0, \bar{P}]$, we take $\lambda_{\text{req}} = \theta \lambda^*(\bar{P}, 0)$ where $\theta \leq 1$ and $\lambda^*(\bar{P}, 0)$ is the optimal throughput at the maximum transmit power \bar{P} (i.e., it is the largest achievable throughput for a given network and power range). For a given power level P , θ translates to ρ (as in problem **P3**) as follows: $\theta = \rho (\lambda^*(P, 0) / \lambda^*(\bar{P}, 0))$. Given these notations, Fig. 9 shows the evolution of the optimal lifetime (by solving **P3**), as a function of the throughput requirement for a fixed power. The different curves correspond to different values of the fixed power. This is also represented in Fig. 10 which depicts the envelope of all the curves in Fig. 9 as a function of θ (the curve marked T^*). The curve marked P^* in Fig. 10 is the transmit power which achieves the optimal lifetime for the corresponding value of θ .

Several remarks are in order. First, from the displacement of

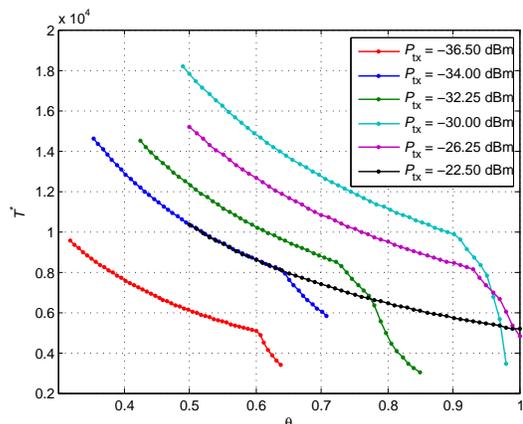


Fig. 9. Optimal Lifetime vs. Throughput Requirement for Fixed Transmit Power.

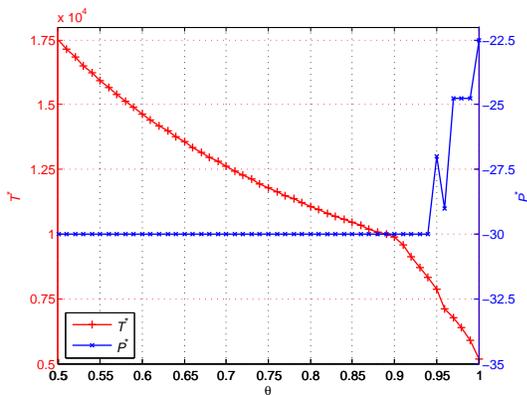


Fig. 10. Lifetime and transmit power curves as function of θ for the 30-node arbitrary network.

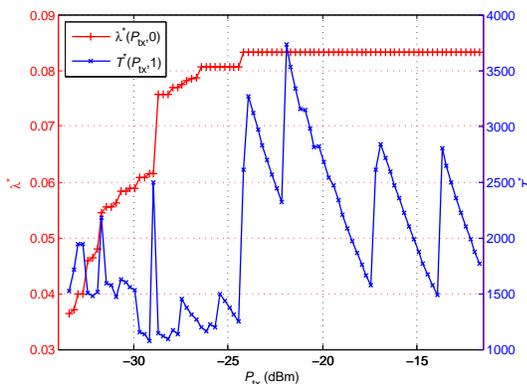


Fig. 11. Throughput and lifetime curves for a 25-node arbitrary network with two power levels and two modulations.

the curves from the origin in Fig. 9, it is clear that intermediate power levels provide better tradeoffs between optimal lifetime and throughput requirement. Secondly, we find that there is a threshold level in the throughput requirement below which reduction of the throughput requirement results in only a proportional improvement in lifetime. But above the threshold, lifetime can improve much better than proportionally, by relaxing the throughput requirement. The threshold observed here corresponds to ρ_0 as defined in Section V-C. Note that the threshold could be larger than ρ_0 because the proportional tradeoff region may extend beyond ρ_0 where $\mathcal{C}_f(\rho)$ is not complete yet. Finally, it can be seen that, for very high P_{tx} , there may be no such threshold as seen for $P_{tx} = -22.5$ dBm. All these findings confirm the insights from Section V-C. Note that all the points on the curve shown in Fig. 10 are *Pareto optimal* [5] with respect to throughput and lifetime. In fact, the curve characterizes the Pareto frontier of the feasible throughput-lifetime allocation region.

C. Multiple Power Offsets and Modulation Levels

Finally, we consider the case of multiple power offsets and modulation levels. We consider a 25-node arbitrary network, similar to that shown in Fig. 2 (b). We do not depict this network due to lack of space. Each node is allowed to choose

between two power levels, P_{tx} and $P_{tx} - 7$ dB, as well as two modulation schemes with data-rates of unity and two. The SINR threshold of the rate two modulation scheme is 3 dB higher than that of the unit rate modulation scheme. We solve the problems TO and P1. The solutions of these problems are plotted against P_{tx} in Fig. 11. As expected, allowing multiple power and modulation levels, does not fundamentally alter the relationship between throughput and lifetime. In particular, note the similarity between Fig. 11 and Fig. 3 (b).

VIII. CONCLUSION

Operating a multi-hop wireless network while achieving both high throughput and high lifetime is a highly desirable design goal. However, since these objectives do often conflict with each other, it is naturally important to identify the tradeoff between them. Towards this end, we have carried out a systematic study of the tradeoff between network throughput and lifetime, by means of the three problem formulations P1, P2 and P3. By focusing on fixed scheduled networks, we have been able to derive a number of analytical results. These together with the numerical results presented throughout the paper shed light on this tradeoff. Our main finding is that the performance of a network in terms of its max-min throughput and max-min lifetime, is intimately connected to the set of independent sets that are available at the reference transmit power level at which the network is operating. If the transmit power can be increased to expand the pool of candidate independent sets, it may improve the throughput and lifetime. Otherwise, the increasing transmit power only serves to drive up the energy consumption which in turn results in decreased lifetime (or decreased throughput, if there is a lifetime constraint). Overall, our insights call for a careful balancing of objectives when designing wireless networks for high throughput and lifetime.

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APPENDIX

Proposition 1: We first parameterize the **P2** problem by fixing the routing variable $\phi = [\phi_f^r]_{f \in \mathcal{F}, r \in \mathcal{R}_f}$. Let $\Phi = \{\phi \mid \sum_{r \in \mathcal{R}_f} \phi_f^r = 1, \phi_f^r \geq 0\}$ and $\lambda^*(\phi)$ be the optimal solution of the parametrized problem (17). According to the Maximum theorem [4], $\lambda^*(\phi)$ is continuous in ϕ . Given the compactness and convexity of Φ , the Weierstrass theorem [3] implies that $\max_{\phi \in \Phi} \lambda^*(\phi)$ exists and it is indeed $\lambda^*(P, T_{\text{req}})$.

Now since $\lambda^*(\phi)$ is optimal (hence feasible), we have the following inequalities from (7) and (13):

$$\lambda^*(\phi) \sum_{l \in q} \sum_{f \in \mathcal{F}} \left(\sum_{r \in \mathcal{R}_f^l} \phi_f^r \right) \leq \sum_{l \in q} \sum_{\zeta \in \mathcal{I}_l(\mathbf{P})} \alpha_\zeta \leq 1 \quad \forall q$$

$$\lambda^*(\phi) \sum_{\substack{(i,j) \in \mathcal{L} \\ (j,i) \in \mathcal{L} \\ f \in \mathcal{F}}} \left(P_{\text{tx}} \sum_{r \in \mathcal{R}_f^{(i,j)}} \phi_f^r + P_{\text{tx}} \sum_{r \in \mathcal{R}_f^{(j,i)}} \phi_f^r \right) \leq \hat{E}_i \quad \forall i \in \mathcal{N}$$

where the first inequality states a necessary condition for schedulability in terms of clique feasibility. Consequently,

$$\lambda^*(\phi) \leq \min_{q, i^l - q} \left[\frac{1}{w_q(\phi)}, \frac{1}{w_i(\phi)} \right]$$

The upper bound (14) is obtained by maximizing over $\phi \in \Phi$ on both sides and applying $\lambda^*(P, T_{\text{req}}) = \max_{\phi \in \Phi} \lambda^*(\phi)$. The proof of the lower bound is omitted; it follows from the sufficient condition for feasible flow rates that can be derived by tightening the clique feasibility constraints by a factor κ depending on the conflict structure [14]. ■

Proposition 2: The right inequality is due to the optimality of $\lambda^*(P, T_{\text{req}})$. For the left inequality, we have

$$\begin{aligned} \lambda^*(\hat{\phi}) &\geq \kappa \cdot \min_{q, i^l - q} \left[\frac{1}{w_q(\hat{\phi})}, \frac{1}{w_i(\hat{\phi})} \right] \\ &= \kappa \cdot \max_{\phi} \left\{ \min_{q, i^l - q} \left[\frac{1}{w_q(\phi)}, \frac{1}{w_i(\phi)} \right] \right\} \\ &\geq \kappa \lambda^*(P, T_{\text{req}}) \end{aligned}$$

where the first inequality is derived by tightening both (7) and (13) with κ , the second inequality is due to the definition of $\hat{\phi}$, and the last inequality follows from (14). Finally, as $\kappa = 1$, if the induced ECG is a perfect graph, we have the equality as required. ■

Proposition 3: The result is rather straightforward from the formulation of TO. Since the set of independent sets remains unchanged between two events at P_n and P_{n+1} , the constraint set of TO remains the same. As a result, the optimal throughput $\lambda^*(P_{\text{tx}}, 0)$ remains constant in $[P_n, P_{n+1})$. By the definitions of independent set and a point P_n of N_p , we have $\mathcal{I}(P_n -) \subset \mathcal{I}(P_n)$ as increasing power for every link is equivalent to reducing the noise power. Therefore, the number of independent sets of TO grows in size upon each event, which suggests a non-negative $\Delta\lambda(p)$. ■

Proposition 4: $\Delta\lambda(p, T_{\text{req}}) \geq 0$ can be shown in the same way as proving $\Delta\lambda(p, 0) \geq 0$. Since the set of independent sets remains unchanged between two events at P_n and P_{n+1} , the constraints (6)–(8) and (10) of **P2** remain the same; the only affected constraint is (13).

In order to prove that the function $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ is continuous in $P_{\text{tx}} \in (P_n, P_{n+1})$, we have to show that for all $\epsilon > 0$ there exists a $\delta > 0$ such that, for $P'_{\text{tx}} \in (P_n, P_{n+1})$, $|P'_{\text{tx}} - P_{\text{tx}}| < \delta \Rightarrow |\lambda^*(P'_{\text{tx}}, T_{\text{req}}) - \lambda^*(P_{\text{tx}}, T_{\text{req}})| < \epsilon$. We only prove the case where $P'_{\text{tx}} > P_{\text{tx}}$; the proof for the other case follows directly. On one hand, given $\epsilon > 0$, we have $\lambda(P'_{\text{tx}}, T_{\text{req}}) = \lambda^*(P_{\text{tx}}, T_{\text{req}}) - \epsilon$ as a feasible solution of **P2**, because we can scale down the variable λ_f accordingly without violating constraints (6)–(8) and (10) and deduce P'_{tx} from (13). This guarantees that $\lambda^*(P'_{\text{tx}}, T_{\text{req}}) - \lambda^*(P_{\text{tx}}, T_{\text{req}}) \geq -\epsilon$. On the other hand, it is impossible that $\lambda(P'_{\text{tx}}, T_{\text{req}}) - \lambda^*(P_{\text{tx}}, T_{\text{req}}) \geq \epsilon$, as it contradicts the optimality of $\lambda^*(P_{\text{tx}}, T_{\text{req}})$. As a result, we have the existence of $\delta = P'_{\text{tx}} - P_{\text{tx}}$ such that $|P'_{\text{tx}} - P_{\text{tx}}| < \delta \Rightarrow |\lambda^*(P'_{\text{tx}}, T_{\text{req}}) - \lambda^*(P_{\text{tx}}, T_{\text{req}})| < \epsilon$.

Finally, $g(p, T_{\text{req}}) \leq 0$ becomes obvious from the above arguments: as $\lambda^*(p', T_{\text{req}}) - \lambda^*(p, T_{\text{req}}) \geq \epsilon$ does not hold for any $\epsilon > 0$ if $p' > p$, the derivative, by definition, is

$$g(p, T_{\text{req}}) = \lim_{p' \rightarrow p} \frac{\lambda^*(p', T_{\text{req}}) - \lambda^*(p, T_{\text{req}})}{p' - p} \leq 0$$

Literally, the function $\lambda^*(P_{\text{tx}}, T_{\text{req}})$ is a decreasing function between “jumps”. ■

Proposition 5: The proof for the continuous part is the same as *Proposition 4*. We only need to show $\Delta\lambda(p, T_{\text{req}})$ can be either positive or negative. By the definitions of independent set under the interference range model and a point P_n of

N_p , we have, on one hand, $\mathcal{I}(P_n-) \not\subseteq \mathcal{I}(P_n)$ as increasing the transmit power for a node i raises the interference range $\sigma R(P_{tx}, z)$ and hence a link l tends to interfere with more links. On the other hand, we have $\mathcal{I}(P_n-) \not\supseteq \mathcal{I}(P_n)$ as a higher transmit power brings more links and hence new independent sets. Therefore, the number of independent sets of **P2** may increase or decrease in size upon each event depending on which of the above subsequences is dominating, which suggests the required results. ■

Proposition 6: According to *Proposition 3*, $\lambda_{\text{req}}(P_{tx}) = \rho\lambda^*(P_{tx}, 0)$ remains constant in (P_n, P_{n+1}) . Moreover, the set of independent sets remains unchanged between two events at P_n and P_{n+1} . Therefore, the constraints (18) and (19) remain the same; the only affected constraint is (21).

In order to prove that the function $T^*(P_{tx}, \rho)$ is continuous in $P_{tx} \in (P_n, P_{n+1})$, we have to show that for all $\epsilon > 0$ there exists a $\delta > 0$ such that, for $P'_{tx} \in (P_n, P_{n+1})$, $|P'_{tx} - P_{tx}| < \delta \Rightarrow |T^*(P'_{tx}, \rho) - T^*(P_{tx}, \rho)| < \epsilon$. We only prove the case where $P'_{tx} > P_{tx}$; the proof for the other case follows directly. On one hand, given $\epsilon > 0$, we have $T(P'_{tx}, \rho) = T^*(P_{tx}, \rho) - \epsilon$ as a feasible solution of **P3**, because we can scale down variable $x_{(i,j)}^f$ accordingly without violating constraints (18) and (19) and deduce P'_{tx} from (21). This guarantees that $T^*(P'_{tx}, \rho) - T^*(P_{tx}, \rho) \geq -\epsilon$. On the other hand, it is impossible that $T^*(P'_{tx}, \rho) - T^*(P_{tx}, \rho) \geq \epsilon$, as it contradicts the optimality of $T^*(P_{tx}, \rho)$. As a result, we have the existence of $\delta = P'_{tx} - P_{tx}$ such that $|P'_{tx} - P_{tx}| < \delta \Rightarrow |T^*(P'_{tx}, \rho) - T^*(P_{tx}, \rho)| < \epsilon$.

Finally, $h(p, \rho) \leq 0$ becomes obvious from the above arguments: as $T^*(p', \rho) - T^*(p, \rho) \geq \epsilon$ does not hold for any $\epsilon > 0$ if $p' > p$, the derivative, by definition, is

$$h(p, \rho) = \lim_{p' \rightarrow p} \frac{T^*(p', \rho) - T^*(p, \rho)}{p' - p} \leq 0$$

Literally, the function $T^*(P, \rho)$ is a decreasing function between “jumps”. ■

Proposition 7: we can easily check that $\frac{\rho_1}{\rho_2} T^*(P_{tx}, \rho_1)$ is an achievable lifetime under $\lambda_{\text{req}}^{(2)}(P_{tx})$: it is the outcome of keeping the optimal configuration for $T^*(P_{tx}, \rho_1)$ and applying duty cycle scaling with a factor $\frac{\rho_2}{\rho_1}$. ■

Proposition 8: According to the assumption, $\mathcal{C}_f(P_{tx}, \rho_1)$ is complete and hence cannot be larger. This ensures that $\mathcal{C}_f(P_{tx}, \rho_1) = \mathcal{C}_f(P_{tx}, \rho_2)$ for any $\rho_2 < \rho_1$. Assume in contradiction that $\rho_2 T^*(P_{tx}, \rho_2) > \rho_1 T^*(P_{tx}, \rho_1)$, then keeping the configuration that achieves $T^*(P_{tx}, \rho_2)$ and applying duty cycle scaling with a factor $\frac{\rho_1}{\rho_2}$ leads to a lifetime larger than $T^*(P_{tx}, \rho_1)$, a contradiction. ■

Proposition 9: We prove the proposition for a more general case where $\Gamma(\lambda, T)$ is a monotonically non-decreasing function of either one of throughput and lifetime when another is fixed. The proof is by contradiction. Without loss of generality, we assume that the optimal solution is only achieved at a point $P_{tx}^* \in (P_n, P_{n+1})$, for some integer n , with optimal values $\Gamma(\lambda(P_{tx}^*), T(P_{tx}^*))$. Now, we fix $T_{\text{req}} = T(P_{tx}^*)$ and reduce P_{tx} to P_n . By *Proposition 4*, the optimal throughput at

P_n is at least $\lambda(P_{tx}^*)$. However, since the power is decreased and $T^*(P_{tx})$ is decreasing in P_{tx} for $P_{tx} \in (P_n, P_{n+1})$ (by *Proposition 6*), the optimal lifetime $T^*(P_n)$, by fixing $\lambda_{\text{req}} = \lambda(P_{tx}^*)$, at P_n is larger than $T(P_{tx}^*)$. As a consequence, $\Gamma(\lambda(P_{tx}^*), T(P_{tx}^*)) \leq \Gamma(\lambda(P_{tx}^*), T^*(P_n))$ given the non-decreasing property of Γ , a contradiction. ■



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