Fueling Wireless Networks Perpetually: A Case of Multi-hop Wireless Power Distribution*

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Abstract—Inspired by the recent invention of a high efficiency Wireless Power Transfer (WPT) technique, we propose in this paper the Perpetual Wireless Networks (PWNs) as a novel wireless networking paradigm. Similar to the conventional wireless (data) access point, a PWN has a power access point, from which electrical power is injected and distributed into the network in a form of multi-hop transfer. Consequently, we lay our focus on this new type of multi-hop flow problems concerning not data but power. We formulate and analyze a set of such power flow problems (some are joint with data flow), and we devise algorithms to solve them. The intriguing insights obtained from solving these optimization problems offer instructive guidance for future studies on real PWN constructions.

I. INTRODUCTION

Wireless networks always have limited lifetime ever since they came into use. As users of wireless nodes expect them to be **tetherless**, i.e., wireless in terms of not only data communications but also power supplies, batteries are unanimously used as the power sources for these nodes. Therefore, network lifetime is heavily restricted by the battery capacity. Though networkers and system designers have been striving for energy efficiency, wireless networks are still far from being long-lived. As battery capacity is finite, power demands for data communications will eventually exhaust the batteries. Subsequently, the networks need to be either re-deployed or re-charged by getting a mobile charger to approach a wireless device or vice versa. Obviously, these subsequences all lead to big inconveniences and potentially huge costs.

The aforementioned issues could be solved if we can charge the network nodes continuously. Apart from the heavily studied energy harvesting mechanisms [7], [12], the recently developed *Wireless Power Transfer* (WPT) technique [4], [5] suggests yet a new potential. The idea of the WPT reported in [4] is based on *magnetic resonant coupling*. In a nutshell, if a power sender and the receiver(s) have the same resonant frequency, the energy can be transferred with rather high efficiency, as the dissipation to extraneous off-resonant objects is negligible. Although this technique is still in experimental phase, research proposals in our networking community already start to reap the potential harvest [6], [14]. However, these proposals still confine themselves to a stereotype of moving chargers to deliver energy to the network nodes.

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In this paper, we aim at fundamentally re-generating the idea of applying WPT to fuel wireless networks. We propose to equip individual nodes with WPT capability and to inject power into the network as flows, similar to data flows in conventional networking sense. Like data dissemination, injected power can be carried through multi-hop distribution. Under the new *Perpetual Wireless Network* (PWN) paradigm, a network may have **infinite** lifetime if the power consumption incurred by data flow can be compensated by the injected power flow.

This fundamentally new wireless networking paradigm also brings us new challenges. Specifically, we need to tackle a new type of network flow problems: power flow problems. Taking into account the inherent broadcast nature of WPT, we formulate a set of power flow problems, and we propose algorithms to solve them based on a detailed analysis on the problem structure. Moreover, using wireless sensor networks (WSNs) as the scenario, we further investigate the joint data and power flow problems. Applying the proposed algorithms, we obtain extensive numerical results on the optimal configurations of power flow and joint data/power flows, which yield instructive insights on practical system constructions. To the best of our knowledge, we are the first to make all these contributions. Our PWN paradigm, on one hand, complements the existing technologies to realize long-lasting wireless networks in an energy efficient manner, and on the other hand, it may redefine various system design aspects (e.g., data routing) given the need for power transfer awareness.

The remaining of our paper is organized as follows. We first discuss the background for WPT and present model definitions for PWNs in Sec. II. We investigate pure power flow problems in Sec. III and extend them to joint data and power flow problems in Sec. IV. In Sec. V, we report the extensive numerical results along with their implications. We finally conclude our paper in Sec. VI.

II. BACKGROUND AND MODELS

In this section, we first overview the background of wireless power transfer, then we introduce the model definition for perpetual wireless networks.

A. Background of WPT

Being an emerging technology, *wireless power transfer* (WPT) based on *resonant magnetic induction* has initiated its commercial use. As it is omni-directional irrespective of

the geometry of the surrounding space, and it can reach to meters without much loss into the off-resonant objects [4], WPT through resonant coupling becomes a proper technique for energy distribution in wireless networks. We may further expect that if multiple receivers are arranged to "attract" the power, a higher power transfer efficiency can be achieved. This has indeed been proven by experiments recently [5]. Therefore, the energy distribution performed in PWN should mostly take a (local) broadcast form in order to gain higher efficiency. But it is intrinsically different from *Wireless Broadcast Advantage* (WBA) [15] in 1) data can be replicated but power cannot, and 2) number of receivers affects the efficiency of power delivery but not that of data delivery.

B. Model Definition for PWNs

Basically, there are two components for a PWN: a power network and a data network. We first present the model definition for pure power network, then we add in data transmission models for the joint flow problems.

1) Model for Power Network: Suppose there is a power distribution network whose nodes are equipped with WPT capabilities, we name it static energy distribution network (SEDN). Given a SEDN with node set V and WPT range $r_i: i \in V$, an edge set E is defined such that $(i, j) \in E \Leftrightarrow$ $d(i,j) \leq r_i$ for $i,j \in V$. Obviously, each edge indicates a potential power transfer link, which is directional and can be asymmetric (unless r_i is identical for all nodes). Although each link should have a capacity (the maximum transferable power) in theory, we omit this constraint in our later formulations, as the current WPT technologies allow for a capacity of tens of Watts while the consumption of wireless devices is often far lower than that. In fact, applying capacity constraints for individual links does not change the complexity of a power flow problem. Each SEDN is equipped with a power access node $s \in V$.¹ It is connected by wire to a power grid, and it injects power into the SEDN.

We denote by $\eta_i < 1$ the efficiency ratio of the power transfer at node *i*, i.e., for power p_i transferred by *i*, only $\eta_i \cdot p_i$ can be collected by a receiver (or receivers). According to Sec. II-A, η_i increases with the number of receivers involved in a transfer. We also make the assumption that, for *m* receivers, $\frac{\eta_i(m)}{m}$ is decreasing in *m*. This is reasonable because $\eta_i(m)$ should be a concave function given that $\eta_i(1) \approx 0.5$ [3] and $\eta_i < 1$. We further assume that the resonant coil of a node can be "turned off" (or made off-resonant) such that the node can stop receiving power. This is important as the essential way of controlling power flow is to schedule the on-off of receiving nodes for each power transfer.

2) Model for Data Network: We specifically assume a WSN under PWN paradigm, where sensor nodes are equipped with both WPT and wireless communication capabilities. The sink $s \in V$ is now in charge of both data collection and power

injection. The assumptions for power flow part are the same as those described in Sec. II-B1.

The data communications are governed by another graph $\tilde{G}(V, \tilde{E})$ where each link $\ell \in \tilde{E}$ is identified by the following physical parameters:

- $o(\ell), t(\ell)$: the origin and the target nodes of ℓ .
- P_{ℓ} : the transmit (tx) power used by the (single) radio of $o(\ell)$. We assume $P_{\ell} \in [0, P_{\max}]$, where P_{\max} is a network-wide upper bound for tx powers.
- c_{ℓ} : the link rate in bits per second. It takes its value from a finite set C. We assume that a particular rate can only be obtained from one modulation/coding scheme that has a unique *signal to noise ratio* (SNR) threshold $\beta(c_{\ell})$.

This virtual link model stems from [8]–[10], it allows us to represent the power control and rate adaptation abilities of modern radios. Any link $\ell \in \tilde{E}$ characterized by $(o(\ell), t(\ell), P_{\ell}, c_{\ell})$ has to meet the SNR condition:

$$\operatorname{SNR}_{\ell} = \frac{G_{\ell} P_{\ell}}{N_0} \ge \beta(c_{\ell}) \tag{1}$$

where G_{ℓ} denotes the channel gain on ℓ and N_0 is the average thermal noise power in the operating frequency band. We also define $f(\cdot)$ as the rate-power conversion, it is a fixed and known map from a link rate c_{ℓ} and the Euclidean distance d_{ℓ} between $o(\ell)$ and $t(\ell)$ to the required tx power P_{ℓ} . Let x_{ℓ} be the data rate over ℓ ; it is constrained by the c_{ℓ} . We also denote by d_i the data generating rate at node i.

III. POWER FLOW PROBLEMS

In this section, we focus on power flow problems in SEDNs. We formulate two power flow problems. We also derive efficient solution techniques for these problems based on a detailed dual analysis.

A. Two Flow Problems

Similar to data flow problems, power flow also has its specific min-cost flow and maximum flow formulations. We present their formulations in the following.

1) Minimizing Total Power (MTP): Given a power demand p_i^d at each node *i*, MTP is meant to minimize the total power injection at *s* while satisfying all demands $\{p_i^d\}_{i \in V}$. Obviously, these power demands correspond to the power required for the PWN (which is supported by the SEDN under consideration) to carry its data traffic loads.

A Direct Formulation. Following the conventional flow conservation law, we get a seemingly simple problem formulation for MTP.

minimize
$$\sum_{j:(s,j)\in E} p_{sj}$$
 (2)

$$\sum_{j:(j,i)\in E} \eta_{ji} p_{ji} - \sum_{j:(i,j)\in E} p_{ij} \ge p_i^d \quad \forall i \in V \setminus \{s\} \quad (3)$$

$$1 > \eta_{ij} \ge 0, \ p_{ij} \ge 0 \qquad \forall (i,j) \in E \quad (4)$$

where p_{ij} is the power flow through a link (i, j). The objective, summing over all nodes that are directly charged by the s, is

¹We use the symbol s as it will become clear later that s can also be the sink of a WSN. Similar to the multi-sink case of WSNs, a SEDN can also have multiple power access nodes. However, we only focus on the single access node case for this initial study.

the total power injection. The constraint (3) requires, for each node, that the difference between all the injected power and all the transferred power to be larger than the power demand. Here η_{ij} represents the equivalent power transfer efficiency ratio for link (i, j). According to Sec. II-B1, this equivalent ratio is a variable affected by two factors: (i) the number of times this link is used in power transfer and (ii) the number of receivers involved in each transfer. Therefore, this is a non-convex problem that is far from trivial.

A Linear Programming (LP) Formulation with Virtual Nodes. Let us introduce some virtual nodes representing different power transfer schedules for a certain node. Specifically, a virtual node $v(i, \{j_1, \ldots, j_{m_v}\})$ denotes a local schedule that transfers power p_v from $v_o = i$ to $v_d = \{j_1, \ldots, j_{m_v}\}$ concurrently, where v_d is a set of m_v nodes within *i*'s WPT range. We use a simple claw graph to illustrate this idea in Fig. 1. Obviously, each virtual node has a corresponding efficiency



Fig. 1. As each virtual node is representing a power transfer schedule, there is an edge between the original source node and each virtual node, and a virtual node is connected to a subset of receiver nodes according to the schedule it represents. For example, the leftmost virtual node indicates a schedule that the source only transfers power to the leftmost receiver.

ratio η_v that is determined by the number of receivers m_v . We define the per-node efficiency ratio $\theta_v = \frac{\eta_v}{m_v}$ for a power transfer schedule represented by v. Then the MTP problem can be formulated into an LP:

[MTP] minimize $\sum_{v:v_o=s} p_v$ (5)

$$\sum_{v:i\in v_d} \theta_v p_v - \sum_{v:i=v_o} p_v \ge p_i^d \quad \forall i \in V \setminus \{s\}$$
(6)

$$p_v \geq 0 \quad \forall v \in \mathcal{S}$$
 (7)

where S is the set of all virtual nodes. The difference between (6) and (3) is that the former uses virtual nodes to represent incoming and outgoing flows. Although this is a standard LP problem, the number of variables involved is exponential in the graph degree Δ , because the number of virtual nodes introduced for each real node *i* is exponential in δ_i , the degree of node *i* in G(V, E). Therefore, directly solving MTP is practically ineffective on all but very small-scale problems; we will need a more efficient solution for it. Fortunately, as will be shown in Sec. III-B2, there exists a polynomial time algorithm to solve this problem.

2) Maximizing Charging Ratio (MCR): While MTP can be deemed as a min-cost power flow problem, we are also interested in a max-flow power problem. However, directly maximizing total received power is meaningless due to the lossy nature of WPT: the trivial solution is to allow only s to transfer to all its neighbors. In other words, the maximization is meaningless without incorporating a certain level of fairness. Therefore, we formulate the problem by following the maximum concurrent flow problem [13].

Using a similar graph extension idea explored in Sec. III-A1, we can directly put MCR into an LP form:

[MCR] maximize
$$\tau$$
 (8)

$$\sum_{v:i\in v_d} \theta_v p_v - \sum_{v:i=v_o} p_v \ge \tau p_i^d \quad \forall i \in V \setminus \{s\}$$
(9)

$$\sum_{v:s=v_o} p_v \leq P \tag{10}$$

$$\tau \ge 0, \ p_v \ge 0 \qquad \forall v \in \mathcal{S}$$
 (11)

where the objective is the maximum *charging ratio* τ that can be achieved for every node.

B. Analysis and Solutions

We first acknowledge that MTP and MCR share a similar problem structure, and they are actually equivalent.

Proposition 1: Let $\sum_{v:v_o=s} p_v^*$ be the optimal value of MTP. If we take the same demands $\{p_i^d\}$ and $\hat{P} = \sum_{v:v_o=s} p_v^*$ as the input of MCR, we obtain an optimal value $\hat{\tau} = 1$. Conversely, let \hat{P} be the total injected power that results in the optimal value of MCR $\hat{\tau} = 1$, then \hat{P} is the optimal value of MTP under the same demands $\{p_i^d\}$, i.e., $\hat{P} = \sum_{v:v_o=s} p_v^*$.

In the following, we rely on a dual analysis to get more insights on the problem structure of MTP, which motivates polynomial time solutions. Note that the analysis and solution are analogous to MCR.

1) Dual Analysis: Let $\lambda = {\lambda_i}$ be the dual variables for (6) and (9), the dual problem for MTP is:

[MTP-Dual] maximize
$$\sum_{i \in V \setminus \{s\}} \lambda_i p_i^d$$
 (12)

$$\theta_v \sum_{i \in v_d} \lambda_i \leq 1 \quad \forall v \in \mathcal{S} : v_o = s$$
(13)

$$\theta_v \sum_{i \in v_d} \lambda_i - \lambda_{v_o} \leq 0 \quad \forall v \in \mathcal{S} : v_o \neq s$$
 (14)

$$\lambda_i \geq 0 \quad \forall i \in V \setminus \{s\} \tag{15}$$

If we deem λ_i as the cost of transferring a unit of power from a node i, $\theta_v \sum_{i \in v_d} \lambda_i - \lambda_{v_o}$ becomes the reduced cost of flowing a unit of power through a virtual node v, or using the power transfer schedule represented by v. Note that the reduced cost incurred by a node receiving power within a schedule v is discounted by θ_v , indicating the loss in WPT. Therefore, for MTP problem to be optimal, the dual problem suggests that the reduced cost by each schedule is non-positive.

2) Solving MTP with Cutting Plane Method: We initialize the algorithm with a feasible schedule $S' = S_0 \subset S$. Solving this restricted problem leads to a set of (restricted) dual solutions λ' . If the problem is not solved to optimal, some schedule $v \in S \setminus S'$ must bear positive reduced cost, i.e., (14) is violated. We simply need to check whether the *separation oracle* [11] $\rho(\lambda) \equiv \max_{v \in S} \rho_v(\lambda) = \max_{v \in S} \left(\theta_v \sum_{j \in v_d} \lambda_j - \lambda_{v_o} \right) = \max_{i \in V} \left[\max_{v:i=v_o} \rho_v(\lambda) \right]$ is positive. The pseudo-code is shown in **Algorithm 1**, which includes an inner descending sorting and an outer linear search. As a byproduct, we can also claim that MTP/MCR can be solved in polynomial time since the separation oracle has a polynomial time complexity $\mathcal{O}(\Delta|V|)$, or equivalently $\mathcal{O}(|V|^2)$.

Algorithm 1: MTP Solver

1 $\mathcal{S}' \leftarrow \mathcal{S}_0$ 2 repeat $\boldsymbol{\lambda}' \leftarrow \mathsf{solveMTP}\left(\{p_i^d\}_{i \in V}, \mathcal{S}'\right)$ 3 for all the $i \in V$ do 4 $\begin{array}{l} \{j_1, \dots, j_{\delta_i}\} \leftarrow \text{sortDescent}\left(\{\lambda'_j\}_{(i,j) \in E}\right) \\ v_d \leftarrow \emptyset; \quad k \leftarrow 1 \end{array}$ 5 6 repeat 7 $\begin{vmatrix} v_d \leftarrow v_d \cup \{j_k\}; & k \leftarrow k+1 \\ \text{until } \rho_i = \theta_v \sum_{j \in v_d} \lambda'_j - \lambda'_i \text{ is maximized}; \end{vmatrix}$ 8 9 10 end $v \leftarrow \arg\max_{(i,v_d)} \left(\{ \rho_i \}_{i \in V, \rho_i > 0} \right); \quad \mathcal{S}' \leftarrow \mathcal{S}' \cup \{ v \}$ 11 12 until S' remains unchanged;

IV. JOINT DATA AND POWER FLOW PROBLEMS

With a better understanding of the power flow problem, we are ready to tackle the joint data and power flow problem. As a PWN loses power due to carrying data traffic while gaining power from its SEDN, there is a necessity to balance the gain and loss. From the optimization point of view, we may either minimize the total injected power required to support a given set of data flows, or maximize data flows under a fixed amount of power injection.

A. Problem Formulations

We directly use the extended graph for the problem formulation. We only need to extend the power transfer graph G while keeping the data communication graph \tilde{G} intact. The following J-MTP is the min-cost version of the joint flow problem.

[J-MTP] minimize
$$\sum_{v:v_o=s} p_v$$
 (16)

$$\sum_{o(\ell)=i} x_{\ell} - \sum_{t(\ell)=i} x_{\ell} \ge d_i \quad \forall i \in V \setminus \{s\}$$
(17)

$$\sum_{v:i\in v_d} \theta_v p_v - \sum_{v:i=v_o} p_v \ge p_i^d \quad \forall i \in V \setminus \{s\}$$
(18)

$$p_i^d - \sum_{o(\ell)=i} \frac{x_\ell}{c_\ell} f(c_\ell, \mathbf{d}_\ell) \ge 0 \quad \forall i \in V \setminus \{s\}$$
(19)

$$x_{\ell} \leq c_{\ell} \quad \forall \ell \in \tilde{E} \tag{20}$$

$$\sum_{o(\ell)=i} \frac{\omega_{\ell}}{c_{\ell}} \leq 1 \quad \forall i \in V$$
(21)

$$x_{\ell} \ge 0, \ p_v \ge 0 \qquad \forall \ell \in \tilde{E}, \forall v \in \mathcal{S}$$
 (22)

Constraints (17) and (18) are data and power flow conservation laws, respectively. (19) states that the total consumed power of a node has to be compensated by the power demanded from the SEDN; the power demand p_i^d is now an optimization variable rather than a fixed value as in MTP. Note that the power consumed in operating a link ℓ depends, besides $f(c_{\ell}, d_{\ell})$, also on the time fraction ℓ is active, hence we compute this fraction by normalizing the flow rate x_{ℓ} against the corresponding link capacity c_{ℓ} . While (20) is a trivial link capacity constraint, (21) needs further elaboration. With the power control and rate adaptation abilities, a node (or the radio of the node) can operate multiple virtual links in a time-division manner (i.e., the node may operate on different physical parameter tuples $(t(\ell), P_{\ell}, c_{\ell})$ at different times). As any flow problem focuses on a snapshot within one time unit and each node only has one radio, the active time fractions for all virtual links sharing the same origin should at most sum to 1. We deliberately omit the interference constraints for data flow, as the current studies focus on power flows.

Similarly, we may also have the max-flow version denoted by *Joint Maximizing Delivery Ratio* (or J-MDR), where we maximize the minimum data delivery ratio.

$$[J-MDR] \quad \text{maximize} \quad \tau' \tag{23}$$

$$\sum_{o(\ell)=i} x_{\ell} - \sum_{t(\ell)=i} x_{\ell} \geq \tau' d_i \quad \forall i \in V \setminus \{s\}$$
(24)

(18), (19), (20), (21)

$$\sum n_{r} < P$$
 (2

$$\sum_{v:s=v_o} p_v \leq P \tag{25}$$

$$\tau' \ge 0, \ x_{\ell} \ge 0, \ p_v \ge 0 \qquad \forall \ell \in E, \forall v \in S$$
 (26)

Here we use τ' as the lower bound of the data delivery ratio. Unlike the direct correspondence between MTP and J-MTP, the relation between MCR and J-MDR is less straightforward, given the implicit function relation between d_i and p_i^d . Nevertheless, it can easily shown that the technique introduced in Sec. III-B2 can also be used to tackle these two problems.

V. NUMERICAL RESULTS AND IMPLICATIONS

Aiming at revealing insights into optimal power flows, we perform extensive numerical computations on the four power flow problems. We report the results in this section, and we also discuss their implications.

A. Setting Parameters

We consider arbitrarily deployed networks with size varying from |V| = 50 to 200. We scale the network area in proportion to the number of nodes so that the node density is always the same. For each network, there is a power access point s, and it also serves as data collection sink for the joint flow problems.

For power flow, the WPT range is set as 4 meters. As existing sensor nodes do not have rate adaptation ability, we take these parameters from IEEE 802.11n. This is a common practice for simulations involving rate adaptation in WSNs (e.g., [2]). With these parameters, the rate-power conversion can be represented as $P_{\ell} = f(c_{\ell}, d_{\ell}) = \beta(c_l) \left(\frac{d_{\ell}}{d_0}\right)^{\alpha} N_0$, where $d_0 = 0.1$ m is the close-in reference distance and $\alpha = 3$ is the path loss exponent.

Our solver for the power flow problems is developed in C++, using GLPK [1] as the LP solver. Each data point in our later plots is obtained by solving 100 problem instances, so we use either empirical cumulative distribution, or boxplot, or error bars (for standard deviations) to represent the variances in these computations.

B. Experiencing MTP

We perform extensive experiments on the MTP problem, in order to reveal the impact of demand distributions on the power flows and the features of optimal power flow in an SEDN. Given the strong correlations between MTP and MCR (*Proposition 1*), we omit the results of MCR for brevity.

What we are first interested in is the impact of the demand distribution pattern on the required injection power. As we have only one power access point in an SEDN, we have three typical demand patterns:

- Uniform: every node raises the same demand.
- Near-High Far-Low (NHFL): nodes closer to the access point have higher demand than nodes that are further.
- Near-Low Far-High (NLFH): nodes closer to the access point have lower demand than nodes that are further.

In Fig. 2, we compare the optimal values of MTP under these three patterns, with the same total demand that is proportional to the network size. We first observe that the power injection required by NHFL is about 2 to 4 dB (1.6 to 2.5 times) of the total demand. This appears to be a much affordable value, compared with the cost of re-deploying or re-charging the network nodes. It is quite intuitive to see that NHFL leads to the lowest injection power, but Uniform performs only marginally better than NLFH.

If the SEDN is used in a PWN with two separated sub-systems, the demands are governed by the wireless data network. Therefore, if energy efficiency is the main objective for this PWN, the deployment and/or operations of the data network should aim at producing an NHFL pattern. If the PWN combines both sub-



Fig. 2. Comparing the minimum injected power for three demand patterns.

systems, then the data flow may naturally result in a certain power demand pattern on the PWN itself. For ad hoc or mesh networks, it is well known that the network center has the highest load (thus highest power demand). For networks with a convergecast pattern (e.g., WSNs), the nodes close to the sink bear the highest load. Therefore, these hotspots are where we should put the power access point.

Next, we want to understand how optimal power transfers behave, mainly in terms of (i) how many schedules are used by each node and (ii) how many destinations are involved in



Fig. 3. Statistics on power transfer schedules in 100-node PWNs.

each schedule. In Fig. 3, we plot the empirical cumulative distribution functions of these two quantities, with respect to the three demand patterns respectively. Two observations can be immediately drawn from the plots:

- 1) The statistics on optimal power transfers are rather independent of the demand distribution pattern.
- Majority (around 85%) of the nodes that indeed transfer energy use only one schedule.

These are good news for practical algorithm designs. A practical power routing mechanism, on one hand, may be designed independent of the demand pattern (though the outcome of performing routing differs). On the other hand, it may incur simple scheduling at each nodes, as only one power transfer schedule is required under most circumstances.

The number of destinations involved in each schedule, shown in Fig. 3(b), appears to be rather concentrated on values around the middle of the maximum and minimum degrees. This shows that the optimal schedules manage to find a balanced broadcast advantage between less loss in transfer (involving more destinations) and less waste in charging (involving only destinations that demand power).

C. Minimizing Power Injection

We hereby compare the energy efficiency of the following four different operation modes of a PWN-WSN:

- MH+MTP: data collection in the WSN is done through min-hop routing with a fixed rate.
- MH-RA+MTP: data collection in the WSN is done through min-hop routing with adaptable rates.
- MCF+MTP: data collection in the WSN is based on the optimal solution of a min-cost (data) flow problem.
- J-MTP: data and power flows are jointly routed based on the optimal solution of J-MTP.

Three comparisons are shown in Fig. 4. Obviously, using min-hop routing with a fixed rate leads to a significantly higher demand in power injection, especially for large networks. Though adding rate adaptation may improve the power efficiency, the injection is still far higher than the minimum injection that is obtained by J-MTP. Interestingly, the power injection resulting from the third mode is only marginally higher than the minimum injection. This latter observation suggests that, in practice, we may apply the decoupled mincost data flow and min-cost power flow (MTP) to operate a



Fig. 4. Comparing total power injected at the sink under four data and power routing modes.

PWN, which requires less intensive computation compared with J-MTP, only at the cost of a slightly worse power efficiency.

D. Maximizing Concurrent Throughput

In this section, we demonstrate the relation between the optimal delivery ratio τ' (obtained from J-MDR) and the optimal charging ratio τ (obtained from MCR). Basically, we solve a sequence of J-MDR with a fixed set of demand $\{d_i\}$ and an increasing power injection P. As a result, we get a sequence of increasing values of τ' . Then we use the data routing mechanisms involved in the first three modes described in Sec. V-C to route the demands $\{\tau'd_i\}$, and we check the deficit in terms of power supply by solving MCR with P and the resulting power demands $\{p_i^d\}$ as input.



Fig. 5. Comparing the charging ratios under a fixed power injection and different data routing mechanisms.

As shown in Fig. 5, the power charging rate for MCF is almost a constant (around 95%, i.e., 5% deficit) when we increase τ' . This again confirms the near optimality of separately optimizing data and power flows, as already shown in Sec. V-C. As expected, min-hop routing performs far worse than MCF, though applying rate adaptation gains slight improvements under large values of τ' . Since min-hop routing is commonly used for data flows due to its simplicity, our results actually argue against it: under the PWN paradigm, some form of min-cost flow has to be applied to route both (yet not necessarily jointly) data and power flows if energy efficiency is a big concern.

VI. CONCLUSION

In this paper, we have proposed Perpetual Wireless Networks (PWNs) as a new wireless networking paradigm, motivated by the recent invention of a high efficiency Wireless Power Transfer (WPT) technique. Rather than re-engineering the conventional mobile energy delivery approach, our proposal has made a significant innovation by delivering power to network nodes through a form of multi-hop wireless transfer. We have studied a new type of multi-hop flow problems concerning not data but power. We have provided novel formulations and algorithms for these problems. Based on the numerical results, we have obtained very constructive insights on real PWN implementations.

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