A Robust Algorithm for Linearly Constrained Adaptive Beamforming

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***Abstract*—A new approach to robust adaptive beamforming for wideband array signals is proposed. General steering vector errors, such as direction-of-arrival mismatch and array positional error, are modeled by "time-delay errors" and compensated for by self-adjusted interpolation filtering. The proposed method effectively overcomes the target–signal cancelation problem without suffering from loss in the degree of freedom for interference rejection, as verified by simulations.**

***Index Terms*—Broadband array, robust adaptive beamforming, time-delay errors.**

I. Introduction

ADAPTIVE array processing has received considerable attention in the past decades due to its wide applications in the fields of wireless communications, speech acquisition, sonar, and so on [1], [2]. To achieve high interference suppression, an adaptive array is able to adjust its beampattern in real time to introduce deep nulls in the directions of arrival (DOA) of strong interferences. The presteered linearly constrained adaptive beamformer (see Fig. 1), also named the Frost processor, which is used to enhance wideband wave signals, e.g., speech signals impinging on a microphone array, has been extensively studied in the literature [3], [4].

Using the conventional Frost algorithm, target–signal cancelation can occur when there exist steering vector errors, including DOA mismatch, positional error, etc. Several robust beamforming techniques have been proposed to solve this problem [5]–[9]. The constraint-based methods [5], [6] are easy to implement, but the constraints introduced can reduce the beamformer's degree of freedom in interference rejection. Recently, a robust generalized sidelobe canceler based on a blocking matrix using constrained adaptive filters was presented [7]. This method is robust to steering vector errors and exhibits quite good interference cancelation capability, but control of the adaptive process of two adaptive modules is difficult. Another type of robust beamformer is based on array calibration [9]. It employs the beamformer output power as an objective function to correct DOA mismatch. This method causes no loss in the degree of freedom in interference rejection, and its computation cost is low, since the calibration is required only when there are detectable changes in signal scenarios, e.g., DOA mismatch. The proposed algorithm in this letter can be regarded as an extension of the work of [9] in two aspects. First, wideband signals (as against narrowband signals discussed in [9]) are treated here. Second, general steering errors (including not only DOA mismatch but also positional errors, etc.) are considered. The novelty of the proposed algorithm is that general steering vector errors mentioned above are represented by "time-delay errors" of array signals and subsequently corrected by time-shift operations using the well-known interpolation function. The adjustment using time-shift operations aims to compensate for steering vector errors by maximizing the beamformer output power locally, which can be done when the target signal just appears or changes.

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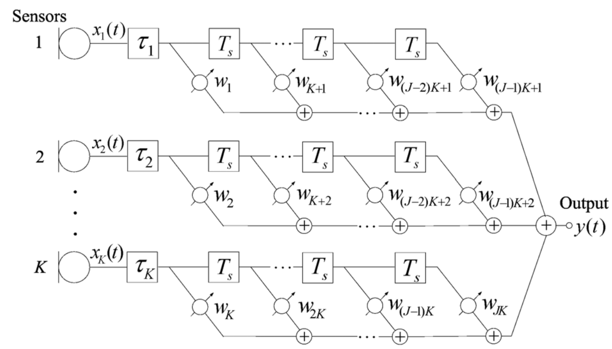

Fig. 1. Wideband beamformer with K sensors and J taps per sensor.

II. Problem Formulation

Consider a presteered wideband beamformer with K sensors and J taps per sensor (Fig. 1). The continuous-time signal received by the kth sensor is designated by $x_k(t), k = 1, 2, \ldots, K$, which can be written as

$$x_k(t) = s(t - \tau_k) + n_k(t), \qquad k = 1, 2, \ldots, K$$

where $s(t)$ is the desired signal from the look direction, τ_k denotes the propagation delay difference of the desired signal at the kth sensor, and $n_k(t)$ represents the totality of interference and noise observed at the kth sensor. The front end of a presteered linearly constrained adaptive beamformer is either digital or analog time delays placed immediately after each sensor, and its function is to steer the array response to the direction of interest. These steering time delays aim to align the desired signal component $s(t - \tau_k)$ at each sensor output exactly in phase. So, it is required that the time delay added to each sensor output is able to completely compensate for the actual delay difference τ_k. These additive time delays are usually calculated based on the knowledge of the DOA of the desired signal and the array steering vectors. However, due to practical imperfections such as steering vector errors, the real delay differences τ_k are impossible to be perfectly compensated for in this way. The misalignment of the desired signal component at each sensor output will cause the desired signal to be regarded as interference and then canceled by the adaptive processor. After the steering delay compensation, the signal at each sensor output becomes

$$x_k(t) = s(t-\Delta_k) + n_k(t+\tau_k-\Delta_k), \quad k=1,2,\ldots,K$$

where Δ_k represents the residual delay difference of the desired signal. It is clear from the above discussion that the desired signal components are perfectly aligned only when $\Delta_k, k=1,2,\ldots,K$, are all equal. The digitalized signal at the kth sensor output is

$$x_k(n) = s(nT_s-\Delta_k) + n_k(nT_s+\tau_k-\Delta_k) \tag{1}$$

where T_s represents the sampling period. The vector of the array signal at time n is defined as in the first equation shown at the bottom of the page. Following the time delay components in Fig. 1 is a multichannel adaptive processor whose weights are iteratively adjusted to minimize output noise/interference power while maintaining a certain frequency response in the look direction. When the steering delays $\Delta_k, k=1,2,\ldots,K$, in (1) are all equal, the optimization problem of the Frost algorithm is formulated as

$$\min_W P = \min_W W^T R_{XX} W \text{ subject to } C_0^T W = F. \tag{2}$$

Here P is the expected beamformer output power, and $(\cdot)^T$ denotes matrix transpose. W is the vector of tap weights, i.e.,

$$W = [\,w_1 \quad \ldots \quad w_{KJ}\,]^T.$$

$R_{XX} = E\left[X(n)X(n)^T\right]$ is the array correlation matrix. C_0 is the constraint matrix and it is defined by

$$C_0 = [\,c_1 \quad \ldots \quad c_j \quad \ldots \quad c_J\,] \tag{3}$$

where

$$c_j = [\,\underbrace{0\ldots0}_{(j-1)K0\text{s}}\ \underbrace{1\ldots1}_{K1\text{s}}\ \underbrace{0\ldots0}_{(J-j)K0\text{s}}\,]^T, \qquad j=1,\ldots J. \tag{4}$$

F is a vector of size $J \times 1$, which determines the frequency response in the look direction.

With the method of Lagrange multipliers, the optimal weight vector W_{opt} of (2) is given by

$$W_{\text{opt}} = R_{XX}^{-1} C_0 \left(C_0^T R_{XX}^{-1} C_0\right)^{-1} F. \tag{5}$$

The optimal beamformer output power is given by

$$P_{\text{opt}} = F^T \left(C_0^T R_{XX}^{-1} C_0\right)^{-1} F. \tag{6}$$

However, when there are steering vector errors, i.e., Δ_k not all equal, the constraint matrix C_0 given by (3) and (4) does not match the true array scenario and can cause serious target–signal cancelation. In Section III, the proposed algorithm aims to use a slightly adjustable constraint matrix C instead of the fixed C_0 so that target–signal cancelation does not occur in the presence of small steering vector errors.

III. Proposed Algorithm

Assume that $X'(n)$ is a signal vector received by the sensor array when there exist steering vector errors. So, the desired signal components in $X'(n)$ are not perfectly aligned in phase, i.e., Δ_k are not all equal in $X'(n)$. Assume also that $X(n)$ is the signal vector related to $X'(n)$ by an accurate time-shift operation that is used to correct the misalignment in Δ_k, i.e., $X(n)$ is the corresponding signal vector of $X'(n)$ except that the desired signal components in $X(n)$ are ideally aligned. Then, the optimal weight vector W_{opt} in (5) with C_0 given by (3) and (4) corresponds only to $X(n)$ but not to $X'(n)$. The time-shift operation between $X'(n)$ and $X(n)$ can be approximately realized through a linear transformation matrix Y, which is given by

$$X'(n) = YX(n). \tag{7}$$

Here, assuming for the moment that $\Delta_k, k=1,2,\ldots,K$, are known, the matrix Y of size $KJ \times KJ$ is defined as in (8), shown at the bottom of the page, where $y_{k,j}$ $(k=1,\ldots,K, j=1,\ldots,J)$ is the $((j-1)K+k)$th row of Y. Each $y_{k,j}$ is a $1 \times KJ$ row vector consisting of subrow vectors $g_1,\ldots,g_J$ as

$$y_{k,j} = [\,g_1 \quad \cdots \quad g_i \quad \cdots \quad g_J\,] \tag{9}$$

where g_i $(i=1,\ldots,J)$ is a $1 \times K$ row vector given by

$$g_i = [\,0 \quad \ldots \quad 0 \quad h_k(i-j) \quad 0 \quad \ldots \quad 0\,] \tag{10}$$

in which the only possibly nonzero element of g_i is at its kth position and denoted by $h_k(i-j)$. Here, $h_k(n)$ are the interpolation filter coefficients that shift a signal along the time axis by Δ_k, i.e.,

$$h_k(n) = \frac{\sin\left[\pi\left(n-\frac{\Delta_k}{T_s}\right)\right]}{\pi\left(n-\frac{\Delta_k}{T_s}\right)}, \quad n=\cdots-1,0,1,\ldots \tag{11}$$

Since $h_k(n)$ has infinite length, to compensate for the time delay, Δ_k completely would require J to be infinite. Nevertheless, (8) is still a good approximation in practice if J is large enough, since the magnitude of $h_k(n)$ diminishes as fast as $1/|n|$.

Minimizing the beamformer output power for $X'(n)$ gives

$$\min_{W'} P' = \min_{W'} W'^T R_{X'X'} W' \tag{12}$$

where W' and P' represent the weight vector and the output power for $X'(n)$, respectively. Using (7), (12) becomes

$$\min_{W'} P' = \min_{W'} W'^T Y R_{XX} Y^T W'. \tag{13}$$

$$X(n) = [\,x_1(n) \quad \ldots \quad x_K(n) \quad x_1(n-1) \quad \ldots \quad x_K(n-1) \quad \ldots \quad x_1(n-J+1) \quad \ldots \quad x_K(n-J+1)\,]^T$$

$$Y = [\,y_{1,1}^T \quad \ldots \quad y_{K,1}^T \quad y_{1,2}^T \quad \ldots \quad y_{K,2}^T \quad \ldots \quad y_{1,J}^T \quad \ldots \quad y_{K,J}^T\,]^T \tag{8}$$

Comparing (13) with (2), the correct constraint matrix for W' is no longer C_0; instead it is given by

$$C = YC_0. \tag{14}$$

Therefore, the new optimization problem is

$$\min_{W'} P' = \min_{W'} W'^T R_{X'X'} W' \text{ subject to } C^T W' = F. \tag{15}$$

In practice, however, since Δ_k is unknown, Y cannot be directly computed using (8)–(11). It follows that C cannot be directly solved using (14). To overcome this difficulty, we assume that Δ_k is small so that the norm of $C - C_0$, denoted by $\|C - C_0\|$, is also small, implying that C lies in the neighborhood of C_0. Furthermore, when the selected C satisfies the real scenario of array signals with general steering vector errors, the beamformer output retains the desired signal with minimal distortion. Otherwise, the desired signal is canceled more or less as interference. So, it is clear that the optimal output power P'_{opt} should have a local maximum with respect to different selections of C. Therefore, a method to find the desired C is to perform a local search in the vicinity of C_0 to maximize the optimal beamformer output P'_{opt}. For example, if the target–signal cancelation is caused by DOA mismatch, the search for C is analogous to a small but important readjustment to the look direction of the presteered beamformer so that this look direction lies in the true DOA of the desired signal [9]. Thus, the criterion for this local search algorithm is given by

$$\max_{C} P'_{\text{opt}} \text{ subject to } \|C - C_0\| \le \delta \tag{16}$$

where δ is a small positive real number used to control the size of the feasible region around C_0. The value of δ is experimentally set based on the norm of steering vector errors. As $P'_{\text{opt}} = F^T (C^T R_{X'X'}^{-1} C)^{-1} F \ge 0$ in view of (6), (16) is equivalent to

$$\min_{C} \frac{1}{F^T (C^T R_{X'X'}^{-1} C)^{-1} F} \text{ subject to } \|C - C_0\| \le \delta. \tag{17}$$

We choose to minimize the reciprocal of the beamformer output power instead of maximizing the output power itself, since the former method is numerically more stable than the latter when using the gradient method. Since $C = YC_0$ and Y is a function matrix of Δ_k, (17) can be rewritten as

$$\min_{\Delta_k} \frac{1}{P'_{\text{opt}}} = \min_{\Delta_k} \frac{1}{F^T (C_0^T Y^T R_{X'X'}^{-1} Y C_0)^{-1} F} \quad \text{subject to } \|YC_0 - C_0\| \le \delta. \tag{18}$$

The problem of (18) is a multidimensional nonlinear optimization problem. In this letter, the gradient method is used to find the optimal Δ_k. Let $Q = C_0^T Y^T R_{X'X'}^{-1} Y C_0$; the partial derivative of $1/P'_{\text{opt}}$ in (18) with respect to Δ_k is given by

$$\frac{\partial}{\partial \Delta_k}\left(\frac{1}{P'_{\text{opt}}}\right) = -\frac{1}{P'^2_{\text{opt}}} \frac{\partial P'_{\text{opt}}}{\partial \Delta_k} \tag{19}$$

where

$$\frac{\partial P'_{\text{opt}}}{\partial \Delta_k} = -F^T Q^{-1} \frac{\partial Q}{\partial \Delta_k} Q^{-1} F \tag{20}$$

and

$$\frac{\partial Q}{\partial \Delta_k} = C_0^T \frac{\partial Y^T}{\partial \Delta_k} R_{X'X'}^{-1} Y C_0 + C_0^T Y^T R_{X'X'}^{-1} \frac{\partial Y}{\partial \Delta_k} C_0. \tag{21}$$

$\partial Y/\partial \Delta_k$ can be calculated using (8)–(11), and its computation is essentially that of $\partial h_k(n)/\partial \Delta_k$, which is given by

$$\frac{\partial h_k(n)}{\partial \Delta_k} = \begin{cases} 0, & n - \frac{\Delta_k}{T_s} = 0 \\ v, & n - \frac{\Delta_k}{T_s} \ne 0 \end{cases} \tag{22}$$

where $v = -\cos[\pi(n - \Delta_k/T_s)]/[T_s(n - \Delta_k/T_s)] + \sin[\pi(n - \Delta_k/T_s)/[\pi T_s (n - \Delta_k/T_s)^2]$.

As the time delay differences Δ_k are all relative quantities compared to each other, it is required to fix one of them, say $\Delta_1 = 0$, and adjust other $\Delta_k, k = 2, 3, \ldots, K$, to find the local maximum of P'_{opt}. The use of the gradient search method requires that the iterative process is convergent. Extensive simulations have shown that the proposed optimization process always converges when steering vector errors are small, although a theoretical proof is not yet available.

The proposed algorithm is now summarized as follows.

Step 1) When new signal data $X'(n)$ is received, the correlation matrix $R_{X'X'}$ is updated by

$$R_{X'X'}^{[n]} = \alpha R_{X'X'}^{[n-1]} + (1-\alpha) X'(n) X'(n)^T, \qquad n = 1, 2, \ldots$$

where α is the update coefficient and close to one.

Step 2) The partial derivatives of $1/P'_{\text{opt}}$ with respect to $\Delta_k, k = 2, 3, \ldots, K$, are calculated using (19)–(22), and then Δ_k are updated as

$$\Delta_k^{[n]} = \Delta_k^{[n-1]} - u \frac{\partial}{\partial \Delta_k}\left(\frac{1}{P'_{\text{opt}}}\right), \qquad n = 1, 2, \ldots$$

where u is a small positive number used to control the convergence speed. $\Delta_1^{[n]}$ is fixed as zero. The initial values $\Delta_k, k = 2, 3, \ldots, K$, are set to be zeros.

Step 3) The constraint matrix is updated by

$$C^{[n]} = Y^{[n]} C_0$$

where $Y^{[n]}$ is computed according to (8)–(11) with the new $\Delta_k^{[n]}$.

Step 4) Check whether $\| C^{[n]} - C_0 \| \le \delta$. If this condition is violated, it is known that the incoming signals are all interferences and noises and, hence, $C^{[n]}$ needs to be reset to C_0. Otherwise, Steps 2)–4) are repeated until $C^{[n]}$ converges to the desired constraint matrix C_{opt}.

Step 5) The traditional iterative algorithm is employed to find the optimal W'_{opt} based on C_{opt} [3].

IV. Simulation Results

In this section, the proposed algorithm is evaluated by computer simulations. A four-element uniform linear microphone array is used in this simulation. The spacing between microphones is 0.04 cm, and the sampling rate is 8 kHz. Bandlimited Gaussian signals (0.3–3.4 kHz) are used as source signals, and the assumed look direction is 0°. The interferences are far from 0°, and −30-dB sensor noises are added. The length of tap line

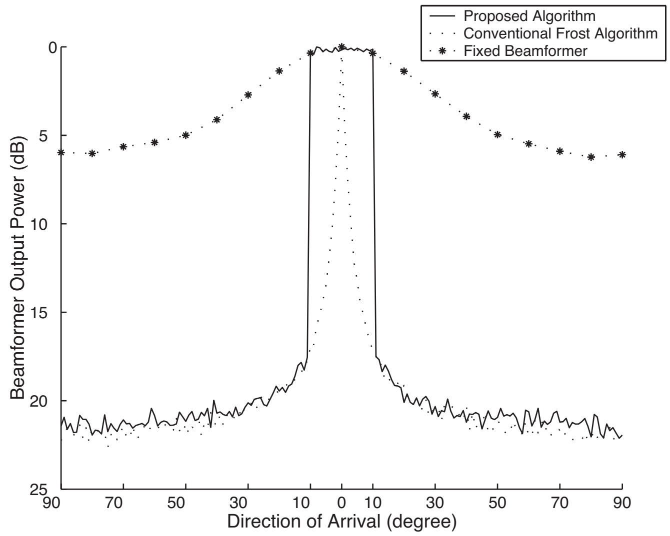

Fig. 2. Acceptance angle of adaptive beamformer.

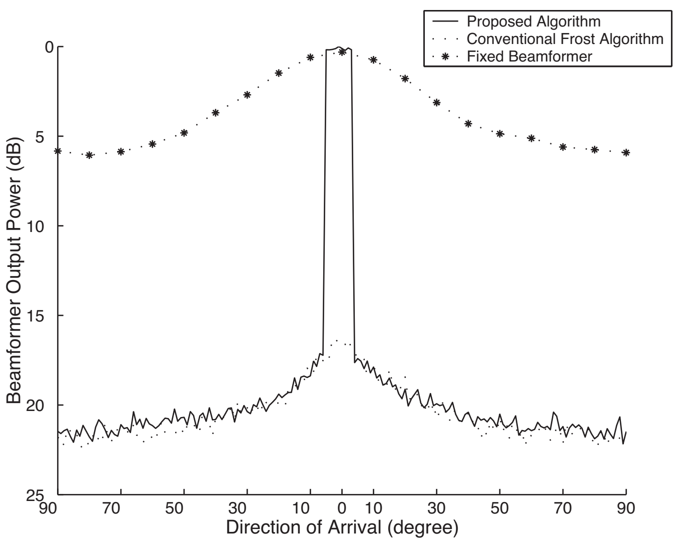

Fig. 3. Beampattern in the presence of array positional error.

for each sensor is $J = 16$. The look direction filter F in (15) is designed to be a lowpass filter with passband (0–3.0 kHz). In the simulations, the step size u is 10^{-8} s, and the allowed maximal iteration times is 200.

The first simulation shows the capability of the proposed algorithm in widening the acceptance angle when there exists DOA mismatch. The beampatterns of the proposed algorithm, the conventional Frost algorithm, and the fixed beamformer are plotted in Fig. 2. It is shown that the acceptance angle of the conventional Frost algorithm is quite narrow, which is significantly widened by the proposed algorithm. Note that the parameter δ in (16) is set to control the range of acceptance angle. Here, $\delta = 5.80$ is used so that the acceptance angles are in the range of $[-10°, 10]°$. The fixed beamformer has a wider main lobe, but its sidelobe level is much higher.

The second simulation demonstrates the performance of the proposed beamformer when the actual positions of array sensors slightly differ from the nominal ones. Assume that the four sensors are placed randomly away from their presumed locations, i.e., $\mathbf{r}(k) = (k-1)d\mathbf{a}_x + \mathbf{r}_n(k)$, $k = 1, \ldots, 4$, $d = 0.04$ m, where $\mathbf{a}_x$ is the unit vector along the x axis, and $\mathbf{r}_n(k)$ is the positional error vector of the kth sensor. In this simulation, $\mathbf{r}_n(k)$ are generated as two-dimensional random Gaussian noises with variance $0.04d^2$. Fig. 3 shows the beampattern of the proposed algorithm in this case. The assumed and actual sensor locations are at (0.0000,0.0000), (0.0400, 0.0000), (0.0800,0.0000), (0.1200, 0.0000), and (0.0045, -0.0004), (0.0367, -0.0103), (0.0827,0.0087), (0.1235, -0.0058), respectively. It is shown that the proposed algorithm is able to avoid target–signal cancelation compared with the conventional Frost algorithm. The proposed algorithm still maintains a certain range of acceptance angle around 0°, although it is narrower than that in Fig. 2. This is because we still use $\delta = 5.80$, which must now tolerate both DOA mismatch and sensor positional error. We could also increase δ to further widen the acceptance angle if required.

V. Conclusion

A new method has been proposed to improve the robustness of a linearly constrained adaptive beamformer against general steering vector errors such as DOA mismatch and sensor positional error. The novelty of this method is the modeling of the constraint matrix by time-shift function so that it can be adaptively adjusted to avoid target–signal cancelation. As this model does not require any knowledge of array manifold functions, it is robust against various types of errors that cause the conventional Frost algorithm to fail. Simulation results have shown that the sensitivity of the adaptive beamformer to steering vector errors can be significantly lowered by the proposed algorithm. With its promising performance, we will further investigate the convergence of the proposed adaptive algorithm and the relative issue of the approximation effect in the interpolation function. In particular, fractional delay interpolation filters may be exploited to improve the computational efficiency [10].

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