thermal FM which is antiphase with respect to the carrier density FM.

Discussion: A small-signal analysis of the laser rate equations yields a relationship between the change in optical frequency $\delta\omega_{opt}$ and the change in photon density δS given by[7]

$$\delta\omega_{opt} \propto \frac{1}{\tau_p \omega_r^2}(j\omega + \gamma_n)\,\delta S \qquad (1)$$

where τ_p is the photon lifetime, ω_r is the relaxation oscillation frequency, ω is the modulation frequency and γ_n is the nonlinear damping factor. Because of the nonlinear damping term in eqn. 1, the FM-AM phase difference is zero at low frequencies, and increases to 90° when $\omega \gg \gamma_n$. Furthermore, since γ_n increases linearly with optical power,[7] the phase difference should increase with frequency at a slower rate as the output power increases. These predictions are borne out by the phase difference data shown in the lower portion of Fig. 3. Also, at a given optical power, the value of γ_n can be determined from the measurement of the FM-AM phase difference with frequency.† Similarly, eqn. 1 accounts for the enhancement of the magnitude of the FM response relative to the AM response, as reported above, resulting in an FM bandwidth which is larger than the AM bandwidth.

Conclusion: The FM and AM responses of a 1530 nm DFB laser in a high-speed driver circuit have been measured for modulation frequencies up to 15 GHz using a novel birefringent fibre interferometer. At frequencies above a few GHz, the FM response was enhanced relative to the AM response. A 12 GHz FM bandwidth was obtained for output powers between 5·5 and 9·0 mW. The phase difference between the FM and AM responses increased from 0 to 90° as the modulation frequency increased. At high optical power levels, the phase difference increased nearly linearly with frequency, with a slope corresponding to a 15–25 ps delay between the FM and AM responses. The data are well explained by the laser rate equations and can be used to determine the nonlinear damping factor γ_n. These results show promise for FSK modulation of DFB lasers at bit rates in the range 10–20 Gbit/s.

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DESIGN OF HIGHLY SELECTIVE TWO-DIMENSIONAL RECURSIVE FAN FILTERS BY RELAXING SYMMETRY CONSTRAINTS

Indexing terms: Filters, Recursive filters, Transfer functions

We show for the design of quadrantally symmetric 2-D fan filters that it is unnecessarily restrictive to prescribe exact quadrantal symmetry, which requires that the denominator of the Z-transform transfer function be product-separable. Superior approximately symmetric fan filter designs can be achieved using nonseparable denominators.

Introduction: A wide variety of useful two-dimensional (2-D) discrete filter transfer functions $H(z_1, z_2)$ possess quadrantal symmetry in their magnitude frequency response; that is, $|H(e^{j\omega_1}, e^{j\omega_2})| = |H(e^{-j\omega_1}, e^{j\omega_2})| = |H(e^{j\omega_1}, e^{-j\omega_2})| = |H(e^{-j\omega_1}, e^{-j\omega_2})|$. Over the past decade, considerable effort has been devoted to exploring such symmetry in 2-D functions, and many useful observations and conditions have since been reported in the literature.[1–3] Of particular interest in this letter are results pertaining to the class of quarter-plane (QP) causal 2-D digital filters described by the real rational Z-transform transfer function given by

$$H(z_1, z_2) \triangleq \frac{N(z_1, z_2)}{D(z_1, z_2)} = \frac{\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn} z_1^{-m} z_2^{-n}}{\sum_{m=0}^{M}\sum_{n=0}^{N} b_{mn} z_1^{-m} z_2^{-n}} \qquad (1)$$

where $N(z_1, z_2)$ and $D(z_1, z_2)$ are relatively prime. It is now well established[1–3] that, for such stable quadrantally symmetric $H(z_1, z_2)$, the denominator $D(z_1, z_2)$ must be product-separable; hence, it must be expressible in the form $D(z_1, z_2) = D_1(z_1)D_2(z_2)$, where $D_1(z_1)$ and $D_2(z_2)$ are strictly Hurwitz polynomials.

In this letter we investigate the design of the widely used class of quadrantally symmetric functions known as fan, or velocity, filters, as shown in Fig. 1. In designing fan filters of this form, we show that the separability constraint can be an unnecessary restriction. We show that, by relaxing the requirement of exact quadrantal symmetry in regions of the 2-D frequency plane where symmetry is unimportant, transfer functions with nonseparable denominators can be used to achieve fan filter designs that are superior to those available with separable denominators.

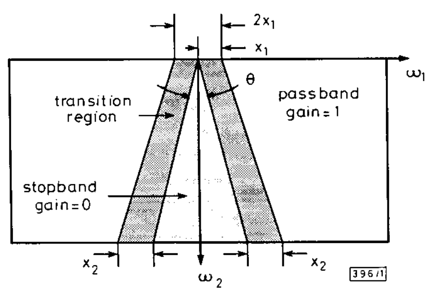

Fig. 1 *Fan filter specification*

Relaxation of quadrantal symmetry: One of the principal benefits of imposing exact quadrantal symmetry on a 2-D function is that stability of the resulting algorithm is guaranteed by ensuring that $D_1(z_1)$ and $D_2(z_2)$ are strictly Hurwitz. However, the full advantage of employing recursion via the term $1/D(z_1, z_2)$ is only obtained if this term plays a dominant role in shaping the overall 2-D magnitude frequency response via the corresponding contributing term $1/|D(e^{j\omega_1}, e^{j\omega_2})|$. Enforcing quadrantal symmetry and therefore separability leads to a contributing term $1/|D_1(e^{j\omega_1})D_2(e^{j\omega_2})|$ which, as a product of 1-D functions, does not lend itself to the required fan shape. Consequently, the primary burden of forming the fan shape falls on the (nonseparable) numerator $|N(e^{j\omega_1}, e^{j\omega_2})|$.

By relaxing the quadrantal symmetry constraint in regions of the 2-D frequency plane where symmetry is not important, such as the transition region, it is possible to achieve significant improvements in design. Consider the specification shown in Fig. 1, where the width of the transition region is defined by the parameters x_1 and x_2. By relaxing quadrantal symmetry in this transition region, a nonseparable denominator $D(z_1, z_2)$ may be used. We propose to approximate the passband and stopband regions as closely as possible while tolerating 'acceptable' behaviour in the transition regions. We define 'acceptable' behaviour in this case as $0 \leq |H(e^{j\omega_1}, e^{j\omega_2})| \leq 1$.

Design examples: The relaxation of quadrantal symmetry in the transition regions is a particularly useful technique for the design of low-order highly selective 2-D fan filters. We choose to design a fan-stop filter having a total angular stopband width $\theta = 20°$. The order of the filter is $(M, N) = (5, 2)$ and the design is carried out using an existing 2-D recursive filter design program,[4] appropriately modified to conform to the relaxed specification in Fig. 1 with $x_1 = 0{\cdot}025\pi$ and $x_2 = 0{\cdot}18\pi$. An expanded view of the stopband of $|H(e^{j\omega_1}, e^{j\omega_2})|$ for the completed design is shown in Fig. 2, and the coefficients $N \triangleq \{a_{mn}\}$ and $D \triangleq \{b_{mn}\}$ in eqn. 1 are given by

$$N = \begin{bmatrix} -0{\cdot}3482633 & 0{\cdot}6782017 & -0{\cdot}2576901 \\ 0{\cdot}9022895 & -1{\cdot}998527 & 0{\cdot}8605637 \\ -0{\cdot}2929108 & 1{\cdot}031212 & -0{\cdot}5784674 \\ -1{\cdot}157559 & 2{\cdot}165731 & -0{\cdot}8327803 \\ 1{\cdot}332481 & -2{\cdot}822859 & 1{\cdot}260246 \\ -0{\cdot}4363195 & 0{\cdot}9466001 & -0{\cdot}4519792 \end{bmatrix}$$
$$D = \begin{bmatrix} 1{\cdot}0 & -1{\cdot}198685 & 0{\cdot}3595865 \\ -2{\cdot}188287 & 2{\cdot}820891 & -0{\cdot}9020295 \\ 0{\cdot}9349396 & -1{\cdot}518957 & 0{\cdot}5785125 \\ 1{\cdot}178034 & -1{\cdot}257805 & 0{\cdot}3193958 \\ -1{\cdot}234083 & 1{\cdot}592515 & -0{\cdot}5103743 \\ 0{\cdot}3274713 & -0{\cdot}4602485 & 0{\cdot}1620259 \end{bmatrix} \qquad (2)$$

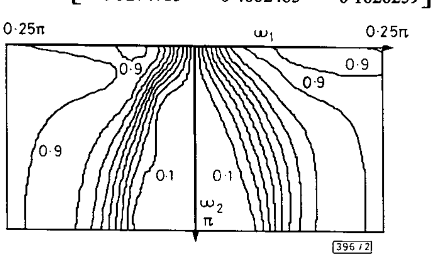

Fig. 2 *Magnitude frequency response of fan filter with nonseparable denominator*

For comparison, the design is repeated for a separable-denominator fan-stop filter, again with $\theta = 20°$. The same 2-D recursive filter design program is used, but with a fan filter specification that imposes quadrantal symmetry in the usual way; that is, quadrantal symmetry is imposed everywhere in the 2-D frequency plane. An expanded view of the stopband for the completed design is shown in Fig. 3.

A comparison of the results in Figs. 2 and 3 clearly shows that the fan filter obtained by relaxing quadrantal symmetry has superior frequency selectivity to the separable-denominator fan filter. To quantify the relative advantage of relaxing transition region symmetry, we use an error function E which is based on the squared error $\varepsilon_i \triangleq [|H(e^{j\omega_1}, e^{j\omega_2})| - M(\omega_1, \omega_2)]^2$, where $M(\omega_1, \omega_2)$ is the ideal transfer function (1 or 0). This error, summed over a grid of passband and

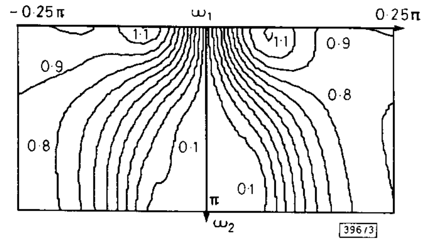

Fig. 3 *Magnitude frequency response of fan filter with separable denominator*

stopband frequencies, $E \triangleq \sum_i \varepsilon_i$, is given as follows for the two designs:

	Separable design error E	Nonseparable design error E
Passband	3·5104	1·4765
Stopband	1·8634	0·5454
Total	5·3739	2·0219

Conclusion: We have shown that the requirement for exact quadrantal symmetry in the design of 2-D quadrantally symmetric fan filters can impose an unnecessary restriction. By relaxing quadrantal symmetry constraints in the transition regions, it is possible to employ filters having nonseparable denominators and near-quadrantal symmetry. We have shown that, for fan filters having a given low order (e.g. 5, 2), the nonseparable design is significantly superior to the separable design.

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EFFICIENT OPERATION OF ARRAY-PUMPED Er^{3+} DOPED SILICA FIBRE LASER AT 1·5μm

Indexing terms: Optial fibres, Lasers and laser applications

We present the results for diode-array pumped Er^{3+} doped silica fibres, which lase near 1550 nm. A maximum output power of 8 mW is obtained with a 13% overall efficiency against the launched pump power.

Introduction: Diode laser pumped operation of the 1·5 μm transition in Er^{3+} doped silica fibres has been reported recently.[1,2] This transition is of particular interest because of the