Two-Parameter Gamma Drop Size Distribution Models for Singapore

Lakshmi Sutha Kumar, Yee Hui Lee, *Member, IEEE*, and Jin Teong Ong

***Abstract*—Gamma model is fitted using the second, fourth, and sixth moments to model the rain drop size distribution (DSD) of Singapore. As the Joss distrometer measures the number of rain drops between the drop diameters from 0.3 to 5 mm, the truncated moment fitting between these drop diameter ranges is also used for modeling the DSD. Gamma DSD requires three-parameter estimation: N_0, the intercept parameter; μ, the shape parameter; and Λ, the slope parameter. The aim of this paper is to find a suitable fixed μ and derive an appropriate $\mu-\Lambda$ relation for the tropical region in order to form a two-parameter gamma model. To find an appropriate μ value, observed DSDs are fitted with different μ values to estimate the rain rates, which are assessed by rain rate observations of the distrometer. Shape–slope relationships are fitted for different categories according to the rain rate and the number of drops. The derived $\mu-\Lambda$ relationships for the Singapore region are compared to the published results from two other regions, and the analysis is presented. Two-parameter gamma models are compared by retrieving the rain rate using the polarimetric radar variables. The effect of truncation on rain rate retrieval is also studied, and the use of the $\mu-\Lambda$ relationship for rain retrieval is recommended for the tropical region. The $\mu-\Lambda$ relation using the truncated moment method for the rain category $R \geq 5$ mm/hr and $rain\,counts \geq 1000$ drops retrieves the rain rates well compared to other $\mu-\Lambda$ relations.**

***Index Terms*—Fixed μ, gamma distributions, rain, rain retrieval, shape parameter, slope parameter, $\mu-\Lambda$ relationship.**

I. Introduction

ACCURATE rain rate estimation requires detailed knowledge of the rain drop size distribution (DSD). Ulbrich [1] has shown that the DSD is best modeled by a gamma distribution and suggested the following form:

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \qquad (1)$$

where $N(D)$ is the distribution of rain drops per diameter interval D to $D + \Delta D$ (in millimeters) in units of $\mathrm{m}^{-3} \cdot \mathrm{mm}^{-1}$, N_0 is the intercept parameter ($\mathrm{mm}^{-1-\mu} \cdot \mathrm{m}^{-3}$), Λ is the slope of the exponential (mm^{-1}), and μ is the dimensionless shape parameter. The moment estimators frequently used to estimate parameters for DSD functions are biased [2]. This bias tends to be stronger when higher order moments are used. The low-order moments for observed drop size data involve substantial uncertainties because of deficiencies in the observations of the smallest raindrops [2]. Therefore, many authors [2]–[5] prefer to work with central moments.

For impact distrometers [such as the Joss–Waldvogel distrometer (JWD), which is the measuring instrument used in this study], there is a so-called "dead time," following the impact of a drop, during which the impact of smaller drops cannot be measured. Therefore, JWD tends to underestimate the number of small drops particularly during a heavy rain event. Kumar *et al.* [6] addressed the dead-time problem of JWD and used truncated gamma models for modeling the DSD. They removed the first four bins, which are likely to have error due to the distrometer's dead time, to calculate the observed moments. However, the equations for the truncated moments are not used in [6]. Recent studies have shown that a large error might be introduced and cause a notable bias of μ or Λ estimation if truncated observations are assumed as the untruncated moments [7]–[9]. Therefore, this paper uses the gamma model using the second, fourth, and sixth moments proposed by Ulbrich and Atlas [7] and the iterative truncated moment fitting between the drop diameter ranges from 0.3 to 5 mm given in [8]. In this paper, the dead-time correction has been applied using the software provided by Distromet Inc. to reduce the effect of dead time. The correction is intended to correct up to 10% of the accuracy. As shown in (1), the gamma DSD is described by three parameters, namely, N_0, μ, and Λ. Rain DSD parameters can be retrieved from a pair of two independent remote measurements [4]. With these two independent remote measurements and an assumed fixed μ or an empirical constraining relationship between the DSD parameters, rain rate can be retrieved.

Fixed μ models are previously used by many researchers [3], [5], [10], [11] for the retrieval of rain rate. Bringi *et al.* [10] fixed $\mu = 3$ in the gamma DSD to estimate rain rate from differential reflectivity (Z_{DR}). Kozu and Nakamura [3] retrieved the rain rate from rain attenuation and reflectivity using the fixed μ model. Their fixed μ range is from four to six for the gamma model using the third, fourth, and sixth moments. The moments third, fourth, and sixth were reapplied for fixed μ values by Tokay and Short [5]. The mode of the shape parameter, μ, was close to six from their data. Rincon and Lang [11] used the fixed μ value of four for the gamma DSD to estimate path average DSD and rain rate. Lakshmi *et al.* [12] tried to find a suitable fixed μ using ten average rain rates from Singapore's drop size data. They found that $\mu = 3$ produces less error in the modeled rain rate with the measured rain rate at the middle rain rates of 23.29–76.15 mm/hr, and $\mu = 4.58$ or 5 produces less error at the high rain rates of 99.55–147.70 mm/hr.

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In this paper, data sets of 14 major rain events are selected to analyze the fixed μ model and the μ–Λ relationships. Most of the rain events in Singapore reach maximum intensity very rapidly; the rain event remains heavy for few minutes; then, it decreased slowly before increasing again. Lower rain rates usually occur during the decreasing period. One rain event refers to the rain from beginning until the end. The events chosen for the analysis have peak rain rates from 28.04 to 191.59 mm/hr. Fixed μ model is analyzed, and the root mean square error in rain rate estimation $(RMSE - R)$ is calculated. Since the fixed μ value differs from region to region, in this paper, the fixed μ value suitable for the tropical region is proposed.

An empirical relation relating any two of the gamma parameters reduces the three-parameter gamma DSD to a two-parameter function. Two-dimensional video distrometer (2-DVD) measurements, made in Florida, shows a high correlation between μ and Λ [4] indicating that these parameters are related. Their gamma DSD is fitted by using the second, fourth, and sixth moments, and a second-order polynomial constraining the relation between μ and Λ is derived. Later, Brandes *et al.* [13] redefined the constraining relation to accommodate heavy rainfall using the 2-DVD measurements made in Florida. He suggested that, prior to curve fitting in the μ–Λ scatter plot, the database should be filtered to exclude rain rates smaller than 5 mm/hr and drop counts smaller than 1000. Zhang *et al.* [14] have used the same μ–Λ relation and have shown that the relation contains useful information and characterize the natural rain DSD variations quite well. He noted that the coefficients of the μ–Λ relation might change with location and season.

Atlas and Ulbrich [15] explained that the μ–Λ correlations proposed by [13], [14] appear to be limited to rainfall events which do not include convective rain; they are biased toward stratiform and transition rains. Therefore, in this paper, three different categories of rain rates, namely, 1 mm/hr $\leq R < 5$ mm/hr (stratiform and transition rain), 5 mm/hr $\leq R <$ 25 mm/hr (stratiform and convective rain), and $R \geq 25$ mm/hr (convective rain), are considered to fit μ–Λ relations. Another category proposed by Brandes *et al.* in [13] is also considered for comparison purposes. He used rain rates greater than 5 mm/hr and rain counts greater than 1000 drops.

Moisseev and Chandrasekar [16] attributed the μ–Λ relation to the effect of moment errors. Although it is shown in [14] that the μ–Λ relation is related to physics as well as moment error, Moisseev and Chandrasekar [16] stated that the correlation was mainly due to the truncation of small raindrops (≤ 0.6 mm) and data filtering. Therefore, this paper uses the truncated moment method [8], where the DSD parameters can be estimated as a function of lower and upper bounds of the drop size spectrum. The μ–Λ relation has been successfully applied for rain retrievals using polarimetric radar measurements of reflectivity (Z_H) and differential reflectivity (Z_{DR}) [13], [17], [18]. Cao and Zhang [18] found that the μ–Λ relation is practically equivalent to the mean function of normalized DSDs proposed by Testud. They concluded that the equivalence between the μ–Λ relation and the Testud's function indicates the physical information in the μ–Λ relation of the constraint-gamma (C-G) model proposed by Zhang *et al.* [4]. Narayana *et al.* [19] studied the variability of the shape–slope $(\mu$–$\Lambda)$ relation using the impact type, i.e., the JWD data measured at Gadanki, India. They used third, fourth, and sixth moments to model the DSD. They reported that the μ–Λ relation is a function of height.

In this paper, we aim to find a suitable fixed μ value and derive an appropriate μ–Λ relation for the tropical country of Singapore. The data were recorded from August 1994 to September 1995, excluding June and July 1995 using a "Joss-type" distrometer RD-69, which is also used in this study. Gamma model parameters are calculated for 14 major rain events using the gamma model. The shape parameter μ is fixed at different constant values, while the DSD is fitted by the gamma model.

An analysis of DSD observations also indicated the existence of a μ–Λ relation. Therefore, in order to find a two-parameter model, a shape–slope relation will be also proposed from the gamma model parameters for Singapore. Rain rates are calculated from fixed μ model, and it is compared with rain rates from measured rain rates in Singapore. The μ–Λ relationship found using Singapore's filtered data is compared with those found in other region. T-matrix calculations are performed for the 1-min integrated DSDs for the entire data set. Polarimetric radar variables from the T-matrix calculations with fixed μ or μ–Λ relations are used to find gamma DSD parameters. The rain rate retrieved using the calculated gamma DSD is compared with the measured rain rate. The fixed μ model and the gamma model using μ–Λ relation are compared to find the best two-parameter gamma models for the Singapore region.

II. Measurement and Formulation of Two-Parameter Gamma Model

A. Measured DSD

The JWD is capable of measuring the drop diameter ranging from 0.3 to > 5 mm with an accuracy of $\pm 5\%$. It distinguishes between drops with time interval of about 1 ms. The total number of drops of diameters ranging from 0.3 to > 5 mm is divided into 20 different bins with 1-min integration time [20].

The rain rate in units of decibels can be calculated from the measured data by

$$R = 10 \times \log_{10}\left(\frac{3600\pi}{6ST}\sum_{i=1}^{20} D_i^3 n_i\right) \tag{2}$$

where n_i is the number of rain drops in the ith channel, D_i is the mean drop diameter in millimeters, $S = 5000$ mm^2 is the sample area, $T = 60$ s is the integration time, and $v(D_i)$ is the terminal velocity of rain drop in meters per second obtained from Gunn and Kinzer. The measured rain DSD, $N(D_i)$ (m$^{-3}\cdot$ mm^{-1}) [20], is expressed by

$$N(D_i) = \frac{n_i \times 10^6}{v(D_i) \times S \times T \times \Delta D_i}. \tag{3}$$

B. Gamma Modeled DSD

The integration of most moment calculations are usually performed from zero to infinite size range as

$$M_k = \int_0^\infty D^k N(D)dD$$
$$= N_0 \Lambda^{-(\mu+k+1)} \Gamma(\mu + k + 1). \quad (4)$$

The three parameters (N_0, μ, or Λ) can be solved from the second, fourth, and sixth moments as proposed by Ulbrich and Atlas [7]. To eliminate Λ and find μ, a ratio is defined as

$$\eta = \frac{M_4^2}{M_2 M_6}. \quad (5)$$

Then, μ can be solved using

$$\mu = \frac{(7 - 11\eta) - \left[(7 - 11\eta)^2 - 4(\eta - 1)(30\eta - 12)\right]^{\frac{1}{2}}}{2(\eta - 1)}. \quad (6)$$

Λ and N_0 can be calculated

$$\Lambda = \left[\frac{M_2(\mu + 3)(\mu + 4)}{M_4}\right]^{\frac{1}{2}} \quad [\mathrm{mm}^{-1}] \quad (7)$$

$$N_0 = \Lambda^{\mu+3} M_2 / \Gamma(\mu + 3) \quad [\mathrm{mm}^{-1-\mu} \cdot \mathrm{m}^{-3}]. \quad (8)$$

The above-mentioned method for estimating DSD parameters is applicable only for untruncated DSD. The DSD parameters are overestimated if the truncated size data are used in the untruncated moment method [8]. For a gamma distribution with a truncated size range, the statistical moments are calculated [8] as

$$M_k = \int_{D_{\min}}^{D_{\max}} D^k N(D)dD$$
$$= N_0 \Lambda^{-(\mu+k+1)} \left[\gamma(\mu + k + 1, \Lambda D_{\max}) - \gamma(\mu + k + 1, \Lambda D_{\min})\right] \quad (9)$$

where $\gamma(\ldots)$ is an incomplete gamma function, and $D_{\min}$ and $D_{\max}$ are 0.3 and 5 mm for Joss distrometer data.

Using the truncated moments shown in (9), moments consistent with truncation can be calculated, and then, the corresponding expressions for DSD parameters can be derived, shown at the bottom of the next page. Equations (10) and (11) constitute joint equations for μ and Λ for the truncated moments that are difficult to separate from each other. An iterative approach is used as explained in [8] to estimate the DSD parameters for the truncated moment method.

III. Results and Discussions

Gamma model parameters, namely, μ, Λ, and N_0, are calculated using (6)–(8), from the distrometer data for the year 1994–1995. Only DSDs having number of rain drops greater than 100 are considered (932 min of data) from 14 rain events. Fig. 1 illustrates scatter plots of the gamma model parameters versus rain rate. It is clear from Fig. 1 that even though the parameters show large variability at lower rain rates, their variability reduces at higher rain rates. The reduction in DSD parameters' variability is mainly attributed to the reduction of mathematical dynamic range of DSD parameters. Of the three parameters plotted in Fig. 1, since the variation in intercept parameter, N_0 in $\mathrm{mm}^{-1-\mu} \cdot \mathrm{m}^{-3}$, is large and in powers of ten, it cannot be kept constant. The other two gamma model parameters can be retrieved from the polarimetric radar variables if the shape parameter is kept constant. As can be seen from Fig. 1, 85% of the shape parameter values are in the range from one to nine. The value of μ is to be adjusted in the iteration process of truncated moment method. Therefore, fixed μ values are analyzed only for the untruncated moment method.

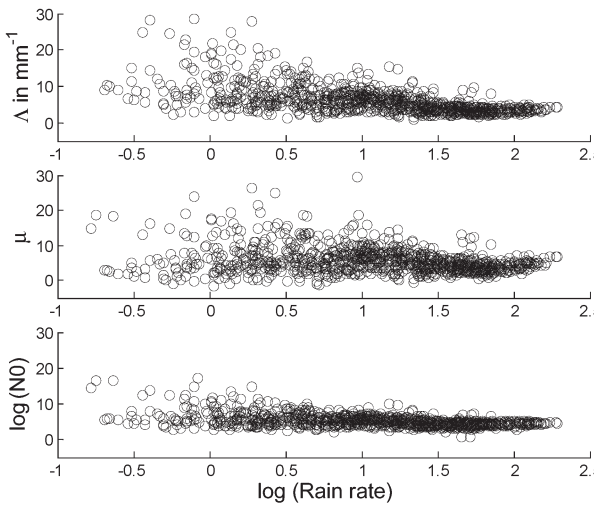

Fig. 1. Scatter plot of gamma model parameters (932 min of data; DSDs having rain drops greater than 100 are only considered).

A. Fixed μ Models

The number of gamma DSD minutes for a particular μ is calculated, and the distribution is plotted in Fig. 2 to study the range of fixed μ values. In Fig. 2, the number of gamma DSD minutes for the particular μ is plotted in terms of counts. For example, the number of gamma DSD minutes for which $\mu = 4$ is 149. From Fig. 2, μ values of 3, 4, and 5 show higher counts, and the peak appears at $\mu = 4$. Although the high counts are at $\mu = 4$, it does not necessarily mean that this is the best value for the fixed μ model. In order to investigate the use of fixed μ gamma models, the shape parameter μ is fixed at -2 to -16 in steps of one. For fixed μ models, μ is fixed at a constant value, while Λ and N_0 are calculated using (7) and (8). Rain rate in units of decibels is then calculated using

$$R = 10 \times \log_{10}\left(6\pi \times 10^{-4} \sum_{i=1}^{20} D_i^3 v(D_i) N(D_i) \Delta D_i\right) \quad (12)$$

where $N(D_i)$ is the fixed μ gamma DSD and ΔD_i is the drop size interval in millimeters.

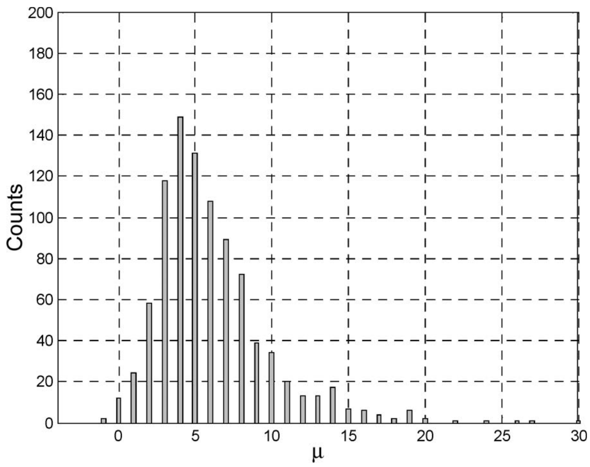

Fig. 2. Distribution of gamma fitted parameter μ. (932 min of data; DSDs having rain drops greater than 100 are only considered).

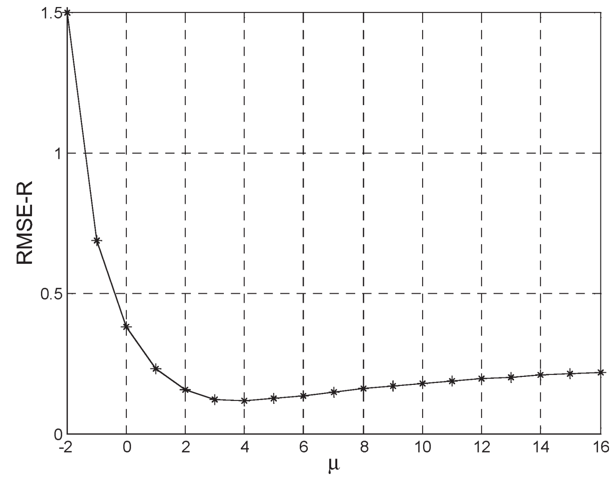

Fig. 3. $RMSE - R$ calculated from measured data (932 min of data; DSDs having rain drops greater than 100 are only considered).

To evaluate the accuracy of the fixed μ gamma models, the $(RMSE - R)$ is used. It is calculated by using

$$RMSE - R = \sqrt{\frac{1}{c}\sum_{i=1}^{c}(R_{\text{calci}} - R_{\text{measi}})^2} \tag{13}$$

where c is the number of data points. In this paper, $c = 932$. R_{calci} is the calculated rain rate using (12) from the fixed μ gamma modeled DSD. R_{measi} is the rain rate calculated from the measured data using (2).

Fig. 3 shows $RMSE - R$ for fixed μ gamma models. As can be seen from Fig. 3, the $RMSE$ values are higher for the fixed μ values less than 2. $RMSE$ values are decreasing rapidly for the increase of μ from -2 to 2. From the μ values of 2 to 4, the $RMSE$ is still decreasing, but at a slower rate. The minimum $RMSE - R = 0.116$ appears for the fixed μ value of 4. Since the $RMSE - R$ for the fixed μ values of 3 and 5 is also near the minimum, the fixed μ values in the range of 3–5 are found to be suitable for Singapore's tropical DSD, and a fixed μ value of 4 which has the maximum DSD minute counts produces the minimum $RMSE - R$ which is more appropriate. The choice of fixed μ ranging from 3 to 5 for Singapore is close to Kozu and Nakamura [3] and Tokay and Short [5] where their fixed μ range is from 4 to 6.

By fixing the shape parameter μ, the three-parameter gamma model now becomes a two-parameter model and can be used for the retrieval of rain rate from radar data. The other two-parameter model, i.e., the μ–Λ relationship which also shows potential [13], [17] for rain rate and reflectivity retrieval with the distrometer estimates, is discussed next.

B. μ–Λ *Relationship*

Fig. 4 shows the scatter plots of the fitted DSD parameters (μ versus Λ) for the DSD of 932 min of data. Fig. 4(a) and (b) is obtained from the untruncated moment method and truncated moment method, respectively. As can be seen from Fig. 4(a) and (b), the variation between the μ and Λ values is large, and there is low correlation between the two parameters. However, the estimated values of μ and Λ obtained using the truncated moment method show better correlation than the estimated values using the untruncated moment method. As given in Fig. 1, the large values of μ and Λ correspond to low rain rates of less than 5 mm/hr. It is reported in [13] that the retrieval of rain rate using μ–Λ relationship agrees well with the measured rain rate in strong convection and higher rain rates. Therefore, the data points are filtered similar to [13], [14], and only the DSDs which have rain rates greater than 5 mm/hr and rain counts greater than 1000 drops are selected.

Fig. 4(c) and (d) shows the shape and slope parameters using the filtered data for the untruncated and truncated moment methods. It is noted that the scatter plot between μ and Λ for the filtered rain cases in Fig. 4(c) and (d) has higher correlation than that without filtering as shown in Fig. 4(a) and (b). Fig. 4(c) and (d) contains only 337 data points but captures 81% of the rainfall amount in Fig. 4(a) and (b). The scatter plots shown in Fig. 4(c) and (d) show less scatter, and the correlation between μ and Λ is higher. A relation between μ and Λ is estimated using a polynomial least squares fit, and it is given as

$$\Lambda = 0.041\,\mu^2 + 0.362\,\mu + 1.644 \tag{14}$$

$$\eta = \frac{[\gamma(\mu+5, \Lambda D_{\max}) - \gamma(\mu+5, \Lambda D_{\min})]^2}{[\gamma(\mu+3, \Lambda D_{\max}) - \gamma(\mu+3, \Lambda D_{\min})] \times [\gamma(\mu+7, \Lambda D_{\max}) - \gamma(\mu+7, \Lambda D_{\min})]} \tag{10}$$

$$\Lambda = \left[\frac{M_2\,[\gamma(\mu+5, \Lambda D_{\max}) - \gamma(\mu+5, \Lambda D_{\min})]}{M_4\,[\gamma(\mu+3, \Lambda D_{\max}) - \gamma(\mu+3, \Lambda D_{\min})]}\right]^{\frac{1}{2}} \tag{11}$$

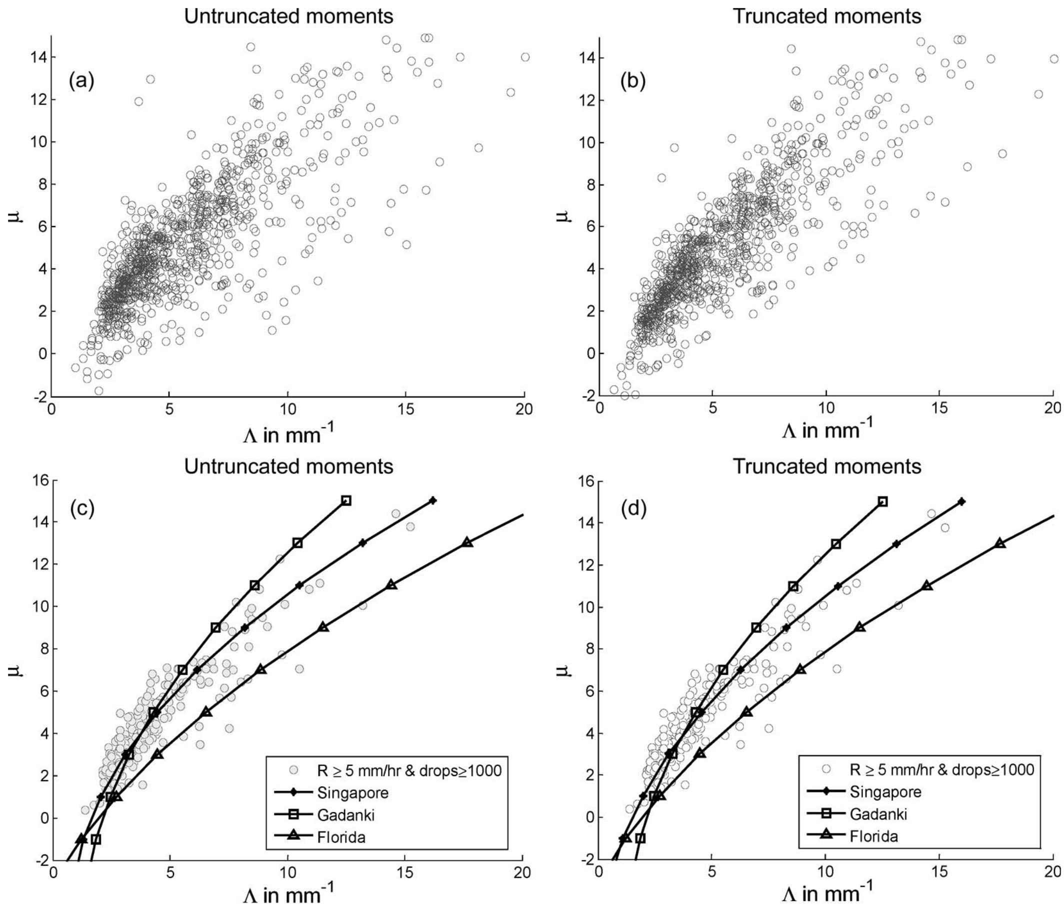

Fig. 4. Scatter plots of μ–Λ values obtained from Singapore's DSD. The curves, obtained from distrometer measurements at Singapore, and distrometer measurements made at Florida (Florida) and Gadanki, India (Gadanki), are overlaid in (c) and (d). (a) Untruncated moment method—without filtering. (b) Truncated moment method—without filtering. (c) Untruncated moment method—with filtering of $rain\ rates \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops. (d) Truncated moment method—with filtering of $rain\ rates \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops.

when untruncated moments are used. In the case of the truncated moment method, the corresponding equation is

$$\Lambda = 0.036\,\mu^2 + 0.432\,\mu + 1.507. \tag{15}$$

Similar to [8], the μ–Λ relations do not change much, but the mean values of μ and Λ change from 4.40 and 4.21 in Fig. 4(c) for the untruncated moment method to 4.09 and 4.06 in Fig. 4(d) for the truncated moment method. The μ–Λ relationships proposed by [13] and [19], denoted as Florida and Gadanki, respectively, are also plotted in Fig. 4(c) and (d) for comparison purposes.

It is clear from Fig. 4(c) and (d) that the trend of the μ–Λ fit of Singapore follows the Florida curve for $\mu > 4$. Even though both curves started near the same points, given the same lambda value, Florida's μ values are lower as compared to the μ values of Singapore. The distance between the two curves increases as the rain rate decreases. The differences in μ–Λ relationship between Singapore and Florida could arise from the use of different type of distrometers at these locations. Another reason for the higher μ values of Singapore given the same lambda value may be due to the type of rain events in both countries. Most of Singapore's rain events are convective. The precipitations used to fit the μ–Λ relationship of Florida may be weaker than the precipitations in Singapore. Seifert [21] compared Florida's μ–Λ relation with the μ and Λ values of different rain events. He stated that μ is much larger in increasing rain than in decreasing rain, resulting in the data points lying above the Florida's μ–Λ relation for the strongest events. He also stated that the values lying somewhat below Florida's μ–Λ relation correspond to the weakest precipitation events with maximum rain rates below 10 mm/hr. This may be the reason for the lower μ values of Florida's fit as compared to Singapore's μ values given the same lambda value.

The Gadanki curve has higher μ values than the Singapore fits given the same lambda value, which indicates that their data consist mainly of convective rain events. Gadanki's μ–Λ relationship in [19] considers a total of 16 rain events of which 5 events are stratiform, 4 events are convective, and 7 events are mesoscale convective systems. Both Gadanki and Singapore curves follow the same trend at most of the higher rain rates and this is because both regions fall in the tropical climate; most of the rain events considered are convective in both data sets. Furthermore, both data sets are collected using the same JWD for the measurements of DSD. However, the Gadanki μ–Λ relationship has slightly higher Λ values for the same μ values than Singapore's curve at higher rain rates.

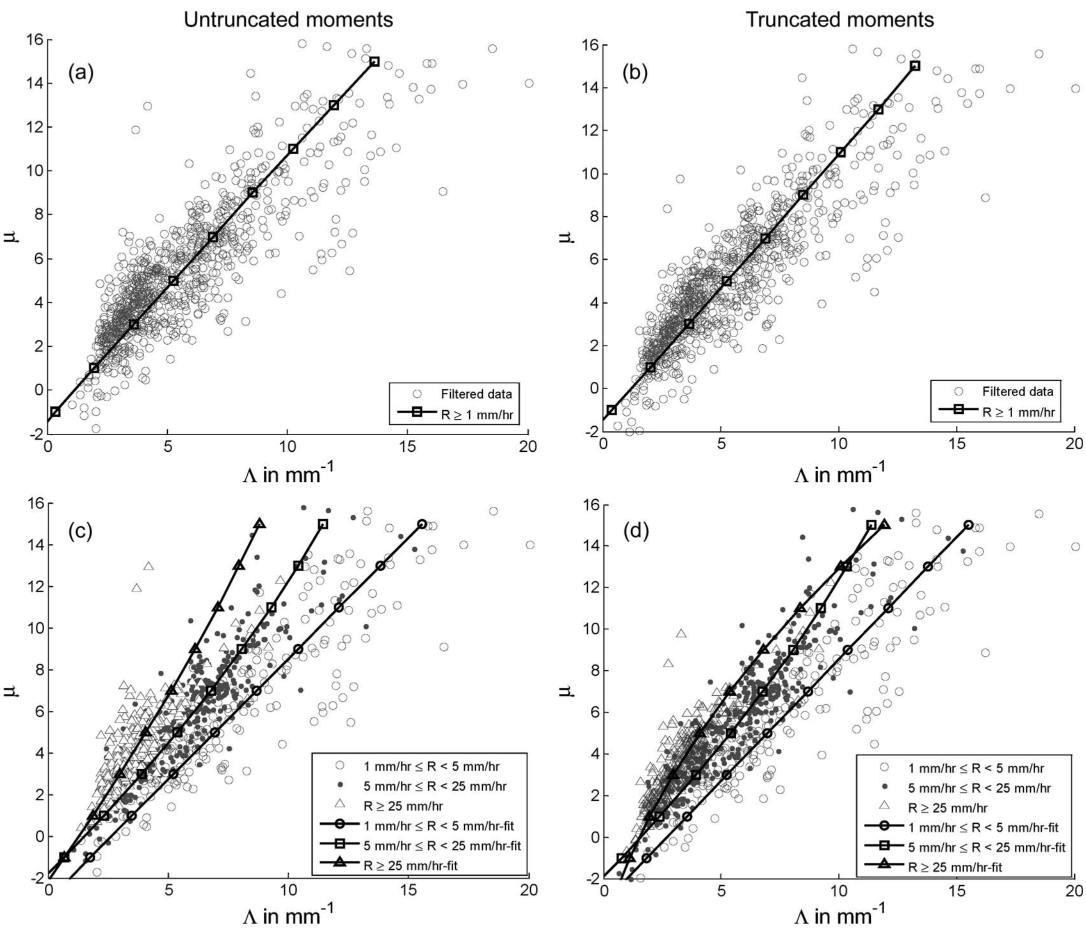

Fig. 5. Scatter plot of $\mu - \Lambda$ values for different rain categories and their corresponding $\mu - \Lambda$ fits. (a) Untruncated moment method—$R \geq 1$ mm/hr. (b) Truncated moment method—$R \geq 1$ mm/hr. (c) Untruncated moment method—1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr. (d) Truncated moment method—1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr.

The $\mu - \Lambda$ relationship obtained for Singapore shows that the data set contains a significant amount of convective rain events. The curve of Florida has very low μ values compared to Singapore and Gadanki fits given the same lambda value indicating weak events in Florida compared to the two regions. This strongly agrees with Zhang's statement [14] that $\mu - \Lambda$ relations vary with the location since each location has different types of rain. Another reason for Florida's fit to have a low μ value given the same lambda value is the use of different distrometers. Florida uses the video distrometer, whereas Gadanki and Singapore use the JWD. Simultaneous observations with impact and video disdrometers have indicated that impact disdrometers like JWD seriously undersample raindrops with diameters less than 1.5 mm [22]. This may be the reason for the over estimation of μ values given the same lambda value at Gadanki and Singapore curves compared to Florida's fit.

In order to examine the appropriate $\mu - \Lambda$ relationship for gamma DSD modeling, at all the rain rates, the $\mu - \Lambda$ relationship is fitted for the category $R \geq 1$ mm/hr. Fig. 5(a) and (b) show the scatter plot of μ and Λ values and polynomial fit of the filtered rain rates greater than 1 mm/hr for the untruncated and truncated moment methods. The spread of the μ and Λ values is still large. Therefore, the $\mu - \Lambda$ relationship is fitted for the categories, namely, 1 mm/hr $\leq R < 5$ mm/hr (stratiform and transition rain), 5 mm/hr $\leq R < 25$ mm/hr (stratiform and convective rain), and $R \geq 25$ mm/hr (convective rain). Fig. 5(c) and (d) shows the scatters of μ and Λ values and polynomial fits of the above-mentioned three rain categories for the untruncated and truncated moment methods, respectively.

It is clear from Fig. 5(c) and (d) that, by splitting the category $R \geq 1$ mm/hr into three different rain categories, an improvement to the fitting can be achieved. The $\mu - \Lambda$ relation of the lower rain rate category, i.e., 1 mm/hr $\leq R < 5$ mm/hr, which has mainly stratiform and transition-type rain, has lower μ values given the same lambda value, whereas the $\mu - \Lambda$ relation of convective rain category $R \geq 25$ mm/hr has higher μ values. The $\mu - \Lambda$ relation of the middle rain category has moderate μ values given the same lambda value in between the other two rain categories. The upward increase of fits from lower rain category to higher rain category clearly indicates that the lower and higher μ values given the same lambda value correspond to stratiform and convective-type rains. It is clear from Fig. 5(c) and (d) that the $\mu - \Lambda$ relations of the rain categories 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr have the same trend and look similar for the untruncated and truncated moment methods. However, when truncation is considered for designing the moment method, the scattering of the μ and Λ values is found to reduce for the rain category $R \geq 25$ mm/hr. Therefore, the fitting for the truncated moment method has reduced μ values given the same lambda

TABLE I
SHAPE–SLOPE RELATIONS FOR DIFFERENT CATEGORY OF RAIN RATES FITTED USING THE UNTRUNCATED MOMENT METHOD

Splitting criteria	$\Lambda = C\mu^2 + B\mu + A$		
	C	B	A
R ≥ 1 mm/hr	0.0012	0.813	1.155
1 mm/hr ≤ R < 5 mm/hr	-0.00098	0.881	2.574
5 mm/hr ≤ R < 25 mm/hr	-0.011	0.827	1.520
R ≥ 25 mm/hr	-0.0057	0.587	1.286
R ≥ 5 mm/hr & drops ≥ 1000	0.041	0.362	1.644
*1 mm/hr ≤ R < 5 mm/hr	X	0.862	2.635

* - Corresponds to linear fit

TABLE II
SHAPE–SLOPE RELATIONS FOR DIFFERENT CATEGORY OF RAIN RATES FITTED USING THE TRUNCATED MOMENT METHOD

Splitting criteria	$\Lambda = C\mu^2 + B\mu + A$		
	C	B	A
R ≥ 1 mm/hr	-0.0014	0.874	0.984
1 mm/hr ≤ R < 5 mm/hr	-0.00078	0.875	2.596
5 mm/hr ≤ R < 25 mm/hr	-0.011	0.816	1.593
R ≥ 25 mm/hr	0.0178	0.428	1.512
R ≥ 5 mm/hr & drops ≥ 1000	0.0356	0.432	1.507
*1 mm/hr ≤ R < 5 mm/hr	X	0.859	2.650

* - Corresponds to linear fit

value as compared to the untruncated moment method for this rain category.

Tables I and II show the rain categories, the type of rain, and the polynomial fit coefficients of the μ–Λ relations of the untruncated and truncated moment methods. As can be seen in Tables I and II, for the rain category 1 mm/hr $\leq R < 5$ mm/hr, the coefficient "C" of the polynomial fits is small, and the values are -0.00098 and -0.00078 for the untruncated and truncated moment methods, respectively. Therefore, this coefficient has minimal effect, and instead of a polynomial fit, a linear fit is proposed for this rain category 1 mm/hr $\leq R < 5$ mm/hr for both the methods. The resultant coefficients from the linear fit are also added at the last row in Tables I and II, respectively. The linear fit has the advantage of being less complex compared to the polynomial fit. For the remaining categories, polynomial fit is preferred.

Fig. 6(a) and (b) shows the μ–Λ relations of Singapore for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, $R \geq 25$ mm/hr, and $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ for the untruncated and truncated moment methods. The μ–Λ relationships of Florida and Gadanki are also plotted in Fig. 6 for comparison purposes. All the μ–Λ relations in Tables I and II use 1-min sampling time. The μ–Λ relation derived for the rain category $R \geq 25$ mm/hr consists of only convective-type rain, and it is above Gadanki curve in Fig. 6(a). However, when truncation is considered, the μ–Λ relation for the rain category $R \geq 25$ mm/hr is closer and follows the same trend of Gadanki curve.

As can be seen from Fig. 6, the μ–Λ relations of the lower rain categories, namely, 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr, are closer to Florida's curve. This indicates that the rain events used by Brandes *et al.* [13] are weaker precipitation events. It is also clear that, even though there are convective points in the Florida curve, most of their points are stratiform and transition points, therefore results in a fit that has lower μ values given the same lambda value. It is observed from Singapore's data that the fit for the rain category $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops has 301 DSD minutes. In those 301 DSD minutes, 43 DSD minutes have rain rates less than 25 mm/hr. These DSD minutes result in a fit with lower μ values given the same lambda value as compared to the fit for the category $R \geq 25$ mm/hr.

Polarimetric radar variables are calculated using the T-matrix code for the rain event. The fixed μ value of four which is appropriate for Singapore DSD and the μ–Λ relations are used with the polarimetric radar variables to estimate the rain rate. The retrieved rain rates are compared with measured rain rates in order to find the best two-parameter models in the next section.

C. *Comparison of Two-Parameter Models*

T-matrix calculations are performed at S-band with 2.72 GHz for the Beard and Chuang drop shape model [23]. The calculations are done at an elevation angle of 1° for a water temperature of 20 °C. The canting angle distribution with zero mean and 10° standard deviation is used for Singapore's tropical climate. Gamma DSD calculated from the Singapore's drop size data is used as an input to the T-matrix code. The calculated polarimetric radar variables, differential reflectivity (Z_{dr}) in decibels, and horizontal reflectivity (Z_{hh}) in mm$^{-6} \cdot$ m^3 are then used as explained in [4] to find the gamma model parameters. The gamma model parameter Λ can be inferred for a specified Z_{dr}, and then, using the inferred Λ and Z_{hh}, the parameter N_0 can be obtained. Shape parameter μ is calculated using the μ–Λ relations for the filtered rain categories.

The rain event on February 26, 1995 is used as an example for the retrieval of rain from polarimetric radar variables. Fig. 7(a)–(c) shows the distrometer-measured rain rates and retrieved rain rates for $\mu = 4$, using the μ–Λ relation for the rain category $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops and using the μ–Λ relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr, respectively. Fig. 7(b) shows the rain rates which are retrieved using the μ–Λ relations in (14) and (15) for the untruncated and truncated moment methods along with the distrometer-measured rain rates. Similarly, Fig. 7(c) shows the retrieved rain rates which are retrieved using the μ–Λ relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R <$ 25 mm/hr, and $R \geq 25$ mm/hr of the untruncated and truncated moment methods along with the distrometer-measured rain rates. Untr $-\ \mu - \Lambda$ fit and Tr $-\ \mu - \Lambda$ fit in the legend of Fig. 7 represent the μ–Λ relations derived using the untruncated and truncated moment methods.

As can be seen from Fig. 7, retrieved rain rates using the fixed value of $\mu = 4$ are overestimated at rain rates greater than around 50 mm/hr and underestimated at lower rain rates. Rain rates retrieved using the μ–Λ relations are closer to the measured rain rates compared to the rain rates retrieved using the fixed μ value of 4. It is clear from Fig. 7(b) that the rain rates retrieved using the truncated moment method μ–Λ

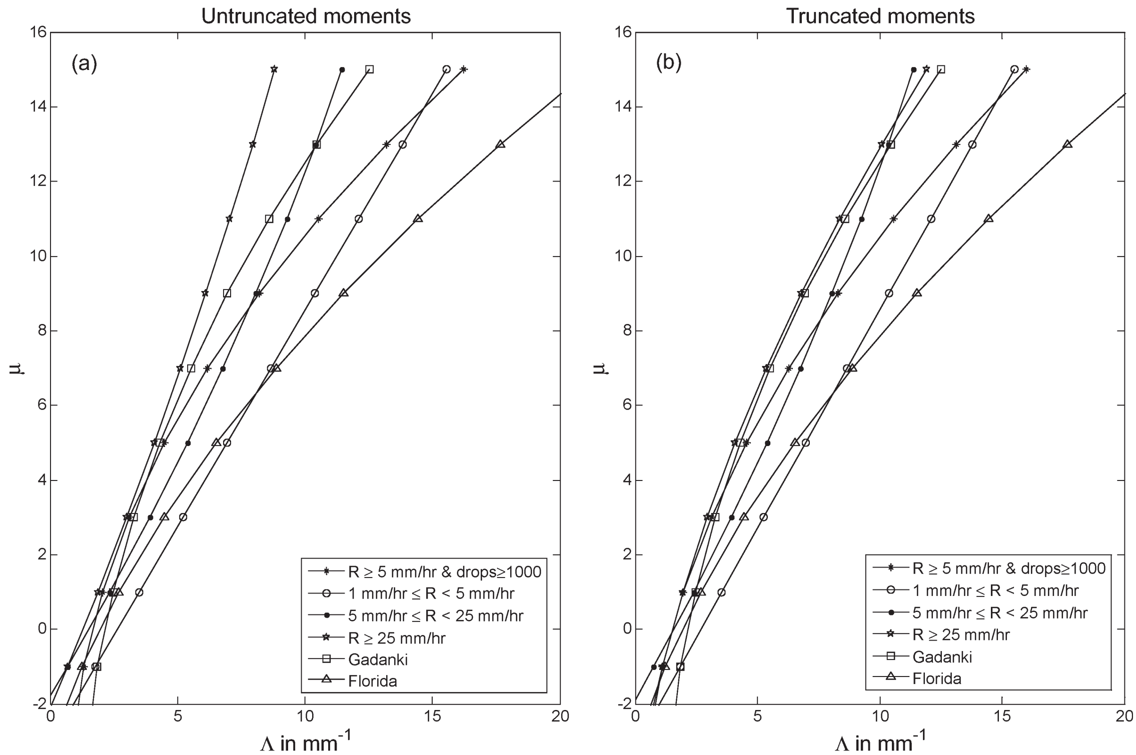

Fig. 6. Singapore's $\mu{-}\Lambda$ relationships for different rain categories, namely, $R \geq 5$ mm/hr and $rain\ counts > 1000$ drops, 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr, along with Florida and Gadanki. (a) Untruncated moment method. (b) Truncated moment method.

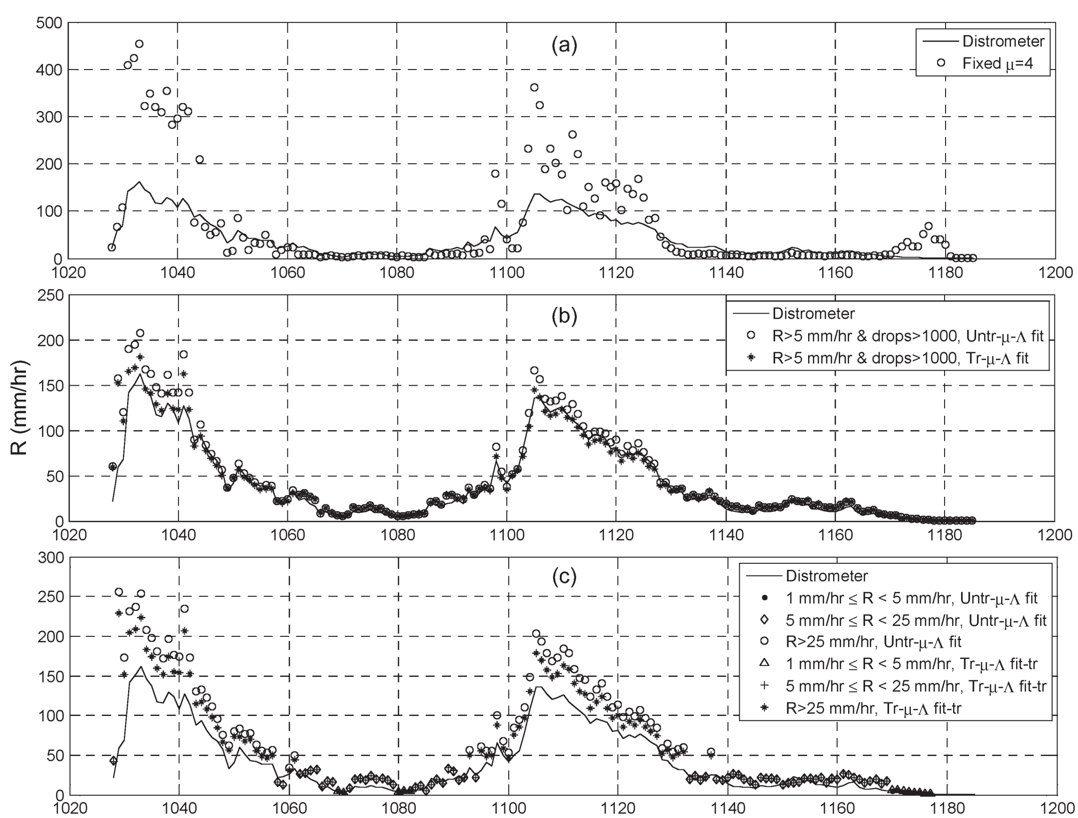

Fig. 7. Comparison of rain rates retrieved from polarimetric radar variables with measured rain rate for the rain event on February 26, 1995. (a) For $\mu = 4$. (b) Using the $\mu{-}\Lambda$ relation for the rain category $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops. (c) Using the $\mu{-}\Lambda$ relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr.

relation for the rain category $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops match well with the distrometer-measured rain rates at all rain rates except for the few points above or around 140 mm/hr. However, the same rain filtering $\mu{-}\Lambda$ relation using the untruncated moment method slightly overestimates at rain rates greater than around 70 mm/hr, and the overestimation is larger particularly at the convective peaks.

The $\mu{-}\Lambda$ relations fitted for the rain categories 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr estimate the rain rates similarly for both the untruncated and truncated moment methods. However, as can be seen from Fig. 7(c), the truncated moment method $\mu{-}\Lambda$ relation fitted for the rain category $R \geq 25$ mm/hr estimates the rain rates closer to the measured rain rates than the untruncated moment method. It is concluded from Fig. 7 that the use of truncated moment method makes the rain retrieval more accurate. Furthermore, the two-parameter model using the $\mu{-}\Lambda$ relation retrieves the rain rates better than the two-parameter model which uses the constant value of μ. Although the rain filtering, namely, 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr, for rain rate retrieval produces accurate results, the rain filtering for $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops is recommended since it produces the most accurate results for rain rate retrieval. The $\mu{-}\Lambda$ relations fitted for different rain categories, namely, 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq$ 5 mm/hr lack data points. This may be the reason for the less accurate retrieval of rain rates by these fits as compared to the $\mu{-}\Lambda$ relation fitted for the rain category $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops.

IV. Conclusion

The two-parameter gamma models have been analyzed using the measured drop size data of Singapore. The μ value of four has been found to be the most appropriate, and the range of μ values from three to five can be used to form the two-parameter gamma model.

The $\mu{-}\Lambda$ relationship has been derived for Singapore for rain rates greater than 5 mm/hr and rain counts greater than 1000 drops. It has been found to be closer to the curve derived for Gadanki, India, since the rain rates in both countries are high. The curve for Florida is further away from the curve of Singapore at lower rain rates. Florida curve has lower μ values given the same lambda value as compared to Singapore's $\mu{-}\Lambda$ relationship. However, the trend of Singapore curve follows the Florida curve for μ values greater than four. The differences in fit may be due to the location, instrument (Singapore and India–JWD; Florida–video distrometer) used for measuring DSD, the selected events used to fit the $\mu{-}\Lambda$ relationship, and the type of gamma model to fit DSD.

The $\mu{-}\Lambda$ relationship is fitted for the categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr, and $R \geq 25$ mm/hr. Lower rain rate categories, namely, 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr, are closer to Florida's curve, and Gadanki's curve is almost similar to the Singapore's $\mu{-}\Lambda$ relation for $R > 25$ mm/hr. This indicates that the rain events selected for fitting Florida curve have more stratiform and transition-type rain, and the rain events selected for fitting Gadanki curve have more convective-type rain. The comparison of retrieved rain rates with measured rain rates has shown that the use of the $\mu{-}\Lambda$ relationship is more accurate than the use of the fixed μ value for the rain retrievals. Although the rain filtering, namely, 1 mm/hr $\leq R < 5$ mm/hr and 5 mm/hr $\leq R < 25$ mm/hr, for rain rate retrieval produces accurate results, the $\mu{-}\Lambda$ relation fitted using the truncated moment method for the rain filtering for $R \geq 5$ mm/hr and $rain\ counts \geq 1000$ drops is recommended since it produces the most accurate results for rain rate retrieval.

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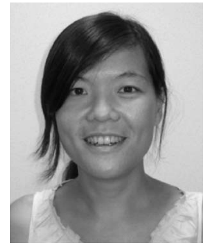

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