Design and Modeling of Nanodevices

Compact Modeling of Nano MOSFETs

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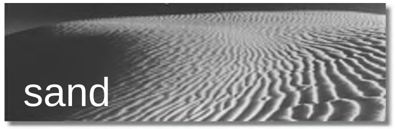
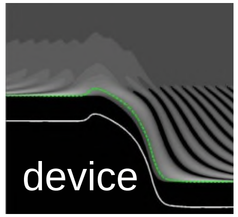
Top-down vs Bottom-up



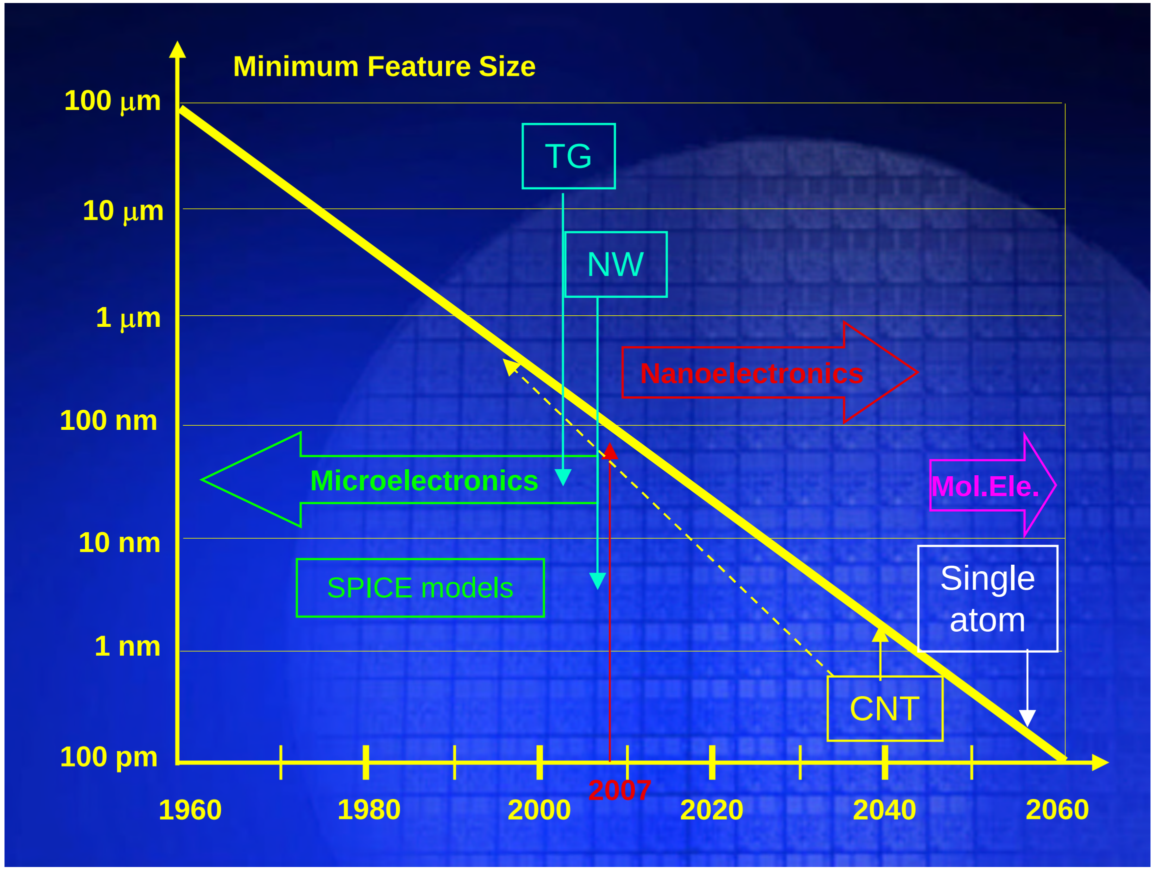
BSIM/SPICE for IC designs ~30yrs

EDA tools for nanodevice ~20yrs



Lithography

Top-down



History:

ICs have been designed by SPICE using BSIM over the past 30 years.

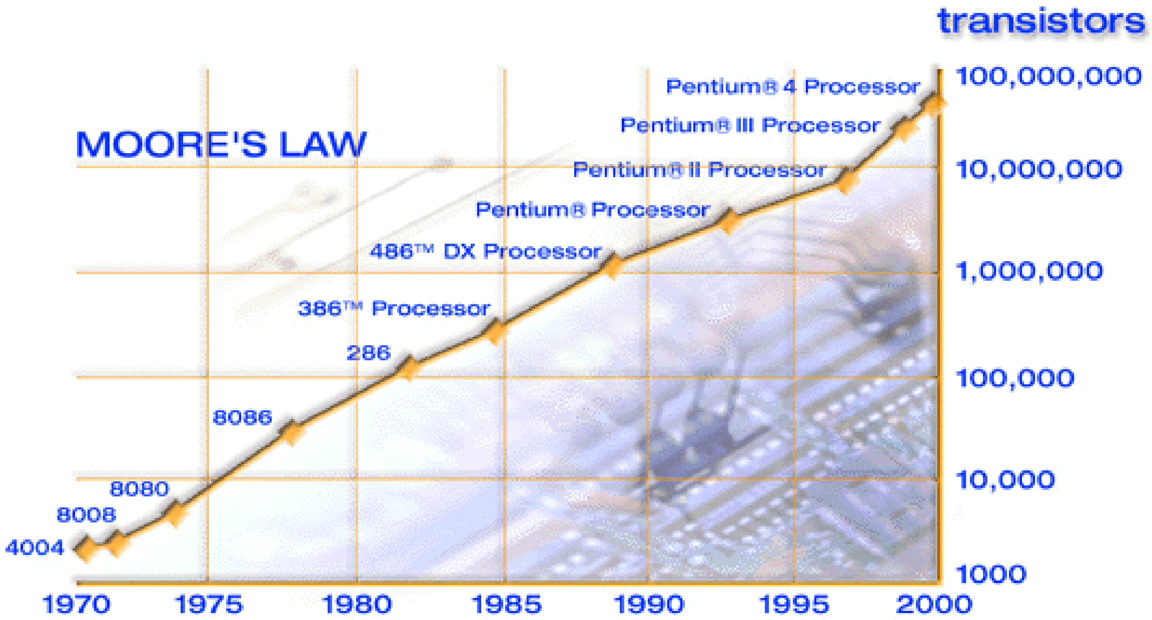
Motivation:

New models for future nano-devices at the atomic scale.

Bottom-up

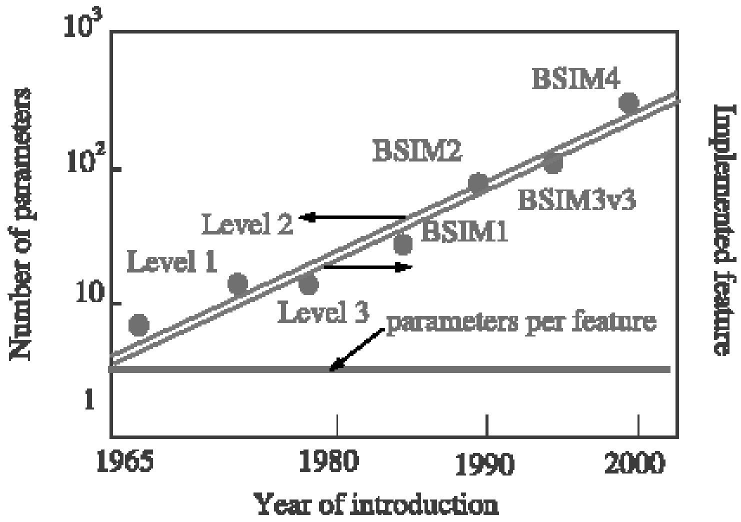
Assembly

"Moore's Law"



Chip complexity will double about every 18 months.

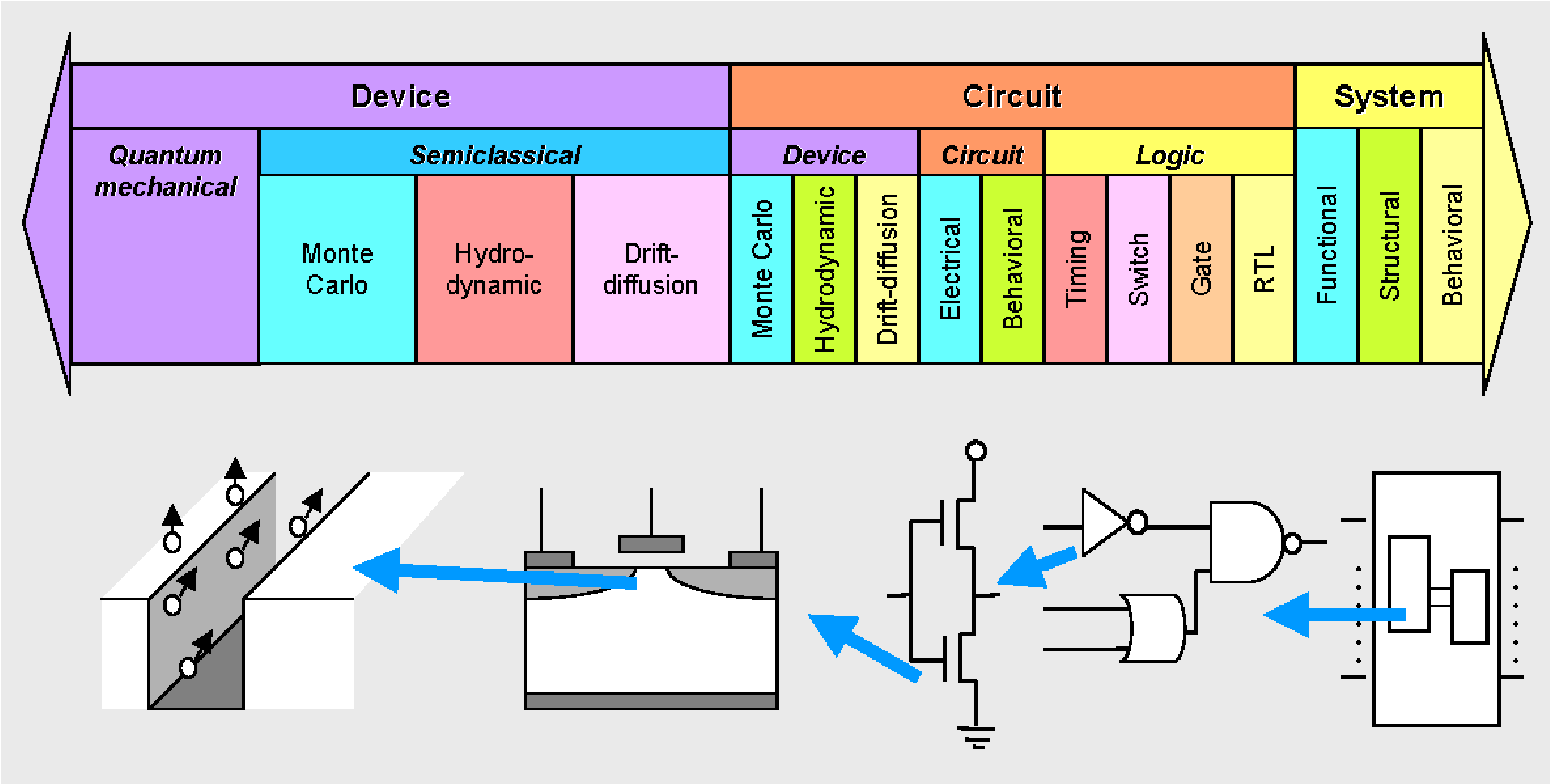
Compact Model Parameters



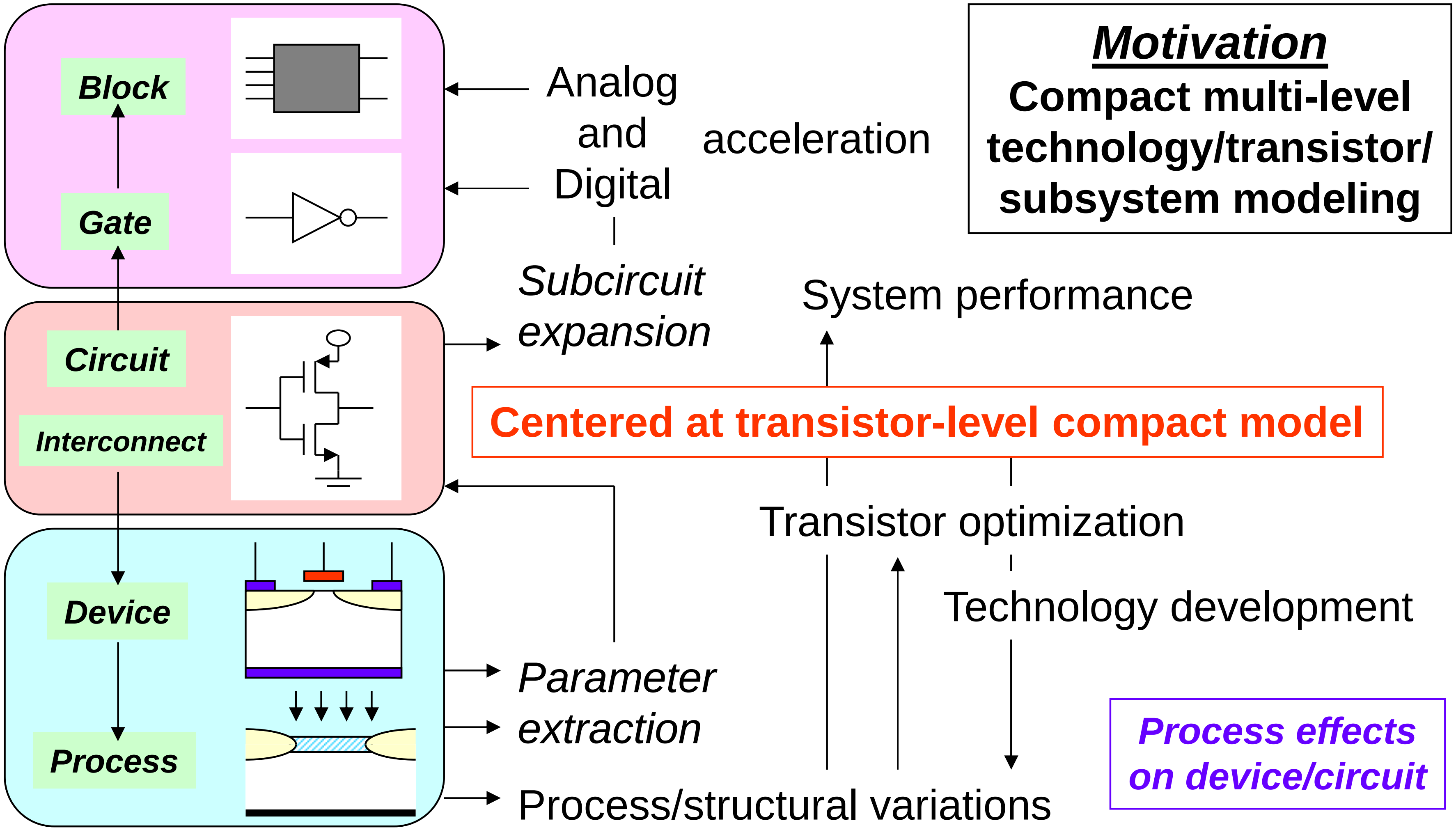
M. Chan, *et al*., *Microelectronics Reliability*, vol. 43, pp. 399-404, 2003.

A disturbing version of "Moore's law" — the number of compact-model parameters doubles about every decade (as a result of "*evolutionary*" development)

Approaches to Analyzing Microelectronic Systems

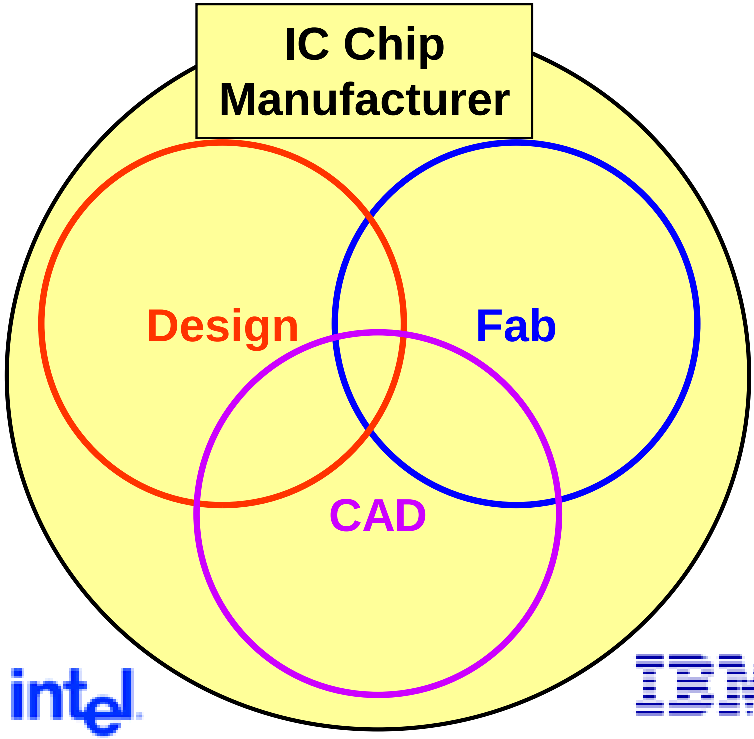


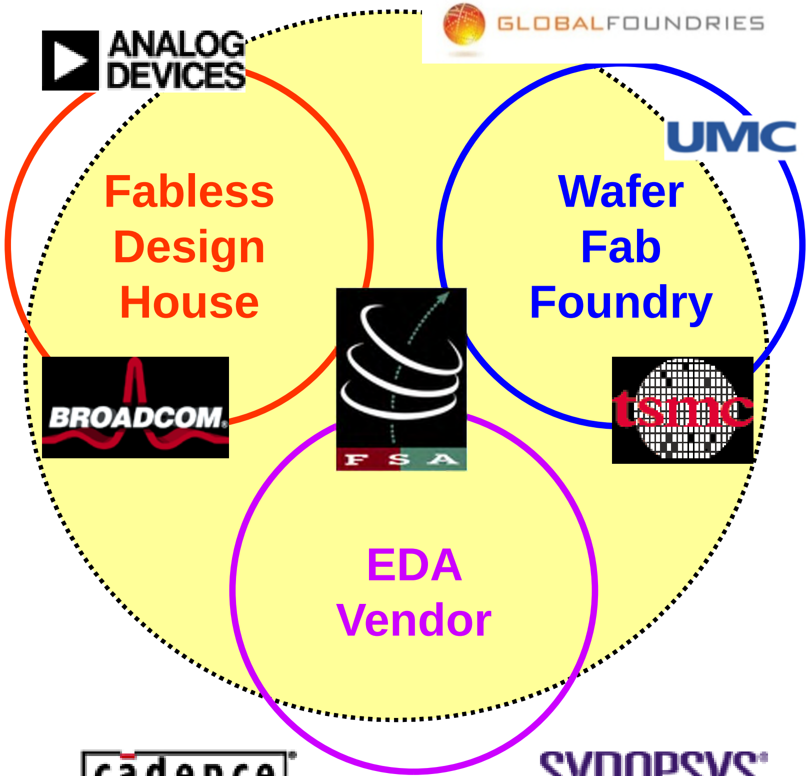
Process – Device – Circuit – Block – System



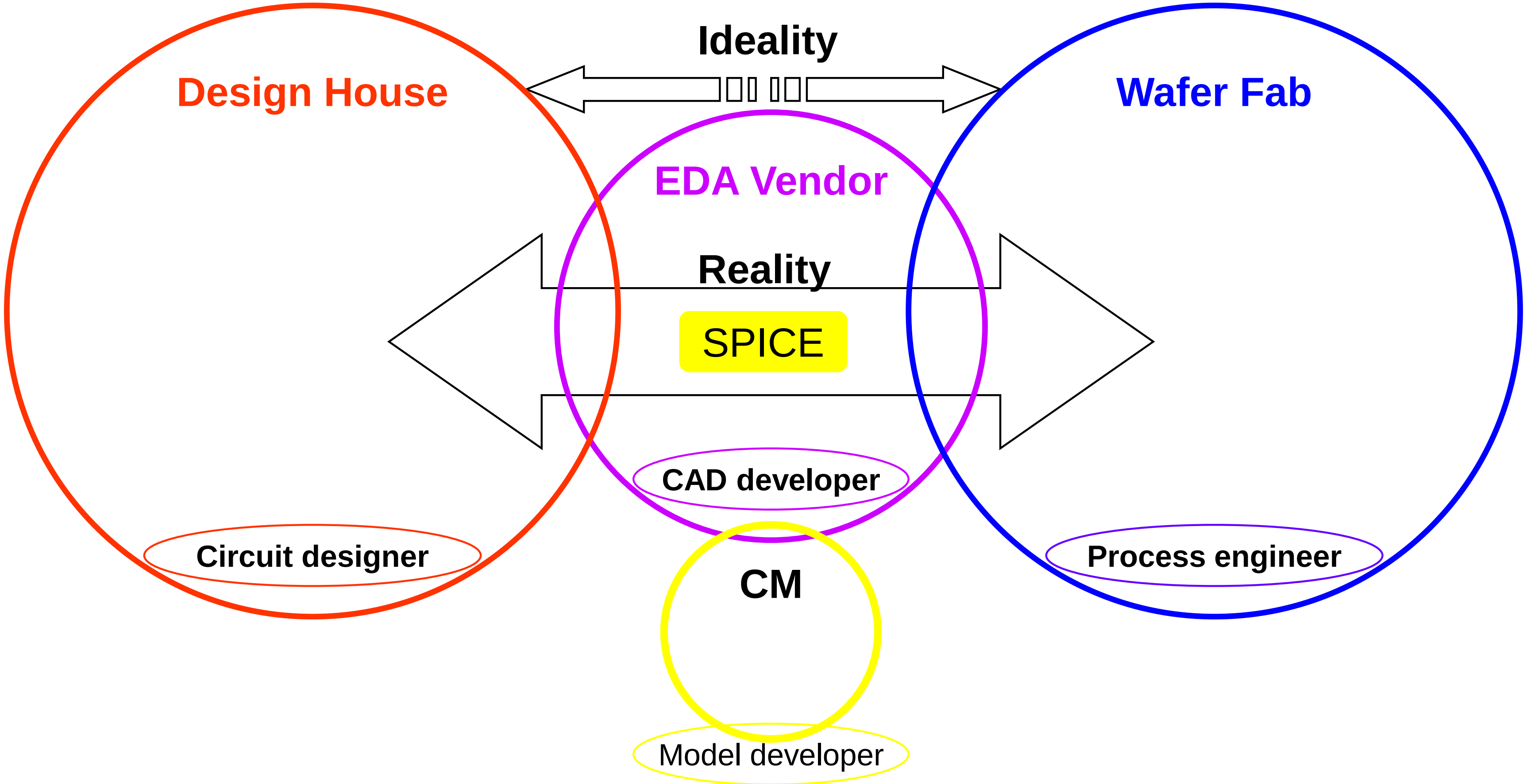
Paradigm Shift in IC Chip Design and Manufacturing

“Vertically”-integrated giant semiconductor manufacturers ⇨ **“Horizontally”-strong foundries and fabless design houses**

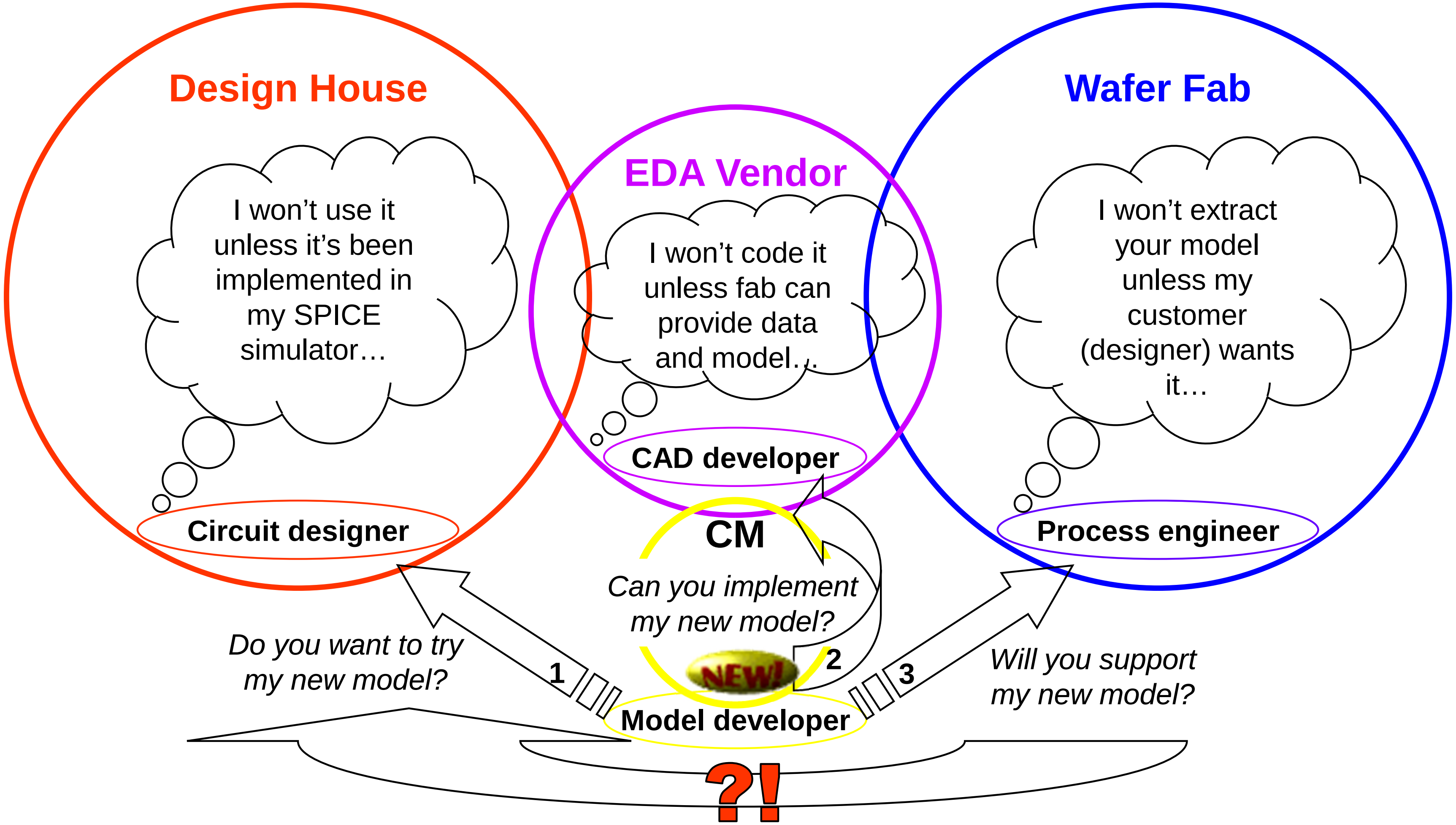




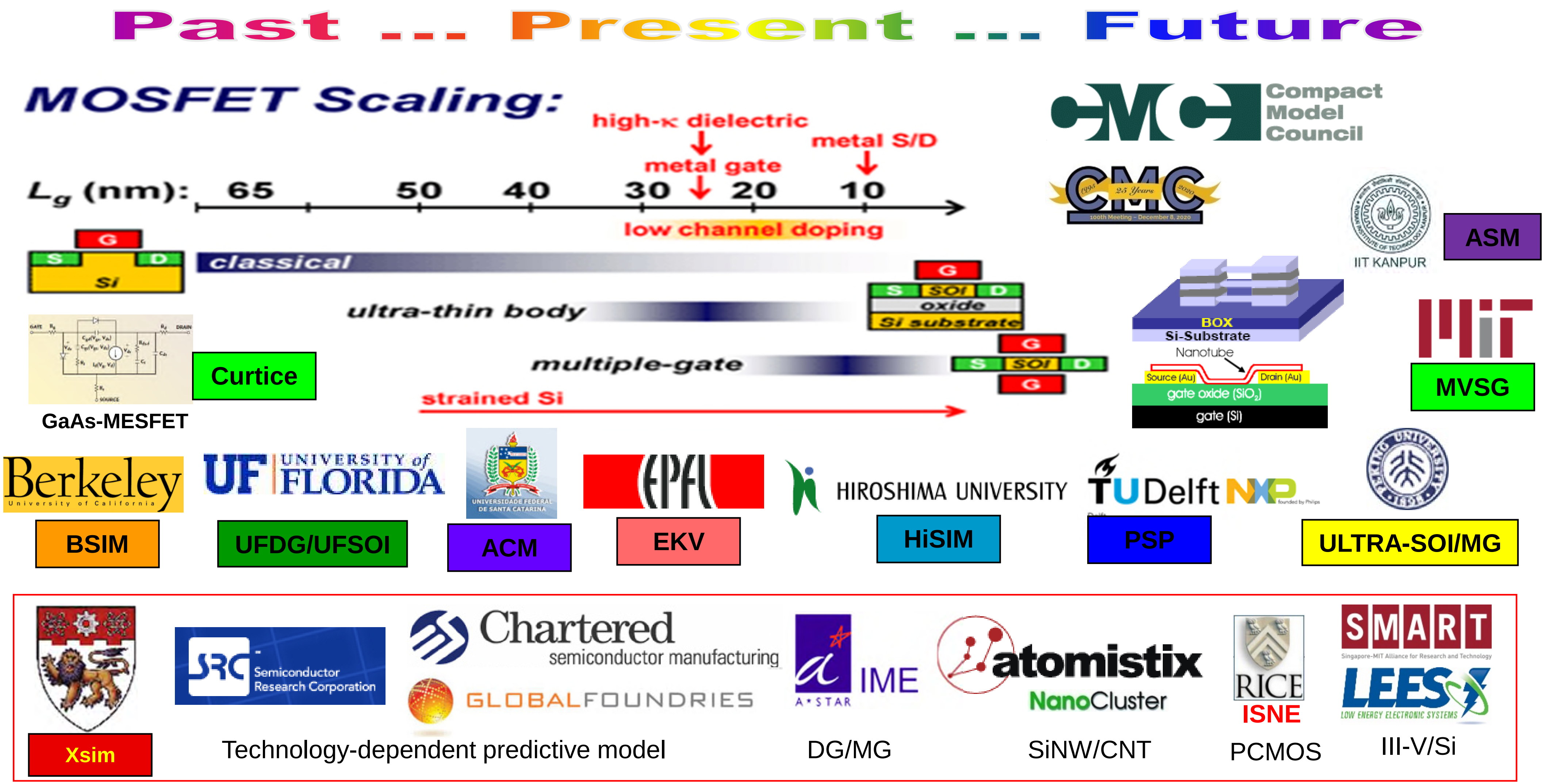
Design–Fabrication Paradigm: Ideality & Reality



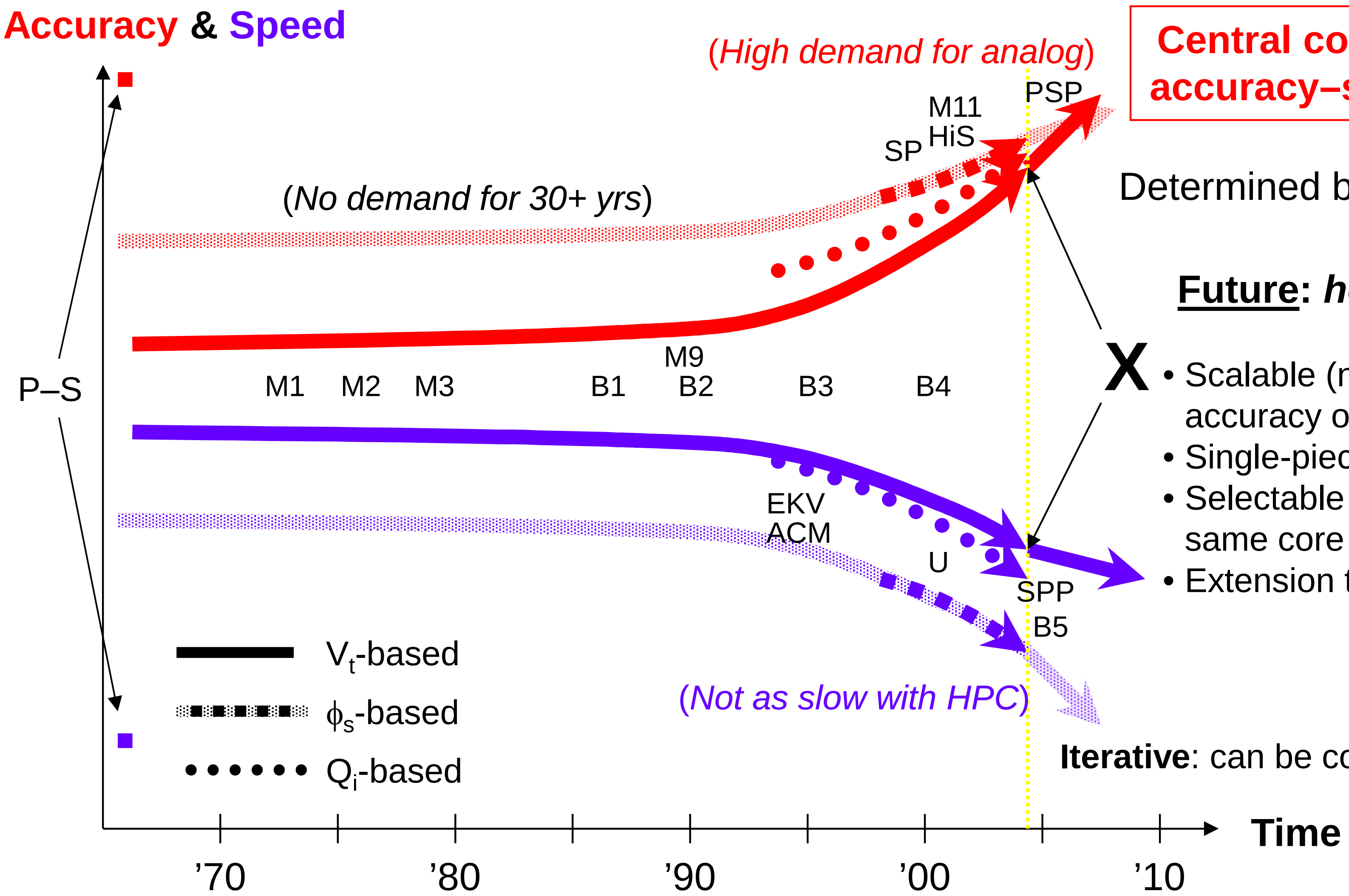
Model Developer's Dilemma



Models and Modeling Groups



Accuracy–Speed Tradeoff: History & Future



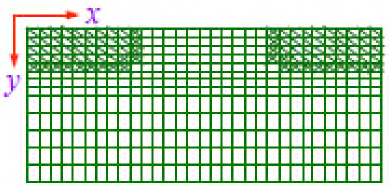
Central concern for CM: accuracy–speed tradeoff

Determined by demand/supply

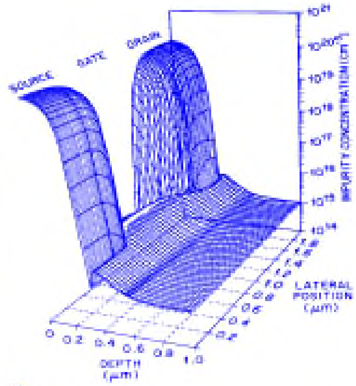
Future: ***how to tradeoff?***

- Scalable (non-binnable): accuracy over geometry
- Single-piece across all regions
- Selectable accuracy within the same core model
- Extension to non-bulk FETs

Role of Compact Model

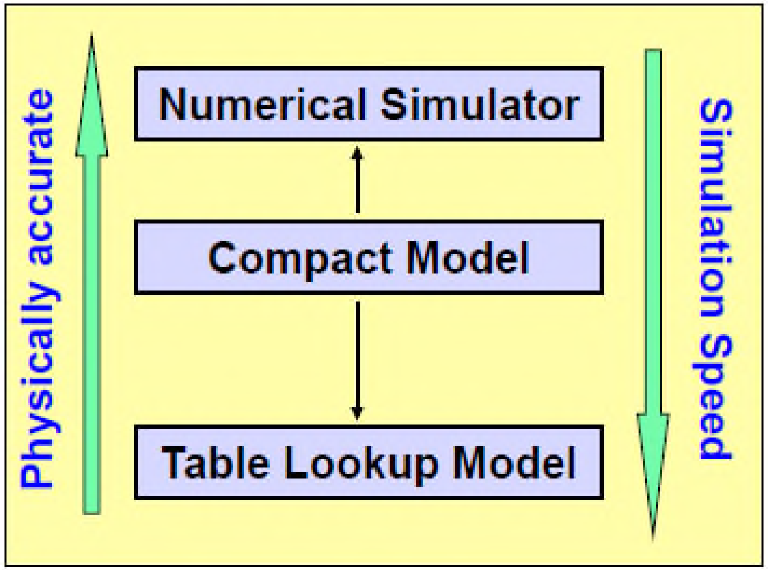


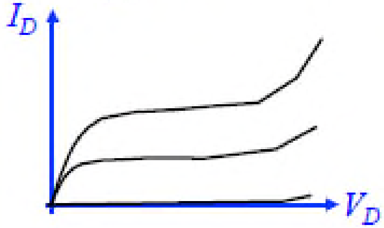
input: - detail physical parameters
- transport models (e.g. mobility)
- boundary conditions



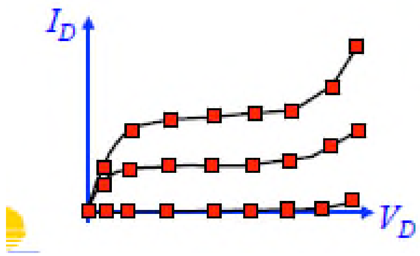
$$I_D = \frac{\mu W C_{ox}}{L}\left[(V_G - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = WC_{ox}\left(V_G - V_T - \frac{V_{DS}}{2}\right)\mu\frac{V_{DS}}{L}$$





V_D	V_G	▪▪▪▪	I_D	I_G	▪▪▪▪
xx.x	xx.x	▪▪▪▪	xx.x	xx.x	▪▪▪▪
⋮			⋮		



input: - large amount of measurement data

Ultimate goal: towards accuracy and simplicity

(Courtesy: M. Chan)

SPICE Circuit Simulation: (Modified) Nodal Analysis

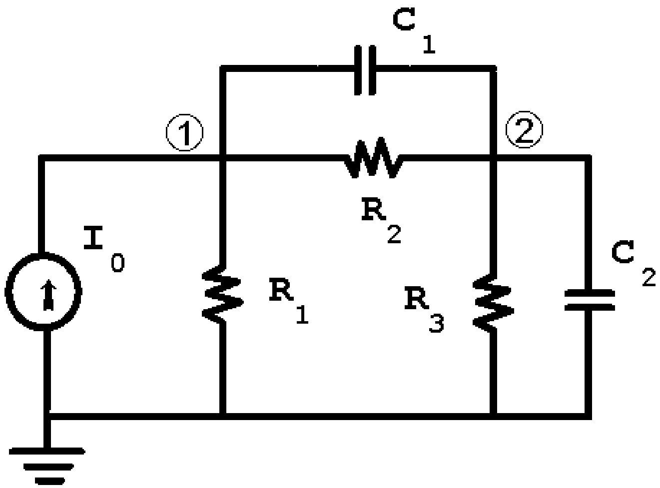
KVL/KCL:

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_1-\left(\frac{1}{R_2}\right)V_2=I_0$$

$$-\left(\frac{1}{R_2}\right)V_1+\left(\frac{1}{R_2}+\frac{1}{R_3}\right)V_2=0$$

DC:

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2}\\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}I_0\\0\end{bmatrix}$$

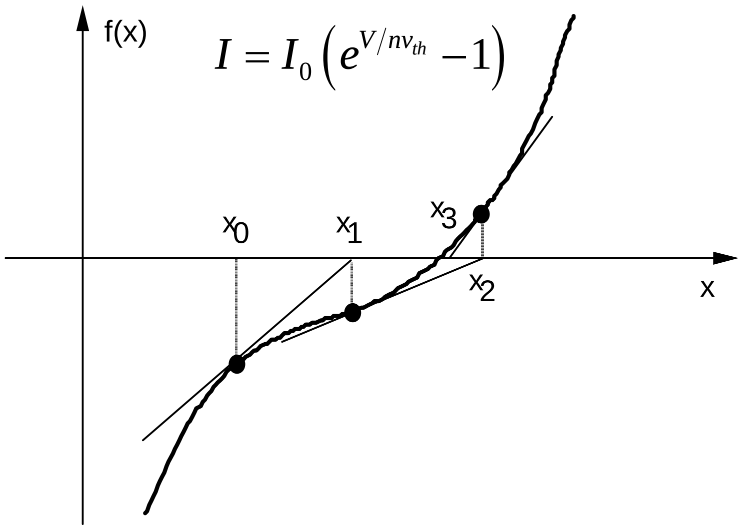


Transient: "companion"

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2}+\frac{2C_1}{h} & -\frac{1}{R_2}-\frac{2C_1}{h}\\ -\frac{1}{R_2}-\frac{2C_1}{h} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{2C_1}{h}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}I_0+i_{C_1}\\-i_{C_1}+i_{C_2}\end{bmatrix}$$

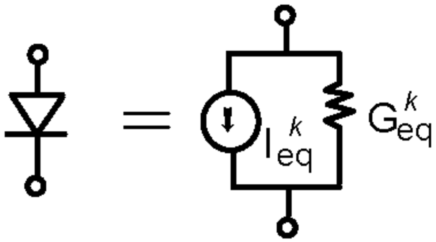
AC:

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2}+j\omega C_1 & -\frac{1}{R_2}-j\omega C_1\\ -\frac{1}{R_2}-j\omega C_1 & \frac{1}{R_2}+\frac{1}{R_3}+j\omega C_1\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}I_0e^{j\theta}\\0\end{bmatrix}$$



Nonlinear: N–R iteration

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

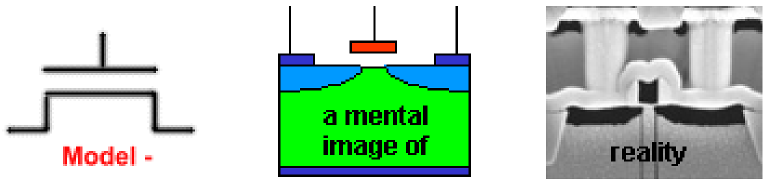


What Is a Model, and Modeling?

John von Neumann

“The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work.”

A model is a mental image of reality

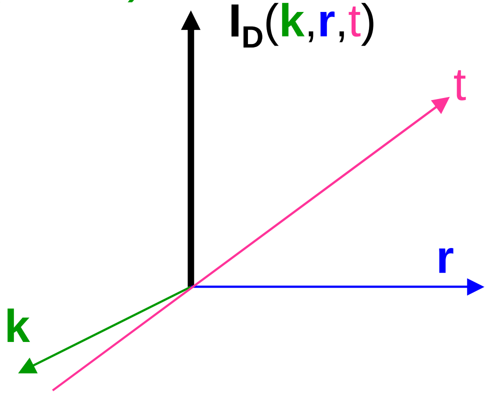


- One can have many different images of the same reality.
- Correct physical approximations and correct mathematical formulations to emulate *ideal* device physical behaviors and corroborate with *real* device characteristics.

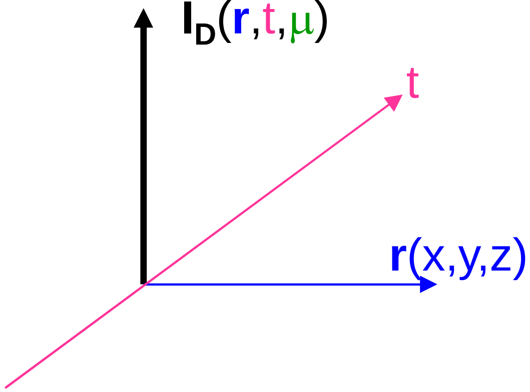
- What does “compact” mean?
- What is “physical” of a model?

Perspective: Compact Modeling for Circuit Simulation

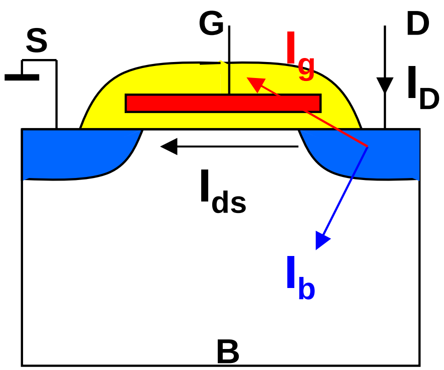
Monte Carlo:
(6-D, t)

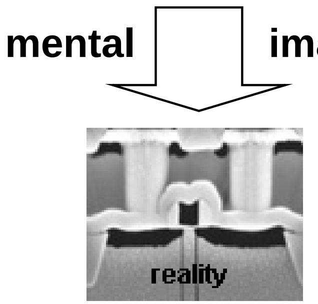


Numerical:
(3-D, t)

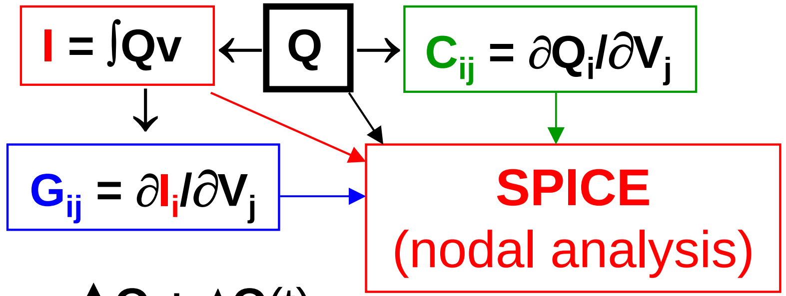


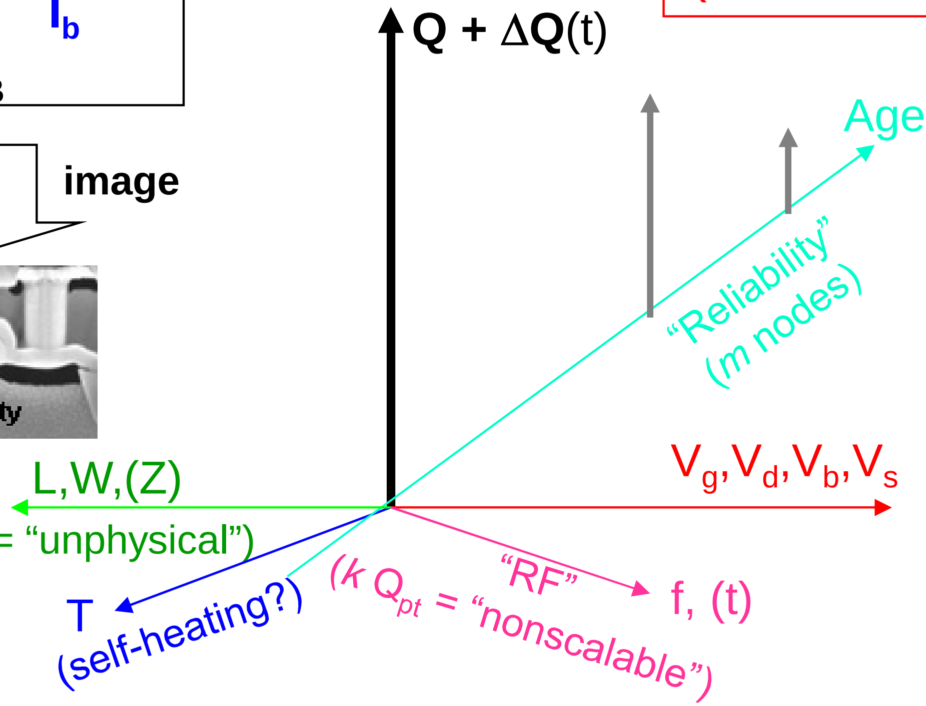
$$I_D(V) = I_{ds} + I_b + I_g$$



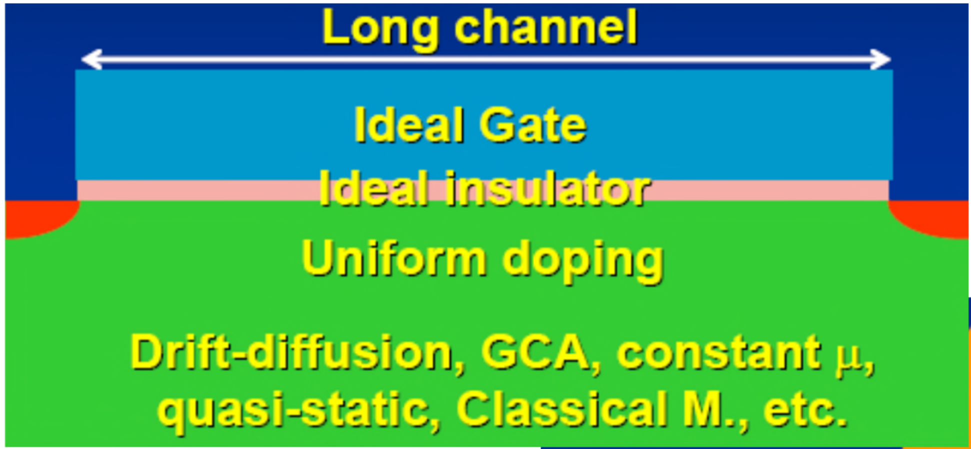


Compact: (0-D, t)



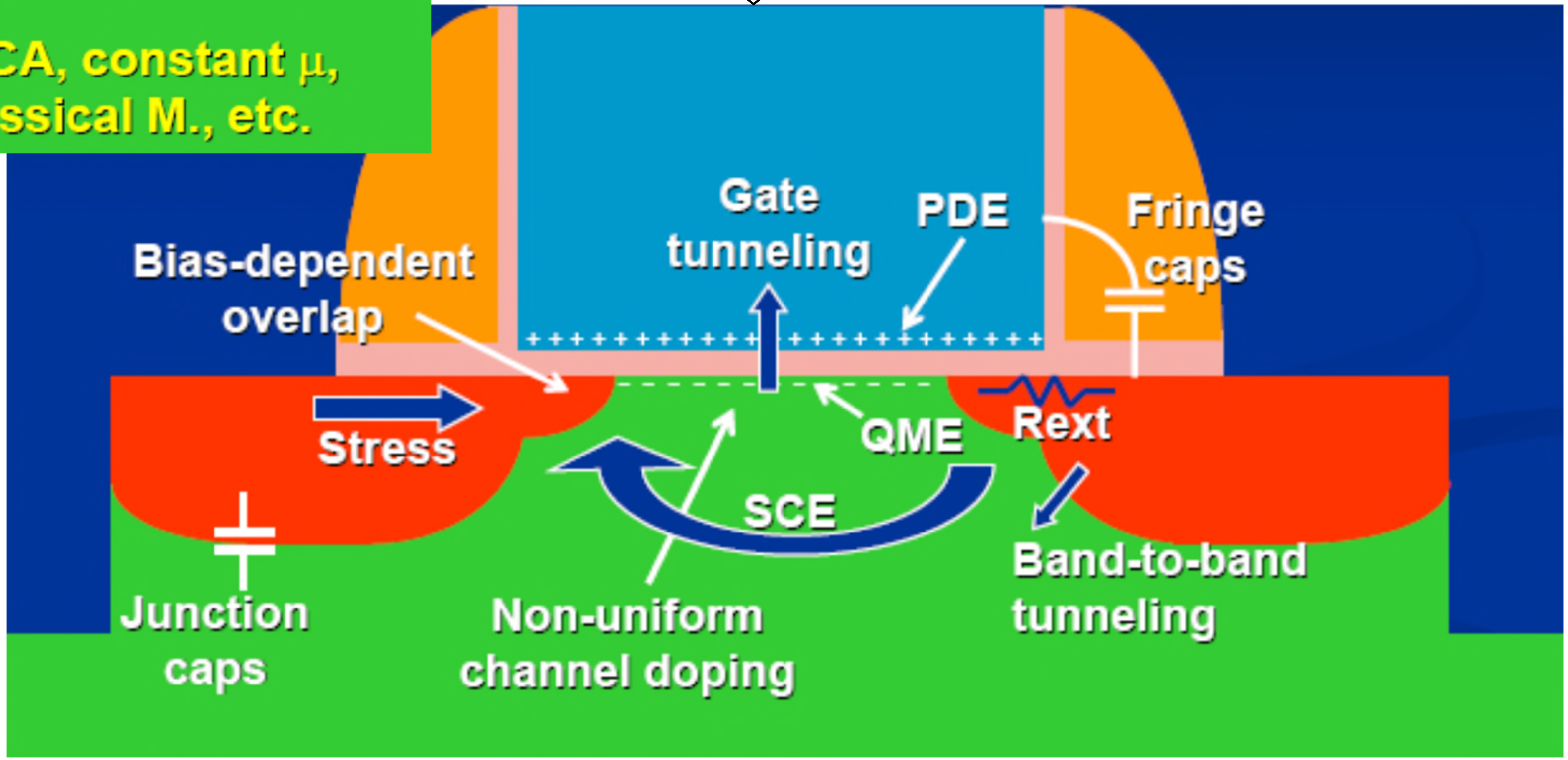


Ideal vs Real MOSFET To Be Modeled



This is what the core model deals with.

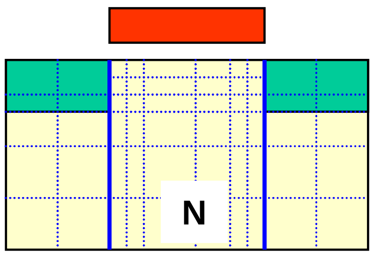
But we need to model this…



(R. Rios, WCM2005)

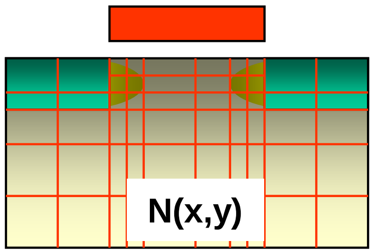
“Binning” vs “Meshing”

- **Binning = piece-wise (in geometry)**
  - **Infinite number of bins = single-device model = nonscalable** (= unphysical **?**)
- **Key difference: “binnable”** (transistor-based) **vs “non-binnable”** (technology-based) **model**
  - **Binnable model**: parameters extracted by ***fitting electrical data*** at fixed geometry
  - **Non-binnable model**: parameters extracted by ***fitting data over geometry*** at fixed bias
- **Compare: Meshing — necessary?** and **physical?**



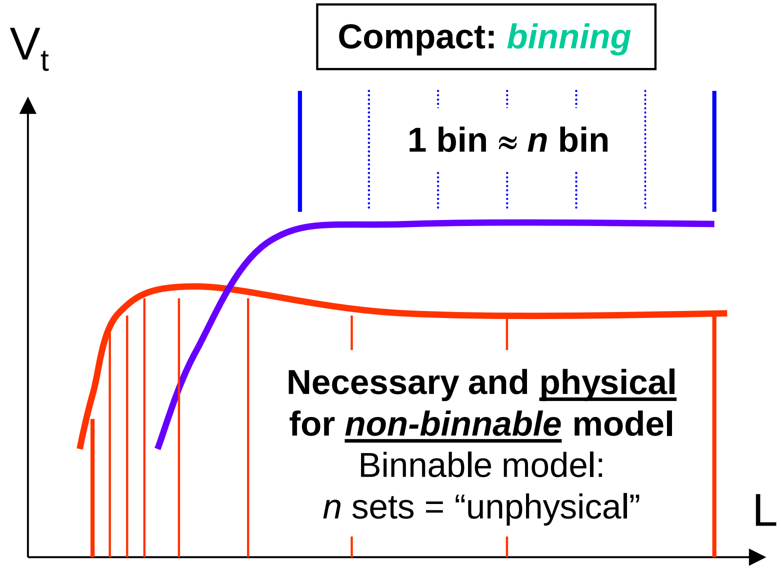
Numerical: *meshing*

Homogeneous:
Meshing unnecessary,
1 mesh ≈ *n* mesh
(ψ_s-model ≈ numerical)

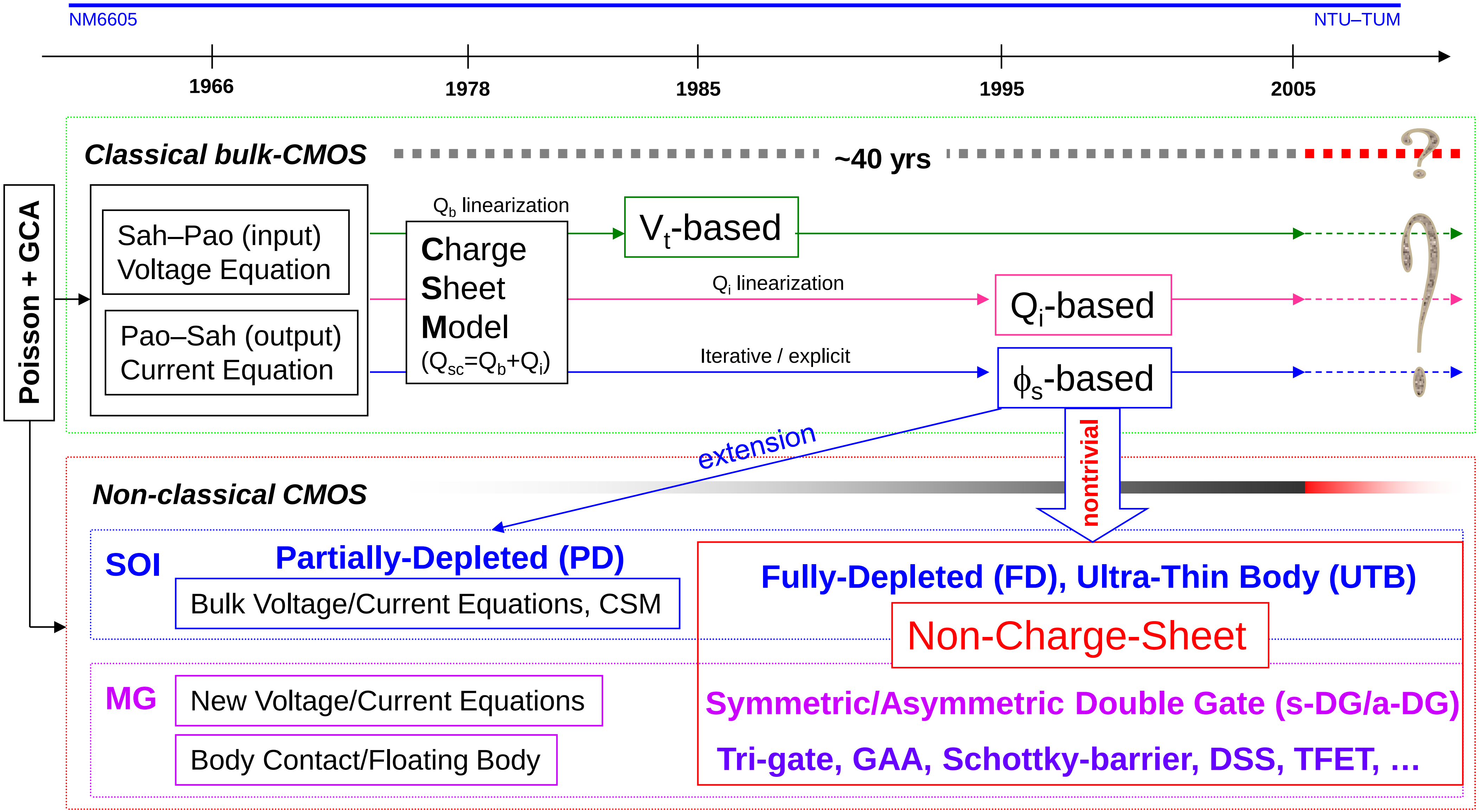


Inhomogeneous:
Meshing necessary,
and physical
(ψ_s-model “less physical”)

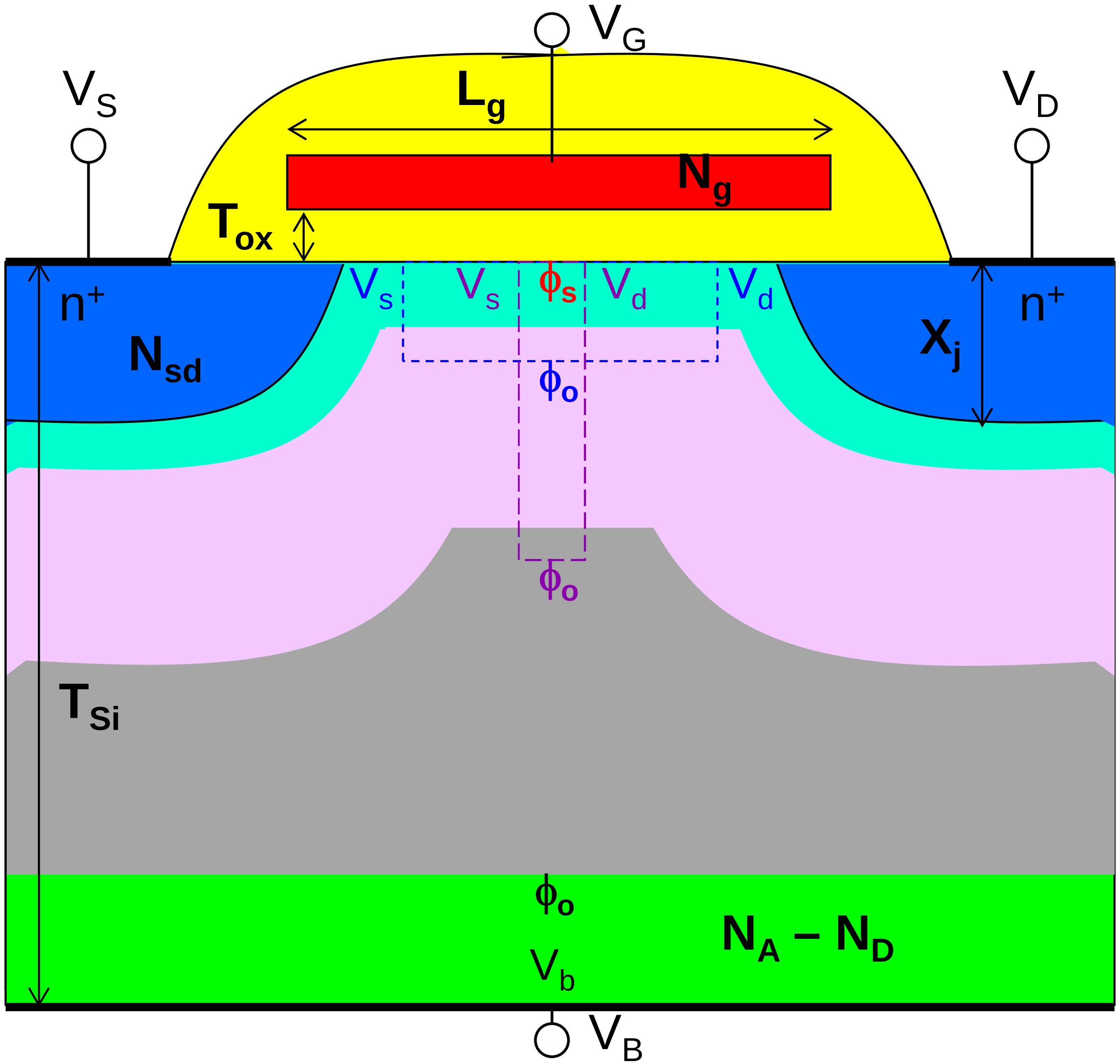
Compact: *binning*

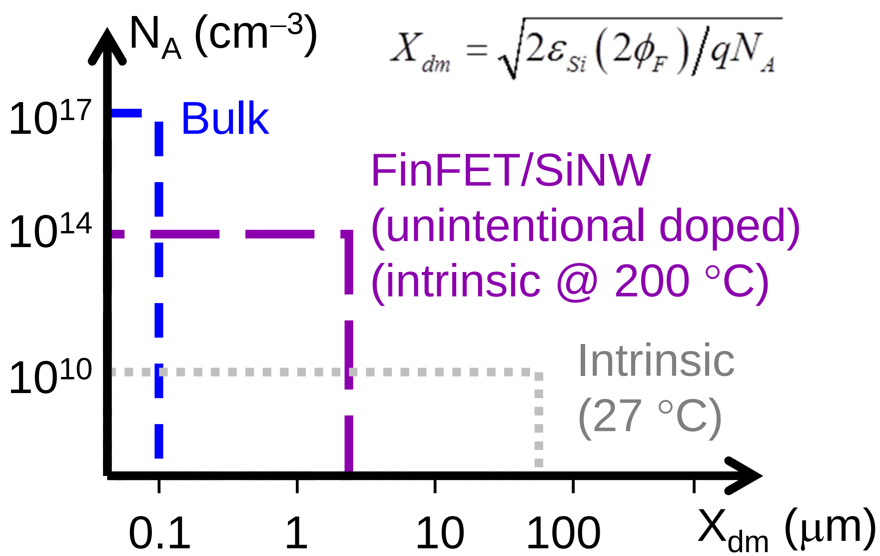


MOSFET Compact Models: History and Future



Conceptual "Core" Bulk-MOS at Various Body Doping





($N_A = N_D = 0$: undoped = "pure" Si)

Bulk: $T_{Si} >> X_{dm}$, with V_B: $\phi_o = V_b = V_B$

SOI: $T_{Si} > X_{dm}$, without V_B: ϕ_o **'floating'**

DG/GAA: $T_{Si}/R << X_{dm}$: **'volume inversion'**

(ϕ_o: 'virtual electrode')

Need for an Extendable Core Model for Future Generation

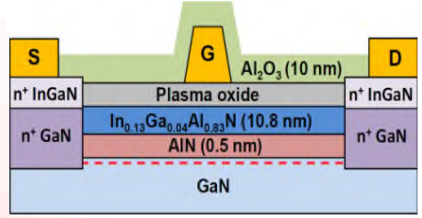
Time: ’60 ’70 ’80 ’90 ’00 ’10 ’20

Pao–Sah

V_t-based models

Q_i-based models

ϕ_s-based models

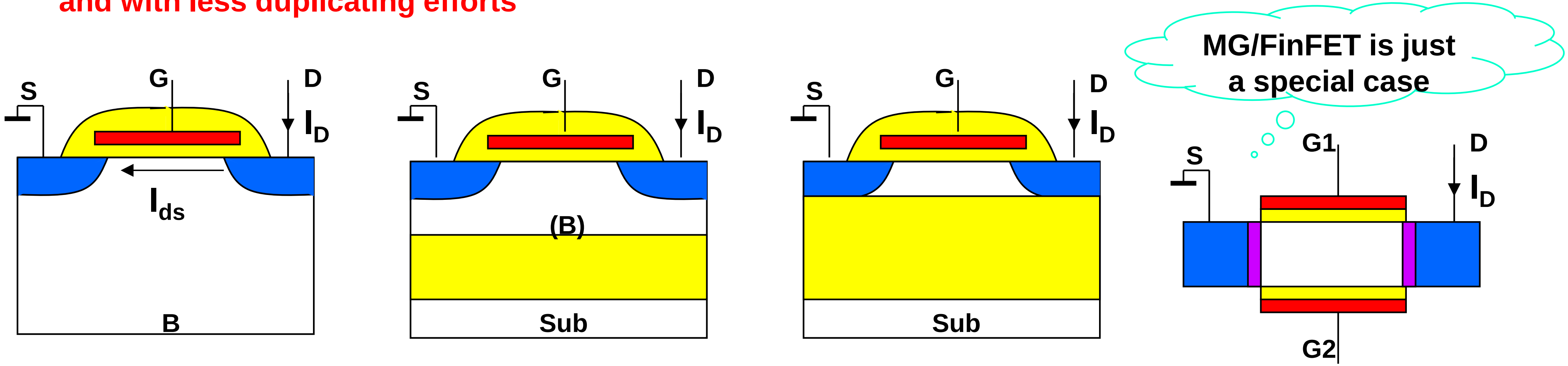


HEMT – leveraging on MOS models

History has witnessed generations of MOS models

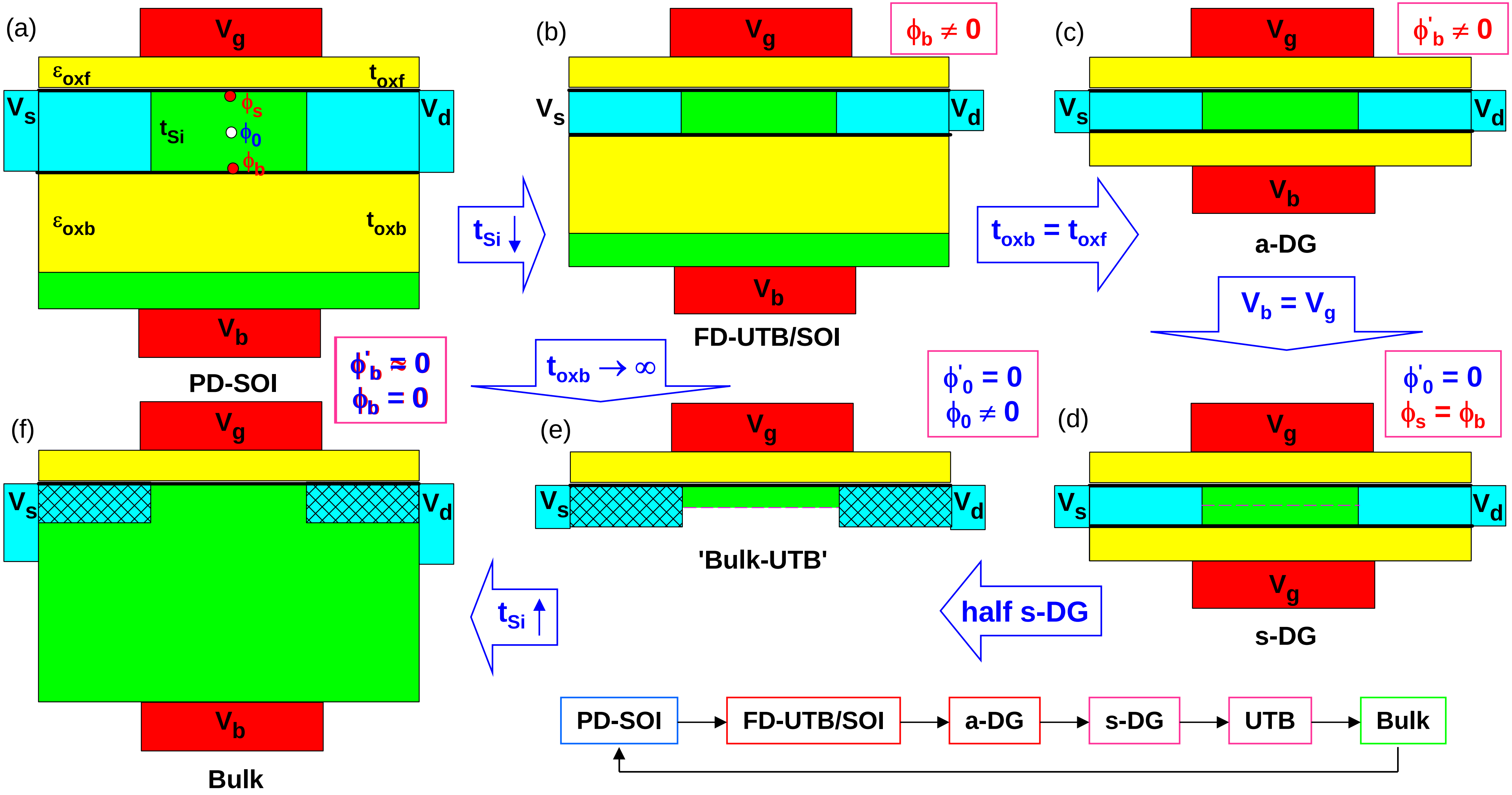
and efforts required from one generation to the next …

— Need for a core model *extendable* to future generations, and with less duplicating efforts

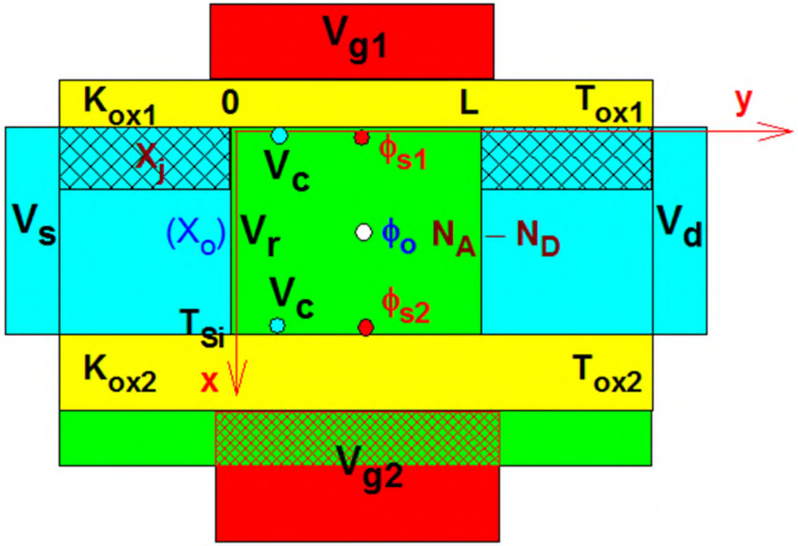


Bulk **PD/DD-SOI** **FD-UTB/SOI** **DG/GAA/SB/DSS**

Seamless Transformation and Unification of MOSFETs



The Generic SOI/DG/GAA MOSFET



Zero-field potential: ϕ_o [$\phi_o'(X_o) = 0$]

Imref-split: $V_{cr} = \phi_{Fn} - \phi_{Fp} = V_c - V_r$

$V_r = V_b$ (BC: body-contacted)
$V_r = V_{min} = \min(V_s, V_d)$ ("FB": w/o BC)

- **Bulk**: special case of s-DG
- **SOI**: special case of ia-DG

Common/symmetric-DG [GAA]

- $V_{g1} = V_{g2} = V_g$: two gates with <u>one bias</u>
- $C_{ox1} = C_{ox2}$: **s-DG** ($X_o = T_{Si}/2$; [R])
- **Full-depletion**: $V_{FD} = V_g(X_d = T_{Si}/2)$
- $C_{ox1} \neq C_{ox2}$: **ca-DG** ($X_o < T_{Si}$)

Independent/asymmetric-DG

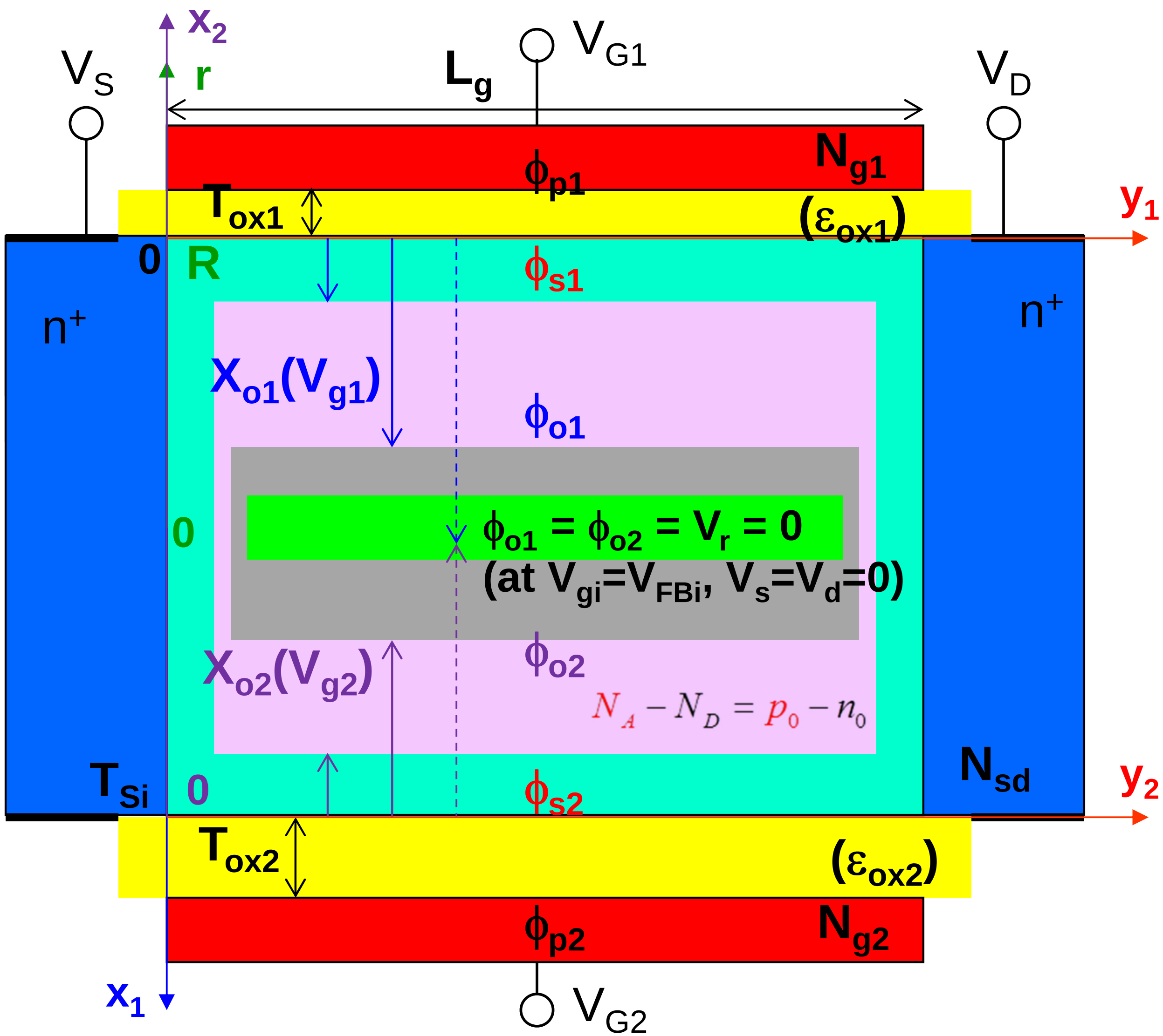
- $V_{g1} \neq V_{g2}$: **ia-DG**, biased <u>independently</u>
- Zero-field location may be outside body
- Consider two "independent" gates; linked through **full-depletion** condition:

$$X_{d1} + X_{d2} = T_{Si}$$

Unification of MOS

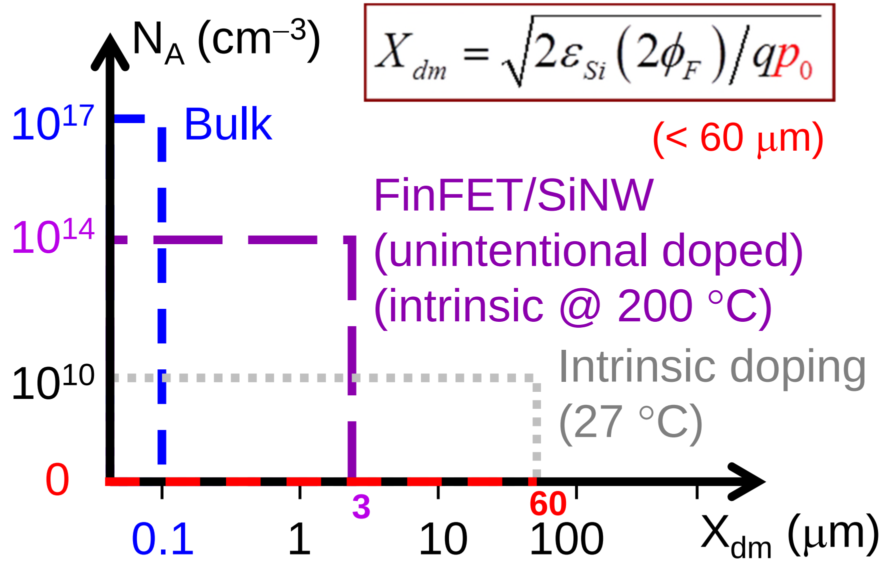
GAA
↑
- SOI ← ia-DG ↔ ca-DG ↔ s-DG → bulk

Generic Double-Gate MOSFET with Any Body Doping



$$X_{dm} = \sqrt{2\varepsilon_{Si}(2\phi_F)/qp_0}$$

(< 60 μm)



($N_A = N_D = 0$: undoped = "pure" Si)

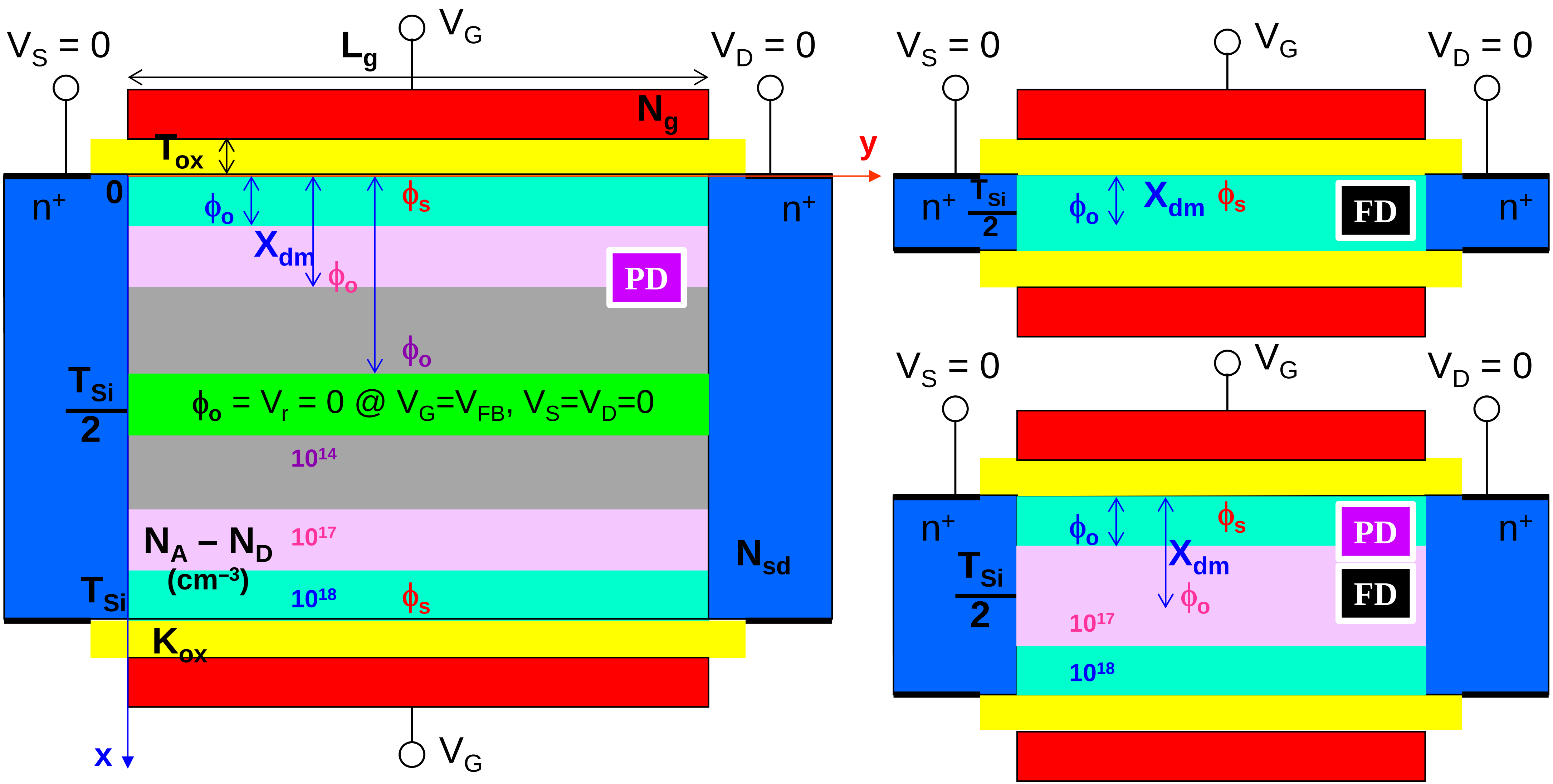
Bulk: $T_{Si} >> X_{dm}$, with BC: $\boldsymbol{\phi_o = V_b = V_B}$

SOI: $T_{Si} > X_{dm}$, without BC: $\boldsymbol{\phi_o}$ **'floating'**

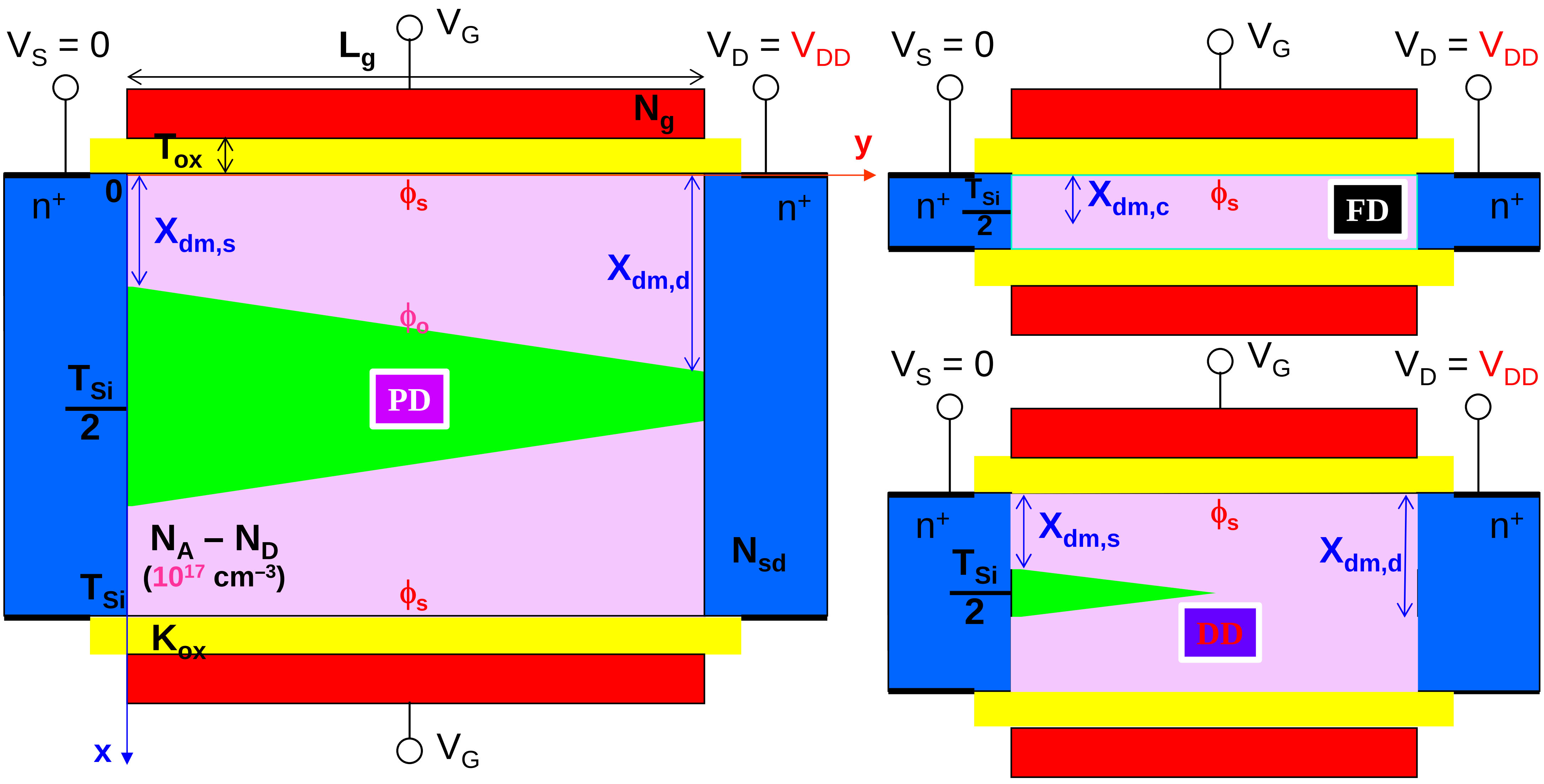
DG/GAA: $\frac{T_{Si}}{2}/R << X_{dm}$: **'volume inversion'**

(ϕ_o: 'virtual electrode')

PD/FD at Various Body Doping/Thickness



Dynamic Depletion (DD) at Various Body Thickness



Symmetric Charge Linearization

❑ Symmetric bulk/inversion charge linearization

$$Q_i(y) \approx \overline{Q_i}\Big|_{\phi_s=\overline{\phi_s}} + \frac{d\overline{Q_i}}{d\phi_s}\Big|_{\phi_s=\overline{\phi_s}}\left(\phi_s - \overline{\phi_s}\right) = -C_{ox}\left[\overline{q_i} - \overline{A_b}\left(\phi_s - \overline{\phi_s}\right)\right]$$

$$\overline{q_i} = V_{gb} - V_{FB} - \overline{\phi_s} - \Upsilon\sqrt{\overline{\phi_s} - V_b} \qquad \overline{A_b} = 1 + \frac{\Upsilon}{2\sqrt{\overline{\phi_s} - V_b}}$$

$$\overline{\phi_s} = \frac{1}{2}\left(\phi_{s,s} + \phi_{s,d}\right) = \frac{\phi_{ds}\left(V_{s,eff}\right) + \phi_{ds}\left(V_{d,eff}\right)}{2}$$

$$V_{c,eff} = \vartheta\left\{V_c, V_{c,sat}, V_{cc',sat}; \delta\right\} \quad \left(c = s, d; c' = d, s\right)$$

❑ Long-channel symmetric current model

$$I_{ds0} = \beta_0\left(\overline{q_i} + \overline{A_b}v_{th}\right)\Delta\phi_s \approx \overline{\beta_0}\left(\overline{q_i} + \overline{A_b}v_{th}\right)V_{ds,eff}$$

$$\beta_0 = \mu_0 C_{ox}\frac{W}{L} \qquad \Delta\phi_s = \phi_{s,d} - \phi_{s,s} \approx \left(2\phi_F + V_{db}\right) - \left(2\phi_F + V_{sb}\right) = V_{ds}$$

$$\overline{\beta_0} = \overline{\mu_{eff\,0}}C_{ox}\frac{W}{L} \qquad \overline{\mu_{eff\,0}} = \frac{1}{2}\left(\mu_{eff\,0,s} + \mu_{eff\,0,d}\right) \qquad \mu_{eff\,0,c} = \frac{\mu_0}{1 + \delta_L\left(V_{c,eff} - V_b\right)/\left(LE_{sat,c}\right)} \quad (c = s, d)$$

$$V_{d,eff} - V_{s,eff} = V_{ds,eff}$$

$$V_{gt,c} = \Upsilon\sqrt{\phi_{s,c} - V_b + v_{th}e^{\left(\phi_{s,c} - 2\phi_F - V_{cb}\right)/v_{th}}} - \Upsilon\sqrt{\phi_{s,c} - V_b}$$

$$\overline{V_{gt}} = \frac{1}{2}\left(V_{gt,s} + V_{gt,d}\right) \qquad E_{eff} = \frac{\zeta_n C_{ox}}{\varepsilon_{Si}}\left(\overline{V_{gt}} + \frac{\zeta_b}{\zeta_n}\Upsilon\sqrt{\overline{\phi_s} - V_b}\right)$$

$$V_{ds,sat} = \frac{V_{gt,s}LE_{sat,s}}{V_{gt,s} + A_{b,s}LE_{sat,s} + 2A_{b,s}v_{th}} = V_{d,sat} - V_s \qquad V_{sd,sat} = \frac{V_{gt,d}LE_{sat,d}}{V_{gt,d} + A_{b,d}LE_{sat,d} + 2A_{b,d}v_{th}} = V_{s,sat} - V_d \qquad E_{sat,c} = \frac{2v_{sat,c}}{\mu_0}$$

Symmetric Linearization of Bulk-Charge Factor for DD

PD **FD**

$$\overline{A_b} \equiv -\left.\frac{\partial q_i}{\partial \phi_s}\right|_{\phi_s=\overline{\phi_s}} = 1 + \frac{\Upsilon}{2\sqrt{\overline{\phi_s} - \overline{\phi_o}}}$$

$$q_i = V_{gf} - \phi_s - \Upsilon\sqrt{\phi_s - \phi_o}$$

$$\overline{\phi_s} = \left(\phi_{s,s} + \phi_{s,d}\right)/2$$

$$\begin{cases} Q_b = -C_{ox}\left[q_{b,acc} + q_{b,sub} - \dfrac{\left(\overline{A_{b,DD}} - 1\right)\Delta\phi_{ds}^2}{12H_{DD}}\right] \\ Q_d = -\dfrac{C_{ox}}{2}\left[\overline{q_i} - \dfrac{\overline{A_{b,DD}}\Delta\phi_{ds}}{6}\left(1 - \dfrac{\Delta\phi_{ds}}{2H_{DD}} - \dfrac{\Delta\phi_{ds}^2}{20H_{DD}^2}\right)\right] \\ Q_s = -\dfrac{C_{ox}}{2}\left[\overline{q_i} + \dfrac{\overline{A_{b,DD}}\Delta\phi_{ds}}{6}\left(1 + \dfrac{\Delta\phi_{ds}}{2H_{DD}} - \dfrac{\Delta\phi_{ds}^2}{20H_{DD}^2}\right)\right] \end{cases}$$

DD

$$\overline{A_{b,DD}} \equiv -\frac{\Delta q_i}{\Delta\phi_{ds}} = \frac{q_{i,s} - q_{i,d}}{\phi_{ds,d} - \phi_{ds,s}}$$

$$q_{i,d} = \vartheta_f\left(V_{gf};\sigma_f\right) - \phi_{ds,d} - \Upsilon\sqrt{\phi_{ds,d} - \phi_{o,d}}$$

$$q_{i,s} = \vartheta_f\left(V_{gf};\sigma_f\right) - \phi_{ds,s} - \Upsilon\sqrt{\phi_{ds,s} - \phi_{o,s}}$$

$$q_{b,sub} = \vartheta_f\left(V_{gf};\sigma_a\right) - \vartheta_f\left(V_{gf};\sigma_f\right) + \Upsilon\sqrt{\overline{\phi_{ds}} - \overline{\phi_o}}$$

$$q_{b,acc} = \vartheta_r\left(V_{gf};\sigma_a\right) - \phi_{acc}$$

$$H_{DD} = \left(\overline{q_i}/\overline{A_{b,DD}}\right) + v_{th}$$

Due to the use of ϕ_{ds} and $\vartheta_f(V_{gf})$, no singularity occurs at flatband

The Poisson–Boltzmann Equation and Solution

$$n = n_i e^{(\psi-\phi_{Fn})/v_{th}}$$

$$p = n_i e^{-(\psi-\phi_F)/v_{th}}$$

$$N_D \approx n_0 = n_i e^{-\phi_{Fn}/v_{th}}$$

$$N_A \approx p_0 = n_i e^{\phi_F/v_{th}}$$

$$\phi_{Fn} = \phi_F + V_{cb}$$

$$v_{th} = kT/q$$

$$E_x = -\frac{d\psi}{dx}$$

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon_{Si}} = -\frac{q(p-n+N_D-N_A)}{\varepsilon_{Si}} = \frac{q}{\varepsilon_{Si}}(n-p+N_A-N_D)$$

$$= \frac{q}{\varepsilon_{Si}}\left[n_i e^{(\psi-\phi_F-V_{cb})/v_{th}} - n_i e^{-(\psi-\phi_F)/v_{th}} + n_i e^{\phi_F/v_{th}} - n_i e^{-(\phi_F+V_{cb})/v_{th}}\right]$$

$$= \frac{qN_A}{\varepsilon_{Si}}\left[e^{(\psi-2\phi_F-V_{cb})/v_{th}} - e^{-\psi/v_{th}} + 1 - e^{-(2\phi_F+V_{cb})/v_{th}}\right] \equiv \frac{qN_A}{\varepsilon_{Si}} G(\psi, V_{cb})$$

GCA:

$$\frac{d^2\psi}{dy^2} << \frac{d^2\psi}{dx^2}$$

$$(x_0 = T_{Si} >> X_{dm})$$

$$\frac{d^2\psi}{dx^2} = -\frac{dE_x}{dx} = -\frac{dE_x}{d\psi}\frac{d\psi}{dx} = E_x\frac{dE_x}{d\psi}$$

B.C.'s:

$$\psi(0,y) = \phi_s(y),\ E_x(0,y) = E_s(y)$$
$$\psi(x_0,y) = 0,\ E_x(x_0,y) = 0$$

$$\frac{E_s^2}{2} = \int_0^{E_s} E_x dE_x = \int_0^{\phi_s}\frac{d^2\psi}{dx^2}d\psi = \frac{qN_A}{\varepsilon_{Si}}\int_0^{\phi_s} G(\psi, V_{cb})d\psi$$

$$F_s(\phi_s, V_{cb}) \equiv \left[\int_0^{\phi_s} G(\psi, V_{cb})d\psi\right]^{1/2}$$

$$E_s^2 = \frac{2qN_A}{\varepsilon_{Si}}\left\{e^{-(2\phi_F+V_{cb})/v_{th}}\left[v_{th}\left(e^{\phi_s/v_{th}}-1\right)-\phi_s\right] + v_{th}\left(e^{-\phi_s/v_{th}}-1\right)+\phi_s\right\} \equiv \frac{2qN_A}{\varepsilon_{Si}}F_s^2(\phi_s, V_{cb})$$

$$F_s(\varphi_s, v_{cb}) = \frac{E_s}{\sqrt{2qN_A/\varepsilon_{Si}}} = \mathrm{sgn}(\varphi_s)\sqrt{v_{th}\left[e^{-(2\varphi_F+v_{cb})}\left(e^{\varphi_s}-1-\varphi_s\right)+\left(e^{-\varphi_s}-1+\varphi_s\right)\right]}$$

$$\varphi_s = \phi_s/v_{th}$$
$$\varphi_F = \phi_F/v_{th}$$
$$v_{cb} = V_{cb}/v_{th}$$

The Complete ("Sah–Pao") Voltage Equation

Gauss law:

$$\varepsilon_{Si}E_s - \varepsilon_{ox}E_{ox} = Q_{ox} \qquad \varepsilon_{ox}E_{ox} = Q_g \qquad -\varepsilon_{Si}E_s = Q_{sc} \qquad C_{ox} = \varepsilon_{ox}/T_{ox}$$

Potential balance:

$$V_{gb} = \phi_{MS} + V_{ox} + \phi_s \qquad V_{FB} = \phi_{MS} - Q_{ox}/C_{ox} \qquad \Upsilon = \sqrt{2q\varepsilon_{Si}N_A}/C_{ox} \qquad E_{ox} = V_{ox}/T_{ox}$$

Poisson

$$E_s = \frac{\varepsilon_{ox}E_{ox} + Q_{ox}}{\varepsilon_{Si}} = \frac{\varepsilon_{ox}(V_{ox}/T_{ox}) + Q_{ox}}{\varepsilon_{Si}} = \frac{C_{ox}(V_{gb} - \phi_{MS} - \phi_s) + Q_{ox}}{\varepsilon_{Si}} = \frac{C_{ox}(V_{gb} - V_{FB} - \phi_s)}{\varepsilon_{Si}}$$

$$V_{gb} - V_{FB} - \phi_s = \operatorname{sgn}(\phi_s)\Upsilon\sqrt{f_\phi}$$

$$= \operatorname{sgn}(\phi_s)\Upsilon\sqrt{\underbrace{v_{th}\exp\left(-\frac{2\phi_F + V_{cb}}{v_{th}}\right)\left[\exp\left(\frac{\phi_s}{v_{th}}\right) - 1\right]}_{(n)} + \underbrace{v_{th}\left[\exp\left(-\frac{\phi_s}{v_{th}}\right) - 1\right]}_{(p)} + \underbrace{\phi_s}_{(N_A)} - \underbrace{\phi_s\exp\left(-\frac{2\phi_F + V_{cb}}{v_{th}}\right)}_{(N_D)}}$$

$$\overbrace{(Q_g + Q_{ox})/C_{ox}}\ \ \overbrace{-Q_{sc}/C_{ox}} = \overbrace{-Q_i/C_{ox} = -(Q_s + Q_d)/C_{ox}} + \overbrace{-Q_b/C_{ox} = -(Q_{acc} + Q_{sub} + Q_{str})/C_{ox}}$$

$$Q_g = -(Q_b + Q_i + Q_{ox})$$

Charge balance

Ward–Dutton partition

one-piece

smoothing

Unified regional

Drain Current: Pao–Sah Double Integral

$$J_{ny}(x,y) = qn\mu_n E_y + qD_n\,\partial n/\partial y = -qn\mu_n\left(\frac{\partial\psi}{\partial y} - \frac{kT}{qn}\frac{\partial n}{\partial y}\right) = -qn\mu_n\frac{\partial}{\partial y}\left(\psi - \frac{kT}{q}\ln\frac{n}{n_i}\right) = -qn\mu_n\,\partial\phi_{Fn}/\partial y = -qn\mu_n\,dV_{cb}/dy$$

$$I_{ds}(y) = -W\frac{dV_{cb}}{dy}\int_0^{t_{Si}} qn(x,y)\mu_n(x,y)\,dx = -W\mu_s(y)Q_i(y)\,dV_{cb}/dy = \text{const.}$$

$$E_y = -\partial\psi/\partial y$$

$$E_x = -\partial\psi/\partial x$$

$$D_n = \mu_n kT/q$$

$$\mu_s(y) = \int_0^{t_i}\mu_n(x,y)n(x,y)\,dx \Big/ \int_0^{t_i} n(x,y)\,dx$$

$$\phi_{Fn} = \psi - v_{th}\ln(n/n_i)$$

$$\mu_0 = \int_{V_{sb}}^{V_{db}}\mu_s(y)Q_i(y)\,dV_{cb} \Big/ \int_{V_{sb}}^{V_{db}} Q_i(y)\,dV_{cb}$$

$$V_{cb}(y) = \phi_{Fn}(y) - \phi_F$$

$$J_{nx} = -qn\mu_n\,dV_{cb}/dx = 0$$

$$n = N_A e^{(\psi-2\phi_F-V_{cb})/v_{th}}$$

$$Q_i(y) \equiv q\int_0^{\infty} n(x,y)dx = -q\int_{\phi_s}^{0} n(\psi,V_{cb})\left(-\frac{dx}{d\psi}\right)d\psi = -q\int_0^{\phi_s}\frac{n(\psi,V_{cb})}{E_x(\psi,V_{cb})}d\psi \approx \frac{-qN_A}{\sqrt{2qN_A/\varepsilon_{Si}}}\int_0^{\phi_s}\frac{e^{(\psi-2\phi_F-V_{cb})/v_{th}}}{\left[\psi + v_{th}e^{(\psi-2\phi_F-V_{cb})/v_{th}}\right]^{1/2}}d\psi$$

$$E_x(x,y) = \operatorname{sgn}(\psi)\sqrt{\frac{2qN_A}{\varepsilon_{Si}}}\left\{e^{-(2\phi_F+V_{cb})/v_{th}}\left[v_{th}\left(e^{\psi/v_{th}}-1\right)-\psi\right] + v_{th}\left(e^{-\psi/v_{th}}-1\right)+\psi\right\}^{1/2} \equiv \operatorname{sgn}(\psi)\sqrt{2qN_A/\varepsilon_{Si}}\,F(\psi,V_{cb})$$

$$I_{ds} = \mu_0\frac{W}{L}\int_{V_{sb}}^{V_{db}}(-Q_i)dV_{cb} = \mu_0\frac{W}{2L}C_{ox}\Upsilon\int_{V_{sb}}^{V_{db}}\int_0^{\phi_s}\frac{e^{(\psi-2\phi_F-V_{cb})/v_{th}}}{\left[\psi + v_{th}e^{(\psi-2\phi_F-V_{cb})/v_{th}}\right]^{1/2}}d\psi\,dV_{cb}$$

$$F(\psi,V_{cb}) \approx F_{di}(\psi,V_{cb}) \equiv \sqrt{\psi + v_{th}e^{(\psi-2\phi_F-V_{cb})/v_{th}}}$$

CSM: Charge-Sheet Model

$$Q_b(y) = -q\int_0^{\infty}(N_A - p)dx = -q\int_{\phi_s}^{0}(N_A - p)\frac{dx}{d\psi}d\psi = -q\int_0^{\phi_s}\frac{N_A - p(\psi, V_{cb})}{E_x(\psi, V_{cb})}d\psi = \frac{-qN_A}{\sqrt{2qN_A/\varepsilon_{Si}}}\int_0^{\phi_s}\frac{1 - e^{-\psi/v_{th}}}{F(\psi, V_{cb})}d\psi$$

Depletion approximation (n = p = 0; also $N_D = 0$):

$$Q_b(y) = \frac{-qN_A}{\sqrt{2qN_A/\varepsilon_{Si}}}\int_0^{\phi_s}\frac{1}{\sqrt{\psi}}d\psi = -\Upsilon C_{ox}\sqrt{\phi_s(y)}, \quad (\phi_s > 0)$$

Potential/charge balance:

$$Q_g(y) = C_{ox}\left[V_{gb} - V_{FB} - \phi_s(y)\right] - Q_{ox} \qquad Q_i + Q_b + Q_{ox} = -Q_g$$

Charge-sheet model (CSM):

$$Q_i(y) = -\left[Q_g(y) + Q_{ox}\right] - Q_b(y) = -C_{ox}\left[V_{gb} - V_{FB} - \phi_s(y) - \Upsilon\sqrt{\phi_s(y)}\right]$$

$$\frac{dQ_i}{dy} = C_{ox}\left(\frac{d\phi_s}{dy} + \frac{\Upsilon}{2\sqrt{\phi_s}}\frac{d\phi_s}{dy}\right) = C_{ox}\left(1 - \frac{\Upsilon^2 C_{ox}}{2Q_b}\right)\frac{d\phi_s}{dy}$$

Sah–Pao ('S–P') voltage equation ($\phi_s > 3v_{th}$):

$$\left[V_{gb} - V_{FB} - \phi_s(y)\right]^2 = \Upsilon^2\left[\phi_s(y) + v_{th}e^{(\phi_s(y) - 2\phi_F - V_{cb}(y))/v_{th}}\right]$$

$$\frac{dV_{cb}(y)}{d\phi_s} = 1 + v_{th}\left[\frac{2(V_{gb} - V_{FB} - \phi_s(y)) + \Upsilon^2}{(V_{gb} - V_{FB} - \phi_s(y))^2 - \Upsilon^2\phi_s(y)}\right]$$

$$= 1 - \frac{v_{th}C_{ox}}{Q_i(y)}\left[1 + \frac{Q_i(y) - \Upsilon^2 C_{ox}}{Q_i(y) + 2Q_b(y)}\right] \approx 1 - \frac{v_{th}C_{ox}}{Q_i(y)}\left[1 - \frac{\Upsilon^2 C_{ox}}{2Q_b(y)}\right]$$

CSM →

$$I_{ds}(y) = -W\mu_s(y)Q_i(y)\frac{dV_{cb}(y)}{d\phi_s}\frac{d\phi_s}{dy}$$

$$= -W\mu_s(y)Q_i(y)\frac{d\phi_s}{dy} + W\mu_s(y)v_{th}\frac{dQ_i(y)}{dy}$$

$$= I_{drift}(y) + I_{diff}(y)$$

Drain Current Model: ϕ_s-based vs Q_i-based

$$I_{ds} = -\mu_0 \frac{W}{L}\int_{\phi_s(0)}^{\phi_s(L)} Q_i(\phi_s)\frac{dV_{cb}}{d\phi_s}d\phi_s \quad \xleftarrow{\phi_s} \quad I_{ds} = \mu_0 \frac{W}{L}\int_{V_{sb}}^{V_{db}}(-Q_i)dV_{cb} \quad \xrightarrow{Q_i} \quad I_{ds} = -\mu_0 \frac{W}{L}\int_{Q_{is}}^{Q_{id}} Q_i \frac{dV_{cb}}{dQ_i}dQ_i$$

CSM: $$Q_i = -C_{ox}\left(V_{gb} - V_{FB} - \phi_s - \Upsilon\sqrt{\phi_s}\right)$$

S–P: $(\phi_s > 3v_{th})$ $$V_{gb} - V_{FB} - \phi_s(y) = -\Upsilon\sqrt{\phi_s(y) + v_{th}e^{(\phi_s(y) - 2\phi_F - V_{cb}(y))/v_{th}}}$$

$$q_i \equiv -Q_i/(C_{ox}v_{th})$$
$$q_b \equiv -Q_b/(C_{ox}v_{th}) = \gamma\sqrt{\varphi_s}$$

$$q_i + q_b + \ln\frac{q_i(q_i + 2q_b)}{\gamma^2} = v_{gb} - v_{fb} - 2\varphi_F - v_{cb}$$

Q_i linearization

UCCM: $$\frac{q_i}{n_q} + \ln\left(\frac{q_i}{n_q}\right) = \frac{v_{gb} - v_{FB}}{n_q} - v_{cb} - n_1$$

$$v_{gb} = V_{gb}/v_{th}$$
$$v_{FB} = V_{FB}/v_{th}$$
$$\gamma = \Upsilon/\sqrt{v_{th}}$$

$$I_{ds} = I_{drift} + I_{diff}$$

$$I_{diff}(y) = W\mu_s(y)v_{th}\frac{dQ_i(y)}{dy}$$

$$I_{drift}(y) = -W\mu_s(y)Q_i(y)\frac{d\phi_s}{dy}$$

$$I_{diff} = \mu_0 \frac{W}{L}v_{th}\int_{\phi_{s0}}^{\phi_{sL}} dQ_i$$

$$I_{drift} = -\mu_0 \frac{W}{L}\int_{\phi_{s0}}^{\phi_{sL}} Q_i d\phi_s$$

$$= \mu_0 C_{ox}\frac{W}{L}v_{th}\left[(\phi_{sL} - \phi_{s0}) + \Upsilon\left(\phi_{sL}^{1/2} - \phi_{s0}^{1/2}\right)\right]$$

$$= \mu_0 C_{ox}\frac{W}{L}\left[(V_{gb} - V_{FB})(\phi_{sL} - \phi_{s0}) - \frac{1}{2}\left(\phi_{sL}^2 - \phi_{s0}^2\right) - \frac{2}{3}\Upsilon\left(\phi_{sL}^{3/2} - \phi_{s0}^{3/2}\right)\right]$$

From S–P:

$$\phi_{s0} = \phi_s(0), \quad V_{cb}(0) = V_{sb}$$
$$\phi_{sL} = \phi_s(L), \quad V_{cb}(L) = V_{db}$$

Q_i-based Current Model

CSM / S–P:

$$Q_i = -C_{ox}\left(V_{gb} - V_{FB} - \phi_s\right) + \Upsilon C_{ox}\sqrt{\phi_s}$$
$$= -\Upsilon C_{ox}\sqrt{\phi_s + v_{th} e^{(\phi_s(y) - 2\phi_F - V_{cb}(y))/v_{th}}} + \Upsilon C_{ox}\sqrt{\phi_s}$$

$$I_{ds} = \mu_0 \frac{W}{L}\int_{V_{sb}}^{V_{db}} (-Q_i) dV_{cb} = -\mu_0 \frac{W}{L}\int_{Q_{is}}^{Q_{id}} Q_i \frac{dV_{cb}}{dQ_i} dQ_i$$

$$I_{ds} = -\mu_0 \frac{W}{L}\int_{Q_{is}}^{Q_{id}} Q_i \left(\frac{1}{n_q C_{ox}} - \frac{v_{th}}{Q_i}\right) dQ_i$$
$$= -\mu_0 \frac{W}{L}\frac{Q_{id}^2 - Q_{is}^2}{2 n_q C_{ox}} + \mu_0 \frac{W}{L} v_{th}\left(Q_{id} - Q_{is}\right)$$

Q_i linearization: $qn_s = -n_q C_{ox}\left(\phi_s - \phi_{sa}\right)$

$$\frac{dQ_i}{dy} = n_q C_{ox}\frac{d\phi_s}{dy}$$

$$n_q = 1 + C_b / C_{ox} \qquad C_b = \frac{\Upsilon C_{ox}}{2\sqrt{\phi_{sa}}}$$

$$Q_{ip} = Q_i\big|_{\phi_s = \phi_p}$$

From UCCM:

$$q_s \equiv -Q_{is} / \left(C_{ox} v_{th}\right)$$
$$q_d \equiv -Q_{id} / \left(C_{ox} v_{th}\right)$$

$$I_{ds} = \mu_0 C_{ox}\frac{W}{L} v_{th}^2 \left(\frac{q_s^2 - q_d^2}{2 n_q} + q_s - q_d\right)$$

CSM / D+D: $Q_i(y) \equiv q\int_0^{\infty} n(x, y) dx \equiv qn_s(y)$

$$I_{ds} = I_{drift} + I_{diff}$$

$$I_{ds}(y) = -W \mu_s Q_i \frac{dV_{cb}}{dy}$$
$$= -W \mu_s Q_i \left(\frac{d\phi_s}{dy} - \frac{v_{th}}{Q_i}\frac{dQ_i}{dy}\right)$$
$$= -W \mu_s Q_i \left(\frac{1}{n_q C_{ox}} - \frac{v_{th}}{Q_i}\right)\frac{dQ_i}{dy}$$

UCCM:

$$\left(\frac{1}{n_q C_{ox}} - \frac{v_{th}}{Q_i}\right) dQ_i = dV_{cb}$$

$$\frac{Q_{ip} - Q_i}{n_q C_{ox}} + v_{th}\ln\left(\frac{Q_i}{Q_{ip}}\right) = V_p - V_{cb}$$

$$\frac{I_{ds}}{I_s} = \left(\frac{q_s^2}{2 n_q} + q_s\right) - \left(\frac{q_d^2}{2 n_q} + q_d\right)$$

$$\left(I_s \equiv \beta_n v_{th}^2, \; \beta_n \equiv \mu_0 C_{ox} W / L\right)$$

$$I_{ds} = I_f - I_r$$

V_t-based Model: Linear (Drift) Current

Source-referenced threshold condition ("pinned" surface potential): $\phi_s(0) = 2\phi_F$

$$\phi_s(y) = \phi_s(0) + V_{cb}(y) = 2\phi_F + V_{sb} + V(y)$$

$$V_{cb}(y) = V_{sb} + V(y),\ (0 \le V \le V_{ds})$$

Bulk-charge linearization:

$$Q_b = -\Upsilon C_{ox}\sqrt{\phi_s} = -\Upsilon C_{ox}\sqrt{2\phi_F + V_{sb} + V(y)}$$

$$\approx -\Upsilon C_{ox}\left(\sqrt{2\phi_F + V_{sb}} + \frac{V(y)}{2\sqrt{2\phi_F + V_{sb}}}\right)$$

Threshold voltage: $V_t = V_{FB} + 2\phi_F + \Upsilon\sqrt{2\phi_F + V_{sb}}$

Bulk-charge factor: $A_b = 1 + \dfrac{\Upsilon}{2\sqrt{2\phi_F + V_{sb}}}$

(For fixed bulk-charge: $A_b = 1$)

$$Q_i = -C_{ox}\left(V_{gb} - V_{FB} - \phi_s\right) - Q_b$$

$$= -C_{ox}\left[V_{gb} - V_{FB} - 2\phi_F - V_{sb} - V(y) - \Upsilon\left(\sqrt{2\phi_F + V_{sb}} + \frac{V(y)}{2\sqrt{2\phi_F + V_{sb}}}\right)\right]$$

$$= -C_{ox}\left[V_{gs} - V_t - A_b V(y)\right]$$

$$I_{ds}(y) = -W\mu_s Q_i(y)\, d\phi_s/dy = WQ_i(y)v,\ \ v = \mu_s E_y,\ \ E_y = -d\phi_s/dy = -dV/dy$$

Linear (drift) current:

($V_{gs} > V_t$)

$$I_{ds} = I_{drift} = \mu_0 \frac{W}{L}\int_0^{V_{ds}} (-Q_i)\,dV = \mu_0 C_{ox}\frac{W}{L}\left(V_{gs} - V_t - \frac{1}{2}A_b V_{ds}\right)V_{ds}$$

V_t-based Model: Subthreshold (Diffusion) Current

Subthreshold surface potential: $\phi_s(y) = \phi_{dd}$

$$Q_b = -\Upsilon C_{ox}\sqrt{\phi_{dd}}$$

$$V_{gb} - V_{FB} - \phi_{dd} = -Q_b / C_{ox}$$

$$\phi_{dd} = \left(-\frac{\Upsilon}{2} + \sqrt{\frac{\Upsilon^2}{4} + V_{gb} - V_{FB}}\right)^2$$

$$I_{ds} = \mu_0 v_{th}\frac{W}{L}\int_{Q_{is}}^{Q_{id}} dQ_i = \mu_0 v_{th}\frac{W}{L}\left(Q_{id} - Q_{is}\right)$$

$$Q_{is} = Q_i\big|_{V_{cb}=V_{sb}} \qquad Q_{id} = Q_i\big|_{V_{cb}=V_{db}}$$

ϕ_s**-based**

$$I_{ds} = \mu_0 C_d v_{th}^2 \frac{W}{L} e^{(\phi_{dd} - 2\phi_F - V_{sb})/v_{th}}\left(1 - e^{-V_{ds}/v_{th}}\right)$$

CSM / S–P:

$$Q_i = -\Upsilon C_{ox}\sqrt{\phi_{dd} + v_{th} e^{(\phi_{dd} - 2\phi_F - V_{cb}(y))/v_{th}}} - Q_b$$

$$Q_i = -\Upsilon C_{ox}\sqrt{\phi_{dd}}\sqrt{1 + \frac{v_{th}}{\phi_{dd}} e^{(\phi_{dd} - 2\phi_F - V_{cb})/v_{th}}} + \Upsilon C_{ox}\sqrt{\phi_{dd}}$$

$$\approx -\Upsilon C_{ox}\sqrt{\phi_{dd}}\left(1 + \frac{v_{th}}{2\phi_{dd}} e^{(\phi_{dd} - 2\phi_F - V_{cb})/v_{th}}\right) + \Upsilon C_{ox}\sqrt{\phi_{dd}}$$

$$= -\Upsilon C_{ox}\frac{v_{th}}{2\sqrt{\phi_{dd}}} e^{(\phi_{dd} - 2\phi_F - V_{cb})/v_{th}}$$

$$C_d(\phi_{dd}) = \frac{\Upsilon C_{ox}}{2\sqrt{\phi_{dd}}} = \sqrt{\frac{q\varepsilon_{Si} N_A}{2\phi_{dd}}} = \frac{\varepsilon_{Si}}{X_{dm}} \qquad X_{dm} = \sqrt{\frac{2\varepsilon_{Si}\phi_{dd}}{qN_A}}$$

$$Q_b(\phi_{dd}) \approx Q_b(\phi_{s0}) + (\phi_{dd} - \phi_{s0})\frac{\partial Q_b}{\partial \phi_{dd}} \qquad -\frac{\partial Q_b}{\partial \phi_{dd}} \equiv C_d$$

$$V_{gb} = V_{FB} + \phi_{dd} - \left[\frac{Q_b(\phi_{s0})}{C_{ox}} - (\phi_{dd} - \phi_{s0})\frac{C_d}{C_{ox}}\right]$$

$$n = 1 + C_d / C_{ox} = 1 + \frac{\Upsilon}{2\sqrt{2\phi_F + V_{sb}}}$$

$$\phi_{dd} - 2\phi_F - V_{sb} = \left(V_{gs} - V_t\right)/n \qquad \left(\phi_{s0} = 2\phi_F + V_{sb}\right)$$

Subthreshold (diffusion) current:

($V_{gs} < V_t$) $\quad \left(V_t' = V_t + V_{off},\ \phi_{s0}' = 1.5\phi_F + V_{sb}\right)$

$$I_{ds} = I_{diff} = \mu_0 C_{ox}\frac{W}{L} v_{th}^2 \frac{C_d'}{C_{ox}} e^{(V_{gs} - V_t')/(n' v_{th})}\left(1 - e^{-V_{ds}/v_{th}}\right)$$

Velocity Saturation and Saturation Current

- **Vertical-field mobility** (empirical) $Q_i = C_{ox}\left(V_{gs} - V_t\right)$ $Q_b \approx C_{ox} V_t$

$$\mu_n = \frac{\mu_0}{1 + \left(E_{eff} / E_{crit}\right)^{\theta}}$$

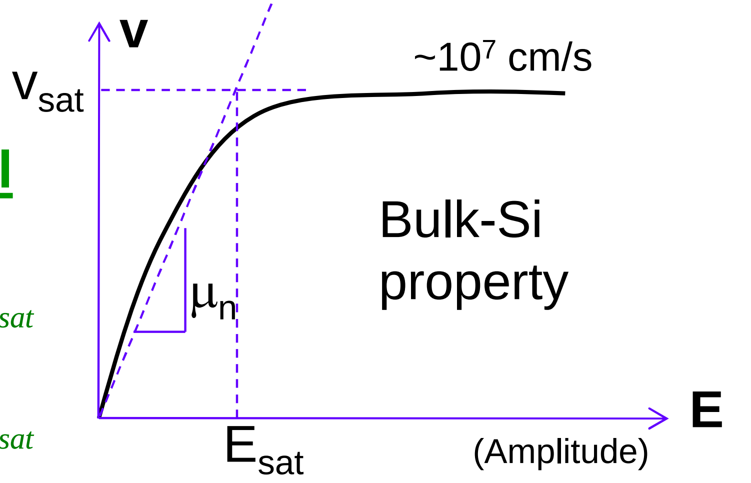
$$E_{eff} = \left(Q_b + \eta Q_i\right) / \varepsilon_{Si} = \frac{V_{gs} + V_t}{6T_{ox}} \quad (\eta = 0.5)$$

- **Saturation field**

$$v_{sat} = \frac{\mu_n E_{sat}}{1 + E_{sat}/E_{sat}} \rightarrow E_{sat} = \frac{2v_{sat}}{\mu_n}$$

Piecewise model

$$v = \begin{cases} \dfrac{\mu_n E}{1 + E/E_{sat}} & E < E_{sat} \\ v_{sat} & E \ge E_{sat} \end{cases}$$



- **Lateral-field mobility**

$$\mu_{eff} = \frac{\mu_n}{1 + V_{ds} / \left(E_{sat} L_{eff}\right)}$$

$$A_b = 1 + \frac{\Upsilon}{2\sqrt{2\phi_F + V_{sb}}}$$

- **Linear current**

$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L}\left(V_{gs} - V_t - \frac{1}{2} A_b V_{ds}\right) V_{ds} \quad (1)$$

- **Saturation current** CSM: $Q_i = -C_{ox}\left[V_{gs} - V_t - A_b V(y)\right]$

$$I_{dsat} = -W v_{sat} Q_{sat} = W v_{sat} C_{ox}\left(V_{gs} - V_t - A_b V_{dsat}\right) \quad (2)$$

(1)(V_{dsat}) = (2): $$V_{dsat} = \frac{E_{sat} L_{eff}\left(V_{gs} - V_t\right)}{V_{gs} - V_t + A_b E_{sat} L_{eff}} \quad (3)$$

(3) → (2):

$$I_{dsat} = W v_{sat} C_{ox} \frac{\left(V_{gs} - V_t\right)^2}{V_{gs} - V_t + A_b E_{sat} L_{eff}} \xrightarrow{L \to 0} \propto \left(V_{gs} - V_t\right)$$

Linear!

Charge-Sharing Model: V_t "Roll-Off"

$$L_{eff} = L_g - 2\sigma X_j$$

❑ Charge-sharing model ("triangle")

➢ ***Without charge-sharing***

$$V_t = V_{FB} - Q_{bm}/C_{ox} + 2\phi_F$$

$$C_{ox} = \varepsilon_{ox}/T_{ox} \qquad Q_{bm} = -qN_A X_{dm}$$

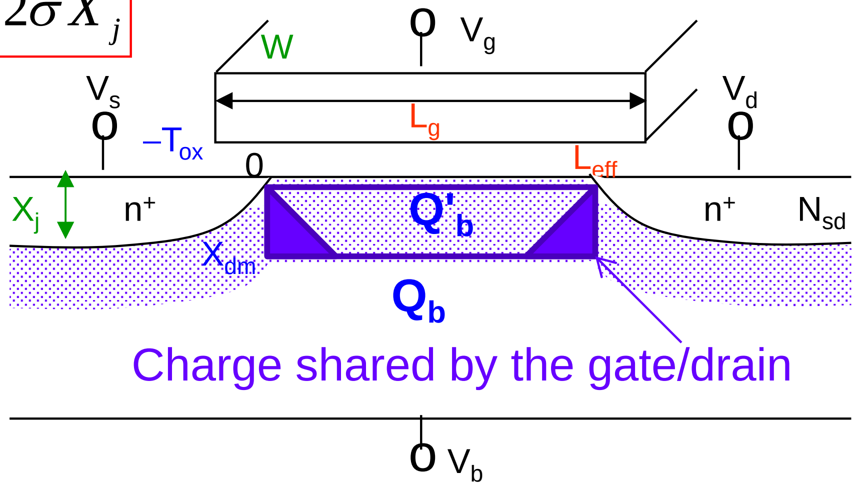
➢ ***With charge-sharing***

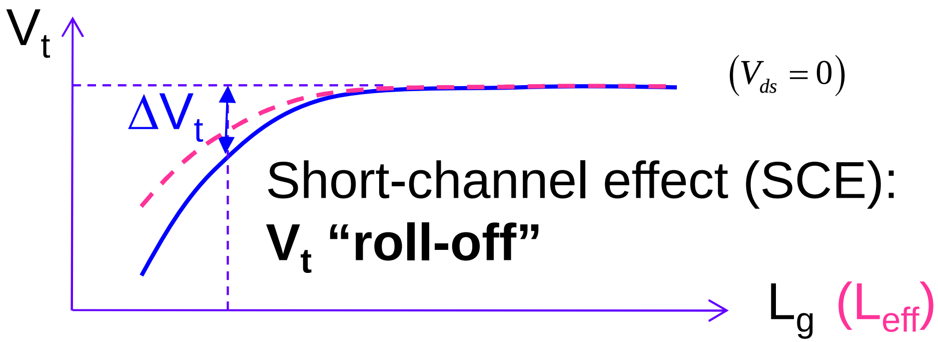
$$X_{dm} = \sqrt{2\varepsilon_{Si}\left(2\phi_F + V_{sb}\right)/qN_A}$$

$$V_t' = V_{FB} - Q_{bm}'/C_{ox} + 2\phi_F$$

$$\Delta V_t \equiv V_t - V_t' = -\frac{Q_{bm}}{C_{ox}}\left(1 - \frac{Q_{bm}'}{Q_{bm}}\right) = -\frac{Q_{bm}}{C_{ox}}\frac{X_{dm}}{L_{eff}}$$

$$= \frac{qN_A X_{dm}}{\varepsilon_{ox}/T_{ox}}\frac{X_{dm}}{L_{eff}} = \frac{2\varepsilon_{Si}}{\varepsilon_{ox}}\frac{\left(2\phi_F + V_{sb}\right)T_{ox}}{L_g - 2\sigma X_j}$$





<u>Simple "Triangle" Model</u>

Total bulk charge:

$$Q_B' = -qN_A W\left(L_{eff} X_d - X_d^2\right)$$

$$Q_B = -qN_A W\left(L_{eff} X_d\right)$$

Bulk charge per unit area:

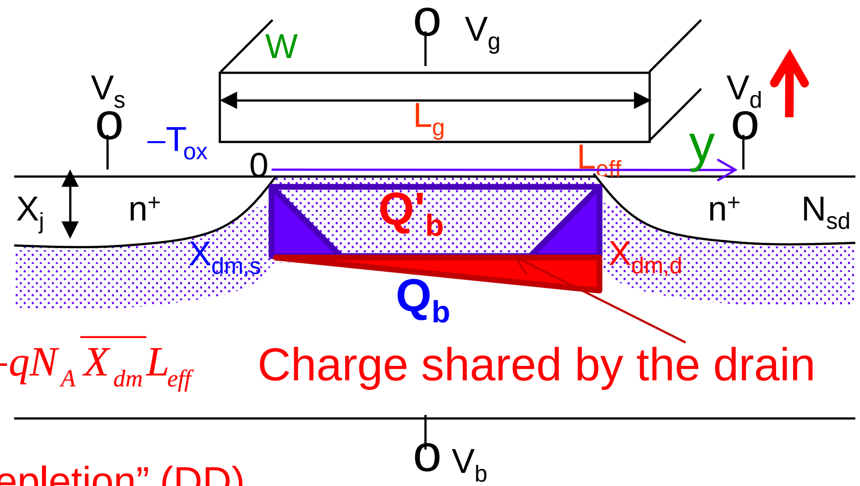
$$\therefore \frac{Q_b'}{Q_b} = \frac{Q_B'}{Q_B} = 1 - \frac{X_d}{L_{eff}}$$

Charge-Sharing Model: V_t “DIBL”

- **Charge-sharing model (“trapezoidal”)**
  - ***Source-end (linear)***: ($V_{ds} = V_{d0}$)

$$X_{dm,s} = \sqrt{2\varepsilon_{Si}\left(2\phi_F + V_{sb}\right)/qN_A}$$

Trapezoidal area = “box”



$$Q_b' = -qN_A \overline{X_{dm}} L_{eff}$$

  - ***Drain-end (saturation)***: ($V_{ds} = V_{dd}$)

$$X_{dm,d} = \sqrt{2\varepsilon_{Si}\left(2\phi_F + V_{db}\right)/qN_A}$$

“Dynamic depletion” (DD)

$$\overline{\Delta V_t}\left(V_{sb}, V_{ds}\right) = \frac{qN_A}{\varepsilon_{ox}/T_{ox}} \frac{\left(\overline{X_{dm}}\right)^2}{L_{eff}} = \frac{\varepsilon_{Si} T_{ox}}{2\varepsilon_{ox}} \frac{\left(\gamma_s\sqrt{2\phi_F + V_{sb}} + \gamma_d\sqrt{2\phi_F + V_{db}}\right)^2}{\left(L_g - 2\sigma_d X_j\right)}$$

  - ***Average depletion width***: (any V_{ds})

$$\overline{X_{dm}} \equiv \frac{X_{dm,s} + X_{dm,d}}{2} \qquad \left(V_{db} = V_{sb} + V_{ds}\right)$$

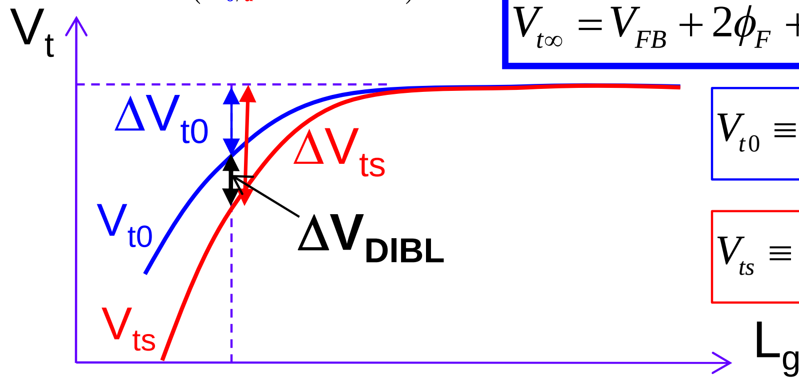
$$= \sqrt{\frac{2\varepsilon_{Si}}{qN_A}} \frac{\left(\gamma_s\sqrt{2\phi_F + V_{sb}} + \gamma_d\sqrt{2\phi_F + V_{db}}\right)}{2}$$

$$\left(\sigma_{0/d} \sim 0.6/0.7\right)$$

$$V_{t\infty} = V_{FB} + 2\phi_F + \Upsilon\sqrt{2\phi_F + V_{sb}}$$

$$V_{t0} \equiv V_{t\infty} - \overline{\Delta V_t}\Big|_{V_{ds}=V_{d0}}$$

$$V_{ts} \equiv V_{t\infty} - \overline{\Delta V_t}\Big|_{V_{ds}=V_{dd}}$$



- **DIBL: Drain-Induced Barrier Lowering**

$$\Delta V_{DIBL}\left(L_g\right) \equiv V_{t0}\left(L_g\right) - V_{ts}\left(L_g\right)$$

Reverse Short-Channel Effect: V_t "Roll-Up" & "Halo"

❑ Empirical RSCE model ("halo")

$$N_{eff} = N_A + \frac{N_{pile}}{\cosh\left(L_{eff}/2l_\beta\right)}$$

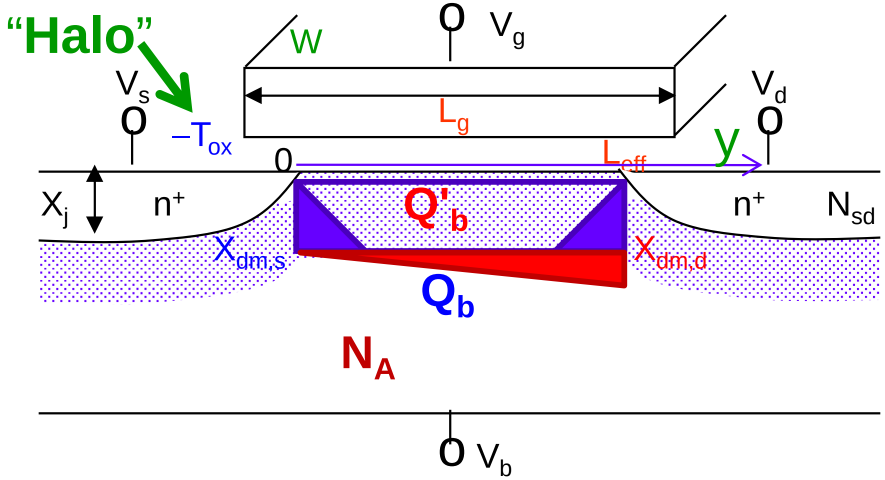
- ***Halo pile-up***: (κ)

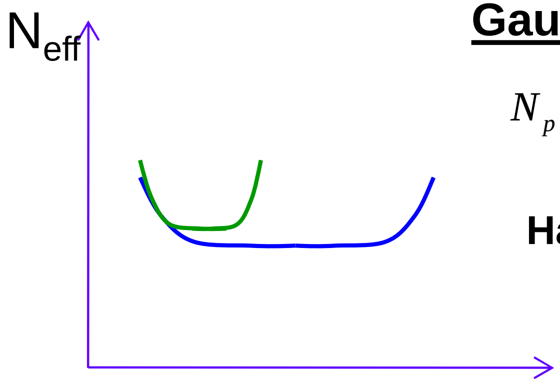
$$N_{pile} = \kappa N_A$$

- ***Halo lateral spread***: (β)

$$l_\beta = \beta\left(2\phi_F + V_{sb}\right)^{0.25} \qquad \phi_F = (kT/q)\ln\left(N_A/n_i\right)$$

- ***Replacing all previous N_A by N_{eff}***

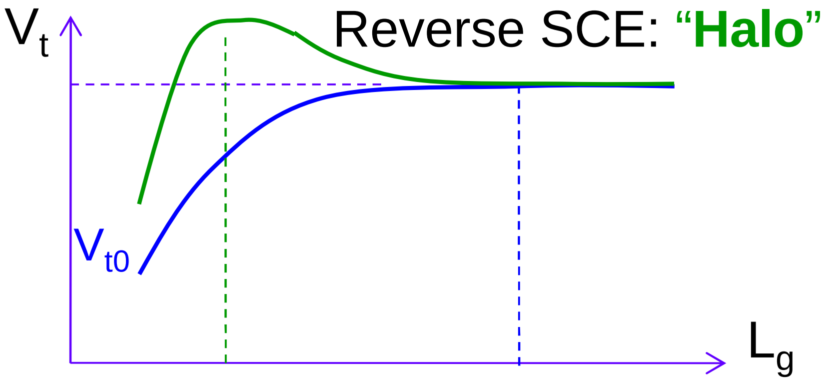




Gaussian halo model

$$N_p(y) = N_{pile} e^{-\left[(y-l_\mu)/l_\beta\right]^2}$$

Halo dose, tilt, energy



$$N_{eff} = \frac{\int_0^{L_{eff}} N_p(y)\,dy}{L_{eff}} = N_A + \frac{\sqrt{\pi}N_{pile}}{L_{eff}/l_\beta}\left[erf\left(\frac{L_{eff}-l_\mu}{l_\beta}\right) + erf\left(\frac{l_\mu}{l_\beta}\right)\right]$$

Summary of Important (Simple) Equations

❑ Effective body doping and related equations

➢ ***Halo doping*** $\phi_F = v_{th} \ln(N_A/n_i)$

$N_{pile} = \kappa N_A$ $l_\beta = \beta(2\phi_F + V_{sb})^{0.25}$

$$N_{eff} = N_A + \frac{N_{pile}}{\cosh(L_{eff}/2l_\beta)}$$

$v_{th} = kT/q = 0.0259V$

$\phi_F = v_{th} \ln\left(\frac{N_{eff}}{n_i}\right)$ $\Upsilon = \frac{\sqrt{2q\varepsilon_{Si} N_{eff}}}{C_{ox}}$

➢ ***Physical quantities*** $L_{eff} = L_g - 2\sigma_d X_j$

$C_{ox} = \varepsilon_{ox}/T_{ox}$ $\varepsilon_{ox} = \kappa_{ox}\varepsilon_o$ $\varepsilon_{Si} = \kappa_{Si}\varepsilon_o$

$V_{FB} \equiv \phi_{MS} - Q_{ox}/C_{ox} = \Phi_M - (\chi + E_g/2 + \phi_F) - qN_{ss}/C_{ox}$

$C_d = \varepsilon_{Si}/X_{dm}$ $n = 1 + C_d/C_{ox}$ **(Physical constants)**

$$X_{dm} = \sqrt{\frac{2\varepsilon_{Si}(2\phi_F + V_{sb})}{qN_{eff}}}$$

Physical parameters:
$L_g, T_{ox}, X_j, \Phi_M, N_{ss}, T$

❑ Threshold voltage

➢ ***Long-channel (1D theoretical model)***

$$V_{t\infty} \equiv V_{gs}\big|_{\psi_s = 2\phi_F + V_{sb}} = V_{FB} + 2\phi_F + \Upsilon\sqrt{2\phi_F + V_{sb}}$$

➢ ***Linear V_{t0}:*** $V_{t0} \equiv V_{t\infty} - \overline{\Delta V_t}\big|_{V_{ds}=V_{d0},\, V_{sb}=0}$

➢ ***Saturation V_{ts}:*** $V_{ts} \equiv V_{t\infty} - \overline{\Delta V_t}\big|_{V_{ds}=V_{dd},\, V_{sb}=0}$

➢ ***Any channel-length and body/drain-bias***

$$V_t(V_{sb}, V_{ds}) \equiv V_{t\infty} - \overline{\Delta V_t} = V_{t\infty}(N_{eff}, T_{ox}) - \frac{\varepsilon_{Si} T_{ox}}{2\varepsilon_{ox}(L_g - 2\sigma_d X_j)}\left(\gamma_s\sqrt{2\phi_F + V_{sb}} + \gamma_d\sqrt{2\phi_F + V_{sb} + V_{ds}}\right)^2$$

Summary of Important (Simple) Equations

(TCAD: W = 1 μm)

❑ Drain current

➢ *Bulk-charge factor*

$$A_b = 1 + \frac{\Upsilon}{2\sqrt{2\phi_F + V_{sb}}}$$

Fitting parameters:

$N_A, \kappa, \beta, \sigma_d, \gamma_{s/d}$

$v_{sat}, \mu_0, E_{crit}, \theta, \delta$

$$E_{eff} = \frac{V_{gs} + V_t}{6T_{ox}}$$

$$E_{sat} = \frac{2v_{sat}}{\mu_n}$$

➢ *Linear*

$$I_{dlin}(V_{gs}, V_{sb}, V_{ds}) = \mu_{eff} C_{ox} \frac{W}{L_{eff}} \left(V_{gs} - V_t - \frac{1}{2} A_b V_{ds} \right) V_{ds}$$

$$\begin{cases} I_{dlin}(V_{gs}, 0, V_{d0}) \\ I_{dlin}(V_{dd}, 0, V_{ds}) \end{cases}$$

➢ *Saturation*

$$I_{dsat}(V_{gs}, V_{sb}, V_{ds}) = W v_{sat} C_{ox} \frac{(V_{gs} - V_t)^2}{V_{gs} - V_t + \delta A_b E_{sat} L_{eff}}$$

$$\begin{cases} I_{dsat}(V_{gs}, 0, V_{dd}) \\ I_{dsat}(V_{dd}, 0, V_{ds}) \end{cases}$$

➢ *Subthreshold*

$$I_{dsub}(V_{gs}, V_{sb}, V_{ds}) = \mu_n C_d v_{th}^2 \frac{W}{L_{eff}} e^{(V_{gs} - V_t)/(n v_{th})} \left(1 - e^{-V_{ds}/v_{th}}\right)$$

$$\begin{cases} I_{dsub0}(V_{gs}, 0, V_{d0}) \\ I_{dsubs}(V_{gs}, 0, V_{dd}) \end{cases}$$

❑ Mobility

➢ *Vertical-field*

$$\mu_n = \frac{\mu_0}{1 + \left(E_{eff} / E_{crit} \right)^{\theta}}$$

➢ *Lateral-field*

$$\mu_{eff} = \frac{\mu_n}{1 + V_{ds} / \left(E_{sat} L_{eff} \right)}$$

❑ *Drive current I_{on}:*

$$I_{on} \equiv I_{dsat}(V_{dd}, 0, V_{dd})$$

❑ *Leakage current I_{off}:*

$$I_{off} \equiv I_{dsub}(0, 0, V_{dd})$$