Concise Current Source Implementation for Efficient 3-D ADI-FDTD Method

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***Abstract*—A concise current source implementation is presented for the unconditionally stable three-dimensional (3-D) alternating direction implicit finite-difference time-domain (ADI-FDTD) method. Unlike the conventional implicit symmetric source scheme applied in both updating procedures, the new implementation involves implicit current formulation in the first procedure only. Note however that the resultant accuracy does not deteriorate but remains to be identical as before. This is achieved using our recent efficient algorithm for the 3-D ADI-FDTD method. Moreover, the current formulations applied in both procedures therein are simplified, along with further reduction of the floating point operations count for the main iterations. Analytical validation of the equivalence among conventional implicit symmetric source scheme and our efficient algorithms with alternative current implementations are discussed. Numerical results provide closer scrutiny of the asymmetry errors, which may still exist even with symmetric source implementation.**

***Index Terms*—Alternating direction implicit finite-difference time-domain (ADI-FDTD) method, unconditionally stable method, computational electromagnetics.**

I. Introduction

There have been several investigations on the excitations and current source implementations for the unconditionally stable alternating direction implicit finite-difference time-domain (ADI-FDTD) method [1]-[7]. Most researchers have agreed that the current source excitation function should be evaluated at discrete time step $n + 1/2$ rather than at other time steps, e.g. $n + 1/4$ and/or $n + 3/4$. Furthermore, the current terms are to be embedded within the implicit (tridiagonal) system instead of being incorporated explicitly. While the ADI-FDTD algorithm comprises two updating procedures, it is found to be more accurate if the excitations are to be applied in both procedures and not just in the first procedure only. Upon using such implicit symmetric source scheme in both procedures, the ADI-FDTD method has been demonstrated to be free of asymmetry error up to the numerical noise level [7].

In this letter, we present a concise alternative current implementation for the source-incorporated ADI-FDTD method. Unlike the previous implicit symmetric source scheme [4]-[7], our new implementation involves implicit current formulation in the *first* procedure only. Note however that the resultant accuracy does not deteriorate but remains to be identical as before. This is made possible using our recent efficient algorithm for the 3-D ADI-FDTD method [8]. Moreover, the current formulations applied in *both* procedures therein are simplified, along with further reduction of the floating point operations (flops) count for the main iterations. Analytical validation of the equivalence among conventional implicit symmetric source scheme and our efficient algorithms with alternative current implementations are discussed. Numerical results provide closer scrutiny of the asymmetry errors, which may still exist even with symmetric source implementation.

Manuscript received

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Digital Object Identifier

II. Concise Current Source Implementation

In this section, we describe the concise implementation of current sources for efficient 3-D ADI-FDTD method. Both electric (J) and magnetic (M) current sources are considered in a lossless isotropic medium with permittivity ϵ and permeability μ. For convenience, we define the notations

$$b = \frac{\Delta t}{2\epsilon}, \quad d = \frac{\Delta t}{2\mu} \tag{1}$$

and ∂_x, ∂_y, ∂_z denote the spatial difference operators for the first derivatives along x, y, z directions respectively. To make the algorithm efficient, we also introduce the tilded field ($\tilde{E}$, $\tilde{H}$) and auxiliary ($\tilde{e}$, $\tilde{h}$) variables. [The former will be related directly to the physical untilded field variables (E, H) later.] The complete algorithm involves two procedures comprising implicit and explicit (including auxiliary) updatings as follows:
(First procedure from n to $n + 1/2$)

$$\tilde{e}^n_\xi = \tilde{E}^n_\xi - \tilde{e}^{n-1/2}_\xi, \quad \xi = x, y, z \tag{2}$$

$$\tilde{h}^n_\xi = \tilde{H}^n_\xi - \tilde{h}^{n-1/2}_\xi, \quad \xi = x, y, z \tag{3}$$

$$\frac{1}{2}\tilde{E}^{n+1/2}_x - \frac{bd}{2}\partial^2_y \tilde{E}^{n+1/2}_x = \tilde{e}^n_x + b\partial_y \tilde{h}^n_z - bJ^{n+1/2}_x - bd\partial_y M^{n+1/2}_z \tag{4a}$$

$$\frac{1}{2}\tilde{E}^{n+1/2}_y - \frac{bd}{2}\partial^2_z \tilde{E}^{n+1/2}_y = \tilde{e}^n_y + b\partial_z \tilde{h}^n_x - bJ^{n+1/2}_y - bd\partial_z M^{n+1/2}_x \tag{4b}$$

$$\frac{1}{2}\tilde{E}^{n+1/2}_z - \frac{bd}{2}\partial^2_x \tilde{E}^{n+1/2}_z = \tilde{e}^n_z + b\partial_x \tilde{h}^n_y - bJ^{n+1/2}_z - bd\partial_x M^{n+1/2}_y \tag{4c}$$

$$\tilde{H}^{n+1/2}_x = 2\tilde{h}^n_x + d\partial_z \tilde{E}^{n+1/2}_y - 2dM^{n+1/2}_x \tag{5a}$$

$$\tilde{H}^{n+1/2}_y = 2\tilde{h}^n_y + d\partial_x \tilde{E}^{n+1/2}_z - 2dM^{n+1/2}_y \tag{5b}$$

$$\tilde{H}^{n+1/2}_z = 2\tilde{h}^n_z + d\partial_y \tilde{E}^{n+1/2}_x - 2dM^{n+1/2}_z. \tag{5c}$$

E. L. Tan, “Concise Current Source Implementation for Efficient 3-D ADI-FDTD Method,” IEEE Microwave and Wireless Components Letters, Vol. 17, No. 11, pp. 748-750, November 2007. DOI: 10.1109/LMWC.2007.908037

(Second procedure from $n+1/2$ to $n+1$)

$$\tilde{e}_\xi^{n+1/2} = \tilde{E}_\xi^{n+1/2} - \tilde{e}_\xi^{n}, \quad \xi = x, y, z \tag{6}$$

$$\tilde{h}_\xi^{n+1/2} = \tilde{H}_\xi^{n+1/2} - \tilde{h}_\xi^{n}, \quad \xi = x, y, z \tag{7}$$

$$\frac{1}{2}\tilde{E}_x^{n+1} - \frac{bd}{2}\partial_z^2\tilde{E}_x^{n+1} = \tilde{e}_x^{n+1/2} - b\partial_z\tilde{h}_y^{n+1/2} \tag{8a}$$

$$\frac{1}{2}\tilde{E}_y^{n+1} - \frac{bd}{2}\partial_x^2\tilde{E}_y^{n+1} = \tilde{e}_y^{n+1/2} - b\partial_x\tilde{h}_z^{n+1/2} \tag{8b}$$

$$\frac{1}{2}\tilde{E}_z^{n+1} - \frac{bd}{2}\partial_y^2\tilde{E}_z^{n+1} = \tilde{e}_z^{n+1/2} - b\partial_y\tilde{h}_x^{n+1/2} \tag{8c}$$

$$\tilde{H}_x^{n+1} = 2\tilde{h}_x^{n+1/2} - d\partial_y\tilde{E}_z^{n+1} \tag{9a}$$

$$\tilde{H}_y^{n+1} = 2\tilde{h}_y^{n+1/2} - d\partial_z\tilde{E}_x^{n+1} \tag{9b}$$

$$\tilde{H}_z^{n+1} = 2\tilde{h}_z^{n+1/2} - d\partial_x\tilde{E}_y^{n+1}. \tag{9c}$$

Equations (2)-(9) constitute the main iterations of the efficient source-incorporated 3-D ADI-FDTD method. To deduce the physical fields for output, we simply relate the tilded and untilded variables (valid at integer and half time steps) as

$$E_\xi = \frac{1}{2}\tilde{E}_\xi, \quad \xi = x, y, z \tag{10}$$

$$H_\xi = \frac{1}{2}\tilde{H}_\xi, \quad \xi = x, y, z. \tag{11}$$

Note that such relations are to be invoked only for certain field components of interest at few desired observation points, or they can be omitted via proper initial normalization. When the intermediate magnetic field variables are not to be output (as is the usual case), the explicit updating equations (5) and (7) can be combined to read

$$\tilde{h}_x^{n+1/2} = \tilde{h}_x^{n} + d\partial_z\tilde{E}_y^{n+1/2} - 2dM_x^{n+1/2} \tag{12a}$$

$$\tilde{h}_y^{n+1/2} = \tilde{h}_y^{n} + d\partial_x\tilde{E}_z^{n+1/2} - 2dM_y^{n+1/2} \tag{12b}$$

$$\tilde{h}_z^{n+1/2} = \tilde{h}_z^{n} + d\partial_y\tilde{E}_x^{n+1/2} - 2dM_z^{n+1/2}. \tag{12c}$$

Similar combination of (9) and (3) (at $n+1$ time step) leads to

$$\tilde{h}_x^{n+1} = \tilde{h}_x^{n+1/2} - d\partial_y\tilde{E}_z^{n+1} \tag{13a}$$

$$\tilde{h}_y^{n+1} = \tilde{h}_y^{n+1/2} - d\partial_z\tilde{E}_x^{n+1} \tag{13b}$$

$$\tilde{h}_z^{n+1} = \tilde{h}_z^{n+1/2} - d\partial_x\tilde{E}_y^{n+1}. \tag{13c}$$

Equations (12)-(13) make the main iterations even simpler and more efficient. In particular, when the difference operators above represent specifically the second-order central-differencing operators on Yee cells, the total flops count for the right-hand sides of main iterations (excluding source) is found to be only 42. This flops count is much less than that (102) of the original 3-D ADI-FDTD method! To recover the magnetic fields occasionally for output, one just needs to call upon, instead of (11),

$$H_\xi^{n+1} = \frac{1}{2}\left(\tilde{h}_\xi^{n+1} + \tilde{h}_\xi^{n+1}\right), \quad \xi = x, y, z. \tag{14}$$

Other implementation details may be referred to [8] including for-looping, tridiagonal system solving, memory reuse etc. Furthermore, there are other possibilities of implementations, including different difference operators, e.g. higher order [9], parameter optimized [10], etc.

III. Discussion and Numerical Result

The concise current implementation above merely involves source incorporation in the first procedure. There is no current term needed to be applied in the second procedure. Such implementation is somewhat similar to the single excitation in only the first procedure of [3], [7], but note that the source magnitude need not be doubled. Moreover, for the (electric) current source, it is incorporated within the implicit system, cf. (4). This may be termed implicit current formulation or in-matrix implementation [4], [7], in contrast to the previous explicit current formulation or out-matrix implementation [1], [3].

It is interesting to find that our concise current implementation in only the first procedure yields exactly the same accuracy as that of the implicit symmetric source scheme in both procedures [4]-[7]. In fact, both source-incorporated ADI-FDTD methods are equivalent as can be proven analytically as follows. From both (8a) and (9b) (multiplied by $1/2$) taken at one time step backward, we have

$$\tilde{e}_x^{n-1/2} = \frac{1}{2}\tilde{E}_x^{n} + \frac{b}{2}\partial_z\tilde{H}_y^{n}. \tag{15}$$

Substituting (15) into the x-component of (2) gives

$$\tilde{e}_x^{n} = \frac{1}{2}\tilde{E}_x^{n} - \frac{b}{2}\partial_z\tilde{H}_y^{n}. \tag{16}$$

From (9c) (multiplied by $1/2$) taken again at one time step backward, the z-component of (3) can be written

$$\tilde{h}_z^{n} = \frac{1}{2}\tilde{H}_z^{n} - \frac{d}{2}\partial_x\tilde{E}_y^{n}. \tag{17}$$

Applying (16)-(17) into (4a) and recognizing the relations (10)-(11), we obtain

$$E_x^{n+1/2} - bd\partial_y^2 E_x^{n+1/2} = E_x^{n} - b\partial_z H_y^{n} + b\partial_y H_z^{n} - bd\partial_y\partial_x E_y^{n} - bJ_x^{n+1/2} - bd\partial_y M_z^{n+1/2}. \tag{18}$$

This can be seen to be the implicit updating equation for the intermediate electric field x-component in the conventional ADI-FDTD method with symmetric source implementation.

To proceed further, we refer to (4a) and ∂_y-operated (5c) to get

$$\tilde{e}_x^{n} = \frac{1}{2}\tilde{E}_x^{n+1/2} - \frac{b}{2}\partial_y\tilde{H}_z^{n+1/2} + bJ_x^{n+1/2}. \tag{19}$$

Substituting (19) into the x-component of (6) gives

$$\tilde{e}_x^{n+1/2} = \frac{1}{2}\tilde{E}_x^{n+1/2} + \frac{b}{2}\partial_y\tilde{H}_z^{n+1/2} - bJ_x^{n+1/2}. \tag{20}$$

From (5b) (multiplied by $1/2$) and the y-component of (7),

$$\tilde{h}_y^{n+1/2} = \frac{1}{2}\tilde{H}_y^{n+1/2} + \frac{d}{2}\partial_x\tilde{E}_z^{n+1/2} - dM_y^{n+1/2}. \tag{21}$$

Applying (20)-(21) into (8a) while noting the tilded and untilded variables relationship, we finally arrive at

$$E_x^{n+1} - bd\partial_z^2 E_x^{n+1} = E_x^{n+1/2} + b\partial_y H_z^{n+1/2} - b\partial_z H_y^{n+1/2} - bd\partial_z\partial_x E_z^{n+1/2} - bJ_x^{n+1/2} + bd\partial_z M_y^{n+1/2}. \tag{22}$$

This correspond to the implicit updating equation for the final electric field x-component in the conventional ADI-FDTD

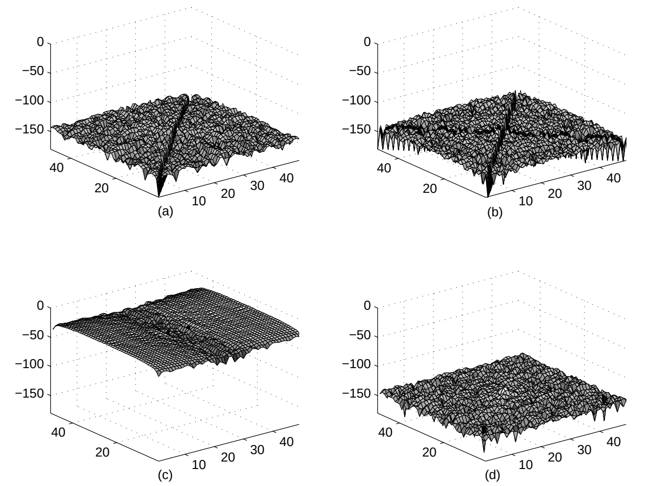

Fig. 1. Asymmetry errors (26)-(27). (a) δ_{a,E_z} for ADI-FDTD; (b) δ_{a,E_z} for Yee-FDTD; (c) δ_{a,H_t} for ADI-FDTD; (d) δ_{a,H_t} for Yee-FDTD.

method with symmetric source implementation. For the other implicit and explicit updating equations, they can also be shown to coincide in the similar manner.

With reference to our recent efficient algorithm for ADI-FDTD method [8], we note that the (electric) current sources are incorporated therein via explicit (auxiliary) formulation or out-matrix implementation in both procedures, cf. [8, eqns. (1),(5)]. This has been made simpler and convenient in the present work via implicit current formulation or in-matrix implementation in only the first procedure. Furthermore, the field and auxiliary variables defined in both algorithms can be related simply by

$$e_\xi^n = \tilde{e}_\xi^n - bJ_\xi^{n+1/2}, \quad h_\xi^n = \tilde{h}_\xi^n - dM_\xi^{n+1/2} \tag{23}$$

$$e_\xi^{n+1/2} = \tilde{e}_\xi^{n+1/2}, \quad h_\xi^{n+1/2} = \tilde{h}_\xi^{n+1/2} \tag{24}$$

$$E_\xi^* = \tilde{E}_\xi^{n+1/2}, \quad H_\xi^* = \frac{1}{2}\tilde{H}_\xi^{n+1/2}. \tag{25}$$

These relations provide further analytical validation of the equivalence among various efficient and conventional source-incorporated ADI-FDTD methods.

For numerical exemplification, let us revisit the free-space cavity of PEC walls meshed with $50 \times 50 \times 5$ uniform grids in [7]. The source J_z is located at the center of cavity (i_s, j_s, k_s) with first-order differentiated Gaussian excitation pulse. In addition to the asymmetry error defined earlier (in dB) with respect to E_z

$$\left[\delta_{a,E_z}\right]_{i,j} = 10\log_{10}\frac{|E_{z(i,j,k_s)} - E_{z(j,i,k_s)}|}{\max|E_z|}, \tag{26}$$

we also introduce the asymmetry error (dB) based on the transverse magnetic fields as

$$\left[\delta_{a,H_t}\right]_{i,j} = 10\log_{10}\frac{|H_{x(i,j,k_s)} + H_{y(j,i,k_s)}|}{\max[|H_x|, |H_y|]}. \tag{27}$$

Such definition considers that ideally the magnetic field components curling around the source at the same radial distance should have the same magnitude. Fig. 1(a) shows a snapshot of δ_{a,E_z} for our concise current implementation using $\Delta t = 6\Delta t_{\text{CFL}}$ after 100 time steps (also tested for more steps with similar findings). Very low error level can be observed, which is similar to that using the symmetric source implementation of [7]. This is expected since both source implementations are equivalent, and they differ mainly in our simplicity (first vs. both procedures) and efficiency (42 vs. 102 flops). Furthermore, our previous current implementation [8] also produces the same results, thus their equivalence is justified again via (23)-(25). On the other hand, the asymmetry error according to δ_{a,H_t} is not small as shown in Fig. 1(c). This implies that there may still exist such asymmetry even with symmetric source implementation of [7] (or equivalently our concise one above or previous one [8]). For comparison, both asymmetry errors are negligible in Yee-FDTD method as dictated by Fig. 1(b) and (d) after 600 time steps of Courant step size Δt_{CFL}.

IV. Conclusion

This letter has presented a concise alternative current implementation for the 3-D ADI-FDTD method. Unlike the previous implicit symmetric source scheme, our new implementation involves implicit current formulation in the first procedure only. Note however that the resultant accuracy does not deteriorate but remains to be identical as before. This has been achieved using our recent efficient algorithm for the 3-D ADI-FDTD method. Moreover, the current formulations applied in both procedures therein have been simplified, along with further reduction of the flops count for the main iterations. Analytical validation of the equivalence among conventional implicit symmetric source scheme and our efficient algorithms with alternative current implementations have been discussed. Numerical results have provided closer scrutiny of the asymmetry errors, which may still exist even with symmetric source implementation.

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