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Position and Force Control for Constrained Manipulator Motion: Lyapunov's Direct Method

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***Abstract*— A design procedure for simultaneous position and force control is developed, using Lyapunov's direct method, for manipulators in contact with a rigid environment that can be described by holonomic constraints. Many manipulators that interact with their environment require taking into account the effects of these constraints in the control design. The forces of constraint play a critical role in constrained motion and are, along with displacements and velocities, to be regulated at specified values. Lyapunov's direct method is used to develop a class of position and force feedback controllers. The conditions for gain selection demonstrate the importance of the constraints. Force feedback has been shown not to be mandatory for closed loop stabilization but it is useful in improving certain closed loop robustness properties.**

I. Introduction

In order to use robot manipulators in many tasks, it is necessary to control both the position and velocity of the end-effector and the constraint force between the end-effector and the environment. Recent research has focused on simultaneous position and force control [2]–[5], [7]–[10], [13], [14]. Many control schemes have been proposed. Raibert and Craig have proposed a hybrid control method [9]; Yoshikawa has extended it to dynamic hybrid control [3] and Khatib has proposed an operational space formulation [13]. But in each of these papers, no explicit conditions were developed to guarantee closed loop stability. A careful stability analysis for such closed loop systems has only recently been given. McClamroch and Wang proposed a modified computed torque controller to achieve stable position and force tracking [5]. Mills and Goldenberg applied the theory for linear descriptor system to achieve stable position and force control [8]. McClamroch and Wang have recently developed conditions for local stabilization using a linear feedback controller [7]. These methods of [5], [7] require construction of a nonlinear coordinate transformation in which the constraints are trivial.

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This paper uses Lyapunov's direct method to develop position and force control laws for constrained manipulators. This method overcomes the difficulties due to the nonlinearities of the robot dynamics and the coupling between the robot dynamics and the holonomic constraints. There is also no need to determine a nonlinear coordinate transformation as in [5], [8]. This approach represents an extension of Lyapunov's direct method to constrained robot systems.

The objective of this paper is to demonstrate that Lyapunov's direct method can form the basis for position and force control design and to present conditions that guarantee closed loop regulation. This paper also presents a case study to show that the control design method is easily applied to develop stabilizing controllers for constrained robot systems. We emphasize that the holonomic (equality) constraints are *always* assumed to be active; situations where constraints may be inactive are beyond the scope of this paper.

The organization of this paper is as follows. In Section II, constrained motion is modeled using a Lagrangian formulation and objectives are defined for position and force control. In Section III, Lyapunov's direct method is used to develop position and force controllers; conditions for gain selection which guarantee closed loop stability are also provided. A pole assignment procedure is also developed. In Section IV, closed loop robustness properties are analyzed and discussed. In Section V, concluding remarks are made.

II. Constrained Dynamics and Control Objectives

The class of robot systems with holonomic constraints has a wide range of potential applications [4], such as a robot manipulator whose end-effector is always in contact with a constraint surface, multirobots holding a common object, etc. It has been argued that these constrained robot systems can be modeled using a Lagrangian formulation expressed by a set of differential-algebraic equations [2], [4].

Let $q \in R^n$ be a generalized configuration vector and $\dot{q} \in R^n$ be a generalized velocity vector. Suppose holonomic constraints on the motion are described by the following m algebraic equations

$$\boldsymbol{\Phi}(q) = 0 \tag{1}$$

where $\boldsymbol{\Phi}^T = [\phi_1, \cdots, \phi_m]$ is at least twice differentiable. The kinetic and potential energy functions are denoted by $K(q, \dot{q}) = 1/2\dot{q}^T M(q)\dot{q}$ and $P(q)$, respectively, where $M: R^n \rightarrow R^{n\times n}$ is a symmetric positive definite inertia matrix, and the potential energy function $P: R^n \rightarrow R$ is at least twice differentiable. A Lagrangian function is defined for this constrained robot system as

$$L(q, \dot{q}) = K(q, \dot{q}) - P(q) \tag{2}$$

so that [2], [4]:

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}} L(q, \dot{q})\right] - \frac{\partial}{\partial q} L(q, \dot{q}) = J^T(q)\lambda + u \tag{3}$$

where $\lambda = (\lambda_1, \cdots, \lambda_m)^T \in R^m$ is a vector of m constraint force and $u \in R^n$ is a vector of control inputs. $J(q)$ is the Jacobian matrix of the constraint function $\boldsymbol{\Phi}(q)$. Using the definition of $L(q, \dot{q})$, the equations of constrained motion can be expressed as

$$M(q)\ddot{q} + \theta(q, \dot{q}) = J^T(q)\lambda + u \tag{4}$$

where

$$\theta(q, \dot{q}) = \left[\frac{d}{dt} M(q)\right]\dot{q} - \frac{\partial}{\partial q}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right] + \frac{\partial}{\partial q} P(q).$$

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The constrained dynamics are described by n second order differential equations (4) and m algebraic equations (1) in terms of $n+m$ variables $\boldsymbol{q}$ and λ. The m vector of variables λ determines the constraint force vector $\boldsymbol{J}^T(\boldsymbol{q})\lambda$.

In this paper, the control design objective is to obtain local simultaneous regulation of both position and constraint force to specified regulation vectors. Regulation vectors are specified by a desired constant position vector $\boldsymbol{q}_d$ and a desired constant force vector $\boldsymbol{f}_d$ that must be consistent with the given constraints in the sense that they satisfy $\boldsymbol{\Phi}(\boldsymbol{q}_d) = 0$ and $\boldsymbol{f}_d = \boldsymbol{J}^T(\boldsymbol{q}_d)\lambda_d$ for some constant vector $\lambda_d \in R^m$. The objective is to achieve local regulation at $(\boldsymbol{q}_d, \lambda_d)$ using a controller with linear feedback of displacement, velocity and constraint force. As will be seen, force feedback is not necessary to achieve force regulation. Local convergence of displacements and forces towards the desired position and force values can be guaranteed by appropriate selection of the feedback gains.

III. Position and Force Regulation Using Lyapunov's Direct Method

In the previous section, we have seen that the constrained dynamics are represented by nonlinear coupled differential-algebraic equations. The nonlinearities and coupling make it a challenging problem to perform analysis and control design for these differential-algebraic equations. For unconstrained robot systems, Takegaki and Arimoto [10], and many others, have successfully applied Lyapunov's direct method to obtain a family of simple controllers for position control. In this section, we use Lyapunov's direct method to develop controllers for simultaneous position and force control of constrained mechanical systems.

A. A Nonlinear Feedback Controller

To achieve regulation of position and force to the specified position and force vectors $(\boldsymbol{q}_d, \lambda_d)$, it is necessary to guarantee the desired values are an equilibrium of the closed loop equations. This can be achieved by the controller:

$$\boldsymbol{u} = \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}) - \frac{\partial}{\partial \boldsymbol{q}} P_d(\boldsymbol{q}) - C\dot{\boldsymbol{q}} \tag{5}$$

where $P_d(\boldsymbol{q})$ is any function that satisfies

$$\frac{\partial}{\partial \boldsymbol{q}} P_d(\boldsymbol{q}_d) = \boldsymbol{f}_d. \tag{6}$$

The closed loop equations are

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + [\frac{d}{dt}M(\boldsymbol{q})]\dot{\boldsymbol{q}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}}[\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}] + \frac{\partial}{\partial \boldsymbol{q}} P_d(\boldsymbol{q}) = \boldsymbol{J}^T(\boldsymbol{q})\lambda - C\dot{\boldsymbol{q}}. \tag{7}$$

Equation (6) requires that the gradient of $P_d(\boldsymbol{q})$ at $\boldsymbol{q}_d$ be parallel to the constraint force vector $\boldsymbol{f}_d$. The $n \times n$ matrix $\boldsymbol{C}$ is assumed to be symmetric and to satisfy $\dot{\boldsymbol{q}}^T \boldsymbol{C} \dot{\boldsymbol{q}} > 0$ for all $\dot{\boldsymbol{q}} \neq 0$ satisfying $\boldsymbol{J}(\boldsymbol{q}_d)\dot{\boldsymbol{q}} = 0$.

A Lyapunov function for the constrained system can be constructed to guarantee local stability of the equilibrium $(\boldsymbol{q}_d, 0)$. In particular, we introduce a function

$$P_{cd}(\boldsymbol{q}) = P_d(\boldsymbol{q}) - P_d(\boldsymbol{q}_d) - \boldsymbol{\Phi}^T(\boldsymbol{q})\lambda_d. \tag{8}$$

This function can be used to form a Lyapunov function for the constrained system as

$$V(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + P_{cd}(\boldsymbol{q}). \tag{9}$$

This is a local positive definite function of $(\boldsymbol{q}, \dot{\boldsymbol{q}})$ at the equilibrium, if the Hessian matrix of $P_{cd}(\boldsymbol{q})$ is positive definite at $\boldsymbol{q}_d$. Its derivative along the solution of equations (7), (1) is

$$\frac{d}{dt}V(\boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T C \dot{\boldsymbol{q}} \leq 0$$

where we have used the identities

$$\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = 0, \ and\ \dot{\boldsymbol{q}}^T[\frac{d}{dt}M(\boldsymbol{q})]\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}^T \frac{\partial}{\partial \boldsymbol{q}}[\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}].$$

These developments lead to the following theorem:

Theorem 1: The closed loop constrained system (1) and (7) is locally asymptotically stable and the position and constraint force asymptotically converge to the specified position and force vector $(\boldsymbol{q}_d, \lambda_d)$ in the sense that

$$\boldsymbol{q}(t) \rightarrow \boldsymbol{q}_d$$
$$\dot{\boldsymbol{q}}(t) \rightarrow 0$$
$$\lambda(t) \rightarrow \lambda_d$$

as $t \rightarrow \infty$ for any $(\boldsymbol{q}(0), \dot{\boldsymbol{q}}(0))$ in a neighborhood of $(\boldsymbol{q}_d, 0)$ and satisfying the constraint equations (1) if $\boldsymbol{C}$ is symmetric and positive definite and if the $n \times n$ symmetric matrix

$$N(\boldsymbol{q}_d, \lambda_d) = [\frac{\partial^2}{\partial \boldsymbol{q}^2} P_d(\boldsymbol{q}_d) - \sum_{j=1}^{m} \lambda_{jd} \frac{\partial^2}{\partial \boldsymbol{q}^2} \phi_j(\boldsymbol{q}_d)] \tag{10}$$

is positive definite.

Proof: It follows from results in [6] that there is a neighborhood of $(\boldsymbol{q}_d, 0)$ in R^{2n} such that: if $(\boldsymbol{q}(0), \dot{\boldsymbol{q}}(0))$ is in this neighborhood and satisfies

$$\boldsymbol{\Phi}(\boldsymbol{q}(0)) = 0$$
$$\boldsymbol{J}(\boldsymbol{q}(0))\dot{\boldsymbol{q}}(0) = 0.$$

There exists a unique solution $(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))$ that satisfies the initial conditions and the differential-algebraic equations (1), (7), at least locally on $[0, t_1)$.

We now examine the function $V(\boldsymbol{q}, \dot{\boldsymbol{q}})$ given by (9) along this solution. It is easily shown that

$$\frac{d}{dt}V(\boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T C \dot{\boldsymbol{q}} \leq 0$$

Since $V(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is locally positive definite near $(\boldsymbol{q}_d, 0)$, it follows that $t_1 = \infty$. Further, suppose the solution satisfies, on some interval, $\dot{\boldsymbol{q}}^T \boldsymbol{C} \dot{\boldsymbol{q}} = 0$; then $\dot{\boldsymbol{q}} = 0$ and $\ddot{\boldsymbol{q}} = 0$ since $\boldsymbol{C}$ is symmetric and positive definite. From (7) and (1):

$$\frac{\partial}{\partial \boldsymbol{q}} P_d(\boldsymbol{q}) - \boldsymbol{J}^T(\boldsymbol{q})\lambda = 0$$
$$\boldsymbol{\Phi}(\boldsymbol{q}) = 0.$$

But since $N(\boldsymbol{q}_d, \lambda_d)$ is positive definite and $\boldsymbol{J}(\boldsymbol{q}_d)$ is full rank, the $(n+m) \times (n+m)$ matrix

$$\begin{pmatrix} N(\boldsymbol{q}_d, \lambda_d) & -\boldsymbol{J}^T(\boldsymbol{q}_d) \\ \boldsymbol{J}(\boldsymbol{q}_d) & 0 \end{pmatrix}$$

is invertible. Consequently, the above equations have the unique solution $\boldsymbol{q} = \boldsymbol{q}_d$ and $\lambda = \lambda_d$ according to the implicit function theorem. Consequently, $(\boldsymbol{q}_d, 0, \lambda_d)$ is the only solution of (1) and (7) which satisfy $dV/dt = 0$. Thus according to LaSalle's Theorem [16, p.158], $\boldsymbol{q}(t) \rightarrow \boldsymbol{q}_d$, $\dot{\boldsymbol{q}}(t) \rightarrow 0$, $\lambda(t) \rightarrow \lambda_d$ as $t \rightarrow \infty$. This completes the proof. Q.E.D.

Note that $(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is not the state of the differential-algebraic equations (1), (7); however, the stated assumptions guarantee the existence of a state realization defined on a smooth $2(n-m)$ dimensional manifold [6]. A Lyapunov function approach could be developed for this state realization but that would require construction of a complicated nonlinear transformation as indicated in [5], [6]. Our contribution in this paper is to construct a Lyapunov function which is used directly for analyzing the stability properties of the differential-algebraic equations (1), (7). The specified Lyapunov function (9) is effective as a consequence of the Lagrangian form of equation (7) and the fact that the contact force in (7) is conservative.

A similar approach, using a Lyapunov function for a constrained system, has been used in [10], [15]. In [10], Takegaki and Arimoto used a nonlinear transformation to obtain a reduced order system and constructed a Lyapunov function for it. In [15], Wen and Murphy also used Lyapunov approach to constrained systems. They considered the cases where the external forces on the end effector are due to environmental dynamics, such as the mass-spring-damper system of nonrigid constraint, or mass object being pushed on a rigid constraint surface. These forces are treated as external disturbances.

The condition for the matrix in (10) clearly demonstrates the effects of the holonomic constraints on the stability of constrained systems. The Hessians of the constraint functions play a critical role in determining the local stability of the constrained system.

We can also introduce additional force feedback loops into the controller given by (5) if the force feedback affects the closed loop dynamics only in the direction normal to the constraint surface at the contact point. This can be achieved by a controller with force feedback loops given by

$$\boldsymbol{u} = \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}) - \frac{\partial}{\partial \boldsymbol{q}} P_d(\boldsymbol{q}) - C\dot{\boldsymbol{q}} + \boldsymbol{J}^T(\boldsymbol{q}) G_f(\boldsymbol{\lambda} - \boldsymbol{\lambda}_d) \tag{11}$$

where G_f is an $m \times m$ force feedback gain matrix. It is straightforward to show that the specified position and force vectors $(\boldsymbol{q}_d, \boldsymbol{\lambda}_d)$ can be asymptotically approached by using the same Lyapunov function (9).

B. An Affine Linear Feedback Controller

The controller (5) is in general a nonlinear feedback controller. The conditions in Theorem 1 serve as general guidelines for control design. In the following, we choose a particular function $P_d(\boldsymbol{q})$ which results in a simple affine linear feedback control law. The control law is also shown to achieve simultaneous regulation of position and force to the specified vector $(\boldsymbol{q}_d, \boldsymbol{\lambda}_d)$.

In this case choose

$$\begin{aligned} P_d(\boldsymbol{q}) = P(\boldsymbol{q}) - P(\boldsymbol{q}_d) - [\frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}_d) - \boldsymbol{J}^T(\boldsymbol{q}_d)\boldsymbol{\lambda}_d]^T(\boldsymbol{q} - \boldsymbol{q}_d) \\ + \frac{1}{2}(\boldsymbol{q} - \boldsymbol{q}_d)^T \boldsymbol{W}(\boldsymbol{q} - \boldsymbol{q}_d). \end{aligned} \tag{12}$$

is easy to check that equation (6) is satisfied. The modified energy function is

$$P_{cd}(\boldsymbol{q}) = P_d(\boldsymbol{q}) - \boldsymbol{\lambda}_d^T \boldsymbol{\Phi}(\boldsymbol{q}). \tag{13}$$

It is easy to verify that $P_{cd}(\boldsymbol{q}_d) = 0$ and $\frac{\partial}{\partial \boldsymbol{q}} P_{cd}(\boldsymbol{q}_d) = 0$. Also the matrix $\boldsymbol{W}$ is chosen to be symmetric such that

$$[\boldsymbol{W} + \frac{\partial^2}{\partial \boldsymbol{q}^2} P_d(\boldsymbol{q}_d) - \sum_{j=1}^{m} \lambda_{jd} \frac{\partial^2}{\partial \boldsymbol{q}^2} \phi_j(\boldsymbol{q}_d)] \tag{14}$$

is positive definite. This is always achievable if $P(\boldsymbol{q})$ and $\boldsymbol{\Phi}(\boldsymbol{q})$ are twice continuously differentiable in a neighborhood of $\boldsymbol{q}_d$.

With this choice, the controller (5) takes the following specific form

$$\boldsymbol{u} = \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}_d) - \boldsymbol{J}^T(\boldsymbol{q}_d)\boldsymbol{\lambda}_d - \boldsymbol{W}(\boldsymbol{q} - \boldsymbol{q}_d) - C\dot{\boldsymbol{q}}. \tag{15}$$

This is an affine linear feedback control law. The first two terms represent a constant bias, and the third and fourth terms represent the feedback of position and velocity errors. Note that if the matrices $\boldsymbol{W}$ and C are diagonal matrices, then the feedback controller is decentralized. Such feedback is useful in implementation of robot control systems where each joint actuator depends only on feedback of its local joint displacement and velocity. The closed loop equations with controller (15) are given as

$$\begin{aligned} M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{\theta}_d(\boldsymbol{q}, \dot{\boldsymbol{q}}) + C\dot{\boldsymbol{q}} + \boldsymbol{W}(\boldsymbol{q} - \boldsymbol{q}_d) \\ = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{\lambda} - \boldsymbol{J}^T(\boldsymbol{q}_d)\boldsymbol{\lambda}_d \end{aligned} \tag{16}$$

where

$$\boldsymbol{\theta}_d(\boldsymbol{q}, \dot{\boldsymbol{q}}) = [\frac{d}{dt} M(\boldsymbol{q})]\dot{\boldsymbol{q}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}}[\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}] + \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}) - \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}_d).$$

Following Theorem 1, we have Corollary 2.

Corollary 2: Consider the closed loop constrained system (16), (1). Position and contact force locally asymptotically converge to the specified position and force vector $(\boldsymbol{q}_d, \boldsymbol{\lambda}_d)$ if the $n \times n$ matrices $\boldsymbol{W}$ and C are symmetric and positive definite and such that the matrix in (14) is symmetric and positive definite.

In the above, simultaneous regulation is achieved for both position and constraint force without using feedback of the constraint force error. However, feedback of the constraint force error can be introduced to tune the constraint force error response and to obtain improved robustness of the closed loop system. A linear feedback controller, including feedback of the constraint force error, is

$$\begin{aligned} \boldsymbol{u} = \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}_d) - \boldsymbol{J}^T(\boldsymbol{q}_d)\boldsymbol{\lambda}_d - \boldsymbol{W}(\boldsymbol{q} - \boldsymbol{q}_d) - C\dot{\boldsymbol{q}} \\ + \boldsymbol{J}^T(\boldsymbol{q}_d) G_f(\boldsymbol{\lambda} - \boldsymbol{\lambda}_d). \end{aligned} \tag{17}$$

The $m \times m$ matrix G_f is assumed to be symmetric and nonnegative definite and is referred to as the force feedback gain matrix. The closed loop equations are given as

$$\begin{aligned} M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{\theta}_d(\boldsymbol{q}, \dot{\boldsymbol{q}}) + C\dot{\boldsymbol{q}} + \boldsymbol{W}(\boldsymbol{q} - \boldsymbol{q}_d) \\ = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{\lambda} - \boldsymbol{J}^T(\boldsymbol{q}_d)\boldsymbol{\lambda}_d + \boldsymbol{J}^T(\boldsymbol{q}_d) G_f(\boldsymbol{\lambda} - \boldsymbol{\lambda}_d) \end{aligned} \tag{18}$$

where

$$\boldsymbol{\theta}_d(\boldsymbol{q}, \dot{\boldsymbol{q}}) = [\frac{d}{dt} M(\boldsymbol{q})]\dot{\boldsymbol{q}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}}[\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}] + \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}) - \frac{\partial}{\partial \boldsymbol{q}} P(\boldsymbol{q}_d).$$

Corollary 3: Consider the closed loop constrained system (18), (1). Position and contact force locally asymptotically converge to the specified position and force vector $(\boldsymbol{q}_d, \boldsymbol{\lambda}_d)$ if the matrices $\boldsymbol{W}$ and C satisfy the conditions stated in Corollary 2 and if the matrix G_f is symmetric, positive semidefinite and sufficiently small.

This statement can be justified by the fact that when G_f is zero, the result reduces to Corollary 2; by continuity, the result is true for sufficiently small G_f. Results have not yet been obtained, using Lyapunov's method, which allow specification of explicit conditions on G_f which guarantee closed loop asymptotic stability. We do know that, in general, the closed loop may become unstable if G_f is too large.

C. Pole Assignment Approach

Another important feature of control design by Lyapunov's direct method for constrained systems is that the feedback gain selection is not unique. This can allow for selection of gain matrices based on specifications of transient response. In this section, we give a pole assignment procedure for the differential-algebraic equations linearized at the desired position and force vectors. Suppose that the $2(n-m)$ pole locations for the linearized equations are specified by a set of self conjugate complex numbers $s_1, s_2, \cdots, s_{2(n-m)}$. Other specifications are the same as before. A pole assignment procedure is indicated to determine selections of the $n \times n$ matrices W and C.

From the results in [12], the linearized differential-algebraic equations of the closed loop system are

$$M(q_d)\delta\ddot{q} + C\delta\dot{q} + K(q_d, \lambda_d)\delta q = J^T(q_d)\delta\lambda \tag{19}$$

$$J(q_d)\delta q = 0 \tag{20}$$

where

$$K(q_d, \lambda_d) = [W + \frac{\partial^2}{\partial q^2}P(q_d) - \sum_{j=1}^{m} \lambda_{jd}\frac{\partial^2}{\partial q^2}\phi_j(q_d)].$$

The following procedure determines feedback gain matrices C and W so that the $2(n-m)$ poles of (19), (20) have the specified complex values.

Procedure

Step 1: Group the desired poles in pairs, e.g., $(s_1, s_2), \cdots, (s_{2(n-m)-1}, s_{2(n-m)})$, where the complex poles are paired with their complex conjugates. Then form the $n - m$ scalar second order polynomials

$$p_1(s) = s^2 + c_1 s + k_1 = (s - s_1)(s - s_2)$$
$$p_2(s) = s^2 + c_2 s + k_2 = (s - s_3)(s - s_4)$$
$$\vdots$$
$$p_{n-m}(s) = s^2 + c_{n-m}s + k_{n-m} = (s - s_{2(n-m)-1})(s - s_{2(n-m)})$$

and the polynomial matrix

$$P_d(s) = \mathrm{diag}[p_1(s), \cdots, p_{n-m}(s)] = I_{n-m}s^2 + C_d s + K_d$$

where $C_d = \mathrm{diag}[c_1, \cdots, c_{n-m}]$ and $K_d = \mathrm{diag}[k_1, \cdots, k_{n-m}]$.

Step 2: Compute the $n \times (n-m)$ matrix V_2 from the singular value decomposition of

$$J(q_d) = U\Sigma V^T,$$

where $V = [V_1, V_2]$.

Step 3: Compute the $(n-m) \times (n-m)$ square matrix S by the decomposition

$$V_2^T M(q_d) V_2 = S^T S.$$

Step 4: Construct a reference matrix polynomial

$$P_r(s) = S^T P_d(s) S = M_r s^2 + C_r s + K_r$$

where $M_r = V_2^T M(q_d) V_2$, $C_r = S^T C_d\, S$, $K_r = S^T K_d S$.

Step 5: Choose C to be symmetric and to satisfy

$$V_2^T C V_2 = C_r.$$

Step 6: Choose W to be symmetric and to satisfy

$$V_2^T W V_2 = K_r - V_2^T[\frac{\partial^2}{\partial q^2}P(q_d)]V_2 + V_2^T[\sum_{j=1}^{m}\lambda_{jd}\frac{\partial^2}{\partial q^2}\phi_j(q_d)]V_2 .$$

The matrices W and C chosen in this procedure are used as described in the previous section to construct a linear feedback controller (15) or (17). This controller guarantees that the desired values (q_d, λ_d) are locally asymptotically approached, with the specified local transient response characteristics. The justification for this procedure is indicated in the Appendix.

Notice that selection of the matrices C, W is not uniquely determined by this procedure; different choices of these matrices give the same specified closed loop pole locations. The freedom in C and W selection can be further studied to achieve regulation of contact force transient in addition to the regulation of displacement transient response.

IV. Robustness Properties

In this section, we study the robustness properties of the closed loop system with the controller (17). The controller (17) is designed to achieve closed-loop asymptotic stability at the desired position and force specifications for any initial condition that is close to the specification and consistent with the constraint functions. In this section, we consider the implications of using this feedback control structure to reduce the effects of external force disturbances and unmodeled errors in the constraint functions. We show that this feedback control structure is able to reduce such effects.

A. Effects of Force Disturbances

Suppose there is an external force disturbance so that the constrained system is described by

$$M(q)\ddot{q} + \theta(q, \dot{q}) = u + J^T(q)\lambda + d \tag{21}$$

$$\Phi(q) = 0 \tag{22}$$

where d represents a constant n-vector force disturbance. Recall that $q = q_d$, $\dot{q} = 0$, $\lambda = \lambda_d$ define the asymptotic final values, when $d = 0$, of the closed loop with the controller (17) and the feedback gain matrices chosen to guarantee local asymptotic stability. The first order approximation is given by

$$M(q_d)\delta\ddot{q} + C\delta\dot{q} + [W + N(q_d, \lambda_d)]\delta q = J^T(q_d)(I_m + G_f)\delta\lambda + d \tag{23}$$

$$J(q_d)\delta q = 0 \tag{24}$$

where we use the short hand notation $N(q_d, \lambda_d)$ defined in (10).

Since the feedback gains C and W are chosen to guarantee that the equilibrium $(q_d, 0)$ of the nominal system (when $d = 0$) is asymptotically stable, the differential-algebraic equation (23), (24) is asymptotically stable if the disturbance d is assumed to be sufficiently small. The steady states are modified under this disturbance so that, as $t \to \infty$

$$\delta q \to \delta q_{ss} \tag{25}$$

where

$$\delta q_{ss} = [W + N(q_d, \lambda_d)]^{-1}\{I - J^T(q_d)[J(q_d)[W + N(q_d, \lambda_d)]^{-1}J(q_d)[W + N(q_d, \lambda_d)]^{-1}\}d$$

and

$$\delta\lambda \to \delta\lambda_{ss} \tag{26}$$

where

$$\delta\lambda_{ss} = -(I_m + G_f)^{-1}\{J^T(q_d)[W + N(q_d, \lambda_d)]^{-1}J^T(q_d)\}^{-1} \times J(q_d)[W + N(q_d, \lambda_d)]^{-1}d.$$

Thus the steady state displacement errors do not depend on the force feedback gain matrix G_f and are inversely proportional to the displacement feedback gain matrix W. The steady state contact force errors do depend on the force feedback gain matrix G_f and are inversely proportional to it. The steady state contact force errors depend on the displacement gain W. "High gain" must be used with care since linearized equations are used to approximate the nonlinear equations in a local neighborhood. Nevertheless, if high gains in the displacement feedback loops maintain closed loop stability, it results in improved steady state displacement accuracy. And high gain in the force feedback loops might result in instability as pointed out in a previous section, but on the other hand, if closed loop stability is maintained, then improved steady state contact force accuracy for additive force disturbance is obtained.

B. Effects of Constraint Uncertainities: Scaling Errors

Suppose that there are uncertainties in the constraint functions so that the constrained system is described by

$$M(q)\ddot{q} + \theta(q,\dot{q}) = u + J^T(q)\lambda \quad (27)$$
$$\Phi(q) = \Delta \quad (28)$$

where Δ represents a constant m vector of constraint scaling errors. The mathematical definition of Δ is clear from (28). We refer to Δ as a scaling error since it defines a local translation near q_d of the constraint manifold. Recall that $q = q_d$, $\dot{q} = 0$, $\lambda = \lambda_d$ define the asymptotic final values, corresponding to $\Delta = 0$. Suppose that the selection of the feedback gain matrices guarantees that the closed loop system with control (17) and $\Delta = 0$ locally asymptotically converges to the desired position and force (q_d, λ_d). A first order approximation of the closed loop system is described by the following linearized equations if Δ is sufficiently small

$$M(q_d)\delta\ddot{q} + C\delta\dot{q} + [W + N(q_d,\lambda_d)]\delta q J^T(q_d)(I_m + G_f)\delta\lambda \quad (29)$$
$$J(q_d)\delta q = \Delta \quad (30)$$

where the notation $N(q_d, \lambda_d)$ is as before. Again the equilibrium is asymptotically stable if the feedback gains are chosen according to the conditions in the previous section and the uncertainty Δ is sufficiently small. The steady state displacement error and contact force error of the closed loop constrained system satisfy, as $t \to \infty$,

$$\delta q \to \delta q_{ss} \quad (31)$$

where

$$\delta q_{ss} = [W + N(q_d,\lambda_d)]^{-1} J^T(q_d) \{J(q_d)[W + N(q_d,\lambda_d)]^{-1} J^T(q_d)\}^{-1}\Delta$$

and

$$\delta\lambda \to \delta\lambda_{ss} \quad (32)$$

where

$$\delta\lambda_{ss} = (I + G_f)^{-1}\{J(q_d)[W + N(q_d,\lambda_d)]^{-1} J^T(q_d)\}^{-1}\Delta.$$

Again we see that the steady state displacement errors do not depend on the force feedback gain matrix G_f and are inversely proportional to the displacement feedback gain matrix W. The steady state contact force errors are inversely proportional to the force feedback gain matrix G_f. Thus, if "high gain" in the displacement feedback loops maintain local stability, it results in improved steady state displacement accuracy for uncertainty in the constraint functions. And if high gain in the force feedback loops maintain closed loop stability, it results in improved steady state contact force accuracy for scaling error in the constraint functions.

C. Effects of Constraint Uncertainties: Rotation Errors

There are other forms of possible modeling uncertainty in the constraint function. Suppose that the actual constraint is given by $\hat{\Phi}(q) = 0$, where the constraint function is approximated, near the desired position q_d, to first order by

$$RJ(q_d)(q - q_d) = 0 \quad (33)$$

where R is $m \times m$ orthogonal matrix. The mathematical definition of R is clear from (33). We refer to R as a rotation since it defines a local rotation near q_d of the constraint manifold. Recall that $q = q_d$, $\dot{q} = 0$, $\lambda = \lambda_d$ define the asymptotic final values corresponding to $R = I$. With the controller (17), the closed loop nonlinear equations are given by

$$M(q)\ddot{q} + \theta(q,\dot{q}) = [\frac{\partial^T}{\partial q}\hat{\Phi}(q)]\lambda - J(q_d)\lambda_d - C\dot{q} - W(q - q_d) + J^T(q_d)G_f(\lambda - \lambda_d)$$

and the linearized closed loop equations are given by

$$M(q_d)\delta\ddot{q} + C\delta\dot{q} + \left[\frac{\partial^2}{\partial q^2}P(q_d) + W - \left[\frac{\partial}{\partial q}(J^T(q)R^T\lambda_d)\right]\right]\delta q = J^T(q_d)(R^T - I)\lambda_d + J^T(q_d)(R^T + G_f)\delta\lambda \quad (34)$$
$$RJ(q_d)\delta q = 0.$$

The gain matrices of the controller (17) are chosen so that the nominal system (when $R = I$) is locally asymptotically stable. If the rotation error matrix $R - I$ is sufficiently small, the equilibrium of the closed loop system remains asymptotically stable and as, $t \to \infty$,

$$q - q_d \to 0 \quad (35)$$
$$\lambda - \lambda_d \to -[R^T + G_f]^{-1}(R^T - I)\lambda_d. \quad (36)$$

However, in contrast to the previous cases, this rotational uncertainty does affect the first order approximation represented by equation (34). Therefore, asymptotic stability of the closed loop is not guaranteed by the controller (17) unless the rotational error is sufficiently small. As in the previous case, high gain in the force feedback loops results in improved steady state contact force accuracy for rotation errors in the constraint functions.

V. Conclusion

A control design method has been proposed for position and force control of mechanical systems with holonomic constraints. A family of controllers is derived using Lyapunov's direct method to achieve the objective of local regulation of both position and force simultaneously. This family includes decentralized controllers in the form of linear feedback of position, velocity and force errors. Lyapunov's direct method has been used successfully here to overcome the difficulties associated with constrained dynamic systems.

It has been shown that force feedback is not necessary to achieve position and force control, but it can be useful to improve closed loop robustness properties.

Appendix

The singular value decomposition $J(q_d) = U\Sigma V^T$ can be used to define the coordinate transformation

$$\delta q = V\delta x,$$

for equations (19), (20), where $V = [V_1, V_2]$. The transformed equations are

$$V_1^T M(q_d)V_2\delta\ddot{x}_2 + V_1^T CV_2\delta\dot{x}_2 + V_1^T K(q_d,\lambda_d)V_2\delta x_2$$

$$= V_1^T J^T(q_d)(I+G_f)\delta\lambda \quad (37)$$

$$V_2^T M(q_d)V_2\delta\ddot{x}_2 + V_2^T C V_2 \delta\dot{x}_2 + V_2^T K(q_d,\lambda_d)V_2\delta x_2 = 0 \quad (38)$$

$$\delta x_1 = 0. \quad (39)$$

From this decomposition, we see that the poles of the linearized differential-algebraic equations (19), (20) are actually the poles of the second order linear differential equation (38). This justifies the procedure for pole assignment given in the text.

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Thermal Tactile Sensing

G. J. Monkman and P. M. Taylor

***Abstract*—The measurement of temperature change as an indication of a materials relative thermal conductivity has often been utilized as a means of tactile sensing. Unfortunately, the long time response of most thermal sensors makes such a technique too slow for normal industrial robotic uses. This paper considers the human tactile performance with particular regard to temperature sensing and introduces a new means by which a usable rise time may be achieved. Two methods, using devices hitherto not utilized for tactile sensing, are demonstrated.**

I. Introduction

According to Harmon [1] the requirement for a tactile sensor to emulate the human finger is a dynamic pressure range of three orders of magnitude with a resolution of 20 by 20 tactels per finger. A spatial resolution of better than 2 mm and a time response within 10 mS with low hysteresis are also desirable. Such a time response may be a little optimistic, even in the case of a physical displacement transducer this represents a displacement velocity of 0.1 m/s for a 1 mm depression. Though perhaps conservative for most robot transitions, this type of velocity is not very representitive of fine grasping actions where much slower movements are more usual.

As one may observe when touching objects with the finger tips, temperature is also an integral part of the human overall tactile sensing strategy. Thermal effects also help determine the "feel" or texture of a surface.

II. The Human Tactile Sense

We cannot consider the finger tips as tactile sensors in isolation without due regard to the human entity as a whole. As pointed out by Lederman [2]: in everyday life our fingers explore their environment, actively pushing against objects to determine their form. Unlike many inanimate compliant membranes, the very nature of flesh allows it to return completely to its original profile after depression very rapidly. Constant blood flow, and other physical movements, help to augment this positive recovery effect.

Much confusion is caused by the surface texture and the relative thermal conductivity of the material in question. Whenever such a sample is encountered, the finger is moved over the surface to give a feel for its surface profile, material stiffness, temperature etc. Furthermore, the human fingers contain at least five different receptor types, many of which are connected in a one-to-many configuration [3]. This must inevitably result in a high degree of cross-talk.

In a simple test, one may observe the inability of the human tactile sense to determine even relative temperatures by touching a selection of objects and then measuring their actual temperature with an accurate thermometer or thermocouple probe. This type of effect will be familiar to most children who have dipped a finger into bowls of water of varying temperature in an attempt to estimate their relative temperatures.

What our fingers detect thermally is not the absolute temperature of a material alone, but also its thermal conductivity and diffusivity.

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