Modelling of Constrained Robot System with Constraint Uncertainties

Yeong Khing Ho,* Danwei Wang, Yeng Chai Soh
School of Electrical & Electronic Engineering, Nanyang Technological University, Singapore, 2263 Republic of Singapore

Received November 7, 1998; accepted September 10, 1999

In this paper, a model for the constrained robot dynamics, incorporating constraint uncertainties is developed. The rigid constraining surfaces are represented by a set of algebraic functions. The uncertainties in the constraint functions reflect the inaccuracy in the descriptions of the constraining surfaces. The constraint uncertainties influence the robot dynamics, giving rise to the need to take into consideration these uncertainties in controller designs. © 2000 John Wiley & Sons, Inc.

1. INTRODUCTION

Constrained robot motion control is concerned with a class of force and position/velocity control of robot motion. In contrast to free motion in space, the constrained robot end effector in this case is interacting with the environment with which it comes into contact. Thus, to study the force control aspect of robots, the dynamics of robotic motion in free space has to be modified to accommodate the new dynamics. In this respect, modelling of the environment is necessary and crucial. Current literature in the field classify the problem into two broad categories[1]: impedance control and hybrid control, dealing, respectively, with environments that are stiff and those that are of infinite stiffness. Impedance modelling is attractive for cases where the dynamics of the environments are important and need to be accounted for explicitly. The published works on impedance control are numerous, chief among these can be found in Refs. 2–4, to cite just a few. Some practical implementations of the impedance control strategy are also reported in Refs. 5–7. On the other hand, environments whose stiffness are infinite are relevant to a class of robot operations where the robot end effectors are constrained to the rigid constraining surfaces. The pioneering works, such as those found in Refs. 8–11, have laid the necessary theoretical ground for the study of constrained robot systems. Generally, the description of environmental constraints on the robot motion is reflected by insisting on the robot movement satisfying certain

* To whom all correspondence should be addressed

© 2000 by John Wiley & Sons, Inc.
CCC 0741-2223/00/010053-09

algebraic constraint functions. The constraint functions so specified can also be viewed as the descriptions of the geometric surfaces. Thus, complex surfaces can be represented by relatively simple algebraic functions. The constraint forces arising from the environmental interactions are then embodied in the dynamics of the robot through a vector of Langrange multipliers.

The reliance on the robot dynamic model for designing controllers means the accuracy of the model parameters has a direct impact on the validity of the designs. Most of the research works on constrained robot systems, however, assume an exact model for the environment. In practice, models are inherently uncertain. The control of such uncertain systems has attracted a lot of attention in the research community. Works that treat the uncertainties in the constrained robot systems were reported in, for instance, Refs. 12 and 13. However, these works do not employ an explicit model in accommodating the constraint uncertainties. Their results may also be restrictive by virtue of their attended assumptions.

This article describes an attempt to model the constrained robot dynamics, incorporating explicitly the constraint uncertainties. An analysis also reviews the influence of the uncertainties in the constraint functions on the constrained dynamics. This has implications in the control of the constrained robot systems.

This article is organized as follows. Section 2.1 outlines the model of constrained robot dynamics with known constraints while Section 2.2 extends the model to the case of constraint functions with uncertainties. Section 3 gives a geometric interpretation of uncertainties in the constraint functions, while Section 4 analyzes the dynamics of the robot system and reviews the influence of the constraint uncertainties on the dynamics.

2. ROBOT DYNAMICS AND CONSTRAINT UNCERTAINTIES

2.1. Model of Constrained Robot Dynamics with Known Constraints

Consider an n-joint robot interacting with a rigid environment. The end effector of the robot exerts a force on the environment while its motion is kinematically confined to the surface of the environment. This constitutes a constrained robot system. The dynamics of such a constrained robot system can be represented by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + f \tag{1}$$

where the joint positions are described by a vector of generalized displacement in robot coordinates, $q \in R^n$, and $\dot{q}$ denotes their time derivatives. M: $R^n \to R^{n\times n}$ represents the inertial matrix which is symmetric and positive definite for all $q \in R^n$. The velocity coupling term is represented by $C(q,\dot{q})$, an $n \times n$ matrix. $g(q) \in R^n$ is the gravity term. The generalized constraint force vector is given by $f \in R^n$ while the vector of control inputs $\tau \in R^n$, all expressed in robot joint space.

To further quantify the vector f, we make the following assumptions about the environment:

Assumption 1. *The environments are assumed to be rigid, smooth surfaces so that contacts with the environmental surfaces are frictionless point contacts.*

Assumption 2. *Throughout the motion of the robot arm, contact with the environment is always maintained.*

Remark 1. A consequence of these assumptions is that the robot end effector is exerting a constraint force orthogonal to the constrained surface at the contact point and this force is a *workless* force.

Next we let $p \in R^j$ denote the generalized position vector of the robot end effector, in coordinates in which constraints on the end effector are defined. These constraints are such that the generalized position vector of the robot end effector is assumed to satisfy an algebraic equation $\zeta(p) = 0$, where ζ: $R^j \to R^m$ is called the constraint function. We assume that the generalized position vector of the robot end effector, in constraint coordinates, can be expressed in terms of the generalized displacement in the robot coordinates according to the algebraic equation $p = H(q)$, where the mapping is H: $R^n \to R^j$. Thus, the constraint function defined by $\phi(q) = \zeta(H(q))$, in robot coordinates, satisfies the constraint equation

$$\phi(q) = 0 \tag{2}$$

Thus, the m constraint functions of Eq. (2) represent the m geometric surfaces constraining the movements of the robot manipulators. The constraints so described are called the holonomic constraints.[14,15] It follows that the generalized constraint forces, f, in robot coordinates, are given by the relation[14,15]

$$f = J^T(q)\lambda \tag{3}$$

where $\lambda \in R^m$ is a vector of generalized multipliers (the Lagrangian multipliers) associated with the constraints, and the Jacobian matrix $J(q) = \partial\phi/\partial q$ denotes the configuration dependent directions of the constraint forces. The following additional assumption is made regarding the m constraint functions:

Assumption 3. *The m constraint functions are linearly independent so that consequently the Jacobian $J(q)$ is of full rank m. The $J(q)$ is also assumed to be bounded.*

Remark 2. Note that each column vector of $J^T(q)$ represents the normal vector to the respective constraining surface.

Equations (1), (2), and (3), thus represent the constrained robot dynamics when the constraint functions (2) are assumed to be known exactly.

2.2. Model of Constrained Robot Dynamics with Constraint Uncertainties

Exact constraint functions imply a precise knowledge about the constraining surfaces. In practice, this may be difficult to achieve. For instance, there may be set-up errors on the work surfaces, or there may even be practical difficulties in giving exact descriptions of the constraining surfaces. Under such circumstances, only a nominal description of the constraining surfaces is all that is available. As constraint functions define the admissible motion space of the robot end effectors, uncertainties in the constraint functions will thus change the admissible motion space. This in turn is expected to affect the dynamics of the constrained system as well. The effects of constraint uncertainties were studied, for instance, by Wang and McClamroch.[12] Their results show that a high gain in the displacement feedback loops results in improved steady state displacement accuracy for scaling errors in the constraint functions. Likewise, high gain in the force feedback loops improved the steady state contact force accuracy. They also investigated the effect of rotation errors in the constraint functions and concluded that such errors does affect the stability of the controller unless the errors are small.[12] The results of these investigations indicate the necessity to incorporate constraint uncertainties in the modelling of constrained robot systems.

In the following, we show a way to incorporate such uncertainties so as to quantitatively describe the modified constrained dynamics arising from such uncertainties in the constraint functions.

First, let us make the following additional assumption:

Assumption 4. *Any functions used to represent uncertainties in the constraint functions are smooth and bounded and that their derivatives exist and are also bounded.*

Remark 3. The uncertainties in the constraint functions are generally unknown. The assumption of smoothness and boundedness automatically excludes surfaces with sharp dents or any irregularities of sharp curvatures.

Next, we assume that for a given constrained robot system, only the nominal constraint functions, $\phi_n(q)$, are available, i.e.,

$$\phi_n(q) = 0 \tag{4}$$

When constraint uncertainty exists, this equation may or may not be satisfied for any robot joint positions q. The actual constraint functions, $\phi_a(q)$, can be written as

$$\phi_a(q) = \phi_n(q) + \Delta(q) = 0 \tag{5}$$

where $\Delta(q)$ is the smooth function representing the uncertainties in the constraints.

The actual constraint forces, f, are now given by

$$f = J_a^T(q)\lambda \tag{6}$$

where

$$\begin{aligned} J_a^T(q) &= J_n^T(q) + \left(\frac{\partial\Delta(q)}{\partial q}\right)^T \\ &= J_n^T(q) + J_\Delta^T(q) \end{aligned} \tag{7}$$

with $J_\Delta^T(q) = (\partial\Delta(q)/\partial q)^T$. The uncertainty functions, $\Delta(q)$, are normally not known. In light of the assumption stated, they are smooth and bounded. Hence, the derivatives $\partial\Delta(q)/\partial q$ exist and are also bounded.

With these developments, the constrained robot dynamics with constraint uncertainties are now represented by the following set of differential−algebraic equations:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + f \tag{8}$$

$$\phi_a(q) = \phi_n(q) + \Delta(q) = 0 \tag{9}$$

$$f = J_a^T(q)\lambda \tag{10}$$

This form of uncertainty modelling first appeared in Refs. 16 and 17.

Constraint forces as expressed in the form of Eq. (10) can be further manipulated. Consider the case of a constrained robot system with *one* constraining surface, i.e., $m=1$. Equation (10) can be written as

$$f=\frac{J^T(q)}{\|J^T(q)\|}\|J^T(q)\|\lambda$$

The vector $J^T(q)/\|J^T(q)\|$ is the unit normal to the constraining surface while the quantity $\|J^T(q)\|\lambda$ is the *magnitude* of the constraint force.

Extending the idea to the case of constrained motion involving two or more constraining surfaces, i.e., $m>1$, we note that the transposed Jacobian of the m constraint functions can be written as

$$J^T(q)=\left[J_1^T(q)\cdots J_i^T(q)\cdots J_m^T(q)\right]$$

where the column vector $J_i^T(q)$ represents the ith normal to the ith surface. The constraint force at the ith constraint surface for joint position q is then given by

$$f_i=\frac{J_i^T(q)}{\|J_i^T(q)\|}\|J_i^T(q)\|\lambda_i$$

where λ_i is the Langrange multiplier, a scalar, associated with the ith surface. The corresponding *magnitude* of the constraint force is then $\|J_i^T(q)\|\lambda_i$.

We can now define a $n\times m$ matrix of unit normals to the m constrained surfaces as

$$N=\left[\frac{J_1^T(q)}{\|J_1^T(q)\|}\cdots\frac{J_i^T(q)}{\|J_i^T(q)\|}\cdots\frac{J_m^T(q)}{\|J_m^T(q)\|}\right] \quad (11)$$

and a vector of *magnitudes* of constrained forces as

$$\Lambda=\left[\|J_1^T(q)\|\lambda_1\cdots\|J_i^T(q)\|\lambda_i\cdots\|J_m^T(q)\|\lambda_m\right]^T \quad (12)$$

Equation (10) can then be re-expressed as

$$f=N\Lambda \quad (13)$$

Constraint forces, expressed in the form of Eq. (13), are useful for the design of controllers where the *magnitudes* of constraint forces are regulated.

3. GEOMETRIC INTERPRETATIONS OF UNCERTAINTIES IN CONSTRAINT FUNCTIONS

Equation (9), together with Eqs. (7) and (10) or (13), represent a general description of the geometric surfaces associated with the robot constrained motion and the attended constraint forces. We can interpret geometrically the significance of the uncertainties $\Delta(q)$. Three situations are considered and their attending types of uncertainties were first investigated by Wang and McClamroch,[12] although their results were limited to local sense.

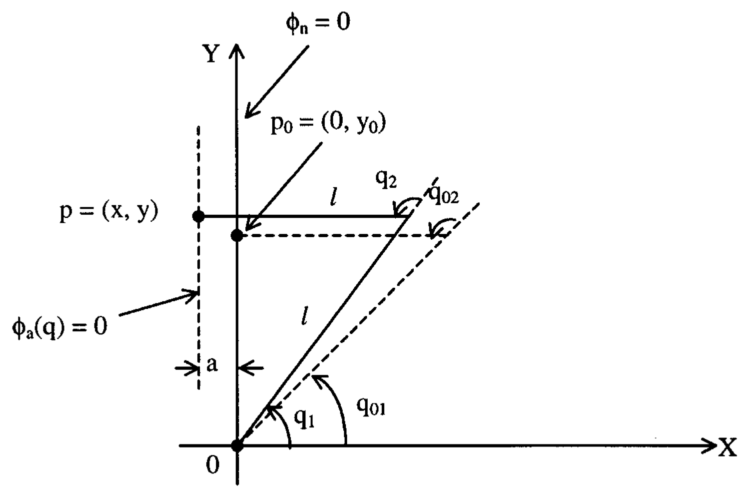

Figure 1. Scaling errors in constraint functions.

3.1. Scaling Errors as Constraint Uncertainties

For ease of explanation, consider a two-link robot with revolute joints, as illustrated in Figure 1. The joint angles are q_1 and q_2. The end of the second link presses against a flat surface, shown by the dotted lines. The point p, represents the current position of the robot end effector and has the coordinates as (x, y), in Cartesian coordinates, or (q_1, q_2), in robot joint coordinates. The point, p_0, is the corresponding position on the nominal surface and has the coordinates (q_{0_1}, q_{0_2}). The nominal constraining surface coincides with the line $x = 0$. From the diagram, we have the following:

Nominal constraint function, $\zeta = x = 0$; therefore, $\zeta(p_0) = 0$. Also, since

$$H(q) = x = l(\cos(q_1) + \cos(q_1 + q_2))$$

then, the nominal constraint function in robot joint coordinates is

$$\phi_n(q) = \zeta(H(q)) = 0 \tag{14}$$

with

$$\phi_n(q_0) = l\big(\cos(q_{0_1}) + \cos(q_{0_1} + q_{0_2})\big) = 0$$

Since the position of the actual surface corresponds to a translational error of a, with respect to the planned position $x = 0$, the scaling error in the constraint function is then a. This scaling error is the uncertainty $\Delta(q)$. The actual surface is thus described by

$$\phi_a(q) = \phi_n(q) + \Delta(q) = 0 \tag{15}$$

For joint angles q_1 and q_2 Eq. (15) is obviously satisfied, but *NOT* the original, nominal constraint function (4). The associated normal to the actual surface, $J_a^T(q)$, is, in this case, parallel to the one normal to the nominal surface at q, i.e., $J_n^T(q)$.

Scaling error occurs at the point where the robot makes contact with the surface whenever the true geometric surface does not intersect with the nominal surface at that point.

3.2. Constraint Uncertainties Manifested as Offset Errors in Surface Normals

Figure 2 shows the case where the nominal surface and the actual surface intersect at the point where the robot end effector is positioned. In this case, both the nominal and the actual constraint functions are satisfied by the robot joint positions, with $\Delta(q) = 0$. However, it is obvious from the figure that the actual surface normal, i.e., $J_a^T(q)$, is different from that of the nominal surface normal, i.e., $J_n^T(q)$. The offset, $J_\Delta^T(q)$ then quantitatively reflects the uncertainties in the constraint functions. The normal vector at q to the actual surface is given by

$$J_a^T(q) = J_n^T(q) + J_\Delta^T(q)$$

where $J_n^T(q) = \partial/\partial q)\phi_n(q)$ and $J_\Delta^T(q) = (\partial/\partial q)\Delta(q)$.

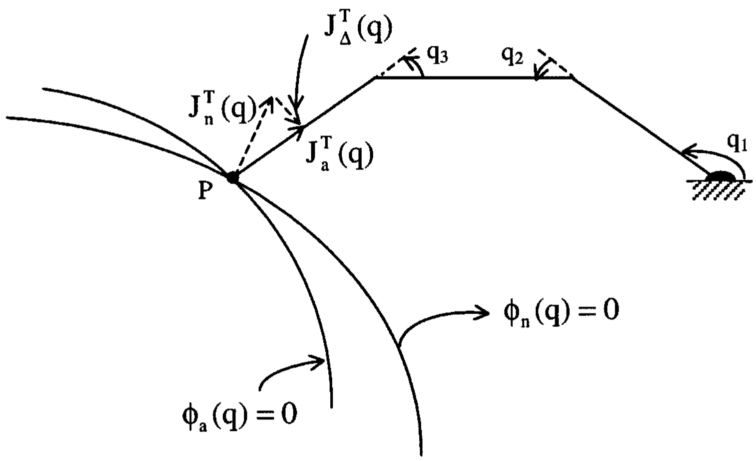

Figure 2. Offset errors in surface normals.

3.3. Constraint Uncertainties—The General Case

In the case that the actual robot end effector position does not correspond to the intersection of the actual and nominal surfaces, we have a combination of the effects of the previous two cases. There is thus a scaling error and an offset in the surface normal. The constraint function uncertainties, $\Delta(q)$, together with $J_\Delta^T(q)$ then completely describe the uncertainties in the constraining surfaces. Figure 3 shows the situation.

Remark 4. These diagram illustrations (Fig. 1, 2, 3) show the constraint case of $m = 1$.

Remark 5. Uncertainties in constraint functions pertaining to constrained robot systems were discussed in Refs. 12, 13, and 18, among others. Most of these works do not treat the uncertainties in general settings. For instance, Ref. 12 uses a linearized robot dynamics model to show the effect of the constrained uncertainties, Ref. 13 models the uncertainties as a rotational error in the surface normals, while Ref. 18 relates the error in an unregulated coordinate of the robot end effector with the uncertainty in the constraint function. Our present analysis shows that there is no necessity to explicitly identify the actual nature of the uncertainties in the constraint functions. Equations (5) and (7) therefore represent a general description of uncertainties in constraint functions associated with robot constrained motion.

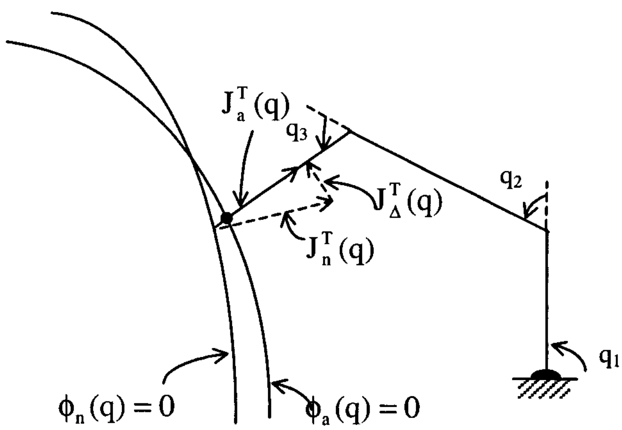

Figure 3. General uncertainties in the constraint functions.

4. ANALYSIS OF THE CONSTRAINED ROBOT DYNAMICS

The equations representing the dynamics of the constrained robot system constitute a singular system of nonlinear differential equations. This singular system presents many difficulties in the analysis of the constrained dynamics. Mills,[18] using the results of Cobb,[19] presented an approach to handle the analysis by first, linearizing the dynamics about the nominal operating point. The linearized dynamics are then decomposed into fast and slow subsystems. The dynamics of the two subsystems are then studied and the appropriate control issues are then resolved. While this analysis reveals some very interesting properties of the descriptor system, it is complicated. The linearization also means that the control is valid only in the neighborhoods of the nominal operating points.

We now look at the problem by another approach. This is the reduction method and has been used by many in various contexts.[8,10,20–22] The approach basically relies on differentiating the algebraic constraint functions sufficient number of times so that an algebraic equation can be formed to solve for the vector of Lagrangian multipliers, λ. With the λ eliminated, the singular system is turned into the regular nonlinear differential equations and the remaining problem becomes one of solving the initial values problem.

4.1. Constrained Robot Dynamics with Known Constraints

From the constraint function $\phi(q) = 0$ we have $J(q)\dot{q} = 0$. Differentiating again gives

$$J(q)\ddot{q} + J(q)\dot{q} = 0$$

Substituting for $\ddot{q}$ from Eq. (1) gives

$$J(q)M^{-1}(q)\{-C(q,\dot{q})\dot{q} - G(q) + \tau + J^T(q)\lambda\} + \dot{J}(q)\dot{q} = 0 \tag{16}$$

or equivalently

$$A(q)\lambda = J(q)M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} - J(q)\dot{q} \tag{17}$$

where the $m \times m$ matrix $A(q)$ is defined as

$$A(q) = J(q) M^{-1}(q) J^{T}(q) \tag{18}$$

By Assumption 3, $J(q)$ is always full rank. Moreover, the inertial matrix, $M(q)$, is symmetric and positive definite. It follows then that $A(q)$ is invertible, giving

$$\lambda = A^{-1}(q) J(q) M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} - A^{-1}(q)\dot{J}(q)\dot{q} \tag{19}$$

or, in a compact form,

$$\lambda = \Psi(q,\dot{q},\tau) \tag{20}$$

Equation (20) is an explicit algebraic expression for λ, in terms of q, $\dot{q}$, and τ. Substituting Eq. (20) into (1) gives

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J^{T}(q)\Psi(q,\dot{q},\tau) \tag{21}$$

Given the initial values, $q(t_0) = q_0$ and $\dot{q}(t_0) = \dot{q}_0$, chosen to be consistent with the constraints, i.e., $\phi(q_0) = 0$ and $J(q_0)\dot{q}_0 = 0$, Eq. (21) can then be solved.

4.2. Constrained Robot Dynamics with Constraint Uncertainties

The above analysis is now extended in a similar manner to the singular systems describing the constrained robot system with uncertainties in the constraint functions. Following a similar process, we have

$$A_a(q)\lambda_a = J_a(q) M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} - \dot{J}_a(q)\dot{q} \tag{22}$$

where $A_a = J_a(q) M^{-1}(q) J_a^{T}(q)$. Expanding $A_a(q)$ by noting that $J_a(q) = J_n(q) + J_\Delta(q)$, we have

$$\begin{aligned} A_a(q) &= J_a(q) M^{-1}(q) J_a^{T}(q) \\ &= (J_n(q) + J_\Delta(q)) M^{-1}(q)(J_n^{T}(q) + J_\Delta^{T}(q)) \\ &= (A_n(q) + \delta A(q)) \end{aligned} \tag{23}$$

where

$$A_n(q) = J_n(q) M^{-1}(q) J_n^{T}(q)$$

and

$$\delta A(q) = J_n(q) M^{-1}(q) J_\Delta^{T}(q) + J_\Delta(q) M^{-1}(q) J_n^{T}(q) + J_\Delta(q) M^{-1}(q) J_\Delta^{T}(q)$$

Writing the right-hand side of Eq. (22) as $B_a(q) = (B_n(q) + \delta B(q))$, and similarly expanding the terms $J_a(q)$ and $\dot{J}_a(q)$, gives

$$\begin{aligned} B_a(q) &= J_a(q) M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} - \dot{J}_a(q)\dot{q} \\ &= (J_n(q) + J_\Delta(q)) M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} \\ &\quad - \left(\dot{J}(q) + \dot{J}_\Delta(q)\right)\dot{q} \\ &= B_n(q) + \delta B(q) \end{aligned} \tag{24}$$

where

$$B_n(q) = J_n(q) M^{-1}(q)\{C(q,\dot{q})\dot{q} + G(q) - \tau\} - \dot{J}_n(q)\dot{q}$$

and

$$\delta B(q) = J_\Delta(q) M^{-1}(q)(C(q,\dot{q})\dot{q} + G(q) - \tau) - \dot{J}_\Delta\dot{q}$$

so that Eq. (22) becomes

$$(A_n(q) + \delta A(q))\lambda_a = B_n(q) + \delta B(q)$$

We can also rewrite λ_a as $\lambda_a = (\lambda_n + \delta\lambda)$. With these we now have

$$(A_n(q) + \delta A(q))(\lambda_n + \delta\lambda) = B_n(q) + \delta B(q) \tag{25}$$

to represent Eq. (22). The terms $A_n(q)$, $B_n(q)$, and λ_n are nothing but those for the nominal systems, i.e., these are the respective terms in Eq. (17). In other words, Eq. (17) can be presented as

$$A_n(q)\lambda_n = B_n(q) \tag{26}$$

Equation (25) can thus be regarded as a perturbed Eq. (26). We can use this to study how the uncertainties in the constraint functions affect the nominal constraint force, λ. For that, we invoke the following theorem from linear analysis[23]:

Theorem 1. *If* $\|\cdot\|$ *denotes any matrix norm, for which* $\|I\| = 1$ *and if* $\|Q\| < 1$, *then* $(I + Q)^{-1}$ *exists, and*

$$\|(I + Q)^{-1}\| \le \frac{1}{1 - \|Q\|}$$

For ease and clarity of presentation, the dependency on q or $\dot{q}$ for the various terms such as A and B are omitted from now on. With this in mind, let us first subtract Eq. (26) from Eq. (25) to obtain

$$A(I + A^{-1}\,\delta A)\,\delta\lambda = \delta B - \delta A\,\lambda$$

Let us now assume that $\rho = \|A^{-1}\|\,\|\delta A\| < 1$ and $\|I\| = 1$. Then, since $\|A^{-1}\,\delta A\| \leq \|A^{-1}\|\,\|\delta A\| < 1$, Theorem (1) implies that $(I + A^{-1}\,\delta A)^{-1}$ exists, and that

$$\|(I + A^{-1}\,\delta A)^{-1}\| \leq (1-\rho)^{-1}$$

Hence, we now have

$$\delta\lambda = (I + A^{-1}\,\delta A)^{-1}\,A^{-1}\,\delta B - (I + A^{-1}\,\delta A)^{-1}\,A^{-1}\,\delta A\,\lambda \tag{27}$$

or

$$\|\delta\lambda\| \leq \frac{\|A^{-1}\|}{(1-\rho)}\|\delta B\| + \frac{\rho}{(1-\rho)}\|\lambda\| \tag{28}$$

From the definition of δB and taking norm, we have the following:

$$\begin{aligned}\|\delta B\| &= \left\| J_\Delta(q)M^{-1}(q)\{C\dot{q} + G - \tau\} - \dot{J}_\Delta(q)\dot{q}\right\| \\ &\leq \|J_\Delta\|\left\|M^{-1}\{C\dot{q} + G - \tau\}\right\| + \|\dot{J}_\Delta\|\,\|\dot{q}\|\end{aligned} \tag{29}$$

Remark 6. By Assumption 4, the bounds on the various quantities in expressions (28) and (29) exist.

Remark 7. Expression (28) clearly shows the influence of the constraint uncertainties on the constraint forces. The expression, together with the inequality expressed by (29), give indications of the upper bounds of the deviations of the constraint forces from those without constraint uncertainty. It is logical to conclude from these expressions that large errors in specifying the constraint functions would result in the constraint forces deviating substantially from the nominal value.

Remark 8. It is interesting to note that $\|\delta\lambda\|$ would be zero under two circumstances: one, of course, is when the uncertainty $\Delta(q)$ is zero, and two, when J_Δ is zero. The latter corresponds to the uncertainties being the offset or translational errors described earlier. Thus, we can conclude that rotational errors of the surface normals are responsible for the deviations of constraint forces from the nominal ones.

Remark 9. The importance of a controller on minimizing the errors arising from constraint uncertainties is evident from the expression for $\|\delta B\|$. Any controller used has to grapple with not only the uncertain term $\|J_\Delta\|$, but also the effect of the nonlinear terms and the gravity term.

5. CONCLUSION

We have developed a model to describe the dynamics of a constrained robot system with constraint uncertainties. The uncertainties in the constraint functions are explicitly expressed in the dynamics of the robot system. The accounts on the geometric interpretations of the uncertainties also show that this mode of modelling of the constraint uncertainties is general. The influence of the uncertainties on the robot dynamics has also become clear.

REFERENCES

1. N.H. McClamroch, Force and impedance control, Robot control—Dynamics, motion planning, and analysis, part 7, M.W. Spong, F.L. Lewis, and C.T. Abdallah, Eds., IEEE Press, New York, 1993, pp. 273–276.
2. R.J. Anderson and M.W. Spong, Hybrid impedance control of robotic manipulators, IEEE J Robotics Automation (1988), 549–556.
3. A.A. Goldenberg, Implementation of force and impedance control in robot manipulators, Proc Conf Robotics Automation, Philadelphia, PA, 1988, pp. 1626–1632.
4. N. Hogan, Impedance control: an approach to manipulation: part I—theory, part II—implementations, part III—applications, J Dynam Syst Meas Contr 107 (1985), 1–24.
5. H. Asada and N. Goldfine, Optimal compliance design for grinding robot tool holders, In Proc Conf Robotics Automation, St. Louis, MO, 1985, pp. 316–322.
6. L. Liu, B.J. Ulrich, and M.A. Elbestawi, Robotic grinding force regulation: design, implementation and benefits, Proc Conf Robotics Automation, Cincinnati, OH, 1990, pp. 258–265.
7. H. West and H. Asada, A method for the control of robot arms constrained by contact with the environment, Proc 1985 American Control Conf, Boston, 1985, pp. 383–386.
8. N.H. McClamroch, Singular systems of differential equations as dynamic models for constrained robot systems, Proc Conf Robotics Automation, San Francisco, 1986, pp. 21–28.
9. N.H. McClamroch, A singular perturbation approach to modeling and control of manipulators constrained by a stiff environment, Proc. 28th Conf Decision and Control, Florida, 1989, pp. 2407–2411.

10. N.H. McClamroch and H. Huang, Dynamics of a closed chain manipulator, Proc. 1985 American Control Conf., Boston, 1985, pp. 50–54.
11. N.H. McClamroch and D. Wang, Feedback stabilization and tracking of constraint robots, IEEE Trans Automat Control 33 (1988), 419–426.
12. D. Wang and N.H. McClamroch, Position and force control for constrained manipulator motion: Lyapunov's direct method, IEEE J Robotics Automat 9 (1993), 83–92.
13. D. Wang, Y.C. Soh, Y.K. Ho, and P.C. Müller, Global stabilization for constrained robot motion with constraint uncertainties, Robotica 16 (1998), 171–179.
14. H. Goldstein, Classical Mechanics, Addison-Wesley, 1950.
15. D.T. Greenwood, Principles of Dynamics, Prentice Hall, second edition, 1965.
16. Y.K. Ho, Modelling and control of constrained robot systems with uncertainties in the constraint functions, PhD dissertation, Nanyang Technological University, School of Electrical & Electronic Engineering, October 1997.
17. Y.K. Ho, D. Wang, and Y.C. Soh, Robust control of manipulators performing constrained motion with uncertainties in the constraint functions, J Robotic Syst 12 (1995), 747–755.
18. J.K. Mills and A.A. Goldengerg, Force and position control of manipulators during constrained motion tasks, IEEE Trans Robot Automat 5 (1989), 30–46.
19. D.J. Cobb, Descriptor variable and generalized singularly perturbed systems: A geometric approach, PhD thesis, University of Illinois, Urbana, 1980.
20. C.W. Gear, B. Leimkubler, and G.K. Gupta, Automatic integration of Euler–Lagrangian equations with constraints, J Comput Appl Math 12 & 13 (1985), 77–90.
21. C.W. Gear and L.R. Petzold, ODE methods for the solution of differential/algebraic systems, SIAM J Numer Anal 21 (1984), 716–728.
22. H. Hemami and B.F. Wyman, Modeling and control of constrained dynamic systems with application to biped locomotion in the frontal plane, IEEE Trans Automat Control 24 (1979), 526–535.
23. P. Lancaster and M. Tismenetsky, The Theory of Matrices, Academic Press, second edition, 1985.