Periodic errors elimination in CVCF PWM DC/AC converter systems: Repetitive control approach

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Abstract: A plug-in digital repetitive learning (RC) controller is proposed to eliminate periodic tracking errors in constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC/AC converter systems. The design of the RC controller is systematically developed and the stability analysis of the overall system is discussed. The periodic errors are forced toward zero asymptotically and the total harmonics distortion (THD) of the output voltage is substantially reduced under parameter uncertainties and load disturbances. Simulation and experimental results are provided to illustrate the validity of the proposed scheme.

1 Introduction

Constant-voltage constant-frequency pulse-width modulated (CVCF PWM) DC/AC converters are widely employed in various AC power-conditioning systems, such as automatic voltage regulators and uninterruptible power supply systems. Output voltage THD is one important index to evaluate the performance of the converters, associated with communication interference, excessive heating in capacitors and transformers etc. Nonlinear loads, causing periodic distortion, are major sources of THD in AC power systems. To minimise THD, several high precision control schemes are proposed for the CVCF PWM DC/AC converters. A deadbeat (or OSAP) controller has been proposed [1–3]. Sliding mode controller (SMC) [4, 5] and hysteresis controller (HC) [6] can overcome parameter uncertainties and load disturbance. However, the deadbeat control is highly dependent on the accuracy of the parameters; random switching pattern of SMC or HC will impose excessive stress on power devices and cause difficulty in lowpass filtering.

Repetitive learning control (RC) law is closely similar to iterative learning control (ILC) [7–11]. Although an ILC system only updates the control input once each cycle and resets the plant at the beginning of each iteration, RC continuously adjusts its control input and needs no reset. The RC method [12], based on the internal model (IM) principle [13], has proposed [14, 15] to achieve high accuracy in the presence of uncertainties for servomechanism. Applications of RC [16] include robots [17], disc drives [18], steel casting process [19], satellites [20]. Without a complete design method and stability analysis of RC system, it has been applied to DC/AC converters [21, 22] with preliminary results.

In this paper, the design of discrete time RC controller is presented systematically. A plug-in RC controller is proposed and developed for the OSAP controlled CVCF PWM DC/AC converters. The stability of overall system is discussed. To show the validity of proposed method, simulation and experimental results are illustrated.

2 Plug-in discrete time repetitive controller

Fig. 1a shows a periodic signal generator. Consider the discrete time RC system shown in Fig. 1b, where \(v_f(z)\) is the reference input signal, \(y(z)\) is the output signal, \(d(z)\) is the disturbance signal, \(e(z)\) is the tracking error signal, \(G_r(z)\) is the transfer function of the plant, \(G_c(z)\) is the repetitive signal generator; plug-in RC controller \(G_p(z)\) is the feedforward compensator, and \(G_{f}(z)\) is the conventional feedback controller. \(G_c(z)\) is chosen so that the following closed-loop transfer function is asymptotically stable.

\[
H(z) = \frac{G_c(z)G_r(z)}{1 + G_c(z)G_r(z)} = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}B(z^{-1})B^*(-z^{-1})}{A(z^{-1})}
\]

where \(d\) is the known number of pure time step delays; \(B^*(-z^{-1})\) is the uncancelable portion of \(B(z^{-1})\); \(B^*(-z^{-1})\) is the cancellable portion of \(B(z^{-1})\).

Based on the internal model (IM) principle [13], the zero-error tracking of any reference input in the steady state can be achieved if a generator of the reference input is included in the stable closed-loop system. Therefore, for a periodic reference input, the RC controller \(G_{f}(z)\) is plugged into the system shown in Fig. 1 as follows [16]:

\[
G_{f}(z) = G_r(z)G_f(z) = \frac{k_z z^{-N_i}}{1 - z^{-N}} G_f(z)
\]

where the repetitive signal generator \(G_c(x)\) and the filter \(G_f(z)\) are chosen as follows:

\[
G_r(z) = \frac{k_z z^{-N_i}}{1 - z^{-N}} = \frac{k_z}{z^N - 1}
\]

\[
G_f(z) = \frac{z^{-N_i}A(z^{-1})B^*(z)}{B^*(z^{-1})b}
\]
where $k_r$ is the repetitive control gain; $N = N_1 + N_2 = \sigma f z$ with $f$ being the reference signal frequency and $f_z$ the sampling frequency; $N_2 = n_2 + d$; $B^{-}(z^{-1})$ is obtained from $B^{-}(z)$ with $z^{-1}$ replaced by $z$; $b$ is a scalar chosen so that $b \geq [B^{-}(1)]^2$; $n_z$ is the order of $B^{-}(z^{-1})$, and $z^{-n_z}$ makes the filter realizable. $G_r(z)$ in (4) is an implementation of the zero phase error tracking controller (ZPETC) design [23].

From Fig. 1, the transfer functions from $y_d(z)$ and $d(z)$ to $v(z)$ in the overall closed loop system are respectively derived as

$$
y(z) = \frac{1 + G_r(z)G_1(z)G_2(z)G_3(z)}{1 + (1 + G_r(z)G_1(z)G_2(z)G_3(z))/H(z)}$$

$$d(z) = \frac{1}{1 + G_r(z)G_3(z)(1 - z^{-N})}$$

From eqns. 5 and 6, it can be concluded that the overall closed-loop system is stable if the following two conditions hold: (i) The roots of $1 + G_r(z)G_3(z) = 0$ are inside the unit circle; and (ii):

$$|1 - k_r z^{-N} G_3(z) H(z)| < 1, \quad \text{for all } z = e^{j\omega}, 0 < \omega < \frac{\pi}{T}$$

(7)

However, in practice, it is impossible to implement the ZPETC filter $G_r(z)$ exactly because of model uncertainty. The unmodelled dynamics $Delta(z)$ be represented in the form of a multiplicative modelling error. Then the relation between the actual system transfer function $H(z)$ and the nominal system transfer function $H(z)$ can be written as

$$H(z) = H(z)(1 + \Delta(z))$$

(8)

Then, using eqns. 1 and 4, the following product can be expressed as

$$G_r(z)H(z) = z^{-N} \frac{B^{-}(z)B^{-}(z)}{b} + \Delta(z)$$

(9)

And it is assumed that there exists a constant $c$ such that $||\Delta(z)|| \leq c$. Therefore, eqn. 7 leads to a conservative stable range for $k_r$ as follows

$$0 < k_r < \frac{2}{\max \|B^{-}(z)B^{-}(z^{-1})/b + \Delta(z)\|} \leq \frac{2}{1 + c}$$

(10)

From Fig. 1, the error transfer function for the overall system can be derived as

$$G_e(z) = \frac{e(z)}{y_d(z) - d(z)} = \frac{1}{1 + G_r(z)G_3(z)(1 - z^{-N})}$$

$$= \frac{1}{1 - z^{-N}(1 - k_r z^{-N} G_3(z) H(z))}$$

(11)

Obviously, if the overall closed-loop system shown in Fig. 1 is asymptotically stable and the angular frequency $\omega$ of the reference input $y_d(t)$ and the disturbance $d(t)$ approaches $\omega_m = 2\pi f z$ for $M = \pi f z$ for even $N$ and $M = (N - 1)/2$ for odd $N$, then $z^{-N} \rightarrow 1$, $\lim_{\omega \rightarrow 0} \|G_r(j\omega)\| = 0$, and thus

$$\lim_{\omega \rightarrow 0} \|e(j\omega)\| = 0$$

(12)

eqn. 12 indicates that zero steady-state tracking error is achieved with the RC controller for any periodic reference input whose frequency is less than half of the sampling frequency. To enhance the robustness of the system, a low-pass filter $Q(z, z^{-1})$ is used in RC controller as follows [17]:

$$Q(z) = k_r Q(z, z^{-1}) z^{-N_1}$$

(13)

where

$$Q(z, z^{-1}) = \frac{\sum_{i=0}^{m} a_i z^i + \sum_{i=1}^{m} a_{-i} z^{-i}}{2 \sum_{i=1}^{m} a_i + a_0}$$

(14)

where $a_i (i = 0, 1, \ldots, m; m = 0, 1, 2, \ldots)$ are coefficients to be designed.

Notice that $Q(z, z^{-1})$ is a moving average filter that has zero phase shift and brings all open-loop poles inside the
3 RC controller for CVCF PWM DC/AC converters

3.1 Modelling CVCF PWM DC/AC converters

The dynamics of the CVCF PWM DC/AC converter (as shown in Fig. 2(a)) can be described as follows [1]:

\[
\begin{bmatrix}
0 & 1 \\
L_a C_a & 0
\end{bmatrix}
\begin{bmatrix}
v_c(k+1) \\
v_{i_a}(k+1)
\end{bmatrix}
+
\begin{bmatrix}
0 \\
1/L_a C_a
\end{bmatrix}
\begin{bmatrix}
v_c(k) \\
v_{i_a}(k)
\end{bmatrix}
\begin{bmatrix}
\varphi_{11} & \varphi_{12} \\
\varphi_{21} & \varphi_{22}
\end{bmatrix}
\begin{bmatrix}
v_c(k) \\
v_{i_a}(k)
\end{bmatrix}
\Delta T(k)
\]

where \(v_c\) is the output voltage, \(i_o\) is the current, \(v_{dc}\) is the DC bus voltage, \(L_a\), \(C_a\), and \(R_a\) are the nominal values of the inductor, capacitor, and load, respectively; as shown in Fig. 2b, the control input \(v_{in}\) is a PWM voltage pulse of magnitude \(v_{dc}\) or \(-v_{dc}\) with width \(\Delta T\) centred in the sampling interval \(T\).

For a linear system \(x = Ax + Bu\), its sampled-data equation can be expressed as \(x(k+1) = e^{AT}x(k) + \int_0^T e^{A(T-r)}Bu(x)\,dr\). Therefore, a sampled-data form for eqn. 16 can be derived as follows:

\[
\begin{bmatrix}
v_c(k+1) \\
v_{i_a}(k+1)
\end{bmatrix}
= \begin{bmatrix}
\varphi_{11} & \varphi_{12} \\
\varphi_{21} & \varphi_{22}
\end{bmatrix}
\begin{bmatrix}
v_c(k) \\
v_{i_a}(k)
\end{bmatrix}
+ \begin{bmatrix}
g_1 \\
g_2
\end{bmatrix}
\Delta T(k)
\]

where coefficients \(\varphi_{ij} = 1 - T^2/2L_a C_a\), \(\varphi_{21} = -T L_a C_a + T^2/2L_a C_a R_a\), \(\varphi_{22} = 1 - T C_a C_a - T^2/2L_a C_a R_a\), \(g_1 = E T/2L_a C_a\), \(g_2 = E T C_a C_a (1 - (T/2C_a R_a))\).

3.2 Problem formulation

Consider the DC/AC converter described by eqn. 17 and its output equation

\[
y(k) = v_c(k)
\]

The objective of the controller is to force the tracking error between \(y(k)\) and its sinusoidal reference \(y_d(k)\) with the period of \(N T\) to approach zero asymptotically.

3.3 Controller design

According to the theory in Section 2, the controller for CVCF PWM DC/AC converter comprises a conventional feedback controller and a plug-in RC controller.

3.3.1 Conventional feedback controller: The ARMA equation for the dynamics (eqns. 17, 18) can be obtained as:

\[
y(k+1) = -p_1 y(k) - p_2 y(k-1) + m_1 u(k) + m_2 u(k-1)
\]

where \(u(k) = \pm \Delta T(k); p_1 = -0.5 (\varphi_{11} + \varphi_{22})\), \(p_2 = \varphi_{11} \varphi_{22} - \varphi_{12} \varphi_{21}\), \(m_1 = g_1\) and \(m_2 = g_2\). If the control law for the plant (eqn. 19) is chosen as

\[
u(k) = \begin{bmatrix}
\frac{1}{m_1}
\end{bmatrix}
\begin{bmatrix}
y_d(k) - m_2 u(k-1) + p_1 y(k) + p_2 y(k-1)
\end{bmatrix}
\]

then \(y(k+1) = y_d(k)\). It yields a deadbeat response \(H(z) = z^{-1}\). Eqn. 20 describes a one sampling ahea preview (OSAP) controller [1].

3.3.2 Plug-in repetitive controller

As well as a sampling time tracking delay, the OSAP controller depends on the model having accurate \(L_a\), \(C_a\), and \(R_a\). In practice, parameter uncertainties \(\Delta L, \Delta C\) and load disturbance \(\Delta R\) yield large tracking errors. Therefore a RC controller is proposed to overcome the periodic disturbance and parameters variation. According to design theory mentioned in Section 2, \(G_c(z) = 1/H(z) = z\) and the RC controller \(G_r(z)\) is proposed

\[
G_r(z) = G_c(z) G_p(z) = \frac{k z^{-N+1} Q(z, z^{-1})}{1 - Q(z, z^{-1}) z^{-N}}
\]

For simplicity of design and analysis, \(Q(z, z^{-1}) = 1\) and \(N = 0\) in this case. In sampled-data form, the RC controller can be expressed as follows:

\[
u_c(k) = u_c(k - N) + k_e e(k - N + 1)
\]

In fact, eqn. 22 is the same as anticipatory learning control law [11].

3.4 Robustness analysis

In view of the uncertainties \(\Delta L, \Delta C\) and \(\Delta R\), the ARMA equation for the actual plant becomes

\[
y(k+1) = -a_1 y(k) - a_2 y(k-1) + b_1 u(k) + b_2 u(k-1)
\]

where \(a_1 = p_1 + \Delta p_1\), \(a_2 = p_2 + \Delta p_2\), \(b_1 = m_1 + \Delta m_1\), \(b_2 = m_2 + \Delta m_2\) are calculated on the basis of the practical parameters \(L = L_a + \Delta L\), \(C = C_a + \Delta C\), \(R = R_a + \Delta R\). When an OSAP controller (eqn. 20) is used.

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Fig. 2 CVCF PWM DC/AC converter and PWM input waveform

a CVCF PWM DC/AC converter
b PWM waveform for \(v_{in}\)

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applied to the plant (eqn. 23), the closed-loop transfer function \( H_z(z) \) without an RC controller becomes

\[
H_z(z) = \frac{(b_1 + b_2z^{-1})}{(z + a_1 + a_2z^{-1})(m_1 + m_2z^{-1}) - (p_1 + p_2z^{-1})(b_1 + b_2z^{-1})}
\]

When \( L = L_o \), \( C = C_o \), \( R = R_o \), a deadbeat response \( H_z(z) = z^{-1} \) is achieved.

In practice, to enhance the robustness, \( Q(z, z^{-1}) \) can set to be \( d_1z + d_0 + d_2z^{-1} \). This doesn't influence the above analysis results. According to the stability analysis in Section 2, the overall system is stabilised if (i) all poles of \( H_z(z) \) in eqn. 24 are inside the unit circle; (ii) \( \|1 - k_z H_z(z)\| < 1/\|Q(z, z^{-1})\| \). \( k_z \) can be larger because of the introduction of \( Q(z, z^{-1}) \).

### 3.5 Simulation and experiment

Our simulation and experimental studies are carried out using the schematic diagram of a controlled DC/AC converter system shown in Fig. 3.

![Stability analysis](image)

**Fig. 4** Stability analysis

- Radius of poles of \( H_z(z) \)
- Maximum \(|e^{\mu} H_z(e^{\mu})|\)

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![Steady-state simulation results with resistance load](image)

**Fig. 5** Steady-state simulation results with resistance load \( R = 4.7 \Omega \)

- a OSAP controlled reference voltage \( v_r(t) \), output voltage \( v_c(t) \), output current \( i_c(t) \)
- b Reference voltage \( v_r(t) \), output voltage \( v_c(t) \), output current \( i_c(t) \) with OSAP plus RC
  - (i) \( v_r(t) \), (ii) \( v_c(t) \), (iii) \( i_c(t) \)
In particular, the DC/AC converter has the parameter values: $C_s = 700 \, \mu F$; $L_s = 600 \, \mu H$; $R_s = 2 \, \Omega$; $C = 800 \, \mu F$; $L = 700 \, \mu H$; $y_s(t)$ is 50 Hz; 10 V (peak) sinusoidal signal; $V_{in} = 20 \, V$; $f = 50 \, Hz$; $f_c = 1/T = 6.25 \, kHz$; $d_0 = 0.9$, $d_1 = 0.05$.

As shown in Fig. 4a, with these above parameter values and when $R > 1 \, \Omega$, all the poles of the closed-loop transfer function $H_s(z)$ in eqn. 24 without the RC controller are located inside the unity circle, the system is stable.

As shown in Fig. 4b, the maximum gain of $|zH_s(z)|$ in frequency domain is no more than 8. According to the stability condition $|1 - k_s z H_s(z)| < 1$ for RC control design, the system with RC controller is stable if $k_s \in (0, 0.25)$. We set $k_s = 0.05$.

### 3.5.1 Simulation results

Figs. 5 and 6 show the simulation results of the only OSAP controlled and RC plus OSAP controlled CVCF PWM DC/AC converter with resistance load and uncontrolled rectifier load, respectively. With OSAP controller, the peaks of tracking error $e(t)$ between output voltage and reference voltage are about 1.8 V in Fig. 5a and about 2.3 V Fig. 6a. Figs. 5b and 6b show the RC controller force the output voltage to approach reference voltage under different loads and significantly reduce the tracking error, respectively.

**Fig. 6** Steady-state simulation results with rectifier load $R_s = 4.70 \, \Omega$, $C_s = 1470 \, \mu F$

- (a) OSAP controlled reference voltage $y_s(t)$, output voltage $v_s(t)$, output current $i_s(t)$
- (b) Reference voltage $y_s(t)$, output voltage $v_s(t)$, output current $i_s(t)$ with OSAP plus RC
- (c) $y_s(t)$, $v_s(t)$, $i_s(t)$

**Fig. 7** Experiment setup

**Fig. 8** Experimental results with resistance load

- (a) Output voltage $v_s(t)$ (upper, 5V/division), output current $i_s(t)$ with OSAP controller
- (b) Output voltage $v_s(t)$ (upper, 5V/division), output current $i_s(t)$ with OSAP plus RC
- (c) Transient response of tracking error $e(t) = y_s(t) - v_s(t)$ with OSAP plus RC

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3.5.2 Experimental results

Experiment setup (Fig. 7) has been built for the converter system shown in Fig. 3. Facilities include DSPACE (DS1102) DSP development toolkits, H-bridge IGBT switches converter and a HP 546002 oscilloscope.

Fig. 8 shows the experimental results of the only OSAP and OSAP plus RC controlled CVCF PWM DC/AC converter under resistance load ($R = 4.7 \Omega$). Output voltage is about 8.2 V with OSAP controller in Fig. 8a; output voltage approaches 10V with OSAP plus RC controller in Fig. 8b. Furthermore, Fig. 8c shows the tracking error $e(t)$ is reduced from about 1.8 V to be less than 0.3 V after about 4 s when RC controller is plugged into the OSAP controlled converter.

Fig. 9 shows the experimental results of the only OSAP and OSAP plus RC controlled CVCF PWM DC/AC converter under uncontrolled rectifier load ($C = 1470 \mu F$, $R = 4.7 \Omega$). Output voltage is about 7.7 V with OSAP controller and is distorted by current surge in Fig. 9a; output voltage approaches 10 V and has less distortion with OSAP plus RC controller in Fig. 9b. Furthermore, Fig. 9c shows the peak of tracking error $e(t)$ is reduced from about 2.3 V to be less than 0.4 V after about 4 s when RC controller is plugged into the OSAP controlled converter.

The experimental transient tracking errors are collected through A/D converters of DS1102 card and TRACE software. The residual tracking errors can be further reduced by improving the sampling frequency $f_c$.

4 Conclusion

A plug-in discrete time repetitive learning control scheme has been proposed for the CVCF PWM DC/AC converter systems. The periodic tracking errors caused by nonlinear load disturbances (such as rectifier load) and parameter uncertainties ($\Delta L$ and $\Delta C$) are eliminated by the plug-in repetitive learning controller. It is shown that the proposed control scheme offers zero error tracking capability for the CVCF PWM DC/AC converter systems under different loads and parameter uncertainties. Minimised output voltage THD and fast response are achieved. Simulation and experimental results are provided to demonstrate the validity of the proposed control scheme.

5 References


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