ground $c^1(x) = 1$, $c^2(x) = 1$, mutual capacitance $c^{12}(x) = 1$, resistances $r^1(x) = 1$, $r^2(x) = 2$ and length $L = 1$ (all the quantities are expressed in normalised units), see Fig. 2. The $R(x)$ and $C(x)$ matrices are

$$R(x) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad C(x) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \tag{10}$$

By means of eqn. 4 we can compute the first three coefficients matrices which appear in eqn. 3 as follows:

$$\alpha_1 = \zeta^{(2)}(x=1) = R \cdot C \cdot \frac{x^2}{2}\Big|_{x=1} = \begin{bmatrix} 1 & -0.5 \\ -1 & 2 \end{bmatrix}$$

$$\alpha_2 = \zeta^{(4)}(x=1) = R \cdot C \cdot R \cdot C \cdot \frac{x^4}{4}\Big|_{x=1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.5 & 0.75 \end{bmatrix}$$

$$\alpha_3 = \zeta^{(6)}(x=1) = R \cdot C \cdot R \cdot C \cdot R \cdot C \cdot \frac{x^6}{6}\Big|_{x=1} = \begin{bmatrix} 0.033 & -0.0417 \\ -0.083 & 0.1167 \end{bmatrix}$$

$$\gamma_1 = \psi^{(1)}(x=1) = C \cdot x|_{x=1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\gamma_2 = \psi^{(3)}(x=1) = C \cdot R \cdot C \cdot \frac{x^3}{3}\Big|_{x=1} = \begin{bmatrix} 1 & -1 \\ -1 & 1.5 \end{bmatrix}$$

$$\gamma_3 = \psi^{(5)}(x=1) = C \cdot R \cdot C \cdot R \cdot C \cdot \frac{x^5}{5}\Big|_{x=1} = \begin{bmatrix} 0.2 & -0.25 \\ -0.25 & 0.35 \end{bmatrix} \tag{11}$$

Then according to the synthesis procedure of eqns. 5 – 9 a three cell lumped net is determined:

$$R_1 = \begin{bmatrix} 1.07 & 0 \\ 0 & 2.14 \end{bmatrix} \quad R_2 = \begin{bmatrix} -0.41 & 0 \\ 0 & -0.82 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.32 \end{bmatrix} \quad C_1 = \begin{bmatrix} 0.93 & -46 \\ -0.46 & 0.93 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -0.13 & 0.066 \\ 0.066 & -0.13 \end{bmatrix} \quad C_3 = \begin{bmatrix} 1.2 & -0.6 \\ -0.6 & 1.2 \end{bmatrix}$$

We tested our reduced three cell lumped model (denoted by R-3C) by comparing it with the iterated lumped structure obtained by discretising each line into 200 sections (denoted by IL-200C). We simulated the output waveforms $V_0^1(t)$, $V_0^2(t)$ of the two lines by introducing the lumped models R-3C and IL-200C into SPICE. Fig. 3 shows these waveforms when one line is driven by a step voltage while the other is short circuited. The proposed reduced order model R-3C evaluates both the delay and the crosstalk with high accuracy. With the employment of R-3C the time simulation is about two orders of magnitude faster.

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Zero tracking error controller for three-phase CVCF PWM inverter

Keliang Zhou and Danwei Wang

A zero tracking error control scheme for three-phase CVCF PWM inverters is proposed. The proposed scheme uses a repetitive controller (RC) to force output line voltages to track a sinusoidal reference signal with zero error. Minimised voltage distortion and a fast response are obtained. The validity of the proposed scheme has been verified by simulations.

Introduction: Total harmonics distortion (THD) is an important index for the evaluation of the performance of three-phase CVCF pulsewidth modulation (PWM) inverters. Nonlinear loads and uncertainties in the parameters, causing periodic tracking errors, are major sources of THD in AC power systems. Many high-precision control schemes for three-phase CVCF PWM inverters, such as the deadbeat controller with observers [1], sliding mode controller [2] and hysteresis controller [3], have been developed. However, they have the disadvantages of high dependence on the accuracy of the plant parameters, or excessive stress on the power device and the difficulty of lowpass filtering caused by random switching patterns. To overcome these disadvantages, a repetitive controller (RC) is proposed to implement robust zero error tracking of a sinusoidal reference input.

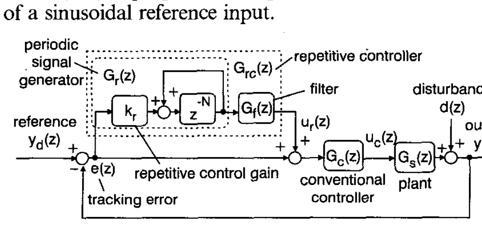

Fig. 1 *Repetitive control system*

Discrete time RC design: As shown in Fig. 1, conventional controller $G_c(z)$ is chosen so that the following closed-loop transfer function is asymptotically stable:

$$H(z) = \frac{G_c(z)G_s(z)}{1 + G_c(z)G_s(z)} = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}B^+(z^{-1})B^-(z^{-1})}{A(z^{-1})} \tag{1}$$

where d is the number of pure time step delays, $B^-(z^{-1})$ is the uncancellable portion of $B(z^{-1})$, and $B^+(z^{-1})$ is the cancellable portion of $B(z^{-1})$. According to the internal model principle, zero error tracking of any reference input in the steady state can be achieved if the generator of the reference input is included in the stable closed-loop system. Therefore, for a periodic reference input signal $y_d(z) = z^{-N}/(1 - z^{-N})y_{d0}(z)$, where $y_{d0}(z) = y_d(0) + y_d(1)z^{-1} + ... + y_d(N-1)z^{-(N-1)}$ represents the first periodic sequence and $N = f/f_c$ with f being the reference input frequency and f_c being the sampling frequency, the RC is proposed as follows [4]:

$$G_{rc}(z) = G_c(z)G_f(z) = \frac{k_r z^{-N}}{1 - z^{-N}} G_f(z) \tag{2}$$

where k_r is the RC gain, the filter

$$G_f(z) = \frac{z^{-n_u} A(z^{-1}) B^-(z)}{B^+(z^{-1}) b}$$

where $B^-(z)$ is obtained from $B^-(z^{-1})$ with z^{-1} replaced by z, $b \geq [B^-(1)]^2$, and n_u is the order of $B^-(z^{-1})$. The error transfer function for the overall closed-loop system is

$$G_e(z) = \frac{e(z)}{y_d(z) - d(z)} = \frac{1}{1 + G_c(z)G_s(z)} \frac{1 - z^{-N}}{1 - z^{-N}(1 - k_r G_f(z) H(z))} \tag{3}$$

where $e(z)$ is the tracking error and $d(z)$ is the disturbance. From eqns. 1 and 3, it can be concluded that the overall closed-loop system is stable if the following two conditions hold: (i) the roots of $1 + G_c(z)G_s(z) = 0$ are inside the unit circle; (ii) $\|1 - k_r G_f(z)H(z)\| < 1$. If the angular frequency ω of $y_d(t)$ and $d(t)$ approaches $\omega_m = 2\pi mf$ (m = 0, 1, ..., $N/2$), then $z^{-N} \to 1$, $\|G_e(j\omega)\| \to 0$, and thus $\lim_{\omega\to\omega_m}\|e(j\omega)\| = 0$. Therefore zero steady-state tracking error is obtained with an RC for any reference input for which the frequency is less than half the sampling frequency.

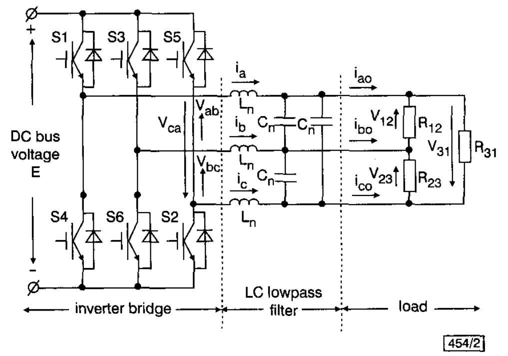

Fig. 2 *Three-phase CVCF PWM inverter with nominal parameters*

$R_{12} = R_{23} = R_{31} = R_n$: resistance loads
i_{ao}, i_{bo}, i_{co}: output currents
V_{12}, V_{23}, V_{31}: output line voltages
L_n, C_n, R_n: nominal parameters

RC for three-phase CVCF PWM inverter: In the α-β co-ordination plane, through a 3/2 transformation, the sampled-data model for the three-phase CVCF PWM inverter (as shown in Fig. 2) can be transformed into two decoupled identical second-order subsystems (see [1]), one of which, for example the α-phase subsystem, becomes [5]:

$$y_\alpha(k+1) = -p_1 y_\alpha(k) - p_2 y_\alpha(k-1) + m_1 u_{\alpha c}(k) + m_2 u_{\alpha c}(k-1) \quad (4)$$

where $y_\alpha(k) = (1/E)v_\alpha(k)$ is the output; $p_1 = -(\varphi_{11} + \varphi_{22})$, $p_2 = \varphi_{11}\varphi_{22} - \varphi_{21}\varphi_{12}$, $m_1 = g_1T/E$, $m_2 = (T/E)(g_2\varphi_{12} - g_1\varphi_{22})$, $u_{\alpha c}(k) = (1/T)u_\alpha(k)$; $T(= 1/f_c)$ is the sampling period; and

$$\varphi_{11} = 1 - \frac{T}{C_nR_n} + \frac{T^2}{2C_n^2R_n^2} - \frac{T^2}{6L_nC_n}$$
$$\varphi_{12} = \frac{T}{3C_n} - \frac{T^2}{6C_n^2R_n} \qquad \varphi_{21} = -\frac{T}{L_n} + \frac{T^2}{2L_nC_nR_n}$$
$$\varphi_{22} = 1 - \frac{T^2}{6L_nC_n} \quad g_1 = \frac{ET}{6L_nC_n} \quad g_2 = \frac{E}{L_n}$$

L_n, C_n, R_n are nominal parameters of the devices.

If the conventional controller $G_\alpha c(z)$ in the sampled-data form is

$$u_{\alpha c}(k) = \frac{1}{m_1}[y_{\alpha d}(k) - m_2u_{\alpha c}(k-1) + p_1y_\alpha(k) + p_2y_\alpha(k-1)] \quad (5)$$

then $y_\alpha(k + 1) = y_{\alpha d}(k)$, where $y_{\alpha d}$ is the sinusoidal reference input. Eqn. 5 represents a one sampling ahead preview (OSAP) controller [5]. It yields deadbeat response $H_\alpha(z) = z^{-1}$ and needs no current sensors. According to eqns. 1 and 2 and $H_\alpha(z) = z^{-1}$, an RC controller $G_{\alpha rc}(z) = k_r z^{-N+1}/(1 - z^{-N})$ is proposed. In sampled-data form, $G_{\alpha rc}(z)$ can be expressed as

$$u_{\alpha r}(k) = u_{\alpha r}(k-N) + k_{\alpha r}e_\alpha(k-N+1) \quad (6)$$

In practice, the inverter parameters and load are $L = L_n + \Delta L$, $C = C_n + \Delta C$, $R = R_n + \Delta R$. The difference equation for the actual plant is

$$y_\alpha(k+1) = -a_1y_\alpha(k) - a_2y_\alpha(k-1) + b_1u_{\alpha c}(k) + b_2u_{\alpha c}(k-1) \quad (7)$$

where a_1, a_2, b_1 and b_2 depend on L, C and R. When the OSAP controller (eqn. 5) is applied to eqn. 7, $H_\alpha(z)$ becomes

$$H_\alpha(z) = \frac{(b_1 + b_2z^{-1})}{(z + a_1 + a_2z^{-1})(m_1 + m_2z^{-1}) - (p_1 + p_2z^{-1})(b_1 + b_2z^{-1})} \quad (8)$$

The overall system is stabilised if: (i) all the poles of eqn. 8 are inside the unit circle; (ii) $\|1 - k_{\alpha rz}H_\alpha(z)\| < 1$. The design of the controller for a β-phase subsystem is the same as that for an α-phase subsystem.

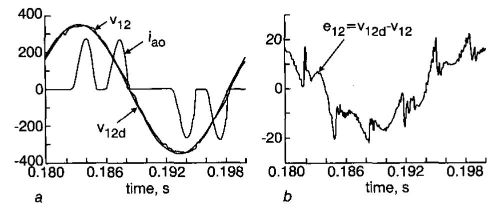

Fig. 3 *Simulated OSAP controlled results with rectifier load*

a Reference line voltage, output line voltage, output current
b Steady-state tracking error

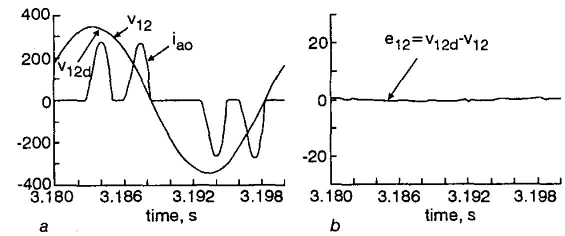

Fig. 4 *Simulated OSAP plus RC controlled results with rectifier load*

a Reference line voltage, output line voltage, output current
b Steady-state tracking error

Simulation: The simulation parameters were C = 500μF, C_n = 450μF, L = 600μH, L_n = 550μH; R_n = 3Ω; the reference line voltages were 200√3V (peak). E = 400V, f = 50Hz and $f_c = 1/T = 10^4$Hz. Based on the above parameters, it was calculated that, if $R > 2.5\Omega$, $H_i(z)$ (i = α, β) is asymptotically stable. In addition, owing to $\max(\|j\omega H_i(j\omega)\|) \simeq 28$, $\|1 - k_rzH_i(z)\| < 1$ leads to $k_{ir} \in (0, 0.07)$. In our case, $k_{\alpha r} = k_{\beta r} = 0.005$. Figs. 3 and 4 show that RC controllers reduce the peaks of tracking errors from 22V to < 0.5V after ~ 3s with phase-controlled rectifier load ($L_l = 1e^{-5}$H, $C_l = 1e^{-2}$ F, $R_l = 4\Omega$). The residue error can be further reduced by improving the sampling frequency f_c.

Conclusion: Theory and simulations have demonstrated that the proposed RC offers zero tracking error capability (minimised line voltage THD) for a three-phase CVCF PWM inverter in the presence of uncertainties in the parameters and periodic load disturbances.

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