Look-Ahead/behind Control of a Car-Like Vehicle

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Abstract

A control scheme is proposed for the trajectory tracking of a car-like vehicle. The control scheme uses a virtual reference point which is in front or in the back of the vehicle for forward or backward motions, respectively. Simulations and experiments are carried out to verify the proposed control scheme.

1 Introduction

Because a well-known work of Brockett [1] identifies that nonholonomic systems cannot be stabilized via a smooth static state feedback, the stabilization of nonholonomic systems has received considerable attention during the past few years. Many sophisticated control schemes were developed, such as nonsmooth feedback laws [2–4], time-varying feedback laws [5, 6], and their combination [7, 8]. However, those control schemes usually involve quite complicated computation. Moreover, they either require unrealistic control effort or produce a slow convergent rate. Consequently, few of them are implementable.

An alternative method to solve the control problem of nonholonomic systems is to divide the control task into two phases, i.e., the trajectory generation phase and the trajectory tracking phase. In the trajectory generation phase, a feasible moving trajectory is generated to overcome the nonholonomic constraints and to avoid the environment obstacles. In the trajectory tracking phase, a feedback control law is developed to stabilized a nonholonomic system to the generated trajectory. Because the generated trajectory is time-varying, the trajectory tracking control law is essentially a time-varying feedback such that, the obstacle of Brockett's theorem is avoided.

This paper addresses a trajectory tracking problem: control a car-like vehicle to track a moving trajectory generated by another real car-like vehicle. Since the desired trajectory is generated by a real car-like vehicle, it avoids the environment obstacles and satisfies the nonholonomic constraints. Unlike most trajectory tracking controllers, which work with feedbacks in global coordinate, the controller in this paper works with feedbacks in the vehicle coordinates. This tracking problem has obvious practical significance when only sensors on the car-like vehicle are used to measure the controlled and target vehicles. For example, encoders on the vehicle are used to collect the vehicle information, and cameras on the vehicle are used to collect the trajectory information.

The tracking control strategy is implemented on an experimental vehicle, called CyCab in our laboratory in Nanyang Technological University. In the experimentation, the recorded data of the same vehicle serve as the desired trajectory, and only encoders on the CyCab are used to collect the feedback information.

This paper is outlined as follows. The tracking problems of a car-like vehicle are formulated in Section 2. The tracking control scheme is derived in Section 3. The simulation results are presented in Section 4. The hardware and system architecture of the CyCab is described and the experimental results are presented in Section 5.

2 Problem Formulation

We will work on the car-like robots, whose configurations are shown in Figures 1-2. The vehicle side P_f of steerable wheels is defined as front and the vehicle side P_b of fixed wheels is defined as back. The tracking problem studied here means that: given a desired trajectory generated by a real vehicle, the car-like vehicle is controlled to follow the desired trajectory using only the feedbacks in the vehicle coordinates. Furthermore, we define that the look-ahead tracking as that the vehicle moves forwards to track the desired trajectory (Figure 1), and the look-behind tracking as that the vehicle moves backwards to track the desired trajectory (Figure 2).

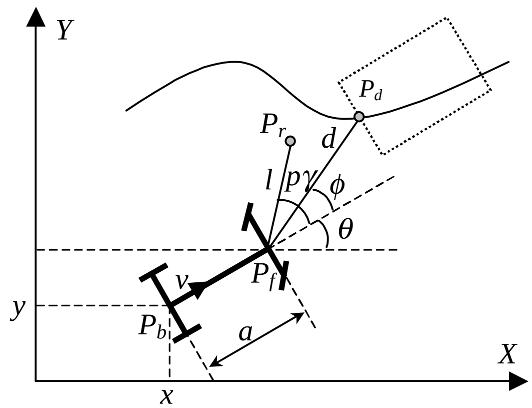

Figure 1: Look-ahead tracking vehicle

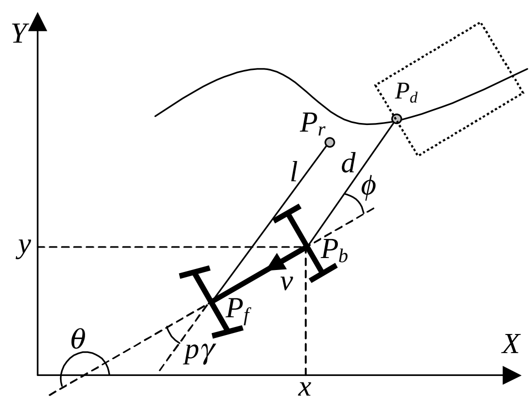

Figure 2: Look-behind tracking vehicle

The kinematic model of the car-like vehicle is

$$\dot{q} = G(q)\mu \tag{1}$$

with

$$q = \begin{bmatrix} x \\ y \\ \theta \\ \gamma \end{bmatrix} \quad \mu = \begin{bmatrix} v \\ \omega \end{bmatrix} \quad G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{a}\tan\gamma & 0 \\ 0 & 1 \end{bmatrix}$$

where, (x, y) are the position coordinates of point P_b in global space. θ is the orientation angle of the vehicle. γ is the steering angle of the steerable wheels. v is the longitudinal velocity at point P_b. ω is the steering rate. a is a constant of the wheelbase.

Let P_d be the current position of the recorded trajectory. P_d can be expressed as

$$\begin{aligned} z_d &= h_d\left(q, d, \phi\right) \\ &= \begin{bmatrix} x \\ y \end{bmatrix} + R^T\left(\theta\right) \begin{bmatrix} \frac{(f+1)a}{2} + fd\cos\phi \\ fd\sin\phi \end{bmatrix} \end{aligned} \tag{2}$$

where, $R(\theta)$ is the standard rotation matrix. d is the distance from the camera to P_d. ϕ is deflection of P_d in the vehicle coordinate. f is a constant that denotes the tracking direction and is given by

$$f = \begin{cases} 1 & for\ look-ahead\ tracking \\ -1 & for\ look-behind\ tracking \end{cases}$$

Then, we define a reference point P_r in front of the motion direction of the car-like vehicle (as show in Figures 1-2), whose position vector is associated with the vehicle configuration and expressed as

$$z = h\left(q\right) = \begin{bmatrix} x \\ y \end{bmatrix} + R^T\left(\theta\right) \begin{bmatrix} a + l\cos\left(p\gamma\right) \\ l\sin\left(p\gamma\right) \end{bmatrix} \tag{3}$$

where, parameters l and p take different values in the look-ahead and look-behind tracking maneuvers.

Now, our control task is to take the reference point P_r to track the current position P_d of the desired trajectory. As discussed in [9,10], this operation will lead the controlled car-like vehicle to follow the desired trajectory.

3 Look-ahead/look-behind Control

The tracking control scheme is based on theoretical results developed in our previous work [9, 10]. Its derivation is presented as follows.

Take (3) as the output of vehicle system (1), its time derivative is

$$\dot{z} = \frac{\partial z}{\partial q}\dot{q} = \frac{\partial z}{\partial q}G(q)\mu = E(q)\mu \tag{4}$$

where, $E(q)$ is the so-called decoupling matrix and can be decomposed as

$$E(q) = R^T(\theta)\bar{E}(\gamma) \tag{5}$$

with

$$\bar{E}(\gamma) = \begin{bmatrix} 1 - \frac{l}{a}\tan\gamma\sin\left(p\gamma\right) & -lp\sin\left(p\gamma\right) \\ \tan\gamma\left(1 + \frac{l}{a}\cos\left(p\gamma\right)\right) & lp\cos\left(p\gamma\right) \end{bmatrix}$$

It is easy to check that, $\bar{E}(\gamma)$ and hence $E(q)$ are regular, if

$$lp \neq 0 \quad and \quad |l-p| < \frac{\pi}{2\gamma_{max}} \quad and \quad |\gamma| < \gamma_{max} < \frac{\pi}{2}$$

Now, we define the output tracking error as

$$\tilde{z} = z - z_d$$

and suppose the desired closed-loop system is

$$\dot{\tilde{z}} + \lambda\tilde{z} = 0 \tag{6}$$

with λ being a positive scalar. Clearly, this desired closed-loop system is exponentially stable.

Then, using $\dot{\tilde{z}} = \dot{z} - \dot{z}_d$ into (6) yields

$$\dot{z} = \dot{z}_d - \lambda\tilde{z} \tag{7}$$

Combining (7) and (4), we get

$$\mu = E^{-1}(q)\dot{z} = E^{-1}(q)\left(\dot{z}_d - \lambda\tilde{z}\right) \tag{8}$$

If all $(q, \tilde{z}, \dot{z}_d)$ are measurable, (8) gives the control scheme in the global coordinate as

$$\mu = E^{-1}(\hat{q})\left[\dot{\hat{z}}_d - \lambda\left(\hat{z} - \hat{z}_d\right)\right] \tag{9}$$

where, $\hat{q}, \hat{z}, \hat{z}_d$ and $\dot{\hat{z}}_d$ are estimations of q, z, z_d and $\dot{z}_d$ respectively.

Next, we derive the control scheme in the vehicle coordinate. Using $\hat{q}$ instead of q in (5) produces

$$E^{-1}(\hat{q}) = \bar{E}^{-1}(\hat{\gamma})R(\hat{\theta}) \tag{10}$$

Using $(\hat{q}, \hat{d}, \hat{\phi})$ instead of (q, d, ϕ) in (2) and taking time derivative, we obtain

$$\dot{\hat{z}}_d = R^T\left(\hat{\theta}\right)F_1\left(\hat{\gamma}, \hat{v}, \hat{d}, \dot{\hat{d}}, \hat{\phi}, \dot{\hat{\phi}}\right) \tag{11}$$

with

$$F_1\left(\hat{\gamma}, \hat{v}, \hat{d}, \dot{\hat{d}}, \hat{\phi}, \dot{\hat{\phi}}\right) = \left[\begin{array}{c} \hat{v}\left(1 - \frac{f\hat{d}}{a}\tan\hat{\gamma}\sin\hat{\phi}\right) + f\dot{\hat{d}}\cos\hat{\phi} - f\hat{d}\dot{\hat{\phi}}\sin\hat{\phi} \\ \hat{v}\tan\hat{\gamma}\left(\frac{(f+1)}{2} + \frac{f\hat{d}}{a}\cos\hat{\phi}\right) + f\dot{\hat{d}}\sin\hat{\phi} + f\hat{d}\dot{\hat{\phi}}\cos\hat{\phi} \end{array}\right]$$

Then, using $(\hat{q}, \hat{d}, \hat{\phi})$ instead of (q, d, ϕ) in (2) and (3), we have

$$\hat{z} - \hat{z}_d = R^T\left(\hat{\theta}\right)F_2\left(\hat{\gamma}, \hat{d}, \hat{\phi}\right) \tag{12}$$

with

$$F_2\left(\hat{\gamma}, \hat{d}, \hat{\phi}\right) = \left[\begin{array}{c} \frac{(1-f)a}{2} + l\cos\left(p\hat{\gamma}\right) - f\hat{d}\cos\hat{\phi} \\ l\sin\left(p\hat{\gamma}\right) - f\hat{d}\sin\hat{\phi} \end{array}\right]$$

Substituting (10)-(12) into (9), and noticing the fact that $R(\theta)$ is an orthogonal matrix, the control scheme in the vehicle coordinate is obtained as

$$\mu = \bar{E}^{-1}(\hat{\gamma})\left(F_1\left(\hat{\gamma}, \hat{v}, \hat{d}, \dot{\hat{d}}, \hat{\phi}, \dot{\hat{\phi}}\right) - \lambda F_2\left(\hat{\gamma}, \hat{d}, \hat{\phi}\right)\right) \tag{13}$$

Note that, control law μ in (13) is a function of feedbacks $(\hat{\gamma}, \hat{v}, \hat{d}, \dot{\hat{d}}, \hat{\phi}, \dot{\hat{\phi}})$. In these feedbacks, $(\hat{v}, \hat{\gamma})$ are feedbacks of vehicle configurations, $(\hat{d}, \dot{\hat{d}}, \hat{\phi}, \dot{\hat{\phi}})$ are feedbacks of the desired trajectory in the vehicle coordinates.

4 Simulation Results

To illustrate the control algorithm (13), a set of simulation results is presented. The conditions for choosing parameter l and p have been proposed in our previous work in [10], i.e., (i) for look-ahead tracking ($f = 1$), choosing $l > 0$ and $p = 1$, and (ii) for look-behind tracking, choosing $l < -a$ and $p(q_d, \mu_d) < p < 0$ (the lower boundary $p(q_d, \mu_d)$ is a negative value depending on the behavior of the desired trajectory).

A forward tracking case is simulated here. In the simulation, the desired trajectory involves two turning motion. We set the wheelbase as $a = 1.2$m. The parameters in (13) are chosen as

$$f = 1, \quad l = 2.5\text{m}, \quad p = 1, \lambda = 10 \tag{14}$$

Initial conditions are given as

$$[\gamma(0), v(0), d(0), \dot{d}(0), \phi(0), \dot{\phi}(0)] = [0, 0, 4, 0, 0, 0] \tag{15}$$

To be comparable with the experimental results in the next section, the integration to steering rate ω has been used such that (v, γ) is applied as control inputs instead of (v, ω). Figure 3 shows the resulting trajectories of the target vehicle and the real vehicle. It is clear that the trajectory of the real vehicle follows the target one very closely. Figures 4-5 give the time evolutions of the feedbacks $(\hat{d}, \hat{\phi}, \hat{v}, \hat{\gamma})$ and the inputs (v, γ).

5 CyCab System and Experimental Results

Experiments of the control scheme (13) are doing on the mobile robot known as the CyCab (Figure 6). The Cycab can carry two persons and has 1.20 meter of wheelbase and 1.10 meter of wheel-thread.

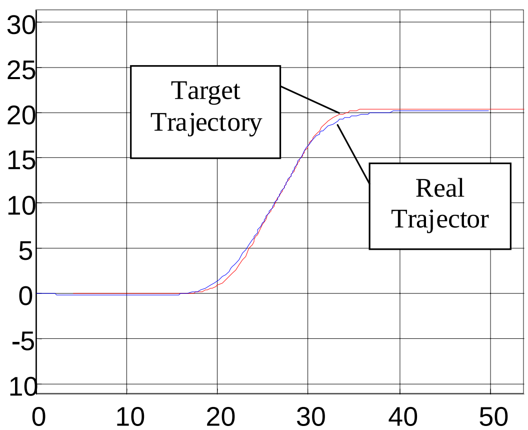

Figure 3: Tracking trajectory

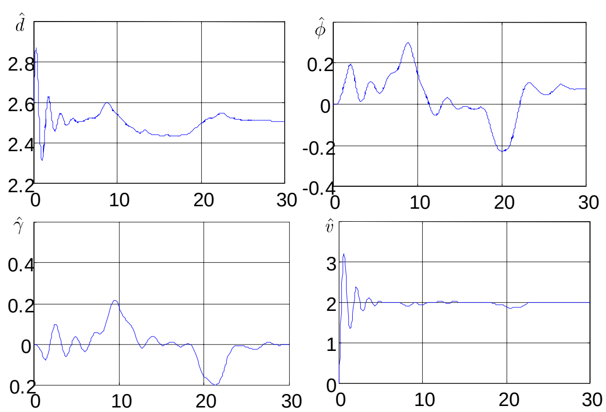

Figure 4: Feedbacks

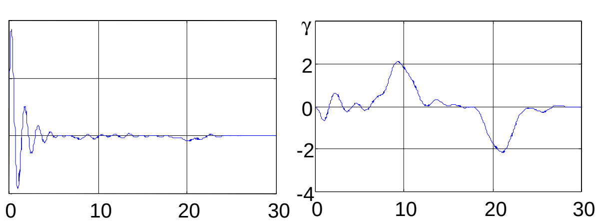

Figure 5: Inputs

Figure 6: CyCab

Its weight is about 350 kilogram. The CyCab has 4 wheels with both front pair wheels and rear pair wheels steerable. Each wheel is equipped with an incremental encoder for velocity feedback and a driving DC motor. Each pair steerable wheels take a 23-degree maximum steering angle and equipped with an absolute encoder for steering angle feedback and a steering system consisted of a DC motor and a hydraulic system.

The control system architecture of the CyCab is shown in Figure 7. It mainly consists of a host computer of Pentium II 450, an onboard MPU system of Motorola 68040 and a Control Interface Board of RSVME 808-1. The onboard MPU system runs ALBATROS Real-time Operating System and controls the entire vehicle. It has two RS232 channels: one is for receiving text-based commands from the host and the other is for outputting data of the encoders (once every 100ms) to the host. The Control Interface Board interfaces with the encoders and the motor controllers. The host computer is the core of the control system. It has multi functions. As the human interface, it gets missions from and shows results to the operator. As a controller, it realizes the control algorithm and issues commands to the onboard MPU system. The multi-thread programs written in Visual C++ and running in the Windows NT environment are applied.

Some experimental results have been made with using the control law (13) in a simulated look-forward tracking case, where the target moves in front of the vehicle. Because the commands for the Motorola 68040 MPU system is required to be velocities and the steering angles, a numerical integration to the steering rate ω is used to calculate the steering an-

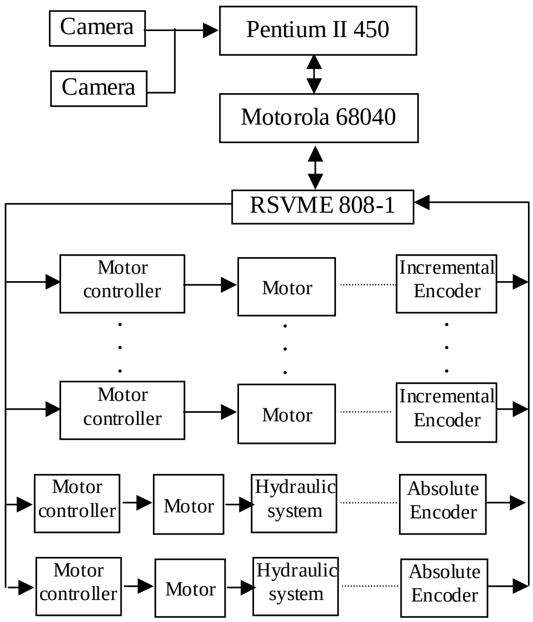

Figure 7: System architecture

gle γ.

In the experiments, parameters in (14) and initial conditions (15) are adopted except $\lambda = 1$. The experimental results are shown in Figures 8-14. Since the real vehicle is behind the desired trajectory about 3.7 meter (look-ahead distance d plus wheelbase a), there do exists errors when the vehicle turns. The tracking locus in Figure 8 shows clearly that the real vehicle follows the desired trajectory very well. Figures 9-12 give the real feedbacks and Figures 13-14 give the real control inputs respectively. All these experimental results of feedbacks and controls are reasonable and agree with the simulation results.

6 Conclusions

A look-ahead/behind tracking controller, which uses only feedbacks in vehicle coordinates, is developed. Experiments are done on the CyCab and the results show the vehicle move forward and backward stably following a vehicle moving in front or behind.

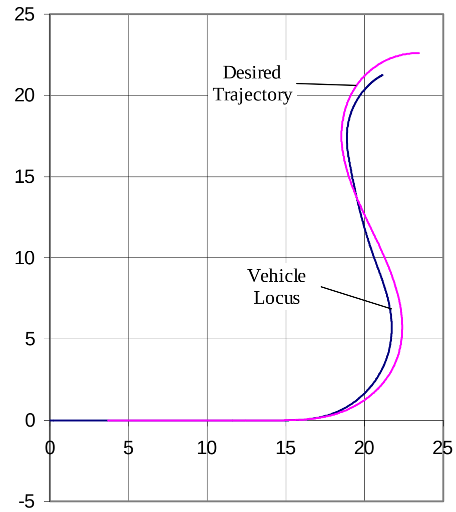

Figure 8: Tracking lucus

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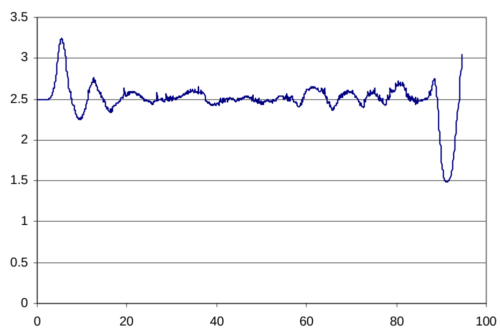

Figure 9: Feedback d

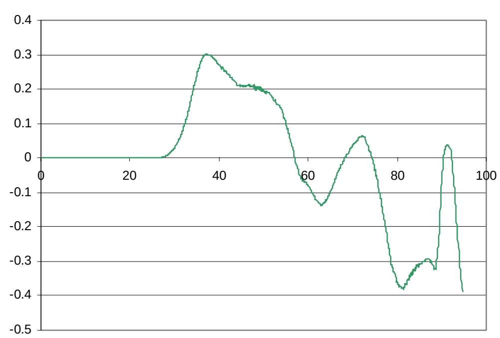

Figure 10: Feedback ϕ

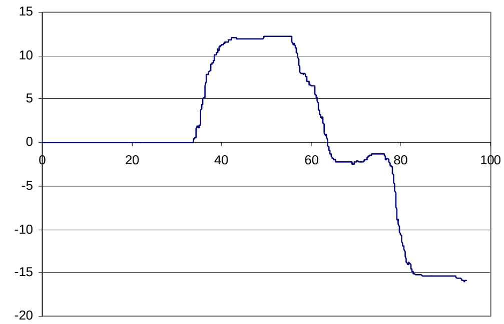

Figure 11: Feedback γ

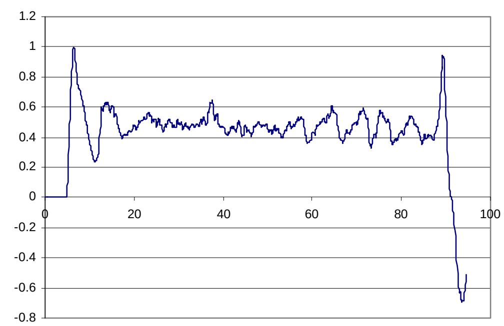

Figure 12: Feedback v

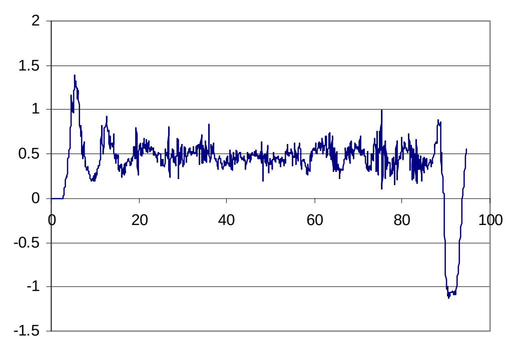

Figure 13: Driving control input

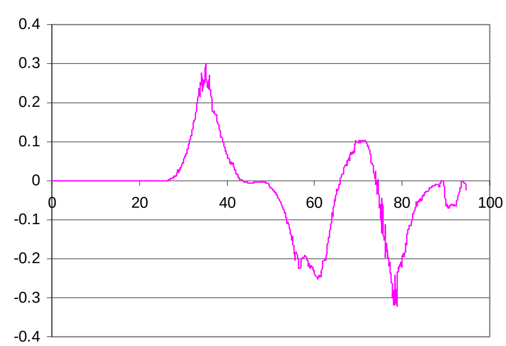

Figure 14: Steering control input γ