NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF CIVIL AND STRUCTURAL ENGINEERING

CV272 - NUMERICAL METHODS

Tutorial 6: Ordinary Differential Equations

1. A stream has been dammed to create a reservoir for water supply and flood control. The rated capacity, *R*, of the reservoir is 4000 m^3. When the actual volume of water in the reservoir, *V*, exceeds *R*, water is discharged through the dam spillway. This discharge, *D* (in m^3/h), over the spillway is given by the following expressions:

$$D = \begin{cases} 0.15(V-R)^{1.5} & \text{if } V > R \\ 0 & \text{if } V \le R \end{cases}$$

About once every 10 years, very heavy rainstorms occur, producing a large inflow, *I*, into the reservoir over several hours. This inflow (in m^3/h) is approximated by the function.

$$I = (80\,te^{-0.25t} + 50)$$

where *t* is time in hours, after the start of the storm and *e* is the natural exponential.

(i) If *I* and *D* are the only two means of water flow into and out of the reservoir respectively, write the simple differential equation relating the rate of increase in the volume of water in the reservoir, *dV/dt*, to the inflow *I* and the discharge *D.*

(ii) Assuming that V = 3960 m^3 when the storm begins, obtain the values of *V* and *D* for each half-hour interval over the first two hours of the storm using the Euler method. Carry out your calculations to one place of decimal.

(Ans. $V(t{=}2)$ = 4092.45 m^3)

2. A sky diver jumps from a plane, and during the time before he opens his parachute the air resistance is proportional to the 3/2 power of his velocity. If it is known that the maximum rate of fall under these conditions is 36 m/s, determine his velocity during the first 1.6 seconds of fall using the Euler and improved Euler methods with Dt = 0.4s. Then, determine how long it takes for the jumper to reach a velocity of 18 m/s. Neglect horizontal drift and assume an initial velocity of zero. Take g = 10 m/s^2.

(Ans. 14.727 m/s; 14.257 m/s; 2.143 s)

3. Using the conditions of Problem 2, determine how long it takes for the jumper to reach a velocity of 18 m/s. Do this by integrating the equation using the Runge-Kutta technique with *Dt* = 0.6s until the velocity exceeds 18 m/s, and then interpolating to obtain the time required to reach a velocity of 18 m/s.
(Ans. 2.1441 s)
Then use the Euler method on the velocity values to determine, approximately, the distance he falls in attaining 18 m/s.
(Ans. 15.607 m)

4. If a cantilever beam of length L is subjected to a uniform load of w kN/m, the equation of its elastic curve is

$$EI\frac{d^2y}{dx^2} = w\left[0.5\left(L^2 + x^2\right) - Lx\right]$$

where x is the position along the beam measured from the fixed end, E is the elastic modulus and I is the second moment of area at that position.

Such a beam of length 2.4 m is subjected to a uniform load $w = 3$ kN/m. The width of the beam is constant but its depth varies linearly along x as shown in Figure Q4. Hence, the value of I along the beam varies proportionally to H^3. At the fixed end, $I = 80 \times 10^6$ mm^4 and the depth is twice that at the free end. The value of E is constant and equals 10,000 N/mm^2.

Using Euler's method, find the value of the deflections, y, (in m, to the nearest 0.001 m) at the positions 1, 2, 3 and 4 shown. (Ans. $y_4 = 0.02$ m)

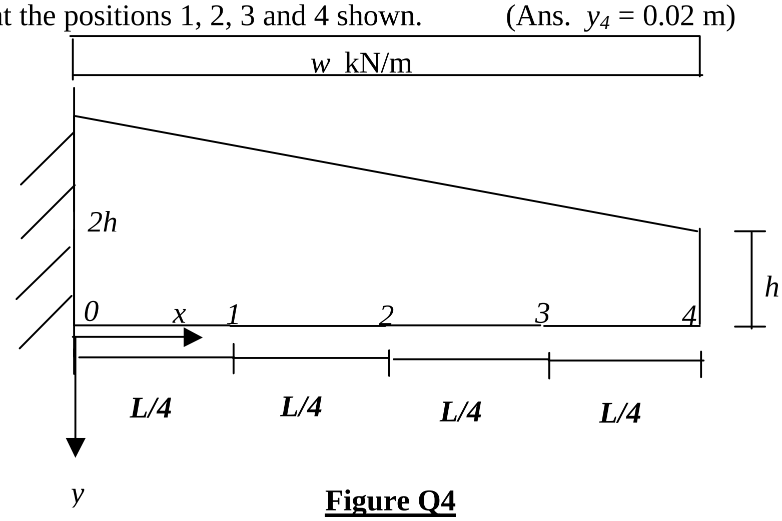

Figure Q4

5. Given: the D.E. $y' = y - 2\sin x$, $y(0) = 1$. Using the Adam-Moulton's predictor-corrector method, find the value of $y(0.4)$ and $y(0.5)$. Employ the Euler method to obtain starting values for your calculations.
(Ans. 1.3309, 1.3796)

TKH/jam
12 July 1999