

**NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF CIVIL AND STRUCTURAL ENGINEERING**

CV272- NUMERICAL METHODS

Tutorial 5: Numerical Integration

1. Figure Q1 shows an idealised stress-strain curve for concrete subjected to compression load. Numerical values of stress (s) and strain (e) are given in Table Q1.
 - (a) Form a difference table for all the data in Table Q1, expressing s as a function of e . Obtain up to and include 4th order differences.
 - (b) Using the Newton-Gregory forward polynomial, fit a polynomial of quartic order ($n=4$) to the recorded points for e at the interval between $e = 0$ and $e = 0.0012$. Express the polynomial in the form of

$$s = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4$$

$$\text{where } s = \frac{(e - e_0)}{0.0003} \text{ and } a_0, a_1, a_2, a_3, a_4 \text{ are constants.}$$

$$(\text{Ans. } P_4(x) = 0.0167s^4 + 0.0333s^3 - 1.5167s^2 + 7.8667s)$$

Table Q1

e	0	3×10^{-4}	6×10^{-4}	9×10^{-4}	1.2×10^{-3}	1.5×10^{-3}	1.8×10^{-3}
s (N/mm²)	0	6.4	10.2	12.2	13.6	14.2	14.2

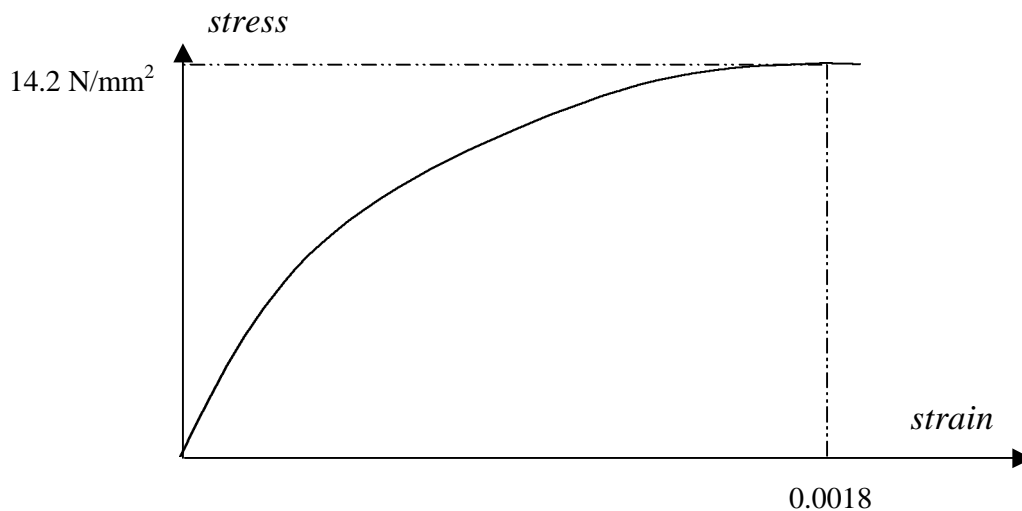


Figure Q1: Idealized Concrete Stress-strain Curve

2. Calculate the area of a quadrant of an ellipse whose semi-axes are a and b ($a = 2b$) by the following methods and compare with the exact result of $\frac{\pi b^2}{2}$.
- (a) Mid-point rule (Ans. $1.732 b^2$)
 (b) Trapezium rule (Ans. b^2)
 (c) Simpson's rule (Ans. $1.488b^2$)
 (d) Gauss's two-point rule (Ans. $1.592b^2$)

3. Estimate the triple integral:-

$$\int_0^{1.2} \int_0^{0.5} \int_0^1 e^{xyz} dx dy dz$$

using

- (a) Simpson's rule (Ans. 0.6128)
 (b) Gauss's two point rule (Ans. 0.6127)
4. Estimate the double integral using 2-point Gaussian integration:

$$\int_1^4 \int_3^4 f(x, y) dy dx$$

where $f(x, y) =$

- (a) xy (Ans. 5.250)
 (b) x^2y (Ans. 8.167)
 (c) x^3y (Ans. 13.125)
 (d) x^4y (Ans. 21.681)

By checking your estimates against the analytical solutions, what conclusions can you reach regarding the accuracy of the method?

TKH/jam
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