NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF CIVIL AND STRUCTURAL ENGINEERING

CV272 – NUMERICAL METHODS

Tutorial 4: Numerical Differentiation

1. Table Q1 is for (1 + log x). Determine estimates of $d(1+\log x)/dx$ at x = 0.15 and 0.23 using (a) two and (b) three terms of the following equation.
(Ans. (a) 2.8625, 1.8775, (b) 2.8775, 1.8808)

$$f'(x_o) = \frac{1}{h}\left(Df_o - \frac{1}{2}D^2 f_o + \frac{1}{3}D^3 f_o - \frac{1}{4}D^4 f_o + \ldots + \frac{1}{n}D^n f_o\right.$$

By comparing to the analytical values, determine the relative errors of each estimate.
(Ans. (a) 1.13%, 0.56%, (b) 0.61%, 0.39%)

Table Q1

x	$1 + \log x$
0.15	0.1761
0.17	0.2304
0.19	0.2788
0.21	0.3222
0.23	0.3617
0.25	0.3979
0.27	0.4314
0.29	0.4624
0.31	0.4914

2. Making use of the Newton-Gregory forward-difference interpolation formula with n = 4, show that the quintic order central difference $f'(x_2)$ is:

$$f'(x_2) = \frac{-f_4 + 8f_3 - 8f_1 + f_o}{12h}$$

with error term $\frac{1}{30}h^4 f^v(x)$, and $x_o < x < x_4$

3. A vehicle going around a bend experiences a side-sway force which depends on the radius of curvature, *r*, of the road at this point. It can be shown that the radius of curvature can be expressed as

$$r = \frac{(1+(f'(x))^2)^{\frac{2}{3}}}{f''(x)}$$

Table Q3

x	0	0.2	0.4	0.6	0.8
$f(x)$	0	0.16	2.56	12.96	40.96

A set of tabular points of the horizontal alignment is shown in Table Q3. Compute the radius of curvature at $x = 0.4$, using

(a) Forward difference (n = 2)
(b) Central difference (n = 2)

Which estimate is more accurate? Explain why.

TKH/jam
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