NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF CIVIL AND STRUCTURAL ENGINEERING

CV272- NUMERICAL METHODS

Tutorial 2: Interpolation II (Evenly and unevenly-spaced data)

1. Given that x_o, ... x_3 are equally-spaced at intervals h, show that Lagrange's interpolation formula with $n = 3$ may be written as

$$f(x) = -\frac{(s-1)(s-2)(s-3)}{3!} f_o + \frac{s(s-2)(s-3)}{2} f_1 - \frac{s(s-1)(s-3)}{2} f_2 + \frac{s(s-1)(s-2)}{3!} f_3$$

where $s = \frac{(x - x_o)}{h}$

2. The bending moments (M) at various points x meters from one end of a beam were recorded in Table Q2.

Table Q2

x (m)	1.0	2.0	4.0	7.0
M (kNm)	109.4	195.0	280.0	135.60

Estimate the bending moment at $x = 2.8$ m (up to 3 decimal places) from

(a) Newton's divided-difference interpolation formula of degree 3. (Ans. 243.873)

(b) Lagrange's interpolation formula, using only the first 3 data points. (Ans. 242.792)

Without recalculating, compute the error in your result for part (b) for ignoring the 4^{th} data print. (Ans. 1.081)

3. The vertical stress s_z under the corner of a rectangular area subjected to a uniform load of intensity q is given by the solution of Boussinesq's equation:

$$s_z = q\, f_z\,(m, n)$$

where q is equal to the load per unit area and

f_z is called the influence value and m and n are dimensionless ratios with $m = a/z$ and $n = b/z$ where a, b and z are dimensions defined in Figure Q3.

The values of f_z at various m and n are given in Table Q3.

If a = 5.6 m, b = 13 m, compute r_z at a depth 10 m below the corner of a rectangular footing that is subjected to a total load of 100 tons, employing a quadratic formula in n and m. (Ans. 0.13897)

Table Q3 - Influence Value f_z

m	n = 1.2	n = 1.4	n = 1.6
0.1	0.02926	0.03007	0.03058
0.2	0.05733	0.05894	0.05994
0.3	0.08323	0.08561	0.08709
0.4	0.10631	0.10941	0.11135
0.5	0.12626	0.13003	0.13241
0.6	0.14309	0.14749	0.15027
0.7	0.15703	0.16199	0.16515
0.8	0.16843	0.17389	0.17739

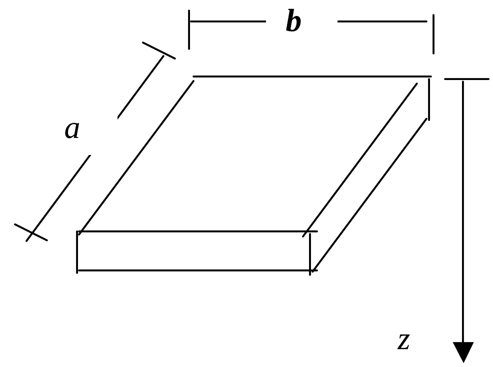

Figure Q3

4.. Table Q4 shows the experimental results of deflections, d, of a 6 meter uniform beam subjected to a non-uniformly distributed load. Due to a faulty dial gauge, the deflection record at point 5 is invalid. Note that x is the distance from one end of the beam.

Table Q4

Point	1	2	3	4	5	6	7
x (m)	0.75	1.50	2.25	3.00	3.75	4.50	5.25
d (mm)	4.5	18.0	30.0	45.0	?	15.0	6.0

(a) Using a 4^{th} degree polynomial, determine the deflection at point 5. (Ans. 38.701)

(b) Strengthening is required for the part of the beam where the deflections exceed 30 mm. Using a 3rd degree polynomial, determine the length of the beam where strengthening is required. For point 5, use the computed deflection in part (a). (Ans. Length= 4.086 – 2.25m)

TKH/jam
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