

**NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF CIVIL AND STRUCTURAL ENGINEERING**

CV272 - NUMERICAL METHODS

Tutorial 1: Interpolation I (Evenly-spaced data)

1. The bending moments M at various points x meters from one end of a beam were recorded in Table Q1. The loading is such that the function $M = f(x)$ should be a polynomial of degree three. One of the M values was wrongly recorded. Form a difference table, identify and correct the error.

Table Q1

$x(m)$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$M(kNm)$	109.4	155.4	195.0	227.73	253.12	271.7	280.0

$x(m)$	4.5	5.0	5.5	6.0	6.5	7.0
$M(kNm)$	280.55	271.88	253.52	225.0	185.85	135.60

Using the corrected Table Q1, estimate the bending moment at $x = 3.3$ m from:

Newton's-Gregory forward difference interpolation formula with $n = 1, 2$ and 3 respectively.
(Ans. 263.67, 264.66, 264.64)

2. (a) Use the data in Table Q2 to estimate the value of y at $x = 0.58$, correct up to 5 significant figures using
(i) quadratic interpolation, and
(ii) cubic interpolation
(Ans. 0.18572, 0.18555)

Table Q2

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.370	0.518	0.697

- (b) Compute the truncation error in case (i) and case (ii) of part (a).
(c) What order of polynomial interpolation would give the exact answer for the data in Table Q2?
(d) What is the exact value of y at $x = 0.58$?
(Ans. 0.18557)

3. (a) If the 2nd degree Gregory-Newton's forward-difference interpolation formula is used to approximate a 3rd degree polynomial $f(x)$, show that the absolute value of error at $s = 1 - a$ is the same as that at $s = 1 + a$, where a is some constant and $s = (x - x_0)/h$, h being the intervals between x_0, x_1, \dots, x_n at which $f(x)$ values are known.
- (b) Table Q3 gives values of bending moment M at various distances x meters from the end of a beam. $M=f(x)$ is known to be a 3rd degree polynomial. What is the maximum absolute value error in $f(x)$ if the 2nd degree Newton's-Gregory forward-difference formula is used to estimate M at intermediate values of x ? (Ans. 0.0302)

Table Q3

$x(m)$	5.0	5.5	6.0	6.5	7.0
$M(kNm)$	271.88	253.52	225.00	185.85	135.60

- (c) Would the absolute errors in estimating M at $x = 5.7$ m be the same for the following cases?
- (i) x_0 taken as 5.0 ; (Ans. 0.0263)
- (ii) x_0 taken as 5.5 (Ans. 0.0301)
4. Find what degree of polynomial can be used to interpolate the data in Table Q4 such that the maximum truncation error is less than 10^{-3} . (Ans. 2nd degree, max error = $-2.95e-4$)

Table Q4

X	1.0	1.1	1.2	1.3	1.4	1.5	1.6
Y	0.2123	0.6717	1.1825	1.7492	2.376	3.0675	3.8279

TKH/jam
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