COKE: Efficient Maintenance of Common Keys in Archives of Continuous Query Results from Deep Websites

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Abstract

In many real-world applications, it is important to create a local archive containing versions of structured results of *continuous queries* (queries that are evaluated periodically) submitted to autonomous database-driven Web sites (e.g., deep Web). Such history of digital information is a potential gold mine for all kinds of scientific, media and business analysts. An important task in this context is to *maintain* the set of *common keys* of the underlying archived results as they play pivotal role in data modeling and analysis, query processing, and entity tracking. A set of attributes in a structured data is a *common key* iff it is a key for all versions of the data in the archive. Due to the data-driven nature of key discovery from the archive, unlike traditional keys, the common keys are *not temporally invariant*. That is, keys identified in one version may be different from those in another version. Hence, in this paper, we propose a novel technique to maintain common keys in an archive containing a sequence of versions of evolutionary continuous query results. Given the current common key set of existing versions and a new snapshot, we propose an algorithm called COKE (common KEY maintenance) which *incrementally* maintains the common key set without undertaking expensive minimal keys computation from the new snapshot. Furthermore, it exploits *certain* interesting evolutionary features of real-world data to further reduce the computation cost. Our exhaustive empirical study demonstrates that COKE has excellent performance and is orders of magnitude faster than a baseline approach for maintenance of common keys.

1 Introduction

Due to the proliferation of database-driven Web sites, there is an enormous volume of structured data on the Web. For instance, a recent study [4] showed that the deep Web contains more than $450,000$ Web databases and these are mostly *structured* data sources ("relational" records with attribute-value pairs) – with a dominating ratio of $3.4 : 1$ versus unstructured sources. Web users typically retrieve relevant data from a deep Web source by submitting an HTML form with valid input values. Search engines, on the other hand, may employ a technique called *surfacing* which automatically submits a large number of queries through the form with valid input values to crawl the content of a deep Web site [17]. In the paper, we address an important problem targeted to deep Web users community instead of search engines.

An important characteristic of deep Web data sources is that they are evolutionary in nature. Consequently, the data content published by a site in response to a query may evolve with time. Hence, Web users may pose *continuous* queries [5,15] (queries that are evaluated periodically on a source) to retrieve relevant data over time. In many applications, it is important to create archives containing previous versions of such evolutionary query results as such history of digital information is a potential gold mine for all kinds of scientific, media and business analysts [18]. For example, one may be interested in finding how the average salaries of people at different cities have changed during 2005 to 2008 to study the effect of subprime mortgage crisis.

The most common strategy to store continuous query results is to first extract structured records from the HTML pages using existing data extraction techniques and then store them in relational table(s) [1, 3, 6]. Consequently, data extracted from a remote Web source Q during a time period τ_1 to τ_n can be represented in the local archive as a sequence of relations $S = \langle R_1, R_2, \ldots, R_n \rangle$. For example, consider two sets of persons' records shown in Figure 1 that are extracted from the results of a continuous query at times τ_1 and τ_2, and stored in relational tables R_1 and R_2, respectively. Each tuple or record in the table represents a person entity in the archive. Note that the *EID* attribute is not part of the records but is used only to facilitate discussions. Notice that *children* and *income* attributes of entity e_4 are updated in R_2; entity e_8 has been inserted in R_2; and entity e_3 in R_1 has been deleted. Figure 2 depicts another example of relational representations of continuous query results from an auction website at two different weeks.

1.1 Motivation

Given a sequence of versions of relation S in the archive, the identification of *common keys* is important for accurate tracking of entities, version management, and query processing over archived relations (we shall elaborate on some of these is-

EID	Name	Birthdate	City	Children	Income
e_1	Alice	1 Jan 80	Los Angeles	1	10000
e_2	Bob	2 Feb 79	Los Angeles	2	20000
e_3	Carol	3 Mar 78	Chicago	3	20000
e_4	Alice	4 Apr 77	Los Angeles	4	20000
e_5	Dave	5 May 76	Houston	2	30000

(a) R_1

EID	Name	Birthdate	City	Children	Income
e_4	Alice	4 Apr 77	Los Angeles	5	40000
e_5	Dave	5 May 76	Houston	2	30000
e_6	Eve	5 May 76	Houston	2	30000
e_7	Bob	6 Jun 75	Los Angeles	5	20000
e_8	Isaac	7 Jul 74	Phoenix	6	50000

(b) R_2

Figure 1: Continuous query results at times (a) τ_1 and (b) τ_2.

EID	Title	Bid	Price	Date	Location
e_1	Apple..Digital	21	350.00	Feb 18 07:54	Florida
e_2	Apple..Grade A	29	202.50	Feb 18 08:23	Iowa
e_3	Apple..New	21	405.00	Feb 18 08:58	N.Y.
e_4	Slightly..16GB	24	227.50	Feb 18 09:21	Illinois
e_5	Apple..Grade A	32	138.00	Feb 18 09:36	Iowa

(a) R_1 (Week *n-1*)

EID	Title	Bid	Price	Date	Location
e_1	Slightly..16GB	24	227.50	Feb 18 09:21	Illinois
e_2	Apple..Grade A	34	152.50	Feb 18 09:36	Iowa
e_3	Apple..Used	21	202.50	Feb 25 10:34	N.Y.
e_4	Apple...World	23	202.50	Feb 25 11:10	N.Y.
e_5	Apple..MA627LL	27	192.50	Feb 20 09:41	Georgia

(b) R_2 (Week *n*)

Figure 2: Continuous query results from an auction site.

sues in Section 2). A key[1] c of a relation R_i in S is a *common key* iff it is a key for all the versions of the relation. That is, c is a key of $R_i \; \forall \; 0 < i \leq |S|$. For example, some of the common keys of R_1 and R_2 in Figure 1 are {*birthdate*, *name*}, {*name*, *children*}, and {*name*, *income*}. In Figure 2, {*date*, *title*, *location*} and {*title*, *bid*, *location*} are two examples of common keys. c is a *minimal common key* iff none of its proper subset is also a common key. For instance, {*date*} and {*title*, *bid*} are the minimal common keys in Figure 2. In this paper, we present an efficient *data-driven* algorithm to identify a set of common keys in S.

Automated identification of *all* common keys in an archive containing versions of continuous query results is a non-trivial problem for several reasons. Firstly, due to privacy reasons, the remote Web site Q may not publish *explicit identifier(s)* (e.g., social security number) of entities. Hence, one may not be able to manually identify such identifiers as keys. Secondly, even if such identifiers do exist (e.g., Vehicle Identification Number (*VIN*)), an explicit list of all keys is often difficult to find manually. Thirdly, identification of common keys should be completely transparent from the Web users and their interaction behaviors with Q should not be affected. Lastly and more importantly, due to lack of availability of the remote database schema of Q, it is not possible for the local archive to take the traditional approach of inferring keys from the schema of Q. Note that this issue is more challenging for typical Web users compared to search engines. The latter may submit *many* queries to retrieve a large number of records with *different* attribute values at different time points in order to crawl Q [17]. In this case, it is easier to determine the common keys quickly. In contrast, the former is only interested in a subset of data in Q as the Web users' goal are *not to crawl* Q. Thus, they may submit relatively *fewer* continuous queries to retrieving fewer attributes values, making it harder to discover common keys quickly.

[1] A set of attributes is a *key* of a relation iff there is no two tuples in this relation with the same value for all attributes in this set.

ID	Web site	Query	No. of Attr.	Avg. no of tuples
D1	www.careerbuilder.com	Information Technology (Category), California (State), Database Administrator (Keywords), 50 (Miles)	12	274
D2	stores.tomshardware.com	MP3 Player (Find Products)	7	554

Figure 3: Real-world data

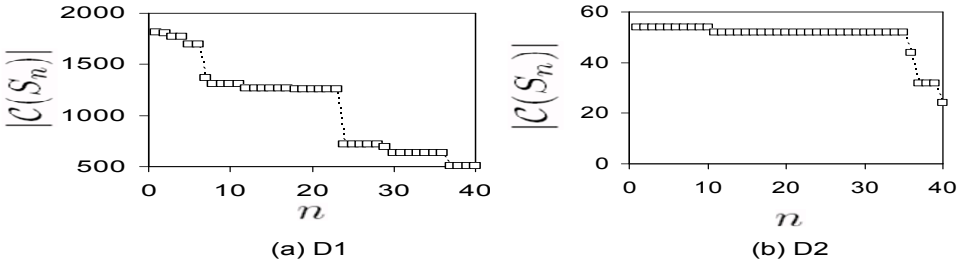


Figure 4: Evolutionary nature of keys.

Evolutionary Nature of Keys. Discovering keys, especially *composite* keys (keys consisting of two or more attributes), is known to be a computationally difficult problem as the number of possible keys increases exponentially with the number of database attributes [24]. To address this problem, recently Sismansis et al. [24] proposed a practical algorithm, called GORDIAN, for discovering composite keys in a collection of real-world entities. Consequently at first glance, it may seem that the common key discovery problem can be solved by first discovering the set of keys $\mathcal{K}(R_i)$ of *any* relation R_i in S where $0 < i \leq |S|$ using an existing data-driven key discovery technique (e.g., GORDIAN [24]). Next, assign $\mathcal{K}(R_i)$ to be the common keys of all the versions in S. Since traditionally keys are temporally invariant, $\mathcal{K}(R_i)$ should remain the same across all versions and therefore looking at any one version is sufficient to choose common keys of S.

Interestingly, the above approach does not work due to the evolutionary nature of keys in the archive. We observed that the keys discovered from R_i by any existing data-driven key discovery technique may *not be temporally invariant*. That is, a key in relation R_i archived at time τ_i may not be a key in R_{i+1} downloaded at time τ_{i+1}. For instance, consider R_2 in Figure 1. The keys $\{birthdate\}$, $\{children, city\}$, and $\{children, income\}$ of R_1 are no more keys in R_2. On the other hand, $\{name\}$ is now a key of R_2. Similarly, in Figure 2 $\{price\}$ is a key in R_1 but not in R_2. Hence, the set of keys discovered from R_i cannot be automatically assigned as the common key set of the entire sequence S.

To get a better understanding of this problem, we experimented with some versions of real-world continuous query results from a set of deep Web sites. Figure 3 shows two representative sites and continuous queries that we analyzed over a time period. A query is shown in the "*Query*" column and the string inside the bracket of each query corresponds to the "*field*" in the search form. Each data set contains

a relational archive of a sequence of query results for a continuous query submitted during 20 November, 2007 and 28 February, 2008. We periodically issued the query every two days and collected 40 snapshots for each of the data sets. The "*No. of Attr.*" column specifies the number of attributes in each record in the results and the "*Avg. no. of tuples*" column specifies the *average* number of records returned by the query at every time point.

Clearly, if an existing common key in the sequence S is not a key in a new relation R_n, then the existing common key set, denoted by $C(S)$, needs to be modified. In other words, "invalidity" of a common key results in evolution of $C(S)$. Observe that whenever a common key is invalid it results in a decrease in the cardinality of $C(S)$. Hence, we empirically observe the evolutionary nature of keys by analyzing the change in cardinality of the common key set over time. Figure 4 graphically elaborates on the evolution of the cardinality of common key sets against the number of relations over time for the two representative data sets. Observe that the number of common keys evolves with time for both the data sets and the frequency of evolution varies widely. For instance, in Figure 4(a) the common keys changed frequently (13 times) whereas in Figure 4(b), it only changed five times. Also, the cardinalities of common key sets for both these sources kept changing throughout the time period. Consequently, the *final common keys may not converge quickly and as a result we may not be able to determined them by observing only first few versions of the query results*.

1.2 Overview

Given the current common key set $C(S)$ of a sequence of relations S and a new relation R_n, at first glance, it may seem that the common key maintenance problem can be solved in two steps. First, find the set of minimal keys of R_n by applying an existing data-driven key discovery technique [16, 24]. The set of all keys can be derived from the set of minimal keys. Second, we can maintain the common keys by computing the intersection of $C(S)$ and keys of R_n. While this approach clearly works, it is computationally expensive as the minimal key discovery step is of high computational cost [24] and we have to execute it for *every* new relation.

Instead, in Section 5 we propose an algorithm called COKE (**CO**mmon **K**ey Maintenanc**E**) which exploits *certain* evolutionary features of real-world data and *incrementally* maintains $C(S)$ after the arrival of R_n *without* computing the set of minimal keys of R_n. Using an efficient *partitioning plan*, COKE first splits R_n based on each $C \in C(S)$ into a set of *non-trivial sub-relations* where each sub-relation has at least two tuples and any two tuples in the same sub-relation have the same value for all attributes in C. Further, any two tuples in two different sub-relations have different value for at least one attribute in C. Observe that if C is no longer a key in R_n, then its non-trivial sub-relation contains at least two tuples. Next for each C and for each non-trivial sub-relation T, COKE creates the *projected sub-relation* from T by removing all the attributes in C from T. Then, it computes the minimal keys of the projected sub-relation using an existing data-driven key

discovery algorithm. Next, the algorithm efficiently computes the set of minimal *proxy keys* of the projected sub-relation over C. A *proxy key* P is a key of the projected sub-relation and $P \supseteq C$. Lastly, the set of minimal common keys of the new sequence is efficiently computed from the proxy keys. Note that COKE can efficiently determine when the common keys remain unchanged in the new version as the non-trivial sub-relation set is empty in this case. Consequently, it discards subsequent steps for computing projected sub-relations and proxy keys.

The above strategy of common keys maintenance is significantly faster than computing the entire set of keys from R_n directly for the following reasons. Firstly, we can ignore the computation of all sub-relations that are *not* non-trivial (i.e., sub-relation containing only one tuple). Interestingly, our analysis of a variety of real-world continuous query results in Section 6 revealed that there are relatively very few non-trivial sub-relations. Consequently, COKE only needs to consider a small number of such sub-relations. Secondly, the number of tuples in each non-trivial sub-relation is often very small. This further improves the performance of COKE by reducing the execution cost of a data-driven key discovery algorithm. We conduct an extensive set of experimental evaluations in Section 7 by comparing the performance of the COKE algorithm with respect to a baseline method on both synthetic and real-world data sets. The experimental results show that COKE is more scalable and orders of magnitude faster than the baseline approach.

The rest of this paper is organized as follows. Section 2 highlights the usefulness of common keys. Section 3 formally introduces the common key maintenance problem. Section 4 presents a naïve approach to solve the common key maintenance problem. We describe the COKE algorithm in Section 5. We report certain interesting evolutionary features of real-world data that COKE exploits in Section 6. We evaluate and compare the performance of our proposed techniques through an extensive set of experiments in Section 7. We review related work in Section 8. Finally, the last section concludes the paper.

2 Usefulness of Common Keys

In this section, we highlight the usefulness of common keys by discussing how evolutionary nature of keys may adversely affect the accuracy of tracking entities and in query processing. In this context, we outline how the knowledge of common keys can help us to alleviate these problems.

Accuracy of tracking entities in a deep Web archive. Monitoring the evolution of deep Web query results is beneficial for several applications such as trend analysis, event tracking and notification, etc. For example, a user may wish to be notified whenever the *price* of any *ipod* drops by 50%. A monitoring algorithm should output high *Quality of Data (QoD)* so that the quality of overall monitoring process can be improved. QoD can be measured in different ways, one of which is *accuracy* of tracking. That is, a tracking algorithm should be able track an entity *correctly* in the query results sequence. For example, consider the results in Fig-

ure 2. Suppose that two entities $e_1 \in R_i$ and $e_2 \in R_j$ are regarded as the same entity if they have same values for the attribute set $\{title, date, location\}$. We refer to such attribute set as *identifier*. If the tracking algorithm is "identifier-aware", then it can track the evolution of the results with high accuracy. For instance, consider the second entity in Figure 2(b). Although it shares the same *title* with the second and fifth entities in Figure 2(a), it is only identical to the fifth entity as the *title*, *date*, and *location* values match for these two entities. Note that the identifiers depends on the continuous queries as *different* queries may retrieve *different* set of attributes from a specific deep Web source. Observe that the values of identifier of an entity e must be identical in two different results' snapshots of a continuous query. That is, if e occurs in R_i and R_j then there must exists a one-to-one mapping between the occurrences of e.

The knowledge of common keys can be used to detect potential identifier(s) when such information is not explicitly available from the source. An identifier must satisfy following two criteria. First, *an identifier must be a common key in S*. For instance, one of the common keys in Figure 2, namely $\{date, title, location\}$ and $\{title, bid, location\}$, *may be* used as an identifier. Second, *the value of a common key of an entity e must be conserved in the historical results sequence of a query*. In other words, if e exists in both R_1 and R_2 then the common key values of e in these two relations must be identical and must not evolve with time. For example, the common key $\{date, title, location\}$ is an identifier for each *auction* entity as the values are conserved in R_1 and R_2 for every entity. Once we are aware of this conserved feature, we can determine that the fourth and fifth entities in R_1 are identical to the first and second entities in R_2, respectively, as they have same common key values. On the other hand, the common key $\{title, bid, location\}$ cannot be an identifier as the value of *bid* of an entity may evolve with time.

Note that traditional techniques for duplicate record detection or reference reconciliation [8, 9] cannot be used to identify identifiers. They assume that the attributes of the records of the same real-world entity may be represented in different ways due to typing error, etc., and *all* attributes of the records can be used to decide whether or not the records refer to the same entity. However, in our problem this is not possible as *some of the attributes of the entities are evolving*.

Query optimization. Consider the archive in Figure 2. Suppose a user wishes to find all *distinct titles* with number of *bids* less than 25 in the last two weeks (R_1 and R_2). To execute this query, the query processor needs to sort the records in R_1 and R_2 based on the attribute $title$ in order to eliminate duplicates. Since $title$ is a key of R_2, the query optimizer may rewrite the SQL query by removing the *distinct* clause. As a result, duplicate elimination can be avoided. However, observe that it is not a key in R_1. Consequently, the above rewriting strategy will generate incorrect results for R_1 (if we assume keys are temporally invariant). In contrast, the knowledge of common keys of R_1 and R_2 can accurately guide the query processor to determine when to avoid duplicate elimination. In the aforementioned example, it cannot be avoided as *title* is not a common key. However, if one wishes to find *distinct* (*title*, *bid*) pairs then duplicate elimination can be avoided as the pair is a

Symbol	Description
S	Sequence of historical relations
R_n	New relation
$\mathcal{K}(R)$	Set of keys of relation R
$\mathcal{MK}(R)$	Set of minimal keys of R
$C(S)$	Set of common keys of S
$\mathcal{MC}(S)$	Set of minimal common keys of S
$\mathcal{P}(S, C)$	Set of proxy keys of S over C
$\mathcal{MP}(S, C)$	Set of minimal proxy keys of S over C
$\mathcal{T}(R, C)$	Set of non-trivial sub-relations of R over C
T^p	The projected sub-relation of $T \in \mathcal{T}(R, C)$

Table 1: Symbols.

common key.

As another example, consider the role keys play in *order optimization*. Suppose a user wishes to group all records by *title* and *price* in the last two weeks (R_1 and R_2). A node in a query plan of this query may require that the input records must be ordered based on attributes *title* first and then *price* (`GROUP BY title, price`). Suppose that the records that are fed into this node are currently ordered based on *title* first and then *location*. Since the required order is not the same as the current order, the query processor needs to sort the records before they are fed into the node. Since $title$ is a key of R_2, the records that are ordered by $title$ are also ordered by attributes $title$ first and then $location$. Additionally, the records that are ordered by $title$ are also ordered by $title$ and *price*. Consequently, the knowledge of keys ensures that the records need not be sorted before feeding them to the node. Unfortunately, the above order optimization cannot be used in the old version R_1 (sorting cannot be avoided). In contrast, the knowledge of common keys R_1 and R_2 can accurately guide the query processor to determine when to avoid sorting during order optimization. In the above example, based on common keys we know that sorting cannot be avoided. However, if the user wishes to groups by *title*, *bid*, and *price* then the knowledge of common keys can guide the query processor to avoid sorting during order optimization.

3 Preliminaries

We begin by discussing our strategy for modeling a sequence of versions of structured continuous query results. Then, we formally introduce the notion of common keys. Finally, we define the problem that we address in this paper. The set of symbols used in this paper is summarized in Table 1.

3.1 Model of Structured Web Data Sequence

We represent structured continuous query results from an autonomous deep Web source Q as a pair (Q, Γ), where $\Gamma = \langle \tau_1, \tau_2, \ldots, \tau_n \rangle$ is the timestamp sequences

recording the times when data was retrieved from Q periodically. We take an *entity view* of the data of Q. We consider query results from Q at time τ_i as primarily a set of entities: $G_{Q_{\tau_i}} = \{G_1, G_2, G_3, \ldots, G_r\}$. Each entity $G_j \in G_{Q_{\tau_i}}$ consists of a set of elements E_j where each element $e \in E_j$ is a pair (a, v) where a is the *attribute* and v is the *literal value* (possibly empty) of e. We say that two entities $G_\ell \in G_{Q_{\tau_i}}$ and $G_m \in G_{Q_{\tau_j}}$ $\forall 1 \leq i < j \leq n$ are *identical*, denoted as $G_\ell = G_m$, if they represent the same real-world entity.

Let A be the set of all attributes for $G_{Q_{\tau_i}}$. Then, $G_{Q_{\tau_i}}$ can be stored in a relational table $R_i(a_1, a_2, \ldots, a_{|A|})$ where $a_k \in A$ $\forall 1 \leq k \leq |A|$ and each record u_j represents an entity $G_j \in G_{Q_{\tau_i}}$. As Q disseminates data periodically over a time period, the collection of historical data of Q can be represented as a sequence $\langle G_{Q_{\tau_1}}, G_{Q_{\tau_2}}, \ldots, G_{Q_{\tau_n}} \rangle$ where $\tau_1 < \tau_2 < \ldots < \tau_n$. Also, we assume that $A_i = A_j$ $\forall 1 \leq i < j \leq n$ where A_i and A_j are the sets of attributes for $G_{Q_{\tau_i}}$ and $G_{Q_{\tau_j}}$, respectively. Consequently, data from Q can be represented as a sequence of relational tables $S = \langle R_1(a_1, a_2, \ldots, a_{|A|}), R_2(a_1, a_2, \ldots, a_{|A|}), \ldots, R_n(a_1, a_2, \ldots, a_{|A|}) \rangle$. In the sequel, the set of attributes (schema) of R_i $(a_1, a_2, \ldots, a_{|A|})$ is omitted if it is understood in the context.

3.2 Common Key

Informally, a key in relation R_i is a *common key* iff it is a key for all the versions of the relation in the sequence S.

Definition 1 *(**Common Key**). Let $S = \langle R_1, R_2, \ldots, R_m \rangle$ be a sequence of relations from a Web source Q with a set of attributes A. Let $C \subseteq A$ and $C \neq \emptyset$. Then, C is a **common key** of S iff C is a key of R_i $\forall 1 \leq i \leq m$. C is a **minimal common key** of S if and only if C is a common key of S and none of its proper subset is also a common key of S.*

We denote the sets of common keys and minimal common keys of S as $\mathcal{C}(S)$ and $\mathcal{MC}(S)$, respectively. For example, consider the relations R_1 and R_2 shown in Figure 1. Let $S = \langle R_1, R_2 \rangle$. The set of attributes $\{name, children\}$ is a common key of S since it is a key of both R_1 and R_2. This set of attributes is also a minimal common key of S since none of its proper subsets (i.e., $\{name\}$ and $\{children\}$) is also a common key of S. Also, $\mathcal{MC}(S) = \{\{name, birthdate\}, \{name, children\}, \{name, income\}\}$. Notice that $\mathcal{MC}(S)$ is a succinct representation of $\mathcal{C}(S)$.

3.3 Common Key Maintenance Problem

Let $S = \langle R_1, R_2, \ldots, R_n \rangle$ be a sequence of relations from a deep Web source Q with a set of attributes A. Note that the keys of R_1 can be computed using an existing data-driven key discovery algorithm. Then for each relation R_i where $i > 1$, we can compute the common keys of the new sequence of relations that incorporates the new relation R_i by examining the common keys of the "old" sequence, and selecting those that are the keys of R_i.

Algorithm 1: Algorithm NAÏVE

Input: $\mathcal{MC}(S_o)$, R_n
Output: $\mathcal{MC}(S_n)$
1 $\mathcal{MK}(R_n) \leftarrow$ call key discovery algorithm on R_n;
2 $\mathcal{MC}(S_n) \leftarrow MIN(\mathcal{MC}(S_o) \otimes \mathcal{MK}(R_n))$;
3 **return** $\mathcal{MC}(S_n)$

Definition 2 ***(Common Key Maintenance Problem)*** *Let $S_o = \langle R_1, R_2, \ldots, R_m \rangle$ be the sequence of relations from source Q at times $\langle \tau_1, \tau_2, \ldots, \tau_m \rangle$. Let R_n be the new relation from Q at time $\tau_n > \tau_m$. Given $\mathcal{MC}(S_o)$ and R_n, the problem of* **common key maintenance** *is to find $\mathcal{MC}(S_n)$ of the updated sequence $S_n = \langle R_1, R_2, \ldots, R_m, R_n \rangle$.*

4 Algorithm NAÏVE

In this section, we present a simple algorithm for computing $\mathcal{MC}(S_n)$. We begin by defining two operators, namely *minimization* and *pairwise union*, that we shall be using subsequently.

Let X be a collection of sets of elements. The *minimization* of X, denoted by $MIN(X)$, is defined as the collection of every set in X whose none of its proper subset is also in X, i.e., $\{x | x \in X \wedge \nexists y \in X \text{s.t } y \subset x\}$. For example, let $X = \{\{name\}, \{birthdate\}, \{name, city\}\}$. Then, $MIN(X)$ is $\{\{name\}, \{birthdate\}\}$. Notice that $\{name, city\}$ is not in $MIN(X)$ since one of its proper subset ($\{name\}$) is in X.

Let X and Y be collections of sets of elements. The *pairwise union* of X and Y, denoted by $X \otimes Y$, is defined as the collection of the union of every set in X with every set in Y, i.e., $\{x \cup y | x \in X \text{ and } y \in Y\}$. For example, let $X = \{\{name\}, \{birthdate\}\}$ and $Y = \{\{name\}, \{city\}\}$. Then, $X \otimes Y$ is $\{\{name\}, \{name, birthdate\}, \{name, city\}, \{birthdate, city\}\}$. It follows from its definition that pairwise union operator is commutative (i.e., $X \otimes Y = Y \otimes X$) and associative (i.e., $X \otimes (Y \otimes Z) = (X \otimes Y) \otimes Z$). For simplicity, we use the notation $\bigotimes_{i=1}^{n} X_i$ as a short form of $X_1 \otimes X_2 \otimes \cdots \otimes X_n$.

4.1 Algorithm

It follows from the definition of common key that the common keys in $C(S_n)$ must be in both $C(S_o)$ and $\mathcal{K}(R_n)$. This implies that $C(S_n)$ can be computed by intersecting $C(S_o)$ and $\mathcal{K}(R_n)$. Based on this, we propose a simple algorithm, called NAÏVE, for computing $\mathcal{MC}(S_n)$, which consists of the following two steps as shown in Algorithm 1. First, we compute $\mathcal{MK}(R_n)$ from R_n by using key discovery algorithm (Line 1). Second, we compute $\mathcal{MC}(S_n)$ from $\mathcal{MC}(S_o)$ and $\mathcal{MK}(R_n)$ by using the following theorem (Line 2).

Theorem 1 Given $\mathcal{MC}(S_o)$ and $\mathcal{MK}(R_n)$, we can compute $\mathcal{MC}(S_n)$ as

$$\mathcal{MC}(S_n) = MIN(\mathcal{MC}(S_o) \otimes \mathcal{MK}(R_n))$$

□

Proof 1 *See Appendix A.*

Example 1 Consider relations R_1 and R_2 shown in Figure 1. Let $S_o = \langle R_1 \rangle$ and $R_n = R_2$. Recall that $\mathcal{MC}(S_o) = \{\{birthdate\}, \{name, children\}, \{name, income\}, \{children, income\}, \{children, city\}\}$. By using key discovery algorithm, we obtain $\mathcal{MK}(R_n) = \{\{name\}\}$. By using Theorem 1, we obtain $\mathcal{MC}(S_n) = \{\{birthdate, name\}, \{name, children\}, \{name, income\}\}$. ∎

The main drawback of the NAÏVE algorithm is that it needs to compute $\mathcal{MK}(R_n)$, which can be expensive, especially when R_n has large numbers of attributes and records. In the next section, we propose a significantly more efficient algorithm, which not only avoids the computation of $\mathcal{MK}(R_n)$, but also exploits evolutionary characteristics of R_n.

5 Algorithm COKE

In this section, we propose an efficient algorithm, called COKE (**CO**mmon **K**ey Maintenanc**E**), for computing $\mathcal{MC}(S_n)$. We restrict this algorithm to in-memory processing where $\mathcal{MC}(S_o)$ and R_n reside in memory. We begin by defining *proxy key*.

Definition 3 *(**Proxy Key**) Given a R with attributes A, let $P \subseteq A$, $Y \subseteq A$, $P \neq \emptyset$ and $Y \neq \emptyset$. Then, P is a **proxy key** of R over Y if and only if P is a key of R and $P \supseteq Y$. P is a **minimal proxy key** of R over Y if and only if P is a proxy key of R over Y and none of its proper subset is also a proxy key of R over Y.*

We denote the sets of proxy keys and minimal proxy keys of R over Y as $\mathcal{P}(R, Y)$ and $\mathcal{MP}(R, Y)$, respectively. For example, consider the relation R_1 shown in Figure 1. The set of attributes $\{name, children\}$ is a proxy key of R_1 over $\{name\}$ since it is a key of R_1 and it is a superset of $\{name\}$. This set of attributes is also a minimal proxy key of R_1 over $\{name\}$ since none of its proper subsets (i.e., $\{name\}$ and $\{children\}$) is also a proxy key of R_1 over $\{name\}$. Also, $\mathcal{MP}(R_1,\{name\}) = \{\{name, children\}, \{name, income\}, \{name, birthdate\}\}$. Notice that $\mathcal{MP}(R, Y)$ is a succinct representation of $\mathcal{P}(R, Y)$ since not only we can compute $\mathcal{P}(R, Y)$ from $\mathcal{MP}(R, Y)$, but also $\mathcal{MP}(R, Y)$ contains much smaller number of sets of attributes than $\mathcal{P}(R, Y)$.

Algorithm 2: Algorithm COKE

Input: $\mathcal{MC}(S_o), R_n$
Output: $\mathcal{MC}(S_n)$
1 $\mathcal{T}(R_n, \cdot) \leftarrow$ SubRelationConst($\mathcal{MC}(S_o), R_n$);
2 $\mathcal{MC}(S_n) \leftarrow$ CommonKeyGen($\mathcal{MC}(S_o), \mathcal{T}(R_n, \cdot)$);
3 **return** $\mathcal{MC}(S_n)$

5.1 Overview

The number of possible common keys is exponential to the number of attributes. To tackle this problem, we break the search space into several sub-search spaces, where each sub-search space corresponds to each minimal common key $C \in \mathcal{MC}(S_o)$, and consists of all the supersets of C. For each sub-search space, we compute its $\mathcal{MP}(R_n, C)$ by first splitting R_n based on C into a set of *non-trivial sub-relations* (denoted by $\mathcal{T}(R_n, C)$). Then, we compute $\mathcal{MP}(R_n, C)$ from $\mathcal{T}(R_n, C)$. After that, from these sets of minimal proxy keys of the sub-search spaces, we compute $\mathcal{MC}(S_n)$ efficiently. We now formally define *non-trivial sub-relations*.

Definition 4 ***(Sub-relation)*** *Give a relation R with attributes A, let $C \subseteq A$ and $C \neq \emptyset$. Let $G = \{T_1, T_2, \ldots, T_n\}$ be a collection of sets of records of R. Then, G is a* ***set of sub-relations*** *of R over C if and only if G satisfies the following four conditions: (a) $R = T_1 \cup T_2 \cup \ldots \cup T_n$, (b) $T_i \cap T_j = \emptyset \; \forall 1 \leq i < j \leq n$, (c) $u[C] = v[C]$ $\forall u, v \in T_i \; \forall 1 \leq i \leq n$, and (d) $u[C] \neq v[C]$ $\forall u \in T_i, v \in T_j \; \forall 1 \leq i < j \leq n$. We call $T_i \in G$ as a* ***trivial sub-relation*** *if $|T_i| = 1$, and as a* ***non-trivial sub-relation*** *if $|T_i| > 1$.*

For example, consider R_1 shown in Figure 1. The collection of sets of records $\{\{e_1, e_4\}, \{e_2\}, \{e_3\}, \{e_5\}\}$ is a set of sub-relations of R_1 over $\{name\}$ since it satisfies all the four conditions above. The sub-relation $\{e_2\}$ is a trivial sub-relation since it contains only one record, while the sub-relation $\{e_1, e_4\}$ is a non-trivial sub-relation since it contains more than one record.

The above idea of computing $\mathcal{MC}(S_n)$ is realized in COKE by the following two phases as shown in Algorithm 2. The *non-trivial sub-relations computation phase* computes sets of non-trivial sub-relations $\mathcal{T}(R_n, \cdot)$ from $\mathcal{MC}(S_o)$ and R_n (Line 1). The *common keys computation phase* then computes $\mathcal{MC}(S_n)$ from $\mathcal{MC}(S_o)$ and $\mathcal{T}(R_n, \cdot)$ (Line 2). For ease of detailed exposition of these phases, we use the following running example.

Example 2 Consider the relations in Figure 1. Let $S_o = \langle R_1 \rangle$ and $R_n = R_2$. Also, $\mathcal{MC}(S_o) = \{\{birthdate\}, \{name, children\}, \{name, income\}, \{children, income\}, \{children, city\}\}$. ∎

Algorithm 3: SubRelationConst

```
Input: MC(S_o), R_n
Output: T(R_n, ·)
1  root ← call PartitioningPlan(MC(S_o));
2  foreach w ∈ root.children do
3      SubRelationStub(R_n, w, {w.label});
4  return T(R_n, ·)
5  procedure: SubRelationStub(T, w, C);
6  G ← partition T based on w.label;
7  if w is a leaf then
8      foreach non-trivial sub-relation T' ∈ G do
9          T(R_n, C) ← T(R_n, C) ∪ {T'};
10 else
11     foreach non-trivial sub-relation T' ∈ G do
12         foreach w' ∈ w.children do
13             SubRelationStub(T', w', C ∪ {w'.label});
```

5.2 Non-trivial Sub-relations Computation

Sub-relation Computation Using Partitioning Plan. One method for computing $\mathcal{T}(R_n, \cdot)$ from $\mathcal{MC}(S_o)$ and R_n is that, for each $C \in \mathcal{MC}(S_o)$ we can partition the records in R_n recursively based on each attribute in C. However, the computation cost can be further reduced if some of the intermediate partitioning steps can be shared among the minimal common keys in $\mathcal{MC}(S_o)$. For example, the minimal common keys {*name*, *children*} and {*children*, *income*} share the attribute *children*, and thus they can share some of the intermediate partitioning steps. We explain our main idea below. First, we compute $\mathcal{T}(R_n, \{children\})$ by partitioning the records in R_n based on the attribute *children*. Second, we compute $\mathcal{T}(R_n, \{name, children\})$ by partitioning the records in each sub-relation in $\mathcal{T}(R_n, \{children\})$ based on the *name*. Third, we compute $\mathcal{T}(R_n, \{children, income\})$ by partitioning the records in each sub-relation in $\mathcal{T}(R_n, \{children\})$ based on *income*.

We formalize the above steps of computing $\mathcal{T}(R_n, \cdot)$ by using a *partitioning plan*. The partitioning plan is a tree in which each of its nodes, except the root node, is labeled with an attribute, and each of its paths from the root to the leaf node represents each minimal common key $C \in \mathcal{MC}(S_o)$, and vice versa. For example, consider the $\mathcal{MC}(S_o)$ and R_n in Example 2. Then, one possible partitioning plan is shown in Figure 5. Notice that the minimal common key {*name*, *children*} is represented as the path of "*children* → *name*", and the path of "*children* → *income*" represents the minimal common key {*children*, *income*} (the root node is not included in the path since it is unlabeled).

Algorithm 3 summarizes the steps of computing $\mathcal{T}(R_n, \cdot)$ using the partitioning plan approach. First, we create a partitioning plan from $\mathcal{MC}(S_o)$ (Line 1). We will

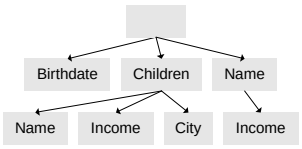


Figure 5: Partitioning plan computed from $\mathcal{MC}(S_o)$.

discuss the method of creating the partitioning plan in the next subsection. The algorithm starts from the root setting R_n as the *current relation*, and recursively visits each of the child nodes (Lines 2-3). At each node, except the root, it partitions the records in the current relation into multiple sub-relations based on the node's label (Line 6). It then checks whether the current node is a leaf node (Line 7). If it is, then it adds all non-trivial sub-relations into $\mathcal{T}(R_n, C)$ where C is the minimal common key represented by the path from the root to the current node (Lines 8-9). If it is not, for each non-trivial sub-relation the algorithm sets it as the current relation, and again recursively visit each of the child nodes (Lines 11-13). Note that the algorithm does not continue the traversal of the partitioning plan for the trivial sub-relations because at the leaf node such sub-relations will not be added into $\mathcal{T}(R_n, C)$. We shall justify the reason for not adding the trivial sub-relations into $\mathcal{T}(R_n, C)$ in Section 5.3.

Lemma 1 *$C \in \mathcal{K}(R_n)$ if and only if $\mathcal{T}(R_n, C) = \emptyset$.*

Proof 2 We first prove that if $C \in \mathcal{K}(R_n)$, then $\mathcal{T}(R_n, C) = \emptyset$. Suppose $C \in \mathcal{K}(R_n)$. Since $C \in \mathcal{K}(R_n)$, by definition of key, no two records in R_n have the same value for all attributes in C. Thus, the partition of the records in R_n based on C produces only trivial sub-relations. Therefore, $\mathcal{T}(R_n, C) = \emptyset$.

We now prove that if $\mathcal{T}(R_n, C) = \emptyset$, then $C \in \mathcal{K}(R_n)$. Suppose $\mathcal{T}(R_n, C) = \emptyset$. Since $\mathcal{T}(R_n, C) = \emptyset$, by definition of sub-relation, the partition of the records in R_n based on C produces only trivial sub-relations. In other words, no two records in R_n have the same value for all attributes in C. Therefore, by definition of key, $C \in \mathcal{K}(R_n)$.

Example 3 The partitioning plan computed from $\mathcal{MC}(S_o)$ is shown in Figure 5. The process of computing $\mathcal{T}(R_n, \cdot)$ using the partitioning plan is shown in Figure 6. Each box in each node represents a sub-relation in that node, and each number in each box represents a record (represented by its *EID*) in that sub-relation.

We illustrate the computation using $\mathcal{T}(R_n, \{children, income\})$ as an example. We start from the root node, set R_n as the current relation, and visit the node *children*. We partition the current relation, i.e., R_n, based on the attribute *children*, and obtain three sub-relations, i.e., $\{e_4, e_7\}$, $\{e_5, e_6\}$, and $\{e_8\}$. Consider the sub-relation $\{e_4, e_7\}$. Since it is non-trivial, we set it as the current relation, and visit the node *income*. We partition the current relation, i.e., $\{e_4, e_7\}$, based on *income*

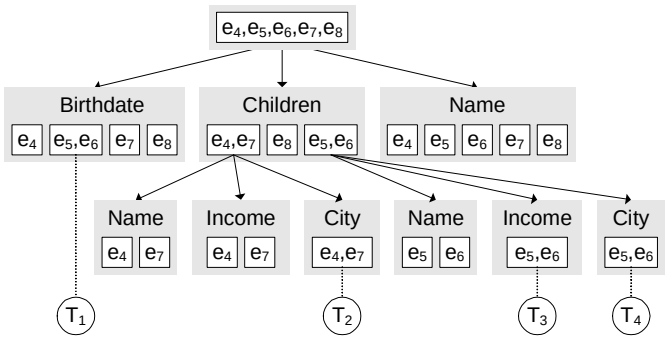


Figure 6: Computation of $\mathcal{T}(R_n, \cdot)$ using the partitioning plan.

and obtain two sub-relations, i.e., $\{e_4\}$ and $\{e_7\}$. Since we are at the leaf node and the sub-relations $\{e_4\}$ and $\{e_7\}$ are trivial, we do not add them into $\mathcal{T}(R_n, \{children, income\})$. Now consider $\{e_5, e_6\}$. Since it is non-trivial, we set it as the current relation, and visit the node *income*. We partition the current relation based on *income* and obtain one sub-relation, i.e., $\{e_5, e_6\}$. Since we are at the leaf node and the sub-relation $\{e_5, e_6\}$ is non-trivial, we add it into $\mathcal{T}(R_n, \{children, income\})$ (denoted as T_3 in Figure 6). Lastly, we consider $\{e_8\}$. Since this is a trivial sub-relation, we prune it. Thus, $\mathcal{T}(R_n, \{children, income\}) = \{T_3\}$. Notice that the node *income*, the child of the node *name*, in the partitioning plan is not traversed since every sub-relation in the node *name* is trivial. Also, the set of sub-relations $\mathcal{T}(R_n, \{children, name\})$ is an empty set and the corresponding minimal common key $\{children, name\}$ is in $\mathcal{K}(R_n)$. This agrees with Lemma 1. The remaining sets of sub-relations are $\mathcal{T}(R_n, \{birthdate\}) = \{T_1\}$ and $\mathcal{T}(R_n, \{children, city\}) = \{T_2, T_4\}$. ∎

Partitioning Plan Construction. Our goal is to construct the partitioning plan such that the computation cost required to construct it and to compute $\mathcal{T}(R_n, \cdot)$ is minimum. This problem is challenging as in one hand, we need to take into account the pruning of the sub-relations and on the other hand, we cannot preprocess R_n too much since the construction of the plan may be too costly. We propose to create a plan that contains the minimum number of nodes and ignore the pruning of the sub-relations. We choose this strategy because it can be computed only from the set of minimal common keys of S_o. Further, a plan that contains lesser number of nodes is likely to reduce the total computation cost.

Theorem 2 *The problem of constructing partitioning plan containing minimum number of nodes from $\mathcal{MC}(S_o)$ is NP-Hard.*

Proof 3 See Appendix B.

We develop efficient heuristics to construct the partition plan. The basic idea is inspired by the FP-Tree construction algorithm [11]. The objective is to assign the attributes that exist in large number of minimal common keys in $\mathcal{MC}(S_o)$ as close

EID	Name	City	Children	Income
e_5	Dave	Houston	2	30000
e_6	Eve	Houston	2	30000

(a) T_1^p

EID	Name	Birthdate	Income
e_4	Alice	4 Apr 77	40000
e_7	Bob	6 Jun 75	20000

(b) T_2^p

EID	Name	Birthdate	City
e_5	Dave	5 May 76	Houston
e_6	Eve	5 May 76	Houston

(c) T_3^p

EID	Name	Birthdate	Income
e_5	Dave	5 May 76	30000
e_6	Eve	5 May 76	30000

(d) T_4^p

Figure 7: Projected sub-relations.

Algorithm 4: PartitioningPlan

Input: $\mathcal{MC}(S_o)$
Output: *root* (root of the partitioning plan)

```
1  root ← new node;
2  foreach C ∈ MC(S_o) do
3      foreach a ∈ C do
4          a.frequency ← a.frequency + 1;
5  foreach C ∈ MC(S_o) do
6      sort the attributes in C according to decreasing frequency;
7  foreach C ∈ MC(S_o) do
8      w ← root;
9      foreach a ∈ C in order of decreasing frequency do
10         w' ← find the node with label a in w.children;
11         if w' does not exist then
12             w' ← new node;
13             w'.label ← a;
14             w.children ← w.children ∪ {w'};
15         w ← w';
16 return root
```

as possible to the root node in order to maximize the number of shared nodes. Consequently, the number of nodes in the plan is minimized. The algorithm is shown in Algorithm 4 and consists of the following three steps. Firstly, it computes the *frequency* of each attribute. The *frequency* of an attribute is defined as the number of minimal common keys in $\mathcal{MC}(S_o)$ that contains the attribute (Lines 2-4). Secondly, it sorts the attributes in each $C \in \mathcal{MC}(S_o)$ based on the descending order of frequency (Lines 5-6). Attributes with same frequency are ordered arbitrarily. Thirdly, for each $C \in \mathcal{MC}(S_o)$, the algorithm creates a path from the root to a leaf node, and share its prefix nodes maximally with other paths having the same prefix (Lines 7-15). We explain this procedure with an example.

Example 4 Consider Example 3. The frequency of *name* is two since there are two minimal common keys (i.e., {*name*, *children*} and {*name*, *income*}) that con-

Algorithm 5: CommonKeyGen

Input: $\mathcal{MC}(S_o), \mathcal{T}(R_n, \cdot)$
Output: $\mathcal{MC}(S_n)$

1 **foreach** $C \in \mathcal{MC}(S_o)$ **do**
2 **foreach** $T \in \mathcal{T}(R_n, C)$ **do**
3 $T^p \leftarrow$ create a projection of T;
4 $\mathcal{MK}(T^p) \leftarrow$ call key discovery algorithm on T^p;
5 $\mathcal{MP}(T, C) \leftarrow \bigcup_{K \in \mathcal{MK}(T^p)} \{K \cup C\}$;
6 **foreach** $C \in \mathcal{MC}(S_o)$ **do**
7 **if** $\mathcal{T}(R_n, C) \neq \emptyset$ **then**
8 $\mathcal{MP}(R_n, C) \leftarrow MIN\left(\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C)\right)$;
9 **else**
10 $\mathcal{MP}(R_n, C) \leftarrow \{C\}$;
11 $\mathcal{MC}(S_n) \leftarrow MIN\left(\bigcup_{C \in \mathcal{MC}(S_o)} \mathcal{MP}(R_n, C)\right)$;
12 **return** $\mathcal{MC}(S_n)$

tain this attribute. The frequencies of *children*, *income*, *birthdate*, and *city* are three, two, one, and one, respectively. Hence, *children* $>$ *name* $>$ *income* $>$ *birthdate* $>$ *city*. After sorting the attributes in each $C \in \mathcal{MC}(S_o)$, we obtain $\mathcal{MC}(S_o)$ = {{*birthdate*}, {*children*, *name*}, {*name*, *income*}, {*children*, *income*}, {*children*, *city*}}. Note that the partitioning plan depends on the ordering of attributes. The partitioning plan constructed from this $\mathcal{MC}(S_o)$ is shown in Figure 5. Notice that the minimal common key {*name*, *income*} is represented as the path *name* $\rightarrow$ *income*. Furthermore, the paths *children* $\rightarrow$ *name*, *children* $\rightarrow$ *income*, and *children* $\rightarrow$ *city* share the prefix node *children*. ∎

Observe that in the aforementioned approach we continue partitioning the new sub-relation until we reach the leaf node of the plan. This is because we want to compute the sets of sub-relations over the minimal common keys where each minimal common key is represented as a path from the root to the leaf node. Hence, if we stop the partitioning before reaching the leaf node, the resulting sets of sub-relations will not be over the minimal common keys.

5.3 Common Keys Computation

We compute the set of minimal common keys of the updated sequence S_n (denoted as $\mathcal{MC}(S_n)$) from the $\mathcal{T}(R_n, \cdot)$ generated in the previous phase. Note that if the common keys remain unchanged in the new sequence then non-trivial sub-relation set is empty (Lemma 1). Hence, in this case Phase 2 does not need to be executed.

Algorithm of Phase 2. The steps for Phase 2 is presented in Algorithm 5. It consists of three key steps as follows. Firstly, for each $C \in \mathcal{MC}(S_o)$ and $T \in \mathcal{T}(R_n, C)$, it computes the set of minimal proxy keys over C ($\mathcal{MP}(T, C)$) from T (Lines 1-5). Next, for each $C \in \mathcal{MC}(S_o)$, the minimal proxy keys in the new

relation (denoted as $\mathcal{MP}(R_n, C)$) is generated from $\mathcal{MP}(\cdot, C)$ (Lines 6-10). Lastly, the algorithm generates $\mathcal{MC}(S_n)$ from $\mathcal{MP}(R_n, \cdot)$ (Line 11). We now elaborate on these steps in turn.

Step 1: Computation of Proxy Keys of Sub-Relations. Given $C \in \mathcal{MC}(S_o)$ it computes $\mathcal{MP}(T, C)$ from $T \in \mathcal{T}(R_n, C)$. First, it creates the *projected sub-relation* T^p from T by removing all the attributes in C from the attributes of T (Line 3). Second, it computes the minimal keys of T^p ($\mathcal{MK}(T^p)$) using an existing key discovery algorithm (Line 4). Third, $\mathcal{MP}(T, C)$ is computed by taking the union of attributes in C and $K \in \mathcal{MK}(T^p)$ (Line 5).

Example 5 We illustrate the computation of $\mathcal{MP}(T_1, \{birthdate\})$. We create the projected sub-relation T_1^p from T_1 by removing the attribute *birthdate* from T_1 as shown in Figure 7(a). By using an existing key discovery algorithm, we obtain $\mathcal{MK}(T_1^p) = \{\{name\}\}$. Then, we can obtain $\mathcal{MP}(T_1, \{birthdate\}) = \{\{name\} \cup \{birthdate\}\} = \{\{name, birthdate\}\}$.

Similarly, we can compute the projected sub-relations of T_2, T_3, and T_4. These are depicted in Figures 7(b)-(d) (denoted as T_2^p, T_3^p, and T_4^p, respectively). The sets of minimal proxy keys of the sub-relations are $\mathcal{MP}(T_2, \{children, city\}) = \{\{name, children, city\}, \{income, chidren, city\}, \{birthdate, children, city\}\}$, $\mathcal{MP}(T_3, \{children, income\}) = \{\{name, children, income\}\}$, and $\mathcal{MP}(T_4, \{children, city\}) = \{\{name, children, city\}\}$. ∎

Step 2: Computation of Proxy Keys of R_n. Given $C \in \mathcal{MC}(S_o)$, in this step our goal is to compute the set of minimal proxy keys of R_n over C (denoted as $\mathcal{MP}(R_n, C)$) from $\mathcal{MP}(\cdot, C)$. Note that $\mathcal{MP}(\cdot, C)$ has already been computed in the preceding step. We consider the following two cases: (a) If $\mathcal{T}(R_n, C) = \emptyset$. Based on Lemma 1, it follows that $C \in \mathcal{K}(R_n)$. Consequently, $\mathcal{MP}(R_n, C) = \{C\}$ (Line 10). (b) If $\mathcal{T}(R_n, C) \neq \emptyset$, then we compute $\mathcal{MP}(R_n, C)$ by using the following theorem (Line 8). Intuitively, for each C we first apply the pairwise union on the proxy key sets of all the non-trivial sub-relations ($\mathcal{MP}(T, C)$). Then, we extract the minimal proxy keys by removing the supersets (using minimization).

Theorem 3 *Let $C \in \mathcal{MC}(S_o)$. Given $\mathcal{MP}(\cdot, C)$,*

$$\mathcal{MP}(R_n, C) = MIN\left(\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C)\right)$$

Proof 4 See Appendix C.

Note that the above theorem holds regardless of whether or not we add trivial sub-relations into $\mathcal{T}(R_n, C)$. This is the reason why we do not add trivial sub-relations in $\mathcal{T}(R_n, C)$ as discussed in Algorithm 3 (Line 8). Also, as the number of elements in $\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C)$ is exponential to the number of sub-relations in $\mathcal{T}(R_n, C)$, we propose a more efficient recursive method based on the following

Parameter	Description	Default value
N_s	# attributes in R_n	20
N_t	# records in R_n	50,000
N_k	# minimal common keys in $\mathcal{MC}(S_o)$	30
N_a	# attributes in each minimal common key in $\mathcal{MC}(S_o)$	5
N_u	# minimal common keys in $\mathcal{MC}(S_o)$ whose set of non-trivial sub-relations is not empty	10
N_p	# non-trivial sub-relations in R_n associated with each minimal common key in $\mathcal{MC}(S_o)$ whose set of non-trivial sub-relations is not empty	20
N_q	# records in each non-trivial sub-relation in R_n associated with each minimal common key in $\mathcal{MC}(S_o)$ whose set of non-trivial sub-relations is not empty	10

Table 2: Parameters for analysis.

lemma. It reduces the number of elements in the intermediate computation by applying minimization operator each time we apply pairwise union operator to remove unnecessary elements from its results.

Lemma 2 *Let* $\{X_1, X_2, \ldots, X_n\}$ *be a collection of sets of elements. Then,*

$$MIN\left(\bigotimes_{i=1}^{n} X_i\right) = MIN\left(X_1 \otimes MIN\left(\bigotimes_{i=2}^{n} X_i\right)\right)$$

Proof 5 See Appendix D.

Example 6 We first illustrate the computation of $\mathcal{MP}(R_n, \{name, children\})$. Since $\mathcal{T}(R_n, \{name, children\}) = \emptyset$, we set $\mathcal{MP}(R_n, \{name, children\}) = \{\{name, children\}\}$. Let us now illustrate $\mathcal{MP}(R_n, \{children, city\})$. Since $\mathcal{T}(R_n, \{children, city\}) = \{T_2, T_4\}$, we compute $\mathcal{MP}(R_n, \{children, city\})$ from $\mathcal{MP}(T_2, \{children, city\})$ and $\mathcal{MP}(T_4, \{children, city\})$. Using Theorem 3, we obtain $\mathcal{MP}(R_n, \{children, city\}) = MIN(\{\{name, children, city\}, \{income, children, city\}, \{birthdate, children, city\}\} \otimes \{\{name, children, city\}\}) = \{\{name, children, city\}\}$. Similarly, we can compute remaining minimal proxy key sets: $\mathcal{MP}(R_n, \{name, income\}) = \{\{name, income\}\}$, $\mathcal{MP}(R_n, \{birthdate\}) = \{\{name, birthdate\}\}$ and $\mathcal{MP}(R_n, \{children, income\}) = \{\{name, children, income\}\}$. ∎

Step 3: Computation of Common Keys of S_n**.** Finally, we compute the set of minimal common keys in S_n ($\mathcal{MC}(S_n)$) from $\mathcal{MP}(R_n, \cdot)$ generated from the preceding step by applying the minimization operator on the sets of proxy keys (Line 11 in Algorithm 5). Note that the minimization operation ensures removal of super keys from this set.

Example 7 We obtain $\mathcal{MC}(S_n) = MIN(\{\{name, children\}\} \cup \{\{name, income\}\} \cup \{\{name, birthdate\}\} \cup \{\{name, children, income\}\} \cup \{\{name, children, city\}\}) = \{\{name, children\}, \{name, income\}, \{name, birthdate\}\}$. ∎

	AMZN1	AMZN2	CNET1	CNET2	DIGG1	DIGG2	EBAY1	EBAY2
Web site	amazon.com		cnet.com		digg.com		ebay.com	
Query	mp3	digital camera	camera	laptop	tech	world	ipod touch	laptop computer
# records in R_1 (10th Nov, 2008)	3051	3708	10100	10100	10200	20400	10489	21950
# records in R_2 (17th Nov, 2008)	3996	3998	10200	10100	10200	20400	10635	21951
# attributes in R_1 and R_2	13	14	24	25	18	18	30	29

Table 3: Real-world data sets.

5.4 Complexity Analysis

In this section, we present the time and space complexities of NAÏVE and COKE algorithms. We summarize the complexities with respect to seven parameters shown in Table 2.

Let $O(\mathbb{F}(R))$ and $O(\mathbb{G}(R))$ be the time and space complexities, respectively, of computing $\mathcal{MK}(R)$ using key discovery algorithm. Then,

Theorem 4 *The time and space complexities of the* NAÏVE *algorithm are* $O(\mathbb{F}(R_n))$ *and* $O(N_s N_t + \mathbb{G}(R_n))$*, respectively.*

Proof 6 See Appendix E.

Theorem 5 *The time and space complexities of the* COKE *algorithm are* $O(N_k N_a N_t \log N_t + N_u N_p \mathbb{F}(T^p))$ *and* $O(N_s N_t + \mathbb{G}(T^p))$*, respectively.*

Proof 7 See Appendix E.

Remark. Recall that we traverse the partitioning plan in a depth first fashion. The space that we require in each node in the partitioning plan is at most N_t. We only store the identifier of the records. Since the depth of the partitioning plan is at most N_a, then the space that we require is at most $N_t N_a$. In contrast, the space that we require to store the new relation is $N_t N_s$. Since $N_s > N_a$, then the space that we require to generate the sets of sub-relations is less than the space required for storing the new relation. Furthermore, the space required to *store* the sets of sub-relations is $N_u N_p N_q$. Note that one record can be in multiple sub-relations in multiple minimal common keys. Since N_u, N_p, and N_q are small, this space is less than the space required to store the new relation.

In COKE, key discovery algorithm is executed against projected sub-relation T^p, whereas in NAÏVE, it is executed against relation R_n. Since T^p has smaller number of attributes and significantly smaller number of records than R_n, the cost of each execution of key discovery algorithm in COKE is much cheaper than in NAÏVE. However, in COKE, key discovery algorithm is executed $N_u N_p$ times, whereas in NAÏVE, it is executed only once. Notice that $N_u N_p$ is the total number of non-trivial sub-relations in R_n. Thus, if $N_u N_p$ is sufficiently large, it is possible that the total cost of execution of key discovery algorithm in COKE to be more expensive than in NAÏVE, which may in overall make COKE slower than NAÏVE. However, as we shall show in the next section, such scenario is improbable in real-world data sets.

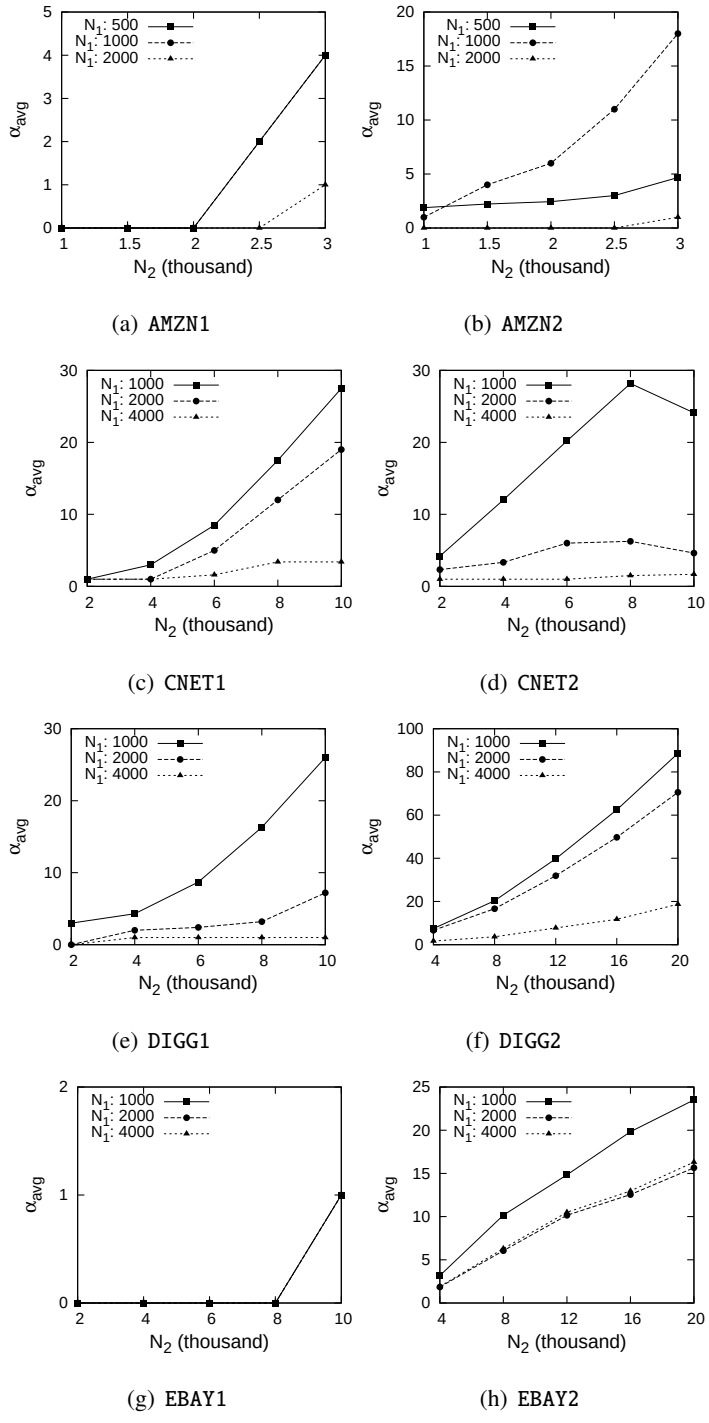


Figure 8: Anorexic nature (α_{avg}) of real-world data sets.

6 Anorexic Nature of Sub-Relations

Recall that we made the assumption that the number of non-trivial sub-relations in R_n and the number of their records are relatively small. This "anorexic" nature of the sub-relations reduces the total execution cost of a key discovery algorithm. Hence, a critical question that needs to be answered is *whether this assumption is practical for evolutionary real-world continuous query results?* In this section, we show that it is indeed the case.

Factors affecting anorexic behavior. In general, there are two factors that affect the extent to which anorexic characteristics are present in relation R_n: (a) the domain size of the minimal common keys in $\mathcal{MC}(S_o)$, and (b) the number of records in R_n. Given two relations R_1 and R_2, we can show the effect of these two factors using the following method. We sample N_1 records from R_1, compute the set of minimal keys of these N_1 records, and use it as $\mathcal{MC}(S_o)$. We can vary the first factor by varying N_1. Specifically, we can increase the domain size of the minimal common keys by increasing N_1. This is because in a relation with large number of records, it is more likely to get duplicate records for sets of attributes with smaller domain size than with larger domain size, and thus, the minimal keys of this large relation are more likely to have larger domain size. Next, we sample N_2 records from R_2, and use it as R_n. We can vary the second factor by varying N_2.

We study the effect of the above factors by analyzing two parameters: α_{avg} and β_{avg}. Let $\widehat{\mathcal{MC}}(S_o)$ be the set of minimal common keys of S_o whose set of non-trivial sub-relations is not empty, i.e., $\widehat{\mathcal{MC}}(S_o) = \{C | C \in \mathcal{MC}(S_o) \text{ and } \mathcal{T}(R_n, C) \neq \emptyset\}$. Then, α_{avg} is defined as the average number of non-trivial sub-relations associated with each minimal common key whose set of non-trivial sub-relations is not empty, i.e.,

$$\alpha_{avg} = \frac{\sum_{C \in \widehat{\mathcal{MC}}(S_o)} |\mathcal{T}(R_n, C)|}{|\widehat{\mathcal{MC}}(S_o)|}$$

β_{avg} is defined as the average number of records in each non-trivial sub-relation, i.e.,

$$\beta_{avg} = \frac{\sum_{C \in \widehat{\mathcal{MC}}(S_o)} \sum_{T \in \mathcal{T}(R_n, C)} |T|}{\sum_{C \in \widehat{\mathcal{MC}}(S_o)} |\mathcal{T}(R_n, C)|}$$

Analysis of α_{avg} and β_{avg}. We applied the above method on real-world data sets shown in Table 3. Each data set is obtained by submitting the query shown in row "*Query*" to the deep Web site shown in row "*Web site*" using the API provided by the site. For each data set, we submitted the same query to the same site on $10th$ and $17th$ November, 2008. Thus, for each data set we have two relations. We use the former relation as R_1, and the latter relation as R_2. The number of attributes and records in these relations are shown in rows "*# attributes*" and "*# records*", respectively. Figure 8 and 9 show the variability of α_{avg} and β_{avg}, respectively, for these data sets. The horizontal axis shows different values of N_2 and each line represents the variation of α_{avg} or β_{avg} for a specific value of N_1.

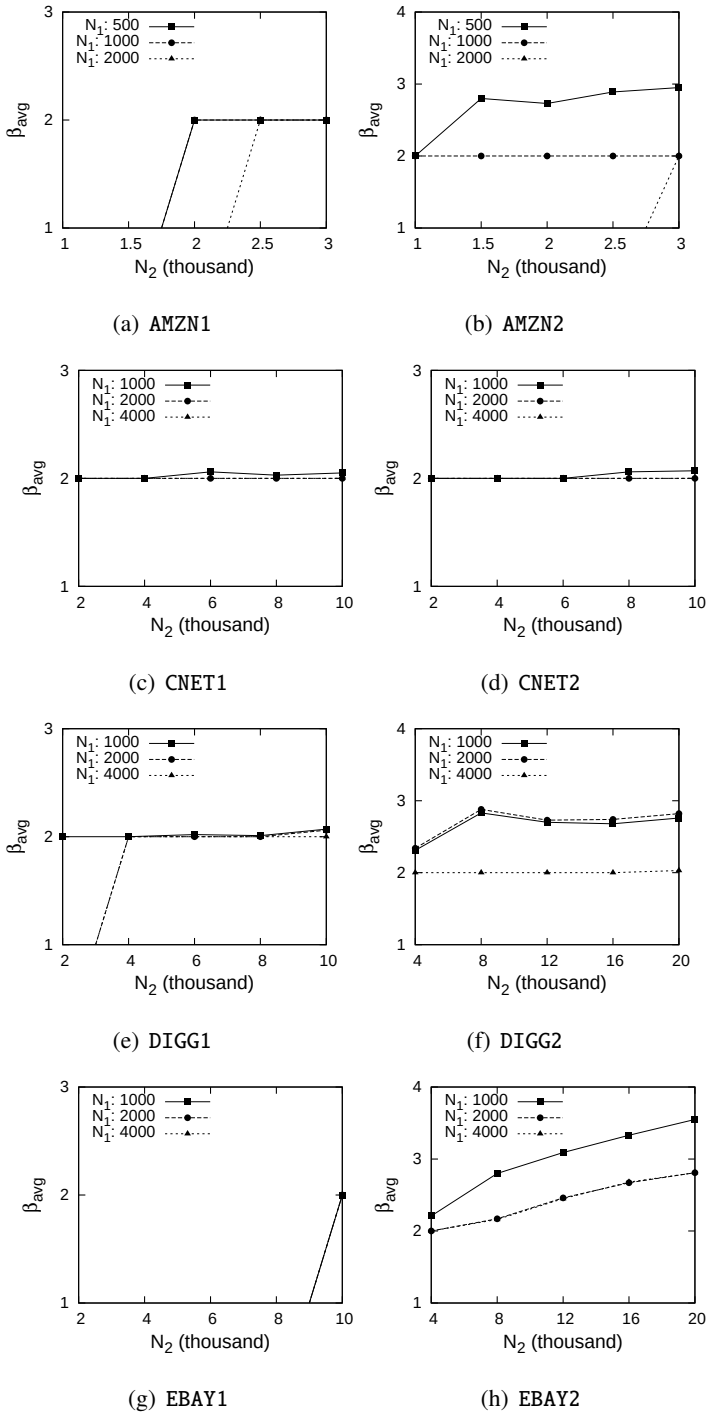


Figure 9: Anorexic nature (β_{avg}) of real-world data sets.

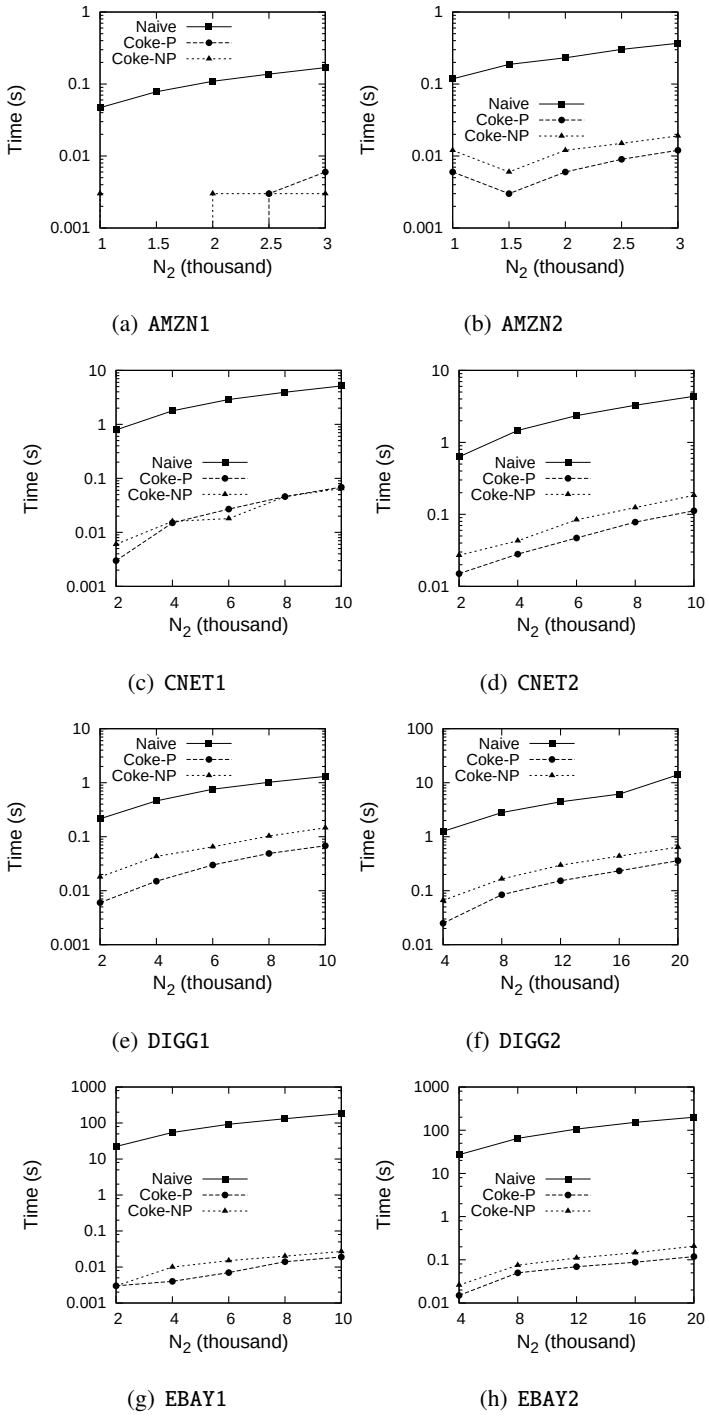
We can see that α_{avg} and β_{avg} tend to decrease and increase with the increase in N_1 and N_2, respectively. This is due to the fact that when the domain size of the minimal common keys is large, it is less likely that there are records in R_n with the same value for all attributes in these minimal common keys. Also, when the number of records in R_n is large, it is more likely that there are records with the same value for all attributes in these minimal common keys.

It is interesting to observe that the number of records in each non-trivial sub-relation not only grows very slowly with respect to the number of records in R_n, but also its value is very small. This is beneficial for COKE as the computation cost of the key discovery algorithm on these sub-relations becomes inexpensive. The number of non-trivial sub-relations however increases roughly linear to the number of records in R_n. This can be expensive in COKE since the number of executions of the key discovery algorithm is linear to the number of non-trivial sub-relations. However, when the domain of the minimal common keys is large, the number of non-trivial sub-relations is relatively small. In many real-world data sets, the domain size of the minimal common keys is large, since the minimal common keys are computed from a relation with large number of records. Hence, we can conclude that evolution of many real-world data sets indeed generates anorexic sub-relations.

7 Performance Evaluation

In this section, we present extensive experimental evaluation of the COKE algorithm by using real-world and synthetic data sets. We have implemented NAÏVE algorithm and two variants of COKE algorithm, namely COKE-P and COKE-NP. COKE-P uses partitioning plan in computing the sets of non-trivial sub-relations, while COKE-NP *does not*. The purpose of implementing COKE-NP is to study the effectiveness of using partitioning plan in computing the sets of non-trivial sub-relations. Recall that the execution time of Phase 1 of COKE is affected by the usage of partitioning plan. Hence, we distinguish between the Phase 1 of COKE-P and COKE-NP. In the sequel, the Phase 1 of COKE-P and COKE-NP are denoted as `Phase 1-P` and `Phase 1-NP`, respectively. The Phase 2 of these two variants, which is not affected by the partitioning plan, is denoted as `Phase 2`.

We use GORDIAN [24] as the key discovery algorithm for NAÏVE as it is efficient for relation with large number of records. We use AGREE SET [16] as the key discovery algorithm for the two variants of COKE as it is efficient for relation with small number of records. The differences between these two algorithms are the following. GORDIAN computes the set of minimal keys by traversing the combinations of attributes, whereas AGREE SET computes the set of minimal keys by enumerating all possible pairs of records. Although GORDIAN uses pruning techniques to eliminate large number of combinations of attributes, it may still suffer from the exponential size of the combinations of attributes. In contrast, although AGREE SET does not suffer from exponential size of the combinations of attributes, it suffers from



(a) AMZN1 (b) AMZN2

(c) CNET1 (d) CNET2

(e) DIGG1 (f) DIGG2

(g) EBAY1 (h) EBAY2

Figure 10: Execution times of NAÏVE, COKE-P, and COKE-NP on real-world data sets.

quadratic size of the pairs of records. Thus, when the relation contains a small number of records and a large number of attributes, AGREE SET is potentially better than GORDIAN. Since the non-trivial sub-relations contain small number of records, we use AGREE SET for the two variants of COKE so that they can handle relations with large number of attributes.

We implemented all algorithms in Java. All experiments were performed on a Windows XP machine with Pentium DC 3.40 GHz processor and 2.99 GB RAM.

7.1 Experiments on Real-World Data Sets

We measured the execution time required by the algorithms to compute $\mathcal{MC}(S_n)$ from $\mathcal{MC}(S_o)$ and R_n.

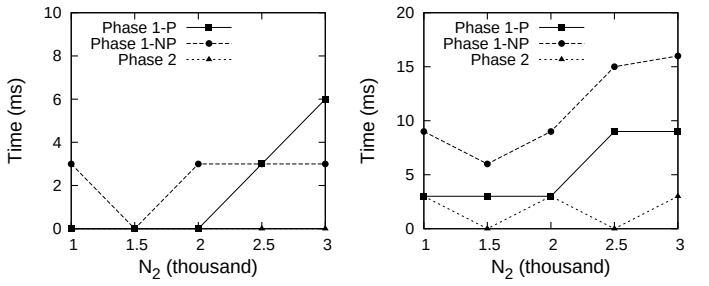
Data sets and experimental setup. Although our proposed algorithms are generic in nature and not tied to any particular domain, we chose a set of deep Web sites to represent real-world data. In particular, eight data sets from various deep Web sites as shown in Table 3 are used for our study. Details related to retrieval of these data sets are discussed in Section 6.

For each data set, we first generate $\mathcal{MC}(S_o)$ by randomly sampling N_1 records from R_1, compute the set of minimal keys of these N_1 records, and use it as $\mathcal{MC}(S_o)$. For each data set, we generate R_n by randomly sampling N_2 records from R_2, and use it as R_n. We set N_1 to 500 for `AMZN1` and `AMZN2` data sets, and 1000 for the remaining data sets. The reason for choosing a relatively small value is to ensure that each minimal common key in $\mathcal{MC}(S_o)$ is associated with many non-trivial sub-relations, each with many records. Notice that this setting actually unfavorable for COKE as it increases the total cost of the execution of the key discovery algorithm.

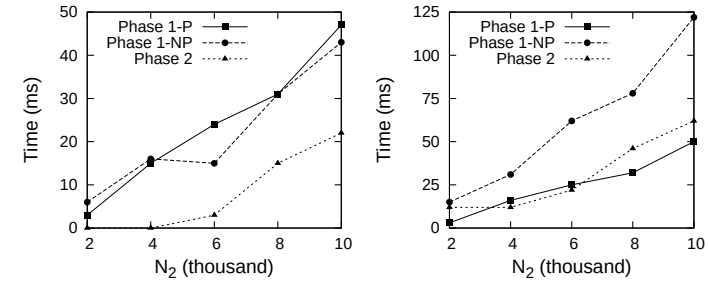
We measured the performance of the algorithms with respect to the number of records in R_n by varying N_2. We ensure that R_n with smaller number of records is a subset of R_n with larger number of records. This is to ensure that the number of non-trivial sub-relations and the number of their records increase as we increase N_2.

Comparison of NAÏVE, COKE-P, and COKE-NP. Figure 10 shows the performances of COKE-P, COKE-NP, and NAÏVE. We can make the following observations. Firstly, COKE-P is orders of magnitude faster than NAÏVE for both data sets. Secondly, it is evident that partitioning plan provides slight benefits to the algorithm. In the next section, using synthetic data, we shall discuss scenarios when partitioning plan provides *significantly large* benefits.

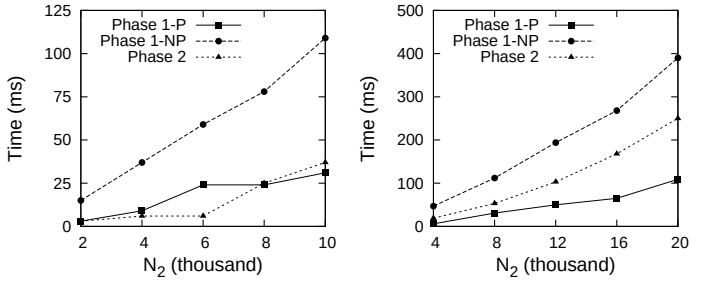
Comparison of the phases of COKE-P and COKE-NP. Figures 11 reports the performances of the two phases. We can see that the execution times of both phases increase roughly linearly with the number of records in R_n. The linear behavior of Phase 1 is due to the fact that increase in number of records increases the number of records that need to be partitioned. Recall that the performance of Phase 2 is influenced by the number of non-trivial sub-relations and their sizes. Hence, as we increase the number of records in R_n, the number of non-trivial sub-relations



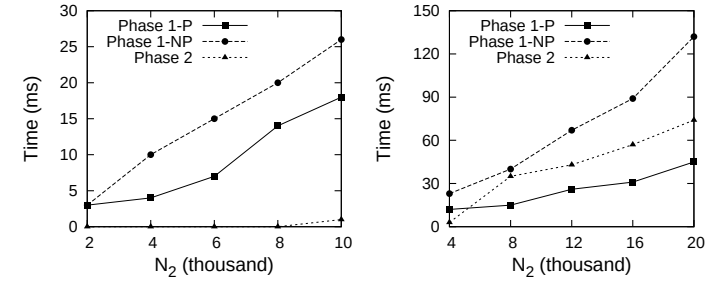
(a) AMZN1 (b) AMZN2



(c) CNET1 (d) CNET2



(e) DIGG1 (f) DIGG2



(g) EBAY1 (h) EBAY2

Figure 11: Execution times of the phases of coKE-P and coKE-NP on real-world data sets.

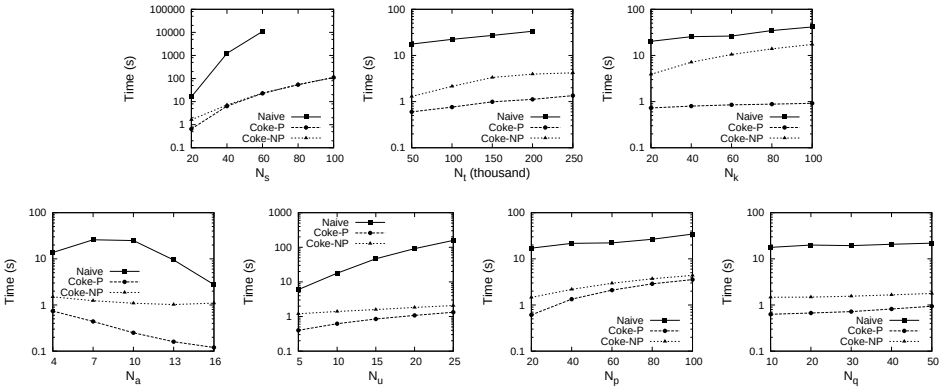


Figure 12: Execution times of NAÏVE, COKE-P, and COKE-NP.

increases roughly linear fashion, and their sizes increase very slowly (Recall from Section 6).

Furthermore, the linear behavior of Phase 2 justifies the usage of AGREE SET as the key discovery technique. Although it is quadratic to the number of records in the non-trivial sub-relations, the linearity of Phase 2 is due to the fact that there is not much variation in the number of records in the non-trivial sub-relations with increasing number of records in R_n. Additionally, the impact of a quadratic algorithm is small since each non-trivial sub-relation contains small number of records.

7.2 Experiments on Synthetic Data Sets

We use synthetic data set to study how the proposed algorithms scale with respect to each of the seven parameters shown in Table 2.

We measured the execution time required by the algorithms to compute $\mathcal{MC}(S_n)$ from $\mathcal{MC}(S_o)$ and R_n with respect to each of the seven parameters by varying one parameter and setting remaining parameters to their default values. The default values of these parameters are given in Table 2. Note that we do not measure the IO cost because all are main memory-based algorithms.

Data sets generation. We synthetically generate $\mathcal{MC}(S_o)$ by using the following procedure. We create N_k minimal common keys. Each of this is created by randomly taking N_a attributes from the attributes of R_n.

We generate R_n by using the following procedure. We create N_t records each with N_s attributes. We set the values of the attributes of these records such that each attribute of R_n is a key of R_n. We randomly take N_u minimal common keys from $\mathcal{MC}(S_o)$. For each of these N_u minimal common keys, we associate it with a set of non-trivial sub-relations, created by randomly taking N_pN_q records from R_n, and dividing them into N_p non-trivial sub-relations each with N_q records. We set the values of the attributes of these N_q records as the following. For each attribute in the minimal common key, we set the values of this attribute to same value. We then

randomly take two attributes that are not in the minimal common key. For each of these attributes, we randomly take three records from N_q records and set the values of this attribute of these records to same value. The reason for taking these two attributes is to enforce that there are at least two super sets of each minimal common key that are not a key of R_n. The objective of this is to vary the minimal common keys in $\mathcal{MC}(S_n)$.

Comparison of efficiency and scalability of NAÏVE and COKE-P. Figure 12 reports the performances of the NAÏVE and COKE-P algorithms with respect to various parameters. It is evident that COKE-P is more efficient and scalable than the NAÏVE algorithm. The superiority of COKE-P can be mostly seen for the parameters N_s and N_u. Note that N_s determines the number of possible combinations of attributes. Although NAÏVE does not suffer from exponential size of the combinations of attributes, it suffers more than COKE-P. The reason for this is because COKE-P executes the key discovery algorithm on non-trivial sub-relations. These sub-relations have smaller number of records as well as smaller number of attributes. Although COKE-P has the overhead of executing the key discovery algorithm multiple times, the total execution time is still orders of magnitude faster than the NAÏVE. Observe that COKE-P is able to handle a data set with 100 attributes in only 110 seconds. Such size of data set is intractable for NAÏVE. Also, the parameter N_u affects the performance of NAÏVE adversely as it needs to find the minimal common keys when it traverses the possible combinations of attributes. However, in COKE-P, the non-trivial sub-relations contain only small number of records, and thus it is cheaper for the key discovery algorithm to find the minimal common keys.

Interestingly, the execution time of the NAÏVE algorithm increases up to a certain value of N_a and then decreases. This is because it uses GORDIAN as the key discovery algorithm. In particular, due to the pruning techniques used by GORDIAN on the lattice structure, it is fast when the minimal keys contain small number of attributes or large number of attributes. In contrast, the execution time of COKE-P decreases with N_a due to decrease in execution time of Phase 2. This is because the number of attributes in the projected non-trivial sub-relation is equal to $N_s - N_a$. Thus, when N_a increases, the projected non-trivial sub-relation contains smaller number of attributes, and thus it is faster for the key discovery algorithm to compute its minimal key set.

Comparison of COKE-P and COKE-NP. We now study the effect of the usage of partitioning plan in the COKE algorithm. Figure 12 reports the advantage of using partitioning plan. Specifically, this advantage can be mostly seen for parameters N_t and N_k. Note that N_t determines the number of records that the algorithms need to partition. Although the execution times of both COKE-P and COKE-NP increase with N_t, COKE-P has smaller rate of increase. This is expected since by using partitioning plan, some of the partitioning can be shared. The parameter N_k determines the number of minimal common keys in $\mathcal{MC}(S_o)$. Similar to N_t, although the execution times of both COKE-P and COKE-NP increase with N_k, COKE-P has a much smaller rate of increase. In practice N_k can be large, since the number of possible combinations of attributes is exponential. In this case it is beneficial to use partitioning

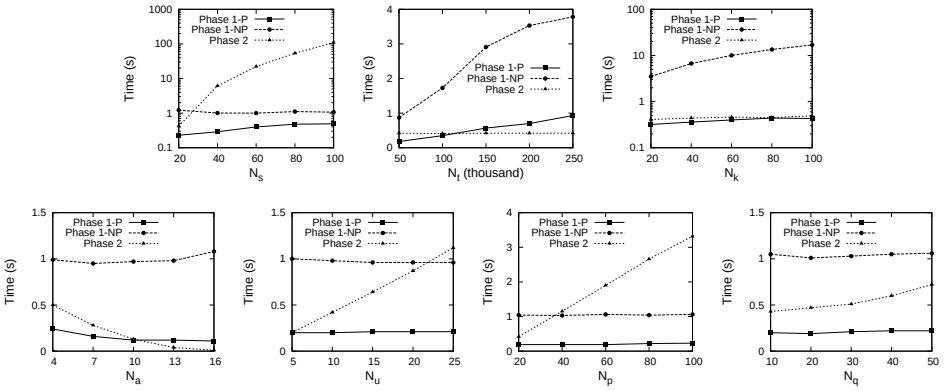


Figure 13: Execution times of the phases of COKE-P and COKE-NP.

plan. The above results also show that our heuristic algorithm for partitioning plan construction is quite effective in reducing the effect of N_t and more importantly N_k.

Comparison of the phases of COKE-P and COKE-NP. Figure 13 reports the performance comparison of different phases in COKE-P and COKE-NP. Observe that the parameter that significantly affect the execution time of Phase 1 is N_t. The parameter that mostly affect the execution time of Phase 2 is N_s. Although the number of possible combinations of attributes is exponential to N_s, the execution time of Phase 2 does not increase exponentially with N_s. This is because COKE-P uses AGREE SET. The other parameters that also affect the execution time of Phase 2 are N_u, N_p, and N_q. The execution time of Phase 2 is linear to N_u and N_p. This is as expected since the number of execution of key discovery algorithm is linearly affected by N_u and N_p. In contrast, the execution time of Phase 2 is super linear to N_q.

8 Related Work

Quality of monitoring the dynamic Web. Monitoring the dynamic Web in response to continuous queries have recently triggered a lot of interest. In this context, optimizing the QoS (Quality of Service) and QoD have been the focus of several research efforts. For example, multi-query optimization has been exploited in [5] to improve the system throughput, optimization of freshness and currency of query results were investigated in [21, 23], and Kukulenz and Ntoulas [14] have studied quality/freshness tradeoff for *bounded* continuous search queries. However, we are not aware of any prior work that studied the discovery of common keys in order to improve accuracy of tracking entities. Note that existing monitoring systems [5, 15, 20] deploy variants of *HTMLDiff* and XML *diff* algorithms to detect and track changes to the underlying data. However, these algorithms are "identifier-oblivious" and as a result they may be confused by similar values for different attributes associated with each entity. Consequently, these algorithms

may adversely affect the QoD by producing erroneous mapping between entities over time. Hence, our work is complimentary to these efforts.

Also, none of the existing efforts [1, 3, 6] in extracting entity-relation tuples from text into relational databases focus on maintenance of common keys.

Discovery of keys and functional dependencies. More germane to our work are efforts related to key discovery problem in structured databases and mining of functional dependencies (FD). GORDIAN [24] and AGREE SET [16] compute the set of minimal keys indirectly by computing the set of maximal non keys and then converting it into the set of minimal keys. There has also been a great deal of work related to mining strict and approximate FDs from the data. TANE [12], FUN [19], and *FD_Mine* [26] take a candidate *generate-and-test* approach where levelwise search strategy is used to explore the search space. They reduce the search space by eliminating candidates using pruning rules. Similar to our approach, TANE uses a partitioning of the tuples with respect to their attribute values to check validity of functional dependencies. It implicitly identifies keys to prune the search space. FastFDs [25] and *Dep-Miner* [16] employ first-depth search and levelwise search strategies, respectively, to discover FDs by considering pairs of tuples. First, a partitioned database is extracted from the initial relation. Then, agree sets are computed and maximal sets are generated using the partitions. Consequently, a minimum FD cover according to these maximal sets is discovered. Several association rules and sampling-based techniques have also been proposed to make FDs less restrictive by allowing some exceptions to the FD rules [13, 22]. Our work differs from these approaches as follows. First, these techniques focus on discovering keys or FDs from *static* data whereas we focus on incrementally maintaining common keys. These techniques do not assume evolving nature of keys or FDs and hence they are not designed to efficiently maintain the common keys. Second, COKE exploits anorexic characteristics of real-world data to devise efficient solution to this problem. Such characteristics are not exploited in traditional key or FD discovery techniques.

9 Conclusions and Future Work

In this paper, we have described a novel technique for efficiently maintaining common keys in a sequence of versions of archived continuous query results from deep Web sites. This is crucial for developing robust techniques for modeling evolutionary Web data, query processing, and tracking entities over time. We have proposed an algorithm called COKE to discover common keys from the archived versions of structured query results represented as relations. It generates minimal common keys without computing the minimal key set of the new relation. Importantly, it exploits certain anorexic properties of real-world data to provide efficient solution to this problem. Our exhaustive empirical study has demonstrated that COKE has excellent real-world performance. We are currently exploring techniques to detect *conserved* common keys to support *identifiers* discovery in order to track entities accurately in the historical query results sequence.

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A Proof of Theorem 1

Theorem 1 states that given $\mathcal{MC}(S_o)$ and $\mathcal{MK}(R_n)$, we can compute $\mathcal{MC}(S_n)$ as

$$\mathcal{MC}(S_n) = MIN(\mathcal{MC}(S_o) \otimes \mathcal{MK}(R_n))$$

For convenience, we denote $\mathcal{MC}(S_o) \otimes \mathcal{MK}(R_n)$ as H. We first define maximization operator which we shall be using for proofs. Let X be a collection of sets of elements, Y be the universe of the elements in the sets in X, and Z be the power set of Y. The *maximization* of X is defined as the collection of every set in Z which at least one of its subsets is in X, i.e., $\{z|z \in Z$ and $\exists x \in X$ such that $x \subseteq z\}$. We denote the maximization of X as $MAX(X)$.

To prove the above theorem, it is sufficient to prove that $C(S_n) = MAX(H)$ since the above theorem can be obtained by applying minimization operator to this statement.

We first prove that $C(S_n) \subseteq MAX(H)$. Consider an element $X \in C(S_n)$. Since $X \in C(S_n)$, based on the definition of common key, it follows that $X \in C(S_o)$ and $X \in \mathcal{K}(R_n)$. Since $X \in C(S_o)$, based on the definition of minimal common key, it follows that there is at least one element $C \in \mathcal{MC}(S_o)$ such that $X \supseteq C$. Since $X \in \mathcal{K}(R_n)$, based on the definition of minimal key, it follows that there is at least one element $K \in \mathcal{MK}(R_n)$ such that $X \supseteq K$. Since $X \supseteq C$ and $X \supseteq K$, it follows that $X \supseteq D$ where $D = C \cup K$. Since $D = C \cup K$, $C \in \mathcal{MC}(S_o)$, and $K \in \mathcal{MK}(R_n)$, based on the definition of pairwise union operator, it follows that $D \in H$. Since $X \supseteq D$ and $D \in H$, based on the definition of maximization operator, it follows that $X \in MAX(H)$. Since $X \in MAX(H)$ whenever $X \in C(S_n)$, it follows that $C(S_n) \subseteq MAX(H)$.

We now prove that $MAX(H) \subseteq C(S_n)$. Consider an element $X \in MAX(H)$. Since $X \in MAX(H)$, based on the definition of maximization operator, it follows that there is at least one element $D \in H$ such that $X \supseteq D$. Since $D \in H$, based on the definition of pairwise union operator, there is at least one element $C \in \mathcal{MC}(S_o)$ and one element $K \in \mathcal{MK}(R_n)$ such that $D = C \cup K$. Since $X \supseteq D$ and $D = C \cup K$, it follows that $X \supseteq C$ and $X \supseteq K$. Since $X \supseteq C$ and $C \in \mathcal{MC}(S_o)$, based on the definition of minimal common key, it follows that $X \in C(S_o)$. Since $X \supseteq K$ and $K \in \mathcal{MK}(R_n)$, based on the definition of minimal key, it follows that $X \in \mathcal{K}(R_n)$. Since $X \in C(S_o)$ and $X \in \mathcal{K}(R_n)$, based on the definition of common key, it follows that $X \in C(S_n)$. Since $X \in C(S_n)$ whenever $X \in MAX(H)$, it follows that $MAX(H) \subseteq C(S_n)$.

B Proof of Theorem 2

We prove this theorem by proving that its decision problem version, *partitioning plan* problem, is NP-Hard. partitioning plan problem can be stated as given $\mathcal{MC}(S_o)$ and an integer N, find whether it is possible to construct a partitioning plan from $\mathcal{MC}(S_o)$ with at most N nodes.

We use a reduction from the *vertex cover* problem [7]. Given an instance of a vertex cover problem consisting of a graph $G = (V, E)$, we transform it into an instance of a partitioning plan problem consisting of a set of minimal common keys $\mathcal{MC}(S_o)$ by converting each edge $(a, b) \in E$ into a minimal common key $\{a, b\} \in \mathcal{MC}(S_o)$. This transformation can, clearly, be performed in polynomial time.

Observe that the partitioning plan constructed from $\mathcal{MC}(S_o)$ consists of exactly three levels since each minimal common key in $\mathcal{MC}(S_o)$ consists of exactly two attributes because its corresponding edge consists of exactly two nodes. We now examine the number of nodes in each level of the partitioning plan. The first and the third level consists of a fixed number of nodes regardless of which attribute, in each minimal common key in $\mathcal{MC}(S_o)$, we assign into the second and the third level. The first level consists of exactly one root node. The third level consists of exactly $|E|$ nodes, where $|E|$ denotes the number of edges in E or, equivalently, the number of minimal common keys in $\mathcal{MC}(S_o)$, since each path from a child-of-root node to a leaf node represents a minimal common key in $\mathcal{MC}(S_o)$ and vice versa. The number of nodes in second level, unlike the other levels, depends on the assignment of the attributes.

We now prove that G has a vertex cover with at most N nodes if and only if it is possible to construct a partitioning plan with at most $1 + |E| + N$ nodes.

We first prove that if it is possible to construct a partitioning plan from $\mathcal{MC}(S_o)$ with at most $1 + |E| + N$ nodes, then G has a vertex cover with at most N. Suppose it is possible to construct a partitioning plan from $\mathcal{MC}(S_o)$ with at most $1 + |E| + N$ nodes. Since there is exactly 1 node in the first level and exactly $|E|$ nodes in the third level, it follows that there are at most N nodes in the second level. Let X be the set of nodes in the second level. Since for each minimal common key $\{a, b\} \in \mathcal{MC}(S_o)$, either $a \in X$ or $b \in X$ or both, it follows that for each edge $(a, b) \in E$, either $a \in X$ or $b \in X$ or both. Since for each edge $(a, b) \in E$, either $a \in X$ or $b \in X$ or both, it follows that X is a vertex cover of G. Since X consists of at most N nodes, it follows that G has a vertex cover with at most N nodes.

We now prove that if G has a vertex cover with at most N nodes, then it is possible to construct a partitioning plan from $\mathcal{MC}(S_o)$ with at most $1 + |E| + N$ nodes. Suppose G has a vertex cover X with at most N nodes. Since X is a vertex cover of G, it follows that for each edge $(a, b) \in E$, either $a \in X$ or $b \in X$ or both. Since for each edge $(a, b) \in E$, either $a \in X$ or $b \in X$ or both, it follows that for each minimal common key $\{a, b\} \in \mathcal{MC}(S_o)$, either $a \in X$ or $b \in X$ or both. We construct a partitioning plan from $\mathcal{MC}(S_o)$ as the following. For each minimal common key $\{a, b\} \in \mathcal{MC}(S_o)$, (1) if $a \in X$ and $b \notin X$, then we assign a into the second level and b into the third level, (2) if $a \notin X$ and $b \in X$, then we assign a into the third level and b into the second level, and (3) if $a \in X$ and $b \in X$, then we assign anyone of them into the second level and the other one into the third level. Note that one of these three cases must happen since either $a \in X$ or $b \in X$ or both. Observe that the number of nodes in the second level is at most the size of X, i.e., N, since we put a node in the second level only if it is in X. Since there is exactly

1 node in the first level, exactly $|E|$ nodes in the third level, and at most N nodes in the second level, it follows that it is possible to construct a partitioning plan from $\mathcal{MC}(S_o)$ with at most $1 + |E| + N$ nodes.

C Proof of Theorem 3

Theorem 3 states that let $C \in \mathcal{MC}(S_o)$; given $\mathcal{MP}(\cdot, C)$, we can compute $\mathcal{MP}(R_n, C)$ as

$$\mathcal{MP}(R_n, C) = MIN\left(\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C)\right)$$

where we assume that we add all trivial sub-relations into $\mathcal{T}(R_n, C)$.

For convenience, we denote $\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C)$ as H. To prove the above theorem, it is sufficient to prove that $\mathcal{P}(R_n, C) = MAX(H)$ since the above theorem can be obtained by applying minimization operator to this statement.

We first prove that $\mathcal{P}(R_n, C) \subseteq MAX(H)$. Consider an element $X \in \mathcal{P}(R_n, C)$. Since $X \in \mathcal{P}(R_n, C)$, based on the definition of sub-relation, i.e., each record belongs to exactly one sub-relation, it follows that for every $T_i \in \mathcal{T}(R_n, C)$, $X \in \mathcal{P}(T_i, C)$. Since $X \in \mathcal{P}(T_i, C)$, based on the definition of minimal proxy key, it follows that there is at least one element $P_i \in \mathcal{MP}(T_i, C)$ such that $X \supseteq P_i$. Since for every $T_i \in \mathcal{T}(R_n, C)$, $X \supseteq P_i$, it follows that $X \supseteq Q$ where $Q = \bigcup_{T_i \in \mathcal{T}(R_n, C)} P_i$. Since $Q = \bigcup_{T_i \in \mathcal{T}(R_n, C)} P_i$ and for every $T_i \in \mathcal{T}(R_n, C)$, $P_i \in \mathcal{MP}(T_i, C)$, based on the definition of pairwise union operator, it follows that $Q \in H$. Since $X \supseteq Q$ and $Q \in H$, based on the definition of maximization operator, it follows that $X \in MAX(H)$. Since $X \in MAX(H)$ whenever $X \in \mathcal{P}(R_n, C)$, it follows that $\mathcal{P}(R_n, C) \subseteq MAX(H)$.

We now prove that $MAX(H) \subseteq \mathcal{P}(R_n, C)$. Consider an element $X \in MAX(H)$. Since $X \in MAX(H)$, based on the definition of maximization operator, it follows that there is at least one element $Q \in H$ such that $X \supseteq Q$. Since $Q \in H$, based on the definition of pairwise union operator, it follows that for every $T_i \in \mathcal{T}(R_n, C)$, there is at least one element $P_i \in \mathcal{MP}(T_i, C)$, such that $Q = \bigcup_{T_i \in \mathcal{T}(R_n, C)} P_i$. Since $X \supseteq Q$ and $Q = \bigcup_{T_i \in \mathcal{T}(R_n, C)} P_i$, it follows that for every $T_i \in \mathcal{T}(R_n, C)$, $X \supseteq P_i$. Since $X \supseteq P_i$ and $P_i \in \mathcal{MP}(T_i, C)$, based on the definition of minimal proxy key, it follows that $X \in \mathcal{P}(T_i, C)$. Since for every $T_i \in \mathcal{T}(R_n, C)$, $X \in \mathcal{P}(T_i, C)$, based on the definition of sub-relation, i.e., every two records in two different sub-relations have different value for at least one attribute in C, it follows that $X \in \mathcal{P}(R_n, C)$. Since $X \in \mathcal{P}(R_n, C)$ whenever $X \in MAX(H)$, it follows that $MAX(H) \subseteq \mathcal{P}(R_n, C)$.

We now prove that Theorem 3 holds when we add no trivial sub-relations into $\mathcal{T}(R_n, C)$. Let us denote the set of sub-relations where we add no trivial sub-relations as $\widetilde{\mathcal{T}}(R_n, C)$. Our aim is to prove that

$$\bigotimes_{T \in \mathcal{T}(R_n, C)} \mathcal{MP}(T, C) = \bigotimes_{T \in \widetilde{\mathcal{T}}(R_n, C)} \mathcal{MP}(T, C)$$

To prove the above statement, it is sufficient to prove that $\mathcal{MP}(T_i, C) \otimes \mathcal{MP}(T_j, C) =$

$\mathcal{MP}(T_i, C)$ for any sub-relation $T_i \in \mathcal{T}(R_n, C)$ and any trivial sub-relation $T_j \in \mathcal{T}(R_n, C)$ since it implies that T_j can be removed from $\mathcal{T}(R_n, C)$.

Since T_j contains only one record, based on the definition of minimal proxy key, it follows that $\mathcal{MP}(T_j, C) = \{C\}$. Consider an element $P \in \mathcal{MP}(T_i, C)$. Since $P \in \mathcal{MP}(T_i, C)$, based on the definition of minimal proxy key, it follows that $P \supseteq C$. Since $P \supseteq C$, it follows that $P \cup C = P$. Since $\mathcal{MP}(T_j, C) = \{C\}$ and for every element $P \in \mathcal{MP}(T_i, C)$, $P \cup C = P$, based on the definition of pairwise union operator, it follows that the statement is true.

D Proof of Lemma 2

Lemma 2 states that given a collection of sets of elements $\{X_1, X_2, \ldots, X_n\}$, we can compute $MIN\left(\bigotimes_{i=1}^{n} X_i\right)$ recursively as

$$MIN\left(\bigotimes_{i=1}^{n} X_i\right) = MIN\left(X_1 \otimes MIN\left(\bigotimes_{i=2}^{n} X_i\right)\right)$$

For convenience, we denote X_1 as P and $\bigotimes_{i=2}^{n} X_i$ as Q. The above theorem can be rewritten as $MIN(P \otimes Q) = MIN(P \otimes MIN(Q))$. To prove the above theorem, it is sufficient to prove that $MAX(P \otimes Q) = MAX(P \otimes MIN(Q))$, since the above theorem can be obtained by applying minimization operator to this statement.

We first prove that $MAX(P \otimes Q) \subseteq MAX(P \otimes MIN(Q))$. Consider an element $a \in MAX(P \otimes Q)$. Since $a \in MAX(P \otimes Q)$, based on the definition of maximization operator, it follows that there is at least one element $b \in P \otimes Q$ such that $a \supseteq b$. Since $b \in P \otimes Q$, based on the definition of pairwise union operator, it follows that there is at least one element $p \in P$ and one element $q \in Q$ such that $b = p \cup q$. Since $q \in Q$, based on the definition of minimization operator, there is at least one element $r \in Q$ such that $r \subseteq q$ and $r \in MIN(Q)$. Let $c = p \cup r$. Since $c = p \cup r$, $p \in P$, and $r \in MIN(Q)$, based on the definition of pairwise union operator, it follows that $c \in P \otimes MIN(Q)$. Since $b = p \cup q$, $c = p \cup r$, and $r \subseteq q$, it follows that $b \supseteq c$. Since $a \supseteq b$ and $b \supseteq c$, it follows that $a \supseteq c$. Since $a \supseteq c$ and $c \in P \otimes MIN(Q)$, based on the definition of maximization operator, it follows that $a \in MAX(P \otimes MIN(Q))$. Since $a \in MAX(P \otimes MIN(Q))$ whenever $a \in MAX(P \otimes Q)$, it follows that $MAX(P \otimes Q) \subseteq MAX(P \otimes MIN(Q))$.

We now prove that $MAX(P \otimes MIN(Q)) \subseteq MAX(P \otimes Q)$. Consider an element $a \in MAX(P \otimes MIN(Q))$. Since $a \in MAX(P \otimes MIN(Q))$, based on the definition of maximization operator, it follows that there is at least one element $b \in P \otimes MIN(Q)$ such that $a \supseteq b$. Since $b \in P \otimes MIN(Q)$, based on the definition of pairwise union operator, there is at least one element $p \in P$ and one element $q \in MIN(Q)$ such that $b = p \cup q$. Since $q \in MIN(Q)$, based on the definition of minimization operator, it follows that $q \in Q$. Since $b = p \cup q$, $p \in P$, and $q \in Q$, based on the definition of pairwise union operator, it follows that $b \in P \otimes Q$. Since $a \supseteq b$ and $b \in P \otimes Q$, based on the definition of maximization operator, it follows that

$a \in MAX(P \otimes Q)$. Since $a \in MAX(P \otimes Q)$ whenever $a \in MAX(P \otimes MIN(Q))$, it follows that $MAX(P \otimes MIN(Q)) \subseteq MAX(P \otimes Q)$.

E Complexity Analysis

We consider the following seven factors in the analysis:

1. N_t: the number of records in R_n.
2. N_s: the number of attributes of R_n.
3. N_k: the number of minimal keys in $\mathcal{MK}(R_n)$. We assume that $\mathcal{MC}(S_o)$ contains N_k minimal common keys. We assume that $\mathcal{MC}(S_n)$ contains N_k minimal common keys. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations, $\mathcal{MP}(R_n, C)$ contains N_k minimal proxy keys. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$, $\mathcal{MP}(T, C)$ contains N_k minimal proxy keys. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$, $\mathcal{MK}(T^p, C)$ contains N_k minimal keys.
4. N_a: the number of attributes in each minimal key in $\mathcal{MK}(R_n)$. We assume that each minimal common key in $\mathcal{MC}(S_o)$ contains N_a attributes. We assume that each minimal common key in $\mathcal{MC}(S_n)$ contains N_a attributes. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations, each minimal proxy key in $\mathcal{MP}(R_n, C)$ contains N_a attributes. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$, each minimal proxy key in $\mathcal{MP}(T, C)$ contains N_a attributes. We assume that for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$, each minimal key $\mathcal{MK}(T^p, C)$ contains N_a attributes.
5. N_u: the number of minimal common keys in $\mathcal{MC}(S_o)$ with non-trivial sub-relations.
6. N_p: the number of sub snapshots in $\mathcal{T}(R_n, C)$ for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations.
7. N_q: the number of records in each sub snapshot in $\mathcal{T}(R_n, C)$ for each $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations.

We use the following additional assumptions:

1. The time complexity of computing $\mathcal{MK}(R)$ of a relation R by using key discovery algorithm is $O(\mathbb{F}(R))$, where $\mathbb{F}$ is a function that depends on the key discovery algorithm used.

2. The space complexity of computing $\mathcal{MK}(R)$ of a relation R by using key discovery algorithm is $O(NA + PQ + \mathbb{G}(R))$, where N is the number of records in R, A is the number of attributes of R, P is the number of minimal keys in $\mathcal{MK}(R)$, Q is the number of attributes in each minimal key in $\mathcal{MK}(R)$, and $\mathbb{G}$ is a function that depends on the key discovery algorithm used. Note that $O(NA)$ is the space complexity of R_n, $O(PQ)$ is the space complexity of $\mathcal{MK}(R)$, and $O(\mathbb{G}(R))$ is the additional space complexity specific to the key discovery algorithm used.

3. The time complexity of computing $MIN(X)$ of a collection of sets of elements X is $O(NA \log N + NMA)$, where N is the number of sets in X, M is the number of sets in $MIN(X)$, and A is the number of elements in each set in X. Note that $O(NA \log N)$ is the time complexity for sorting the sets of elements in X, and $O(NMA)$ is the time complexity for removing the sets whose proper subset is also in X.

4. The space complexity of computing $MIN(X)$ of a collection of sets of elements X is $O(NA)$, where N is the number of sets in X, and A is the number of elements in each set in X.

5. The time complexity of computing $X \otimes Y$ of two collections of sets of elements X and Y is $O(N^2A)$, where N is the number of sets in X and Y, and A is the number of elements in each set in X and Y.

6. The space complexity of computing $X \otimes Y$ of two collections of sets of elements X and Y is $O(N^2A)$ where N is the number of sets in X and Y, and A is the number of elements in each set in X and Y.

Time Complexity of NAÏVE Algorithm. The time complexity of each of the steps in NAÏVE algorithm is:

1. Computing $\mathcal{MK}(R_n)$. The time complexity of this step is $O(\mathbb{F}(R_n))$.

2. Computing $\mathcal{MC}(S_n)$. The time complexity of computing the pairwise union is $O(N_k^2 N_a)$. The time complexity of computing the minimization is $O(N_k^3 N_a)$ since the set of pairwise union contains $O(N_k^2)$ elements. In total, the time complexity of this step is $O(N_k^3 N_a)$.

Thus, the time complexity of NAÏVE is $O(\mathbb{F}(R_n) + N_k^3 N_a)$. For small N_k and N_a, the time complexity becomes $O(\mathbb{F}(R_n))$.

Space Complexity of NAÏVE Algorithm. The space complexity of R_n is $O(N_t N_s)$. The space complexity of $\mathcal{MC}(S_o)$ is $O(N_k N_a)$. Hence, the space complexity of each of the steps in NAÏVE algorithm is:

1. Computing $\mathcal{MK}(R_n)$. The space complexity of computing $\mathcal{MK}(R_n)$ is $O(\mathbb{G}(R_n))$. The space complexity of $\mathcal{MK}(R_n)$ is $O(N_k N_a)$. In total, the space complexity of this step is $O(\mathbb{G}(R_n) + N_k N_a)$.

2. Computing $\mathcal{MC}(S_n)$. The space complexity of the set of pairwise union is $O(N_k^2 N_a)$. The space complexity of $\mathcal{MC}(S_n)$ is $O(N_k N_a)$. In total, the space complexity of this step is $O(N_k^2 N_a)$.

Thus, the space complexity of NAÏVE is $O(N_t N_s + \mathbb{G}(R_n) + N_k^2 N_a)$. For small N_k and N_a, the space complexity becomes $O(N_t N_s + \mathbb{G}(R_n))$.

Time Complexity of COKE Algorithm. The time complexity of each of the steps in COKE is:

1. *Computing* $\mathcal{T}(R_n, \cdot)$. The time complexity of each of the sub-steps is:

   (a) *Constructing the partitioning plan.* The time complexity of computing the frequencies of the attributes is $O(N_k N_a)$. The time complexity of sorting the attributes in the minimal common keys is $O(N_k N_a \log N_a)$. The time complexity of creating the paths for the minimal common keys is $O(N_k N_a \log N_k)$ since we assume that the time complexity of finding a child of a node with a desired label is $O(\log N_k)$. In total, the time complexity of this sub-step is $O(N_k N_a \log N_k + N_k N_a \log N_a)$.

   (b) *Computing* $\mathcal{T}(R_n, \cdot)$. The time complexity of partitioning the records in each node is $O(N_t \log N_t)$ since there are $O(N_t)$ records that must be partitioned. In total, the time complexity of this sub-step is $O(N_k N_a N_t \log N_t)$ since there are $O(N_k N_a)$ nodes in the partitioning plan.

   Thus, the time complexity of this step is $O(N_k N_a \log N_k + N_k N_a \log N_a + N_k N_a N_t \log N_t)$.

2. *Computing* $\mathcal{MC}(S_n)$. The time complexity of each of the sub-steps is:

   (a) *Computing* $\mathcal{MP}(T, C)$. The time complexity of computing $\mathcal{MK}(T^p)$ by using a key discovery algorithm is $O(\mathbb{F}(T^p))$. The time complexity of computing $\mathcal{MP}(T, C)$ is $O(N_k N_a)$. In total, the time complexity of this sub-step is $O(\mathbb{F}(T^p) + N_k N_a)$. Thus, the time complexity of computing $\mathcal{MP}(T, C)$ for all $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$ is $O(N_u N_p \mathbb{F}(T^p) + N_u N_k N_a N_p)$.

   (b) *Computing* $\mathcal{MP}(R_n, C)$. The time complexity of computing each pairwise union $O(N_k^2 N_a)$. The time complexity of computing each minimization is $O(N_k^3 N_a)$ since we assume that the result of minimization contains $O(N_k)$ elements. In total, the time complexity of this sub-step is $O(N_k^3 N_a N_p)$ since there are $O(N_p)$ pairwise union and minimization computations. Thus, the time complexity of computing $\mathcal{MP}(R_n, C)$ for all $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations is $O(N_u N_k^3 N_a N_p)$.

(c) *Computing* $\mathcal{MC}(S_n)$. The time complexity of computing the union is $O(N_k^2 N_a)$. The time complexity of computing the minimization is $O(N_k^3 N_a)$ since the union contains $O(N_k^2)$ elements. In total, the time complexity of this sub-step is $O(N_k^3 N_a)$.

Thus, the time complexity of this step is $O(N_u N_p \mathbb{F}(T^p) + N_u N_k^3 N_a N_p)$.

Thus, the time complexity of CoKE is $O(N_k N_a \log N_a + N_k N_a N_t \log N_t + N_u N_p \mathbb{F}(T^p) + N_u N_k^3 N_a N_p)$. For $N_t \log N_t \geq \log N_a$ and $N_t \log N_t \geq N_u N_k^2 N_p$, the time complexity is $O(N_k N_a N_t \log N_t + N_u N_p \mathbb{F}(T^p))$.

Time Complexity of CoKE Algorithm. The space complexity of R_n is $O(N_t N_s)$. The space complexity of $\mathcal{MC}(S_o)$ is $O(N_k N_a)$. The space complexity of each of the steps in CoKE is:

1. *Computing* $\mathcal{T}(R_n, \cdot)$. The space complexity of each of the sub-steps is:

   (a) *Constructing the partitioning plan.* The space complexity of the frequencies of the attributes is $O(N_s)$. The space complexity of the labels of the nodes in the partitioning plan is $O(N_k N_a)$ since there are $O(N_k N_a)$ nodes in the partitioning plan. The space complexity of storing the children of the nodes in the partitioning plan is $O(N_k^2 N_a)$ since there are $O(N_k N_a)$ nodes in the partitioning plan and there are $O(N_k)$ children in each node. In total, the space complexity of this sub-step is $O(N_s + N_k^2 N_a)$.

   (b) *Computing* $\mathcal{T}(R_n, \cdot)$. The space complexity of the records during the depth-first traversal of the partitioning plan is $O(N_t N_a)$ since there are $O(N_t)$ records and the maximum depth of the traversal is $O(N_a)$. The space complexity of $\mathcal{T}(R_n, \cdot)$ is $O(N_u N_p N_q)$ since the space complexity of each $\mathcal{T}(R_n, C)$ is $O(N_p N_q)$. In total, the space complexity of this sub-step is $O(N_t N_a + N_u N_p N_q)$.

   Thus, the space complexity of this step is $O(N_s + N_k^2 N_a + N_t N_a + N_u N_p N_q)$.

2. *Computing* $\mathcal{MC}(S_n)$. The space complexity of each of the sub-steps is:

   (a) *Computing* $\mathcal{MP}(T, C)$. The space complexity of computing $\mathcal{MK}(T^p)$ is $O(\mathbb{G}(T^p))$. The space complexity of $\mathcal{MK}(T^p)$ is $O(N_k N_a)$. The space complexity of $\mathcal{MP}(T, C)$ for all $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations and $T \in \mathcal{T}(R_n, C)$ is $O(N_u N_k N_a N_p)$. In total, the space complexity of this sub-step is $O(\mathbb{G}(T^p) + N_u N_k N_a N_p)$.

   (b) *Computing* $\mathcal{MP}(R_n, C)$. The space complexity of computing each pairwise union is $O(N_k^2 N_a)$. The space complexity of computing each minimization is $O(N_k N_a)$ since we assume that the result of minimization

contains $O(N_k)$ elements. The space complexity of $\mathcal{MP}(R_n, C)$ for all $C \in \mathcal{MC}(S_o)$ with non-trivial sub-relations is $O(N_u N_k N_a)$. In total, the space complexity of this sub-step is $O(N_k^2 N_a + N_u N_k N_a)$.

(c) *Computing* $\mathcal{MC}(S_n)$. The space complexity of the union is $O(N_k^2 N_a)$. The space complexity of $\mathcal{MC}(S_n)$ is $O(N_k N_a)$. In total, the space complexity of this sub-step is $O(N_k^2 N_a)$.

Thus, the space complexity of this step is $O(\mathbb{G}(T^p) + N_u N_k N_a N_p + N_k^2 N_a)$.

Thus, the space complexity of CoKE is $O(N_t N_s + N_t N_a + N_u N_p N_q + \mathbb{G}(T^p) + N_u N_k N_a N_p + N_k^2 N_a)$. Since $N_s \geq N_a$ and for $N_t \geq N_u N_p N_q$, $N_t \geq N_u N_k N_p$, and $N_t \geq N_k^2$, the space complexity is $O(N_t N_s + \mathbb{G}(T^p))$.