Supervisory Control: Advanced Theory and Applications

Su Rong

Course Information (1)

- Duration of This Course
  - 22/04/2010 – 17/06/2010
- Course Schedule
  - one lecture per week: Thursday 08:45 – 10:30 (6 lectures)
  - one exercise session (before mid-term exam) on 11/05/2010
- Grading Policy
  - home assigments (10%)
  - one mid-term written exam (1.5 hour, 30%) on 20/04/2010
    - Each student must pass the exam (≥60%) before the grade can be counted in
    - A student can take a second test if he/she fails the first one
  - one final project (60%) : choose your own or pick one from a given list

Course Information (2)

- Lecturers
  - Dr. R. Su
    - office: WH0.113
    - email: r.su@tue.nl
  - Dr.ir. J.M. van de Mortel-Fronczak
    - office: WH0.121
    - email: J.M.v.d.Montel@tue.nl
- Prerequisite
  - 2IT15 - Automaten en procestheorie (aanbevolen)
  - 4K420 - Supervisory machine control (aanbevolen)
  - 5JJ50 - Rekennetwerken (aanbevolen)

Emphasis of 4K460

- On how to use results of each supervisor synthesis approach.
- Not on why those results are correct.

I won’t give mathematical proofs in my lectures!

Introduction to Supervisory Control Theory

Outline

- Introduction to Supervisory Control
- Ramadge-Wonham Supervisory Control Theory
- Example – A Pusher-Lift System
- Primary Goals of 4K460

The Concept of Discrete Event Systems (DES)

- A DES is a structure with ‘states’ having duration in time, ‘events’ happening *instantaneously* and *asynchronously*.
  - States: e.g. machine is idle, is operating, is broken down, is under repair
  - Events: e.g. machine starts work, breaks down, completes work or repair

- State space discrete in time and space.

- State transitions ‘labeled’ by events.

The Motivation of Developing Supervisory Control Theory (SCT) for DES (till 1980)

- Control problems ***implicit*** in the literature (enforcement of resource constraints, synchronization, ...)

But

- Emphasis on modeling, simulation, verification
- Little formalization of control synthesis
- Absence of control-theoretic ideas
- No standard model or approach to control

Related Areas

- Programming languages for modeling & simulation
- Queues, Markov chains
- Petri nets
- Boolean models
- Formal languages
- Process algebras (CSP, CCS)

“Great” Expectations for SCT

- System model
  - Discrete in time and (usually) space
  - Asynchronous (event-driven)
  - Nondeterministic
    - support transitional choices
- Amenable to formal control synthesis
  - exploit control concepts
- Applicable: manufacturing, traffic, logistic,...

Relationship with Systems Control Concepts

- State space framework well-established:
  - Controllability
  - Observability
  - Optimality (Quadratic, H_∞)

- Use of geometric constructs and partial order
  - Controllability subspaces
    - Supremal subspaces!

Ramadge-Wonham SCT (1982)

- Automaton representation
  - state descriptions for concrete modeling and computation
- Language representation
  - i/o descriptions for implementation-independent concept formulation
- Simple control “technology”

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RW paradigm is based on *languages*, but implemented on *finite-state automata*

Basic Concepts of Languages

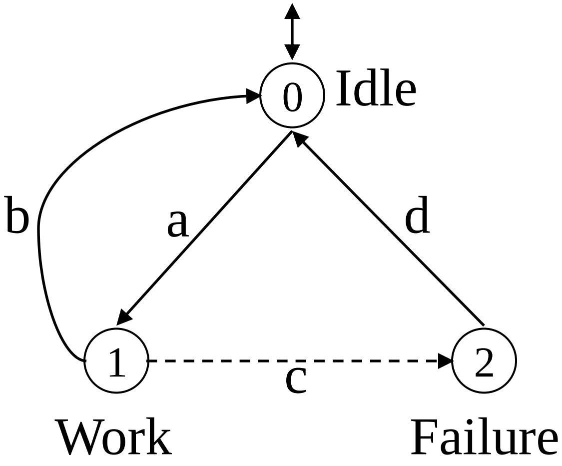
- Given an alphabet Σ (e.g. $\Sigma = \{ a , b , c , d \}$)
  - A string is a finite sequence of events from Σ, e.g. $s = ababa$
  - $\Sigma^+ :=$ { all strings generated from Σ }, $\Sigma^* := \Sigma^+ \cup \{\varepsilon\}$
    - ε is called the *empty* string: $s\varepsilon = \varepsilon s = s$
  - Given $s_1, s_2 \in \Sigma^*$, s_1 is a *prefix* substring of s_2, if $(\exists t \in \Sigma^*)\ s_1 t = s_2$
    - We use $s_1 \leq s_2$ to denote that s_1 is a prefix substring of s_2
  - A language $W \subseteq \Sigma^*$: most time we require W to be *regular*
  - The *prefix closure* of a language W is : $\overline{W} := \{s \in \Sigma^* \mid (\exists s' \in W) s \leq s'\}$
    - W is *prefix closed* if $W = \overline{W}$

Finite-State Automaton (FSA)

- A finite-state automaton is a 5-tuple $G = (X, \Sigma, \xi, x_0, X_m)$, where
  - X : the state set
  - Σ : the alphabet
  - x_0 : the initial state
  - X_m : the marker state set (or the final state set)
  - $\xi : X\times\Sigma \to X$: the transition map
    - ξ is called a *partial* map, if it is not defined at some pair $(x,\sigma)\in X\times\Sigma$.
    - Otherwise, it is called a *total* map.
    - Extension of the transition map: $\xi : X\times\Sigma^* \to X : (x,s\sigma) \mapsto \xi(x,s\sigma) := \xi(\xi(x,s),\sigma)$

The Famous “Small Machine” Model

- $G = (X , \Sigma , \xi , x_0 , X_m)$
  - $X = \{ 0 , 1 , 2 \}$
  - $\Sigma = \{ a , b , c , d \}$
  - $x_0 = 0$
  - $X_m = \{ 0 \}$



a : starts work

b : finishes work

c : machine fails

d : machine is repaired

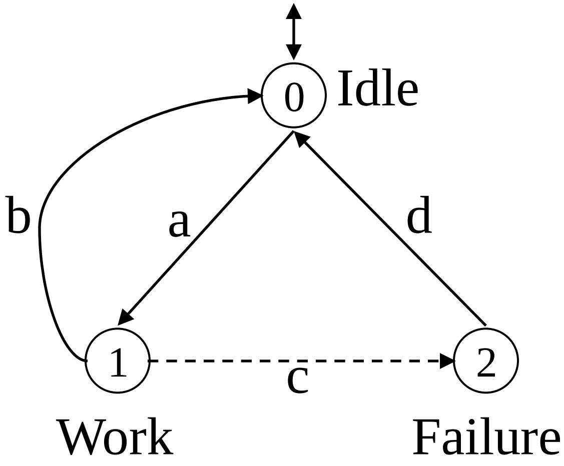
Connection between Language and FSA

- Give a FSA $G = (X , \Sigma , \xi , x_0 , X_m)$,
  - *closed* behavior of G:

    $$L(G) := \{s \in \Sigma^* | \xi(x_0,s) \text{ is defined}\}$$

  - *marked* behavior of G, i.e. the language *recognized* by G,

    $$L_m(G) := \{s \in L(G) \mid \xi(x_0,s) \in X_m\}$$



- G is *nonblocking*, if $\overline{L_m(G)} = L(G)$.
- A language is *regular*, if it is recognizable by a FSA.
  - We can use Arden's rule to derive a language from a FSA.

Natural Projection over Languages

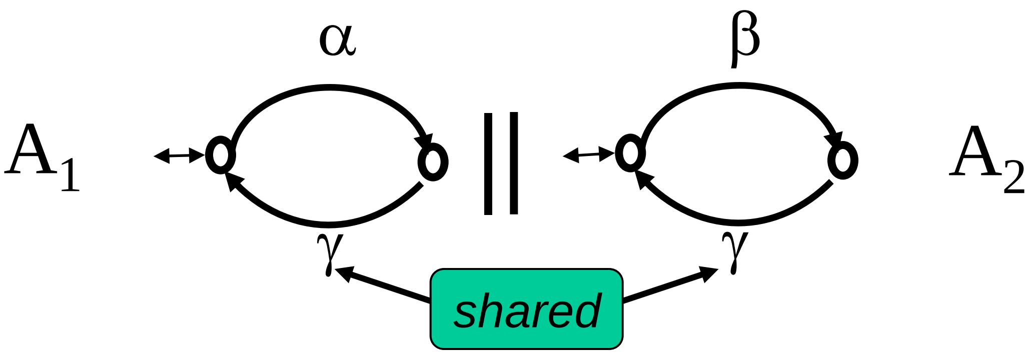
- Given Σ and $\Sigma' \subseteq \Sigma$, $P{:}\Sigma^* \to \Sigma'^*$ is a *natural projection* if
  - $P(\varepsilon) = \varepsilon$
  - $(\forall \sigma \in \Sigma)\ P(\sigma) = \begin{cases} \sigma & \text{if } \sigma \in \Sigma' \\ \varepsilon & \text{if } \sigma \notin \Sigma' \end{cases}$
  - $(\forall s\sigma \in \Sigma^*)\ P(s\sigma) = P(s)P(\sigma)$
- The inverse image map of P is $P^{-1} : \mathrm{pwr}(\Sigma'^*) \to \mathrm{pwr}(\Sigma^*)$ with

$$(\forall A \subseteq \Sigma'^*)\ \ P^{-1}(A) := \{s \in \Sigma^* \mid P(s) \in A\}$$

$$\mathrm{a\,b\,c\,a\,c\,c\,d} \xrightarrow[P]{\Sigma = \{a, b, c, d\}\ \ \Sigma' = \{a, d\}} \mathrm{a} \quad \mathrm{a} \quad \mathrm{d}$$

Synchronous Product over Languages

- Builds a more complex automaton



- with more complex language

$$L_m(A_1) \parallel L_m(A_2) = P_1^{-1}(L_m(A_1)) \cap P_2^{-1}(L_m(A_2))$$

expressed by natural projections

$$P_i: (\Sigma_1 \cup \Sigma_2)^* \rightarrow \Sigma_i^* \quad (i = 1,2)$$

The synchronous product is *commutative* and *associative* !

Implement Synchronous Product by Automaton Operation

- Let $G_1 = (X_1, \Sigma_1, \xi_1, x_{0,1}, X_{m,1})$ and $G_2 = (X_2, \Sigma_2, \xi_2, x_{0,2}, X_{m,2})$,
- Let

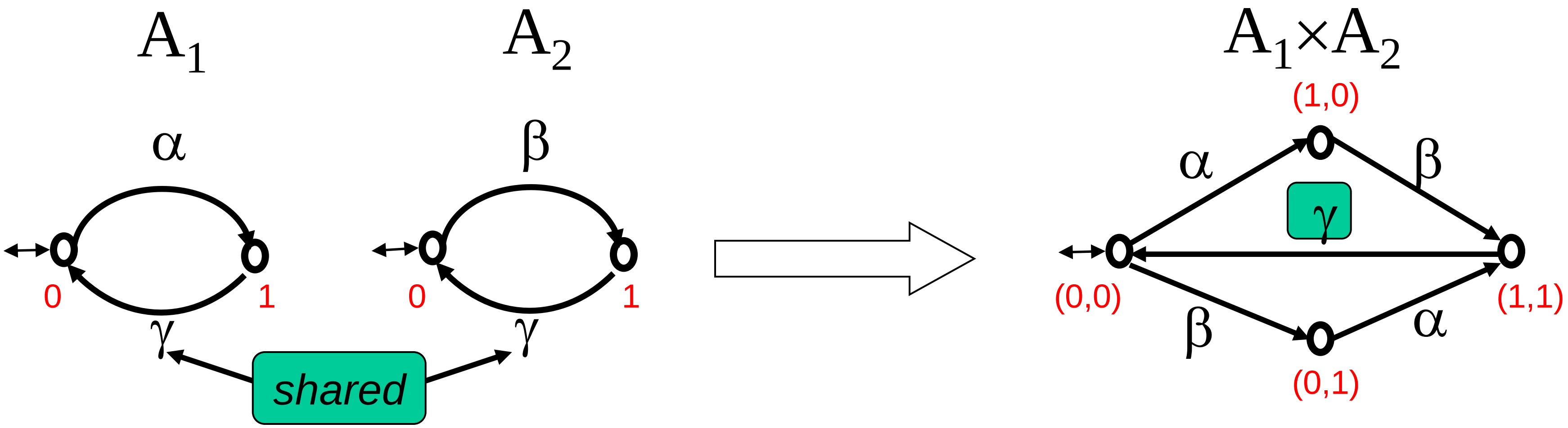
$$G_1 \times G_2 = (X_1 \times X_2, \Sigma_1 \cup \Sigma_2, \xi_1 \times \xi_2, (x_{0,1}, x_{0,2}), X_{m,1} \times X_{m,2})$$

where

$$\xi_1 \times \xi_2((x_1, x_2), \sigma) := \begin{cases} (\xi_1(x_1, \sigma), x_2) & \text{if } \sigma \in \Sigma_1 - \Sigma_2 \\ (x_1, \xi_2(x_2, \sigma)) & \text{if } \sigma \in \Sigma_2 - \Sigma_1 \\ (\xi_1(x_1, \sigma), \xi_2(x_2, \sigma)) & \text{if } \sigma \in \Sigma_1 \cap \Sigma_2 \end{cases}$$

- Result:
  - $L(G_1) \| L(G_2) = L(G_1 \times G_2)$
  - $L_m(G_1) \| L_m(G_2) = L_m(G_1 \times G_2)$

For Example



Automaton product implements synchronous product!

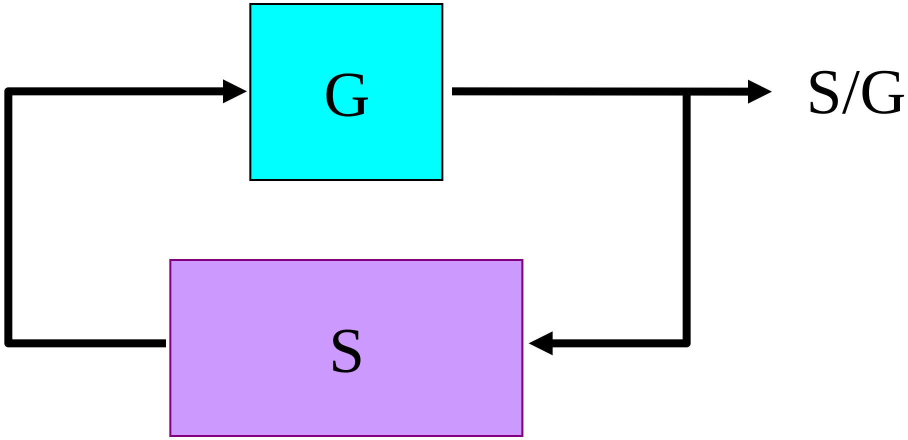
Properties of Projection and Synchronous Product

- [Chain Rule] Given Σ_1, Σ_2 and Σ_3, suppose $\Sigma_3 \subseteq \Sigma_2 \subseteq \Sigma_1$.
  - Let $P_{12}:\Sigma_1^* \rightarrow \Sigma_2^*$, $P_{23}:\Sigma_2^* \rightarrow \Sigma_3^*$ and $P_{13}:\Sigma_1^* \rightarrow \Sigma_3^*$ be natural projections
  - Then $P_{13} = P_{23}P_{12}$

- [Distribution Rule] Given $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$, let $\Sigma' \subseteq \Sigma_1 \cup \Sigma_2$.
  - Let $P:(\Sigma_1 \cup \Sigma_2)^* \rightarrow \Sigma'^*$ be the natural projection. Then
    - $P(L_1 \parallel L_2) \subseteq P(L_1) \parallel P(L_2)$
    - $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma' \Rightarrow P(L_1 \parallel L_2) = P(L_1) \parallel P(L_2)$

We now talk about control …

The Control Architecture



enable/disable events in Σ_c

$\Sigma = \Sigma_c \cup \Sigma_{uc}$

$\Sigma = \Sigma_o \cup \Sigma_{uo}$

Σ_c:=controllable alphabet

Σ_o:=observable alphabet

- Given a plant G and a requirement SPEC, compute a supervisor S
  - $L_m(S/G) := L_m(S) || L_m(G) \subseteq L_m(G) || L_m(SPEC)$
  - S should not disable the occurrence of any uncontrollable event
  - S should make a move only based on observable outputs of G
  - S/G is nonblocking

General Control Issues

Q1 : Is there a control that enforces both **safety**, and **liveness (nonblocking)**, and which is **maximally permissive** ?

Q2 : If so, can its design be **automated** ?

Q3 : If so, with **acceptable computing effort** ?

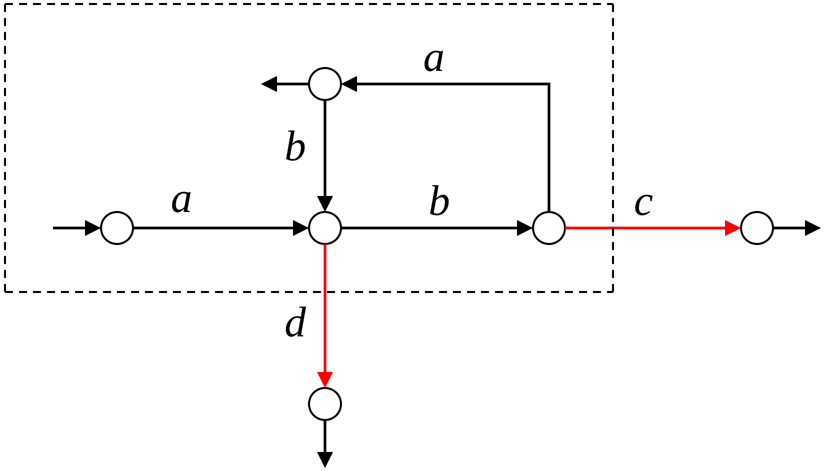
Solution to Question 1

- Fundamental **definition**

A sublanguage $K \subseteq L_m(G)$ is ***controllable*** (w.r.t. G) if

$$\overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}$$

  - "Once in $\overline{K}$, you can't skid out on an uncontrollable event."



$\Sigma = \{a,b,c,d\}$
$\Sigma_c = \{a,c,d\}$
$\Sigma_{uc} = \{b\}$

Supremal Controllable Sublanguage

- Given a plant G and a specification SPEC (both over Σ), let

  $\mathcal{C}(G,SPEC):=\{K \subseteq L_m(G) \cap L_m(SPEC) | K$ is controllable w.r.t. G$\}$
- $\mathcal{C}(G,SPEC)$ is a poset under set inclusion and closed under arbitrary union
  - The largest element is called the *supremal* controllable sublanguage,

Fundamental Result

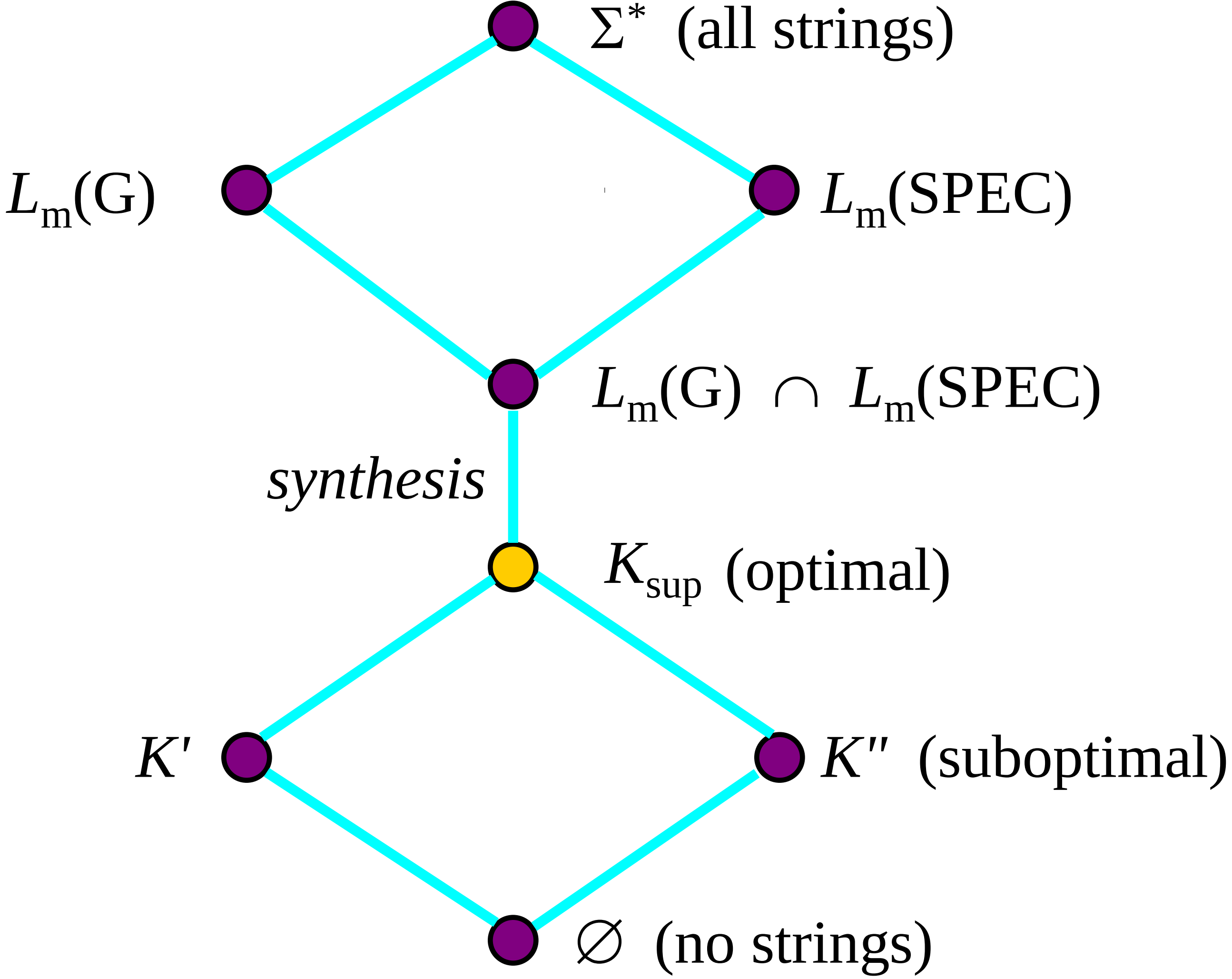
- There exists a (unique) ***supremal*** controllable sublanguage

$$K_{\text{sup}} \subseteq L_{\text{m}}(\text{G}) \cap L_{\text{m}}(\text{SPEC})$$

  - SPEC is an automaton model of a specification

- Furthermore K_{sup} can be effectively computed.

Lattice View of Solution to Question 1



Σ^* (all strings)

$L_m(G)$

$L_m(SPEC)$

$L_m(G) \cap L_m(SPEC)$

synthesis

K_{sup} (optimal)

K'

K'' (suboptimal)

$\varnothing$ (no strings)

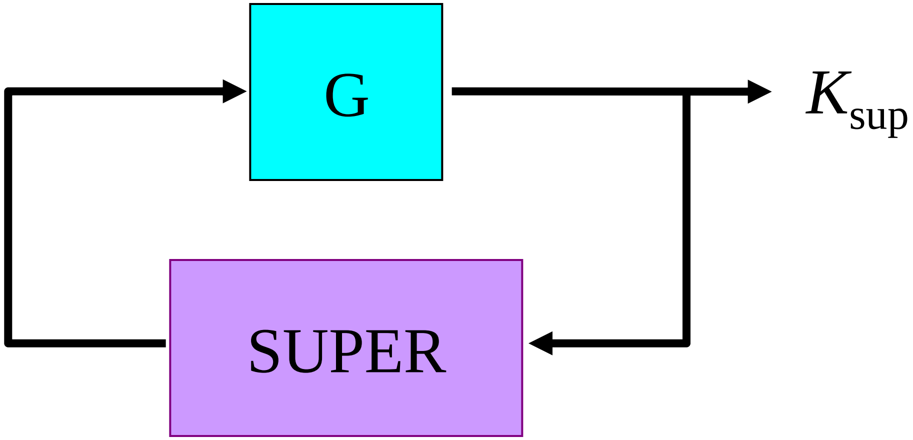
Solution to Question 2

- Given G and SPEC, compute K_{sup}

$$K_{\text{sup}} = L_m(\text{SUPER})$$

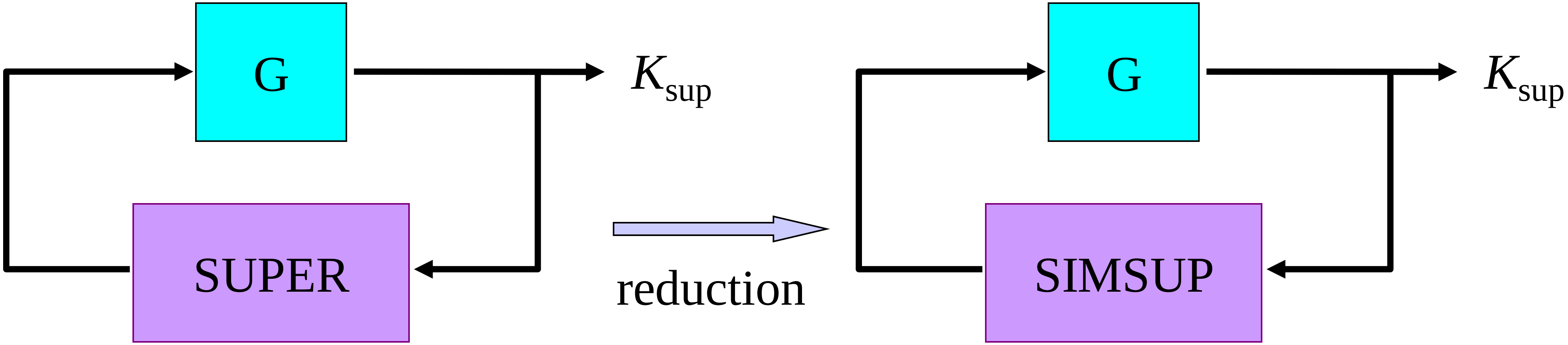
$$\text{SUPER} = \textbf{Supcon}\,(\text{G}\,,\,\text{SPEC})$$

- Given SUPER, implement K_{sup}



enable/disable events in Σ_c

Supervisor Reduction



SUPER and SIMSUP is *control equivalent* if

- $L(G)) \cap L(SUPER) = L(G)) \cap L(SIMSUP)$
- $L_m(G)) \cap L_m(SUPER) = L_m(G)) \cap L_m(SIMSUP)$

Supervisor Reduction

- Controlled behavior has *state size*

$$\|L_m(\mathrm{SUPER})\| \le \|L_m(G)\| \times \|L_m(\mathrm{SPEC})\|$$

- Compute ***reduced, control- equivalent*** SIMSUP, often with

$$\|L_m(\mathrm{SIMSUP})\| \ll \|L_m(\mathrm{SUPER})\|$$

- In TCT:
  - CONSUPER = Condat(G,SUPER)
  - SIMSUP = Supreduce(G,SUPER,CONSUPER)

A solution to Question 3 is *modular*/*distributed*/*hierarchical* control

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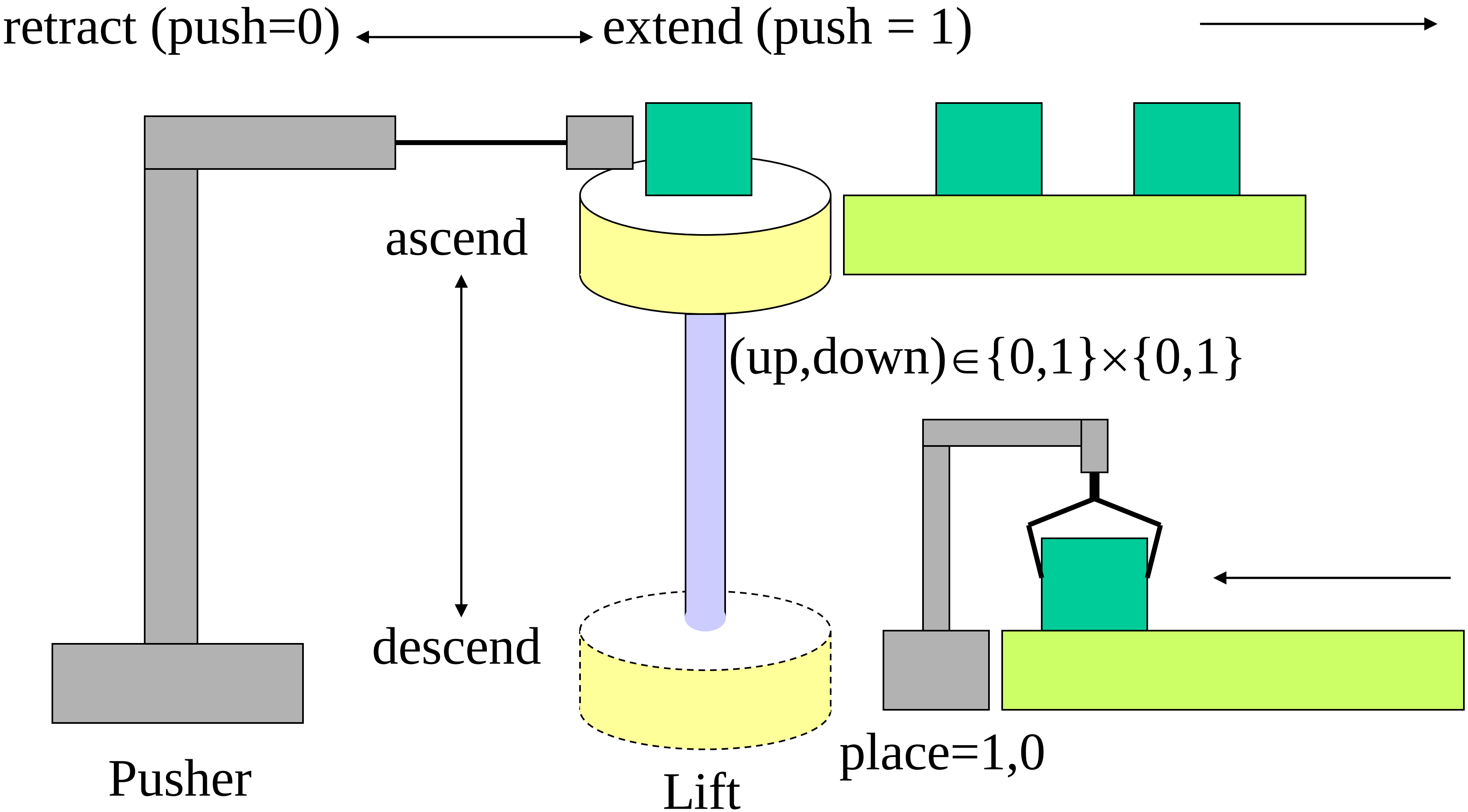
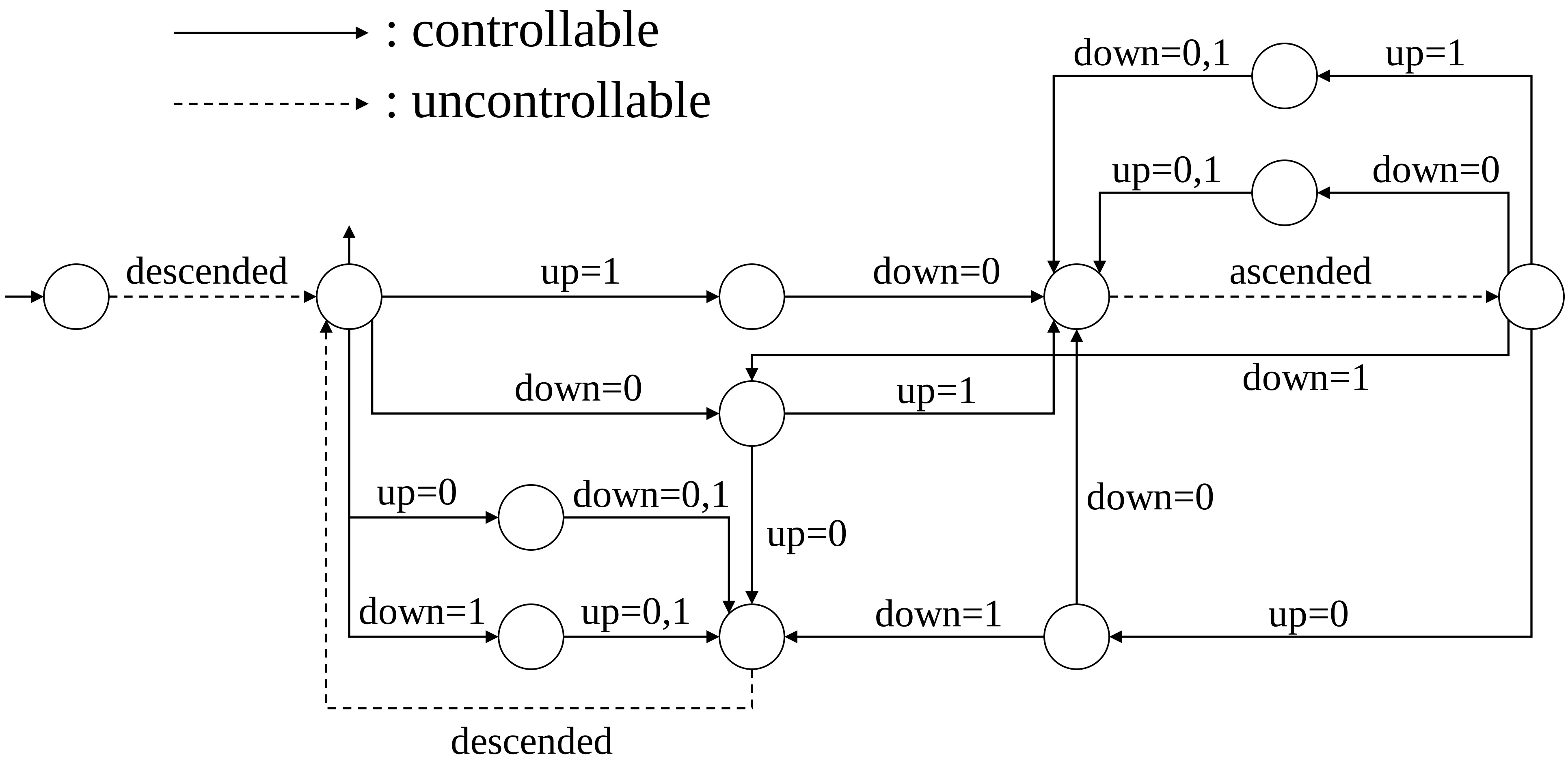
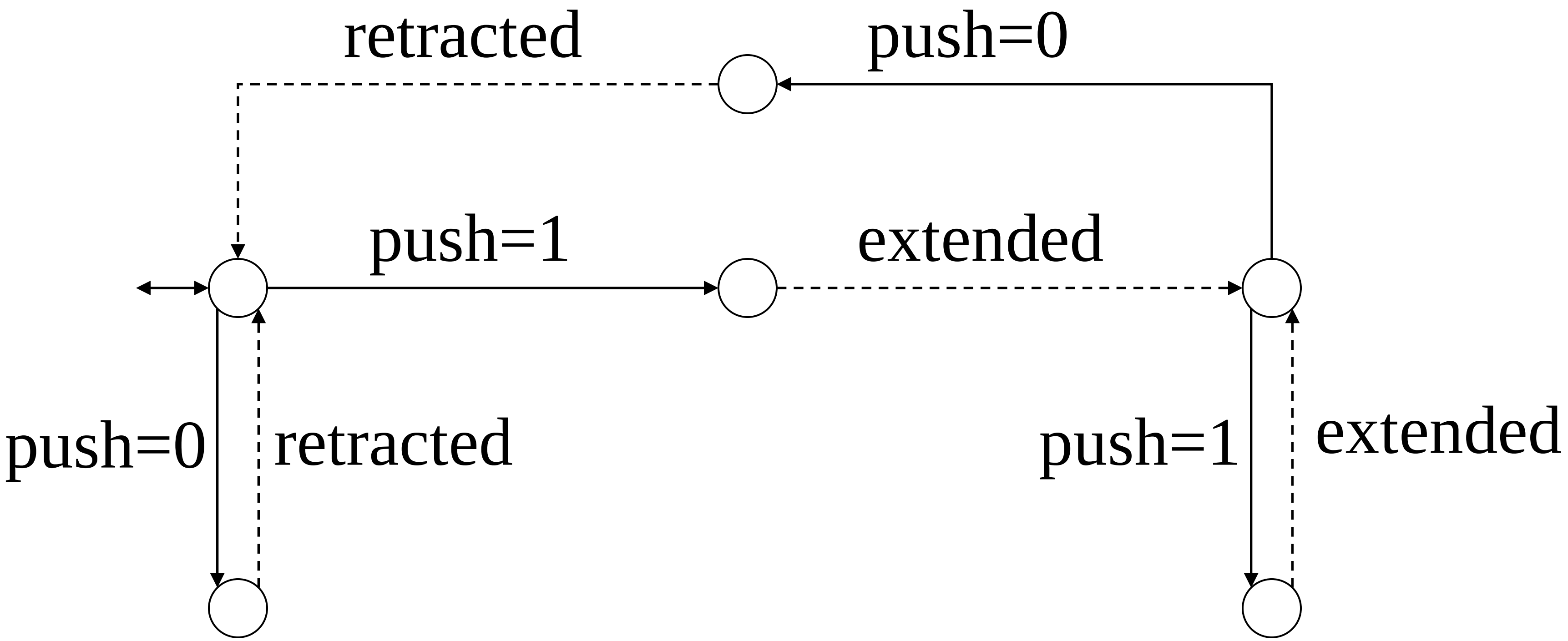
A Pusher-Lift System



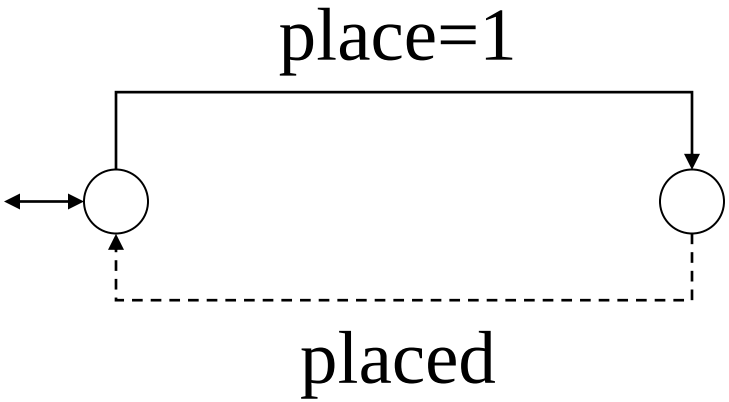
Lift Model G_{lift}



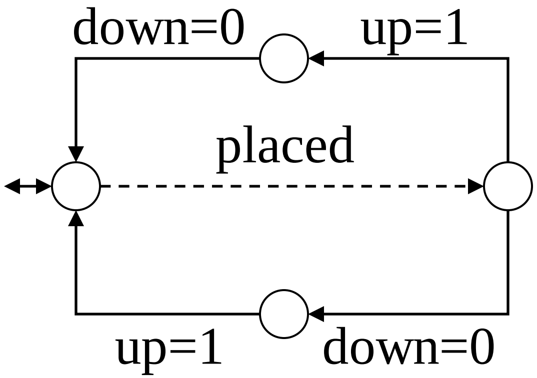
Pusher Model G_{pu}



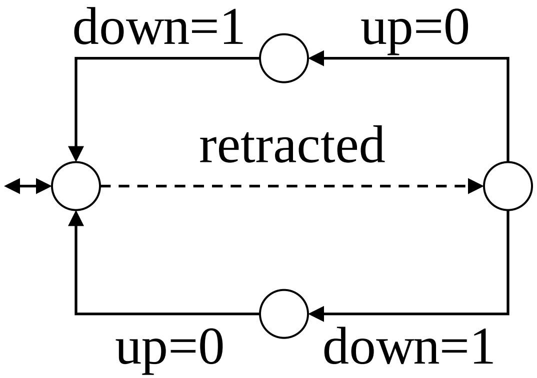
Product Model G_{pro}



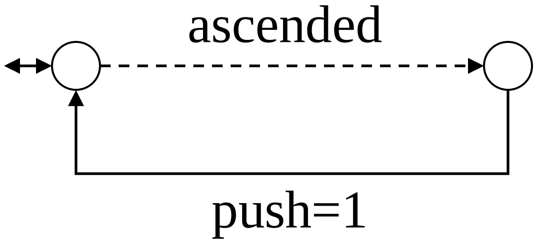
Specifications



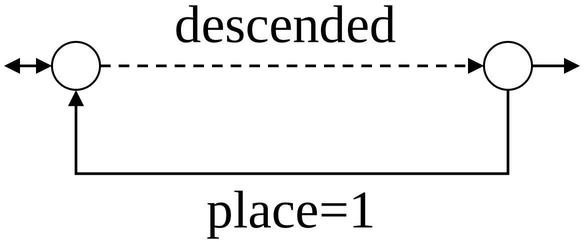
E_1



E_3



E_2



E_4

Monolithic Method – Supervisor Synthesis

- Plant: $G = G_{lift,lo} \times G_{pu} \times G_{pro}$ (use Sync in TCT (240 , 956))
- Specification:
  - $E = E_1 \times E_2 \times E_3 \times E_4$ (64 , 288)
  - $E = \mathrm{Selfloop}(E_1 \times E_2 \times E_3 \times E_4, \Sigma - (\Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Sigma_4))$
- SUPER = Supcon(G , E) (636 , 1369)
- SUPER = Condat(G , SUPER) : controllable
- SIMSUPER = Supreduce(G,SUPER,SUPER) (99 , 476 ; slb=51)

Some Remarks

- Advantages of RW SCT
  - It is conceptually simple
  - Many real systems can be modeled in this framework
- Disadvantages of RW SCT
  - The computational complexity is very high for large systems
  - The implementation issues are not explicitly addressed
    - A procedure of signals→events (supervisory control)→signals is needed.
  - *Performance issues* are not well addressed
    - “Bad” behaviors are forbidden, but no specific “good” behavior is enforced.

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Goals of 4K460

- To introduce several techniques that are aimed to handle the complexity issue involved in supervisor synthesis.
  - Modular control
  - Distributed control
  - Hierarchical control
  - State-feedback control

- To deal with supervisory control under partial observations.

- To address a certain type of performance.

Basic Functions of Supervisor Synthesis Package

Developed by A.T. Hofkamp and R. Su
Systems Engineering Group
Department of Mechanical Engineering

Eindhoven University of Technology

Create Automata

Automaton: B1.cfg

```
[automaton]
states = 0, 1, 2, 3, 4
alphabet = tau, R1-drop-B1, R1-pick-B1, R2-drop-B1, R2-pick-B1
controllable =  R1-drop-B1, R1-pick-B1, R2-drop-B1, R2-pick-B1
observable =  R1-drop-B1, R1-pick-B1, R2-drop-B1, R2-pick-B1
transitions = (0, 1, tau), (1, 2, R1-drop-B1), (2, 1, R2-pick-B1),
              (1, 3, R2-drop-B1), (3, 1, R1-pick-B1), (1, 4, R2-pick-B1),
              (1, 4, R1-pick-B1), (2, 4, R1-drop-B1), (3, 4, R2-drop-B1)
marker-states = 1
initial-state = 0
```

Check Size of Automaton

make_get_size.py

```
[user@host ~] $ make_get_size
Please input model (.cfg): B1.cfg
Number of states: 5
Number of transitions: 9
```

Automaton Product

make_product.py

```
[user@host ~]$ make_product
Please input list of your input automata (comma-seperated list of automata): B1.cfg, B2.cfg
Please input product automaton (.cfg): B1-B2.cfg
Mon Mar 16 10:33:51 2009: Must do 1 product computations.              (memory=9052160 bytes)
Mon Mar 16 10:33:51 2009: Product #1 done: 17 states, 65 transitions   (memory=9052160 bytes)
Mon Mar 16 10:33:51 2009: Computed product                             (memory=9052160 bytes)
                Number of states: 17
                Number of transitions: 65
Mon Mar 16 10:33:51 2009: Product is saved in B1-B2.cfg                (memory=9076736 bytes)
```

Automaton Abstraction

make_abstraction.py

```
[user@host ~]$ make_abstraction
Please input source automaton (.cfg): B1-B2.cfg
Please input list of preserved events (comma-seperated list of event names): tau, R1-drop-B1
Please input name of the abstraction (.cfg): B1-B2-abstraction.cfg
Mon Mar 16 10:40:54 2009: Computed abstraction     (memory=8364032 bytes)
                          Number of states: 5
                          Number of transitions: 14
Mon Mar 16 10:40:54 2009: Abstraction is saved in B1-B2-abstraction.cfg
              (memory=8409088 bytes)
```

Sequential Automaton Abstraction

make_sequential_abstraction.py

```
[user@host ~]$ make_sequnetial_abstraction
Please input list of your input automata (comma-seperated list of automata): B1.cfg, B2.cfg
Please input list of preserved events (comma-seperated list of event names): tau, R1-drop-B1
Please input abstraction (.cfg): B1-B2-sequential-abstraction.cfg
Mon Mar 16 13:01:23 2009: Started	(memory=8249344 bytes)
Mon Mar 16 13:01:23 2009: #states after adding 1 automata: 5		(memory=8257536 bytes)
Mon Mar 16 13:01:23 2009: #states and #transitions after abstraction: 4, 9(memory=8265728 bytes)
Mon Mar 16 13:01:23 2009: #states of 2 automata: 5; #states and #transitions of product: 13 51
		(memory=8278016 bytes)
Mon Mar 16 13:01:23 2009: #states and #transitions after abstraction: 5, 14(memory=8294400 bytes)
Mon Mar 16 13:01:23 2009: Abstraction is saved in B1-B2-sequential-abstraction.cfg
		(memory=8327168 bytes)
```

Natural Projection

make_natural_projection.py

```
[user@host ~]$ make_natural_projection
Please input source automaton (.cfg): B1-B2.cfg
Please input list of preserved events (comma-seperated list of event names): tau, R1-drop-B1
Please input name of the abstraction (.cfg): B1-B2-natural-projection.cfg
Mon Mar 16 10:46:04 2009: Computed projection      (memory=8376320 bytes)
                    Number of states: 3
                    Number of transitions: 3
Mon Mar 16 10:46:04 2009: Projected automaton is saved in B1-B2-natural-projection.cfg
          (memory=8417280 bytes)
```

Check Language Equivalence

Make_language_equivalence_test.py

```
[user@host ~]$ make_language_equivalence_test
Please input first model (.cfg): B1-B2-abstraction.cfg
Please input second model (.cfg): B1-B2-natural-projection.cfg
Language equivalence HOLDS
```

Supervisor Synthesis

make_supervisor.py

```
[user@host ~]$ make_supervisor
Please input plant model (.cfg): plant.cfg
Please input specification model (.cfg): spec.cfg
Please input supervisor (.cfg): supervisor.cfg
Mon Mar 16 12:49:59 2009: Computed supervisor     (memory=14548992 bytes)
                          Number of states: 140
                          Number of transitions: 288
Mon Mar 16 12:49:59 2009: Supervisor saved in supervisor.cfg    (memory=14536704 bytes)
```

Nonconflict Check

make_nonconflicting_check.py

```
[user@host ~]$ make_nonconflicting_check
Please input list of your input automata (comma-seperated list of automata): plant.cfg, supervisor.cfg
Mon Mar 16 12:56:21 2009: Started      (memory=14954496 bytes)
Mon Mar 16 12:56:21 2009: #states after adding 1 automata: 926       (memory=14954496 bytes)
Mon Mar 16 12:56:24 2009: #states and #transitions after abstraction: 926, 3919
            (memory=15073280 bytes)
Mon Mar 16 12:56:24 2009: #states of 2 automata: 139; #states and #transitions of product: 166 380
            (memory=15073280 bytes)
Mon Mar 16 12:56:24 2009: #states and #transitions after abstraction: 3, 6(memory=15036416 bytes)
ok
```

Check Controllability

make_controllability_check.py

```
[user@host ~]$ make_controllability_check
Please input plant model (.cfg): plant.cfg
Please input supervisor model (.cfg): supervisor.cfg
States with disabled controllable events:
   (1, 1): {R2-pick-B2, R3-pick-B2}
   (4, 2): {R2-drop-B2}
   (5, 3): {R3-drop-B2, R2-pick-B2, R3-drop-P33, R3-drop-B3}
   (10, 4): {R3-drop-B3, R2-drop-B2, R3-drop-P33}
   …………
   (799, 121): {R2-pick-B2, R3-pick-B2}

Supervisor is correct (no disabled uncontrollable events)
```

Compute Feasible Supervisor

make_feasible_supervisor.py

```
[user@host ~]$ make_feasible_supervisor
Please input plant model (.cfg): plant.cfg
Please input supervisor model (.cfg): supervisor.cfg
Please input feasible supervisor filename (.cfg): feasible_supervisor.cfg
Mon Mar 16 13:09:43 2009: Computed supervisor      (memory=10522624 bytes)
                          Number of states: 82
                          Number of transitions: 196
Mon Mar 16 13:09:43 2009: Supervisor saved in feasible_supervisor.cfg
     (memory=10547200 bytes)
```

Batch Operation

Batch_Operation.py

```
**************************************************************************************
#!/usr/bin/env python
from automata import frontend

#Compute product
frontend.make_product('B1.cfg, B2.cfg', 'B1-B2.cfg')

#Compute automaton abstraction
frontend.make_abstraction('B1-B2.cfg', 'tau,R1-drop-B1', 'B1-B2-abstraction.cfg')

#Compute supervisor
frontend.make_supervisor('plant.cfg', 'spec.cfg', 'supervisor.cfg')

#Check controllability
frontend.make_controllability_check('plant.cfg', 'supervisor.cfg')
```