Iterative Truncated Arithmetic Mean Filter and Its Properties

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***Abstract*—The arithmetic mean and the order statistical median are two fundamental operations in signal and image processing. They have their own merits and limitations in noise attenuation and image structure preservation. This paper proposes an iterative algorithm that truncates the extreme values of samples in the filter window to a dynamic threshold. The resulting nonlinear filter shows some merits of both the fundamental operations. Some dynamic truncation thresholds are proposed that guarantee the filter output, starting from the mean, to approach the median of the input samples. As a by-product, this paper unveils some statistics of a finite data set as the upper bounds of the deviation of the median from the mean. Some stopping criteria are suggested to facilitate edge preservation and noise attenuation for both the long- and short-tailed distributions. Although the proposed iterative truncated mean (ITM) algorithm is not aimed at the median, it offers a way to estimate the median by simple arithmetic computing. Some properties of the ITM filters are analyzed and experimentally verified on synthetic data and real images.**

***Index Terms*—Edge preservation, image noise attenuation, median approximation, median filter, nonlinear filter.**

I. Introduction

The linear finite-impulse response (FIR) filter is widely used for various purposes in signal and image processing due to its simplicity in realization. The output of all FIR filters is a weighted arithmetic mean of the signal points within the filter window. The arithmetic mean takes a central role in various FIR filters. It is well known that the simple mean filter is optimal for attenuating Gaussian noise, which is the most frequently occurring noise in practice. However, linear filters cannot cope with the nonlinearities of the image formation model and do not take account of the nonlinearities of human vision. The abrupt change in the gray level, such as edges and boundaries, carries important information for both human and machine visual perception. All linear filters tend to blur edges and to destroy fine image details. Filters having good edge preservation properties are highly desirable for image processing. Therefore, nonlinear techniques with edge preservation emerged very early in image filtering and have had a dynamic development in the past three decades.

The median filter [1], originating from the robust estimation theory and well studied in the literature, is a popular nonlinear filter. Its statistical and deterministic properties have been thoroughly studied from a theoretical point of view [2]. Although it is simple in formulation, the median filter yields good edge preservation and impulsive noise suppression characteristics that are highly desirable in image processing. This is evidenced by the amount of research work published [2] and the widespread deployment of the median filter in a variety of applications. Its disadvantages, mainly the inflexibility in the filter structure, the destruction of fine image details, and its relatively poor performance in attenuating additive Gaussian noise and other short-tailed noise, have led to the development of various modifications and extensions of the fundamental median filter.

In order to provide more flexibility in the design of median filters, the weighted median filters were introduced [3]–[5], and a steerable weighted median filter [6] has been recently developed. The median filter was extended to various rank-order-based filters, such as the lower–upper–middle filters [7], [8], the fuzzy rank filters [9], [10], and the rank-conditioned rank-selection filters [11]. To tackle the problem of the destruction of image details, a lot of image detail-preserving filters were proposed, such as multistage median filters [12], [13], FIR–median hybrid filters [14], truncation filters [15], and various noise adaptive switching median filters based on some noise detection mechanisms [16]–[18].

The poor performance of the median filter relative to the mean filter in attenuating Gaussian noise and other short-tailed noise leads to another important bunch of developments. Most of them essentially make compromises between the mean and median filters. Such filters include the L filter, the STM filter [2], the α-trimmed mean (αT) filter, [19], [20], the mean–median (MEM) filter [21], [22], and the median affine (MA) filter [23]. Their characteristics can be tailored to the noise probability distribution. These filters form a family with properties smoothly varying between the two limiting cases, i.e., the mean and median filters. However, their robustness to different kinds of images is controlled by some free parameters. Choosing the optimal value of the parameters to make them well adaptive to the image is not an easy task, although some efforts were made [22]–[24]. The linear combination of the mean and median filters (in MEM filters) may not be an optimal solution to attenuate noise of different degrees of impulsiveness. The αT filter discards samples strictly relying on their rank and ignoring their dispersion. This leads to inefficiency and loss of fidelity [23]. The modified trimmed mean (MTM) filter is very sensitive to the small variation of samples located close to the threshold [23]. On the other hand, the soft-limiting character of the MA filter [23] limits the filter's power in attenuating the strong impulsive noise. The commonality of the aforementioned filters is that they all require two types of operations, namely, arithmetic computing and data sorting. In terms of computation and realization, the data-sorting algorithm is totally different

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from and much more complex than arithmetic computing. For multivariate signals such as color images, finding the vector median is very time consuming, and sorting the vector data is intractable. In contrast, the arithmetic computation can be very easily extended to the vector space.

These observations motivate us to explore a more effective and efficient way for noise attenuation where the optimal solution is neither the mean nor the median. First, if the median is not the optimal solution, why must the filter rely on it using data sorting that is intractable in some applications? A simple second mean after truncating the data samples far away from the first mean has shown much more robustness than the first mean for the fingerprint ridge frequency estimation [25]. Second, the problems of the αT and MTM filters analyzed in [23] can be circumvented by truncating the extreme values to the threshold instead of discarding these samples. Third, in the strong impulsive environment, the hard truncation of all extreme values to a small threshold is more effective than softly weighting them according to their dispersion (in MA filters).

Instead of linearly combining the mean and the median (in MEM filters) or sorting the data followed by averaging samples nearest to the computed median (in αT and MTM filters) or averaging all samples weighted according to their distances from the median (in MA filters), this paper proposes a filter that performs simple truncated arithmetic averaging iteratively. Without sorting the samples, it approaches the median filter. Stopping the iteration early, the proposed filter owns merits of both the mean and median filters. Based on our findings in this paper about the upper bounds of the deviation of the median from the mean, proper dynamic truncation thresholds are proposed that guarantee the filter output, starting from the mean, to approach the median for any data distribution. Some stopping criteria are suggested to facilitate edge preservation and noise attenuation within just a few iterations. Although the proposed iterative truncated mean (ITM) filter is not aimed at the median, it offers a way to estimate the median by simple arithmetic computing.

Although the myriad (LogCauchy) filter does not require data sorting, it is designed for a specific noise distribution, namely, the α-stable distribution, and its performance highly depends on the tunable "linearity parameter" [26]. Similar to the ITM filter, the myriad filter also needs an iterative algorithm, whose computational complexity is, however, much greater than the ITM filter. In this paper, the optimal myriad (OM) filter [26] that uses the optimal linearity parameter determined by the parameters of the noise distribution is experimentally compared with the proposed filter that does not use any prior knowledge of noise.

II. Iterative Truncated Arithmetic Mean Filter

In general, a filter output is the result of an operation on a group of inputs within a filter window. Suppose the filter window contains n inputs residing in a data set $\mathbf{x}_0 = \{x_i\}$, $1 \leq i \leq n$. The mean and median filters, respectively, produce outputs $\mu_0 = \text{mean}(\mathbf{x}_0)$ and

$$\phi = \arg\min_{\varphi} \sum_{i=1}^{n} |x_i - \varphi|. \tag{1}$$

In general, the outputs of mean filter μ_0 and median filter ϕ are different. Some merits and limitations of these two types of filters complement each other. It is therefore desirable in many applications that a filter owns the merits of both the mean and median filters. In terms of computation and realization, the median filter requires some data selection or a sorting algorithm that is totally different from and much more complex than the arithmetic operations used in the mean filter.

Our goal is to build an iterative filter based on simple arithmetic operations, which owns merits of both the mean and median filters. Changing the stopping criteria of the iteration, the filter can produce an output closer to the arithmetic mean or closer to the median. The filter output is the result of the proposed ITM algorithm.

A. Outline of the Proposed ITM Algorithm

Starting from $\mathbf{x} = \mathbf{x}_0$, the proposed ITM algorithm consists of three steps.

Outline of the ITM algorithm:

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1) Compute the arithmetic mean, i.e.,

$$\mu = \text{mean}(\mathbf{x}). \tag{2}$$

2) Compute dynamic threshold τ and truncate input data set $\mathbf{x} = \{x_i\}$ by

$$x_i = \begin{cases} \mu + \tau, & \text{if } x_i > \mu + \tau \\ \mu - \tau, & \text{if } x_i < \mu - \tau \end{cases}, \qquad 1 \leq i \leq n. \tag{3}$$

3) Return to step 1) if stopping criterion $\mathcal{S}$ is violated. Otherwise, terminate the iteration.
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The type-1 ITM filter output is given by

$$y_{t1} = \text{mean}(\mathbf{x}). \tag{4}$$

Letting $\mathbf{x}_r = \{x_i \| x_i - \mu| < \tau\}$ and $|\mathbf{x}_r|$ be the number of elements in set $\mathbf{x}_r$, the type-2 ITM filter output is given by

$$y_{t2} = \begin{cases} \text{mean}(\mathbf{x}_r), & \text{if } |\mathbf{x}_r| > \xi \\ \text{mean}(\mathbf{x}), & \text{otherwise.} \end{cases} \tag{5}$$

Parameter ξ, $\xi > 0$, is used to avoid an unreliable mean when too few input points remain in $\mathbf{x}_r$. In general, ξ is a portion of n, for example, $\xi = n/4$

Unlike the αT and MTM filters that discard the samples of extreme values, the ITM algorithm truncates them to threshold τ. This circumvents the problems of the αT and MTM filters analyzed in [23] and ensures the median of the truncated data set unchanged if a proper dynamic truncation threshold τ is applied. After the iteration, we may choose output y_{t1}, which is called the ITM1 filter (4), or output y_{t2}, which is called the ITM2 filter (5). Although the median as the output may not necessarily outperform the mean in all cases, it is desirable that filter outputs y_{t1} and y_{t2} approach the median of the original inputs given a sufficient large number of iterations. By choosing proper stopping criteria based on the requirement of the application, the filter owns merits of both the mean and median filters. Thus, the goal of the proposed ITM algorithm is to reduce the dynamic range of the inputs iteratively so that the arithmetic mean of the truncated inputs approaches the median.

The most critic technique to achieve this goal is to find a proper dynamic truncation threshold τ. One necessary condition of such a threshold is that it should be large enough to keep the median always within the dynamic range of truncated inputs $\mathbf{x}$. This gives us the lower bound of the truncation threshold to keep the median of $\mathbf{x}$ unchanging in the truncation process. Another necessary condition is that it should be small enough to ensure that the ITM algorithm never idles if $\mu \neq \phi$ for all possible distributions of the data set. This gives us the upper bound of the truncation threshold.

B. Finding the Dynamic Truncation Threshold

As a necessary preliminary of the study, two subsets are defined as

$$\mathbf{x}_h \triangleq \{x_i | x_i \in \mathbf{x},\, x_i > \mu\} \tag{6}$$

$$\mathbf{x}_l \triangleq \{x_i | x_i \in \mathbf{x},\, x_i \leq \mu\}. \tag{7}$$

Let n_h, n_l, μ_h, and μ_l denote, respectively, the numbers and the means of the elements in these two subsets. Obviously, $\mathbf{x}_h \cup \mathbf{x}_l = \mathbf{x}, n_h + n_l = n$ and

$$\begin{aligned} n_h\mu_h + n_l\mu_l &= n\mu \\ &= n_h\mu + n_l\mu. \end{aligned} \tag{8}$$

If we define $\delta_h \triangleq \mu_h - \mu$ and $\delta_l \triangleq \mu - \mu_l$, (8) becomes

$$n_h\delta_h = n_l\delta_l. \tag{9}$$

Now, we are able to explore the possible dynamic truncation thresholds.

One candidate could be $\min[\max(x_i - \mu), \max(\mu - x_i)]$ as it keeps all samples on one of the two sides of mean μ unchanged. Although this threshold is definitely larger than the lower bound, it is also larger than the upper bound because the ITM algorithm idles if $\max(x_i - \mu) = \max(\mu - x_i)$. If we choose $\max(\delta_h, \delta_l)$, the ITM algorithm still may idle, for example, in the case that all x_i on the corresponding side of μ have a constant value. As $\max(\delta_h, \delta_l)$ is still too large, one may think of $\min(\delta_h, \delta_l)$ as the truncation threshold, which ensures the ITM algorithm no idling if $\mu \neq \phi$. Unfortunately, $\min(\delta_h, \delta_l)$ is smaller than the lower bound of the truncation threshold. It is not very difficult to give a data set example that the distance between the median and the mean is larger than $\min(\delta_h, \delta_l)$.

This paper proposes three possible dynamic truncation thresholds that satisfy the two necessary conditions. They are the average of δ_h and δ_l, i.e.,

$$\tau_1 = \frac{1}{2}(\delta_h + \delta_l) \tag{10}$$

the sample standard deviation σ, i.e.,

$$\tau_2 = \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} \tag{11}$$

and the mean absolute deviation of the samples from the mean, i.e.,

$$\tau_3 = \frac{1}{n}\sum_{i=1}^{n}\left(|x_i - \mu|\right). \tag{12}$$

We will prove the validity of the aforementioned three simple statistics as the dynamic truncation threshold based on our findings of the following theorems and propositions, some of which are additional by-products of this paper as they could be useful elsewhere.

Theorem 1: The distance between the median and the mean of any finite data set is never greater than the mean absolute deviation of the data from the mean, i.e., letting $\tau = \tau_3$, we have

$$|\phi - \mu| \leq \tau. \tag{13}$$

Proof: The absolute deviation of a data set $\mathbf{x}$ can be expressed as[1]

$$\begin{aligned} \tau &= \frac{1}{n}\left[\sum_{x_i>\mu}(x_i - \mu) + \sum_{x_i\leq\mu}(\mu - x_i)\right] \\ &= \frac{1}{n}\left[n_h(\mu_h - \mu) + n_l(\mu - \mu_l)\right] \\ &= \frac{1}{n}(n_h\delta_h + n_l\delta_l). \end{aligned} \tag{14}$$

Substituting (9) into (14) yields

$$\begin{aligned} \tau &= 2\frac{n_h}{n}\delta_h \\ &= 2\frac{n_l}{n}\delta_l. \end{aligned} \tag{15}$$

Let $n_{\tau h}$ denote the number of x_i larger than $\mu + \tau$ and $n_{\tau l}$ denote the number of x_i smaller than $\mu - \tau$. Obviously, for $n_{\tau h} \neq 0$, we have

$$n_{\tau h}\tau < \sum_{x_i>\mu+\tau}(x_i - \mu) \leq \sum_{x_i>\mu}(x_i - \mu). \tag{16}$$

Since

$$\sum_{x_i>\mu}(x_i - \mu) = n_h\delta_h \tag{17}$$

(16) becomes

$$n_{\tau h} < \frac{n_h\delta_h}{\tau}. \tag{18}$$

Substituting (15) into (18) yields

$$n_{\tau h} < \frac{n}{2}. \tag{19}$$

(This is obviously true for $n_{\tau h} = 0$.)
According to the definition of the median, (19) means

$$\phi \leq \mu + \tau. \tag{20}$$

Using $n_{\tau l}$ and a way very similar to (16)–(19), we have

$$\phi \geq \mu - \tau. \tag{21}$$

Combining (20) and (21) completes our proof of inequality (13) and, hence, Theorem 1.

[1] $\tau \neq 0$ is assumed in the proof. Obviously, $\phi = \mu$ if $\tau = 0$.

Theorem 1 guarantees the median of any finite data set $\mathbf{x}$ unchanged in the truncation process of the ITM algorithm if τ_3 is chosen as the dynamic truncation threshold. The following Theorem 2 ensures that the truncation process always reduces the dynamic range of $\mathbf{x}$ so that the ITM algorithm with threshold τ_3 never idles for any data distribution if the mean of $\mathbf{x}$ deviates from its median.

Theorem 2: For any finite data set, there exists at least one sample whose distance from the sample mean is greater than the mean absolute deviation of the samples from the mean if the sample median deviates from the sample mean, i.e., letting $\tau = \tau_3$

$$\exists x_i,\ x_i \in \mathbf{x}, \quad \text{that } |x_i - \mu| > \tau, \qquad \text{if } \mu \neq \phi. \tag{22}$$

Proof: According to the definition of the median, $n_h \neq n_l$ if $\mu \neq \phi$. Thus, from (15), we have $\delta_h \neq \delta_l$ and

$$\tau < \max(\delta_h, \delta_l). \tag{23}$$

From the definition of δ_h, δ_l, and (14), it is obvious that

$$\exists x_i,\ x_i \in \mathbf{x}, \quad \text{that } |x_i - \mu| \geq \max(\delta_h, \delta_l). \tag{24}$$

Therefore, we have

$$\exists x_i,\ x_i \in \mathbf{x}, \quad \text{that } |x_i - \mu| > \tau. \tag{25}$$

This completes our proof of inequality (22) and, hence, Theorem 2.

Now, we can explore whether τ_1 (10) can be also the truncation threshold. From (14) and (15), it is not difficult to see

$$\frac{1}{2}(\delta_h + \delta_l) \geq \tau_3. \tag{26}$$

It means that inequality (13) of Theorem 1 holds true for $\tau = \tau_1$. Obviously, inequality (23) holds true for $\tau = \tau_1$ and so does inequality (22) of Theorem 2. Therefore, we can also choose τ_1 (10) as the truncation threshold as it meets the two necessary conditions.

As for τ_2 (11), it is not difficult to see from the definition of the standard deviation that inequality (22) of Theorem 2 holds true for $\tau = \tau_2$. In the probability theory, Chebyshev's inequality [27] can derive a theorem: For continuous probability distributions having an expected value and a median, the difference between the expected value and the median is never greater than one standard deviation. This theorem about the ensemble values of a probability distribution, however, does not mean that the same is true about the sample mean, sample median, and sample standard deviation of a finite data set. To find the theorem on the finite data set analogous to that for continuous probability distributions, we develop the following Proposition 1 about the relation between the standard deviation and the mean absolute deviation.

Proposition 1: For any finite data set, the sample standard deviation is larger than or equal to the sample mean absolute deviation of the data from their mean, i.e.,

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} \geq \frac{1}{n}\sum_{i=1}^{n}\left(|x_i - \mu|\right). \tag{27}$$

Proof: One of Jensen's inequalities [28] shows

$$\varphi\left(\frac{\sum y_i}{n}\right) \geq \frac{\sum \varphi(y_i)}{n} \tag{28}$$

where $\varphi(\cdot)$ is any concave function.

The square root $\sqrt{\cdot}$ is a concave function. Letting $y_i = (x_i - \mu)^2$, we have

$$\sqrt{\frac{\sum(x_i - \mu)^2}{n}} \geq \frac{\sum |x_i - \mu|}{n}. \tag{29}$$

This completes our proof of Proposition 1.

A corollary of Theorem 1 and Proposition 1 is Theorem 3.

Theorem 3: The distance between the sample median and the sample mean of any finite data set is never greater than one sample standard deviation, i.e.,

$$|\phi - \mu| \leq \sigma. \tag{30}$$

Therefore, τ_2 (11) is also a valid truncation threshold as it meets the two necessary conditions.

We see that the mean absolute deviation from the mean is the tightest upper bound of the distance between the median and the mean among the three possible quantities proposed in this paper, namely, $\tau_3 \leq \tau_1$ and $\tau_3 \leq \tau_2$. We have not found any other arithmetic result on a finite data set smaller than it and satisfying (13) of Theorem 1. A smaller truncation threshold, in general, leads to faster convergence of the ITM algorithm. Therefore, $\tau = \tau_3$ is applied in this paper as the dynamic truncation threshold of our ITM algorithm, although a larger one may lead to a slightly better filtering performance in some cases.

The following Proposition 2 shows the decreasing truncation threshold $\tau = \tau_3$ during the ITM iteration and its upper bound.

Proposition 2: The ITM algorithm decreases truncation threshold $\tau(k)$ (12) monotonically to zero if the mean deviates from the median. The upper bound of $\tau(k)$ against the number of iterations k is shown by the inequality

$$\tau(k) < \tau(1)\left(1 - \frac{1}{n^2}\right)^{k-1}, \qquad \text{if } \mu \neq \phi. \tag{31}$$

Proof: For symbolic simplicity, index k is omitted wherever no ambiguity is caused. From (15), we have

$$\begin{aligned}\delta_h + \delta_l &= \frac{1}{2}n\tau(k)\left(\frac{1}{n_h} + \frac{1}{n_l}\right)\\ &= \frac{n^2}{2n_h n_l}\tau(k).\end{aligned} \tag{32}$$

Due to the data truncation in the previous iteration, we have $\delta_h(k) + \delta_l(k) > 2\tau(k-1)$, if $\mu \neq \phi$. Therefore

$$\begin{aligned}\tau(k) &= \frac{2n_h n_l}{n^2}(\delta_h + \delta_l) < \frac{4n_h n_l}{n^2}\tau(k-1)\\ &= \frac{(n_h + n_l)^2 - (n_h - n_l)^2}{(n_h + n_l)^2}\tau(k-1)\\ &= \left[1 - \left(\frac{\Delta n}{n}\right)^2\right]\tau(k-1).\end{aligned} \tag{33}$$

Since $1 \leq (\Delta n)^2 \leq n^2$ if $\mu \neq \phi$, we have inequality (31), from which it is straightforward that

$$\lim_{k \to \infty} \tau(k) = 0, \qquad \text{if } \mu \neq \phi.$$

This completes our proof of Proposition 2.

C. Stopping Criteria

One possible stopping criterion $\mathcal{S}_1$ to ensure output y_{t1} close to the median is to meet the condition

$$\mathcal{S}_1(\varepsilon_1): \ \Delta n \triangleq |n_h - n_l| \le \varepsilon_1. \tag{34}$$

It is easy to see that, if criterion $\mathcal{S}_1$ with $\varepsilon_1 = 1$ is met, the truncated mean is the median for an even number of samples. For an odd number of samples, the nearest sample on one of the two sides of the truncated mean is the median.

However, in some specific cases, stopping criterion $\mathcal{S}_1$ needs a large number of iterations to be met or even can never be met with a finite number of iterations. One example is the image step edge covered by the filter window where all inputs equal to either one or the other of the two constant values. On the other hand, in this case, ITM2 filter output y_{t2} is the median of inputs just after one iteration. Thus, we can take $\mathcal{S}_1$ as a sufficient but not necessary stopping condition.

A simple way to limit the number of iterations k is to set a predefined maximum number ε_2. The stopping criterion is then

$$\mathcal{S}_2(\varepsilon_2): \ k \ge \varepsilon_2. \tag{35}$$

In general, ε_2 depends on the number of input points n in the filter window. However, it may not be a linear function of n. In this paper, $\varepsilon_2 = 2\sqrt{n}$ is chosen from experience.

A more efficient stopping criterion to handle the horizontal or vertical step edge could be

$$\mathcal{S}_3(\varepsilon_3): \ \Delta n_\tau \triangleq |n_{\tau h} - n_{\tau l}| \ge \varepsilon_3. \tag{36}$$

However, the number of pixels on the opposite edge side of the filter center varies from $\sqrt{n}$ to $(n - \sqrt{n})/2$. Obviously, the setting of $\varepsilon_3 = (n - \sqrt{n})/2$ only solves the problem when the filter center is on the edge. On the other hand, the setting of $\varepsilon_3 = \sqrt{n}$ may lead to an immature stop. Therefore, an auxiliary constrain is necessary, such as

$$\mathcal{S}_4: \ \Delta n_\tau(k) = \Delta n_\tau(k-1). \tag{37}$$

A sophisticated stopping criterion $\mathcal{S}$ could be some combination of the above, such as

$$\mathcal{S} = \mathcal{S}_1(\varepsilon_1) \vee \mathcal{S}_2(\varepsilon_2) \vee \mathcal{S}_3(\varepsilon_3) \vee [\mathcal{S}_3(\varepsilon_4) \wedge \mathcal{S}_4] \tag{38}$$

where $\varepsilon_3 \ge \varepsilon_4$, for example, $\varepsilon_3 = (n - \sqrt{n})/2$ and $\varepsilon_4 = \sqrt{n}$.

It must be mentioned that the aforementioned possible stopping criteria are for general cases. It is very difficult, if not impossible, to find a stopping criterion optimal for all types of images and noise. There must be more efficient or effective stopping criteria for some specific applications or data sets.

III. Properties of the ITM Filters

The proposed ITM filters start from the arithmetic mean and move toward the median of inputs $\mathbf{x}_0$. Some of their properties listed here are apparent, and the others are corollaries of the theorems and propositions presented in the last section.

Property 1: ITM1 and ITM2 filter outputs y_{t1} and y_{t2} both converge to the median of the samples in the filter window, i.e.,

$$\lim_{k\to\infty} y_{t1} = \lim_{k\to\infty} y_{t2} = \phi \tag{39}$$

where k is the number of iterations.

Proof: From Theorem 1, we have $|\mu - \phi| \le \tau$. Therefore, $\lim_{k\to\infty} y_{t1} = \lim_{k\to\infty} y_{t2} = \phi$ if $\lim_{k\to\infty} \tau = 0$ because $\mu - \tau \le y_{t1}, y_{t2} \le \mu + \tau$. From Proposition 2, we have $\lim_{k\to\infty} \tau = 0$ if $\mu \ne \phi$.

For $\mu = \phi$, $y_{t1} = \mu = \phi$. In this case, $\lim_{k\to\infty} \tau \ne 0$ only if $\tau(k+1) = \tau(k)$ occurs. This happens only if there is no sample outside the truncation boundary. From the definition of τ (12) we see that, in this case, $|x_i - \mu| = \tau \ \forall x_i, x_i \in \mathbf{x}$, and hence, $y_{t2} = y_{t1} = \phi$ from the definition of y_{t2} (5). This completes the proof of (39) and Property 1.

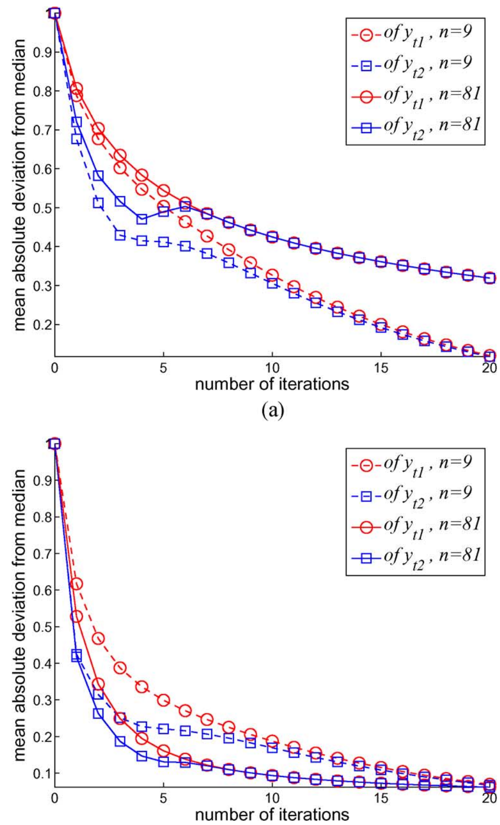

Fig. 1. Average absolute deviations over 100 000 filter outputs from the median against the number of iterations. They are normalized by those of the mean filters. The filter inputs are random numbers drawn from the (a) Gaussian and (b) exponential distributions.

Fig. 1 shows the average absolute deviations over 100 000 filter outputs y_{t1} and y_{t2} from median ϕ against the number of iterations. They are normalized by the average absolute deviations of mean μ_0 from the median. The inputs of the filters are random numbers drawn from a Gaussian distribution and an exponential distribution. A small filter size of $n = 3 \times 3 = 9$ and a large size of $n = 9 \times 9 = 81$ are tested. Filters of size 81 in the symmetric Gaussian environment converge slower than the other three cases in Fig. 1 because the deviation is normalized by that of the mean. The mean of 81 Gaussian random numbers is much closer to the median than the other three cases in Fig. 1. With the setting of $\xi = n/4$, the points of the curves showing the largest difference between the ITM1 and ITM2 filters indicate the number of iterations where around 75% of the samples have been truncated.

Fig. 1 just shows examples that the ITM filters converge to the median filter. In fact, our objective is not outputting the median. As shown in the experiments later, the proposed ITM1 and ITM2 filters with just a few iterations outperform both the mean and median filters in many applications.

Property 2: The ITM2 filter of size in odd number preserves image step edges with any number of iterations.

Proof: An image step edge is defined as an intrinsic 1-D spatial function, which is constant along one orientation (dimension) and a step function along the dimension orthogonal to the former, called the edge profile. If the filter mask covers only one side of the edge, the ITM2 filter does not change the constant gray value of pixels. If the filter mask covers both sides of an image step edge, it contains more pixels on the edge side where the mask center resides than on the other edge side. Therefore, all pixels on this edge side are within the truncation bound, and those on the other side are out of it from the first iteration. Hence, the ITM2 filter output is the average of the pixels on only one side of the edge where the filter mask center resides. This completes the proof.

However, the ITM2 filter output is different from that of the median filter, although both preserve image step edges. The former is the arithmetic average of pixels on one edge side, whereas the latter is the median of pixels distributed on both edge sides. Therefore, we certainly expect a better noise attenuation capability of the ITM2 filter than the median filter for a noise-contaminated step edge, which is evidenced by the experiments later.

Although the ITM1 filter does not own Property 2, it blurs the step edge lighter than the mean filter with just a few iterations. Fig. 2(a) shows a step edge profile. The filter size of $n = 11 \times 11 = 121$ is applied. After the first iteration, the outputs of the ITM2 filter are the same as those of the median filter. The diamond and circle marked lines are, respectively, the outputs of the mean filter and the ITM1 filter after only five iterations.

Although the ITM filters do not preserve other smooth edges, they produce much lighter blur effect than the mean filter. A smooth edge profile is modeled by a logsig function $f(t) = 1/(1+\exp(-\alpha t))$, whose smoothness is controlled by α. The profile approaches a step function if $\alpha \to \infty$ and approaches a very flat slope function or almost a constant if $\alpha \to 0$. Fig. 2(b) shows that y_{t1} blurs the edge much lighter than the mean filter and y_{t2} is very close to the median filter.

In addition to edge preservation, another apparent property of the ITM filters is that they preserve the homogeneous area of the image. i.e., $y_{t1} = y_{t2} = c$, if $\forall i, x_i = c$.

Property 3: The ITM filters are invariant to scale and bias, i.e., if $\mathbf{z} = \{\alpha x_i + c\}\ \forall x_i, x_i \in \mathbf{x}$, we have

$$y_{t1}(\mathbf{z}) = \alpha y_{t1}(\mathbf{x}) + c \tag{40}$$

$$y_{t2}(\mathbf{z}) = \alpha y_{t2}(\mathbf{x}) + c \tag{41}$$

where α and c are two constants. The proof is trivial and, hence, omitted.

Property 4: The ITM2 filter with any number of iterations removes impulses $\mathbb{D}_1$ from homogeneous area $\mathbb{D}_2$, i.e.,

$$x_i = \begin{cases} c_1, & \text{for } x_i \in \mathbb{D}_1, |\mathbb{D}_1| < n/2 \\ c_2, & \text{for } x_i \in \mathbb{D}_2, |\mathbb{D}_2| > n/2 \end{cases}, \qquad \mathbb{D}_1 \cup \mathbb{D}_2 = \mathbf{x}_0 \tag{42}$$

where $c_1 \neq c_2$, and $|\mathbb{D}_1|$ and $|\mathbb{D}_2|$ are the numbers of elements in sets $\mathbb{D}_1$ and $\mathbb{D}_2$, respectively.

Proof: If $|\mathbb{D}_1| < n/2$, $|\mathbb{D}_2| > n/2$, and $c_1 \neq c_2$, all pixels $x_i = c_1$ are out of the truncation bound, and all pixels $x_i = c_2$ are within it from the first iteration of the filter. Hence, ITM2 filter outputs $y_{t2} = c_2$ from the first iteration of the filter. This completes the proof.

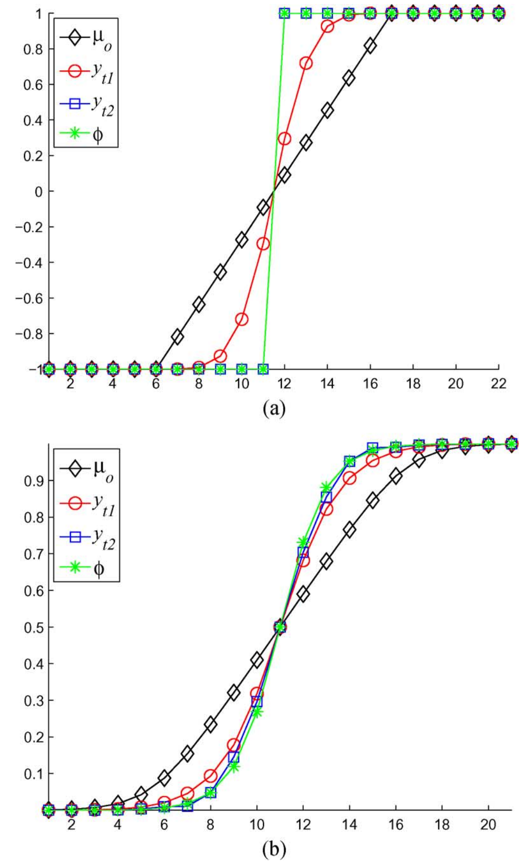

Fig. 2. Profile outputs of the median, mean, ITM1, and ITM2 filters of size $n = 11 \times 11$ after five iterations for the (a) step edge and (b) smooth edge.

IV. Experimental Studies

No parameter of the proposed filter is optimized for a specific data set or noise distribution. The same parameters are applied to the ITM filters throughout all experiments. The stopping criterion of the ITM algorithm is fixed as $\mathcal{S} = \mathcal{S}_1(1) \vee \mathcal{S}_2(2\sqrt{n}) \vee \mathcal{S}_3[(n-\sqrt{n})/2] \vee [\mathcal{S}_3(\sqrt{n}) \wedge \mathcal{S}_4]$ and $\xi = n/4$ is fixed for the ITM2 filter. Better filtering performances than those shown in this paper will be obtained if the aforementioned parameters are adjusted for different data sets. All noise applied in this paper has independent and identically distributed and zero mean. The standard deviation of Gaussian noise is denoted by σ_n. Six sets of experiments are reported here.

The first two sets of experiments test the filters' noise attenuation capability in a homogeneous region, and the next two sets test the filters' overall performance in image structure preservation and noise attenuation. The ITM filters are compared with the mean, median, αT [19], [20], and MEM [21], [22] filters. The mean absolute error (MAE) over 100 000 independent outputs is used as the performance indicator for synthetic data, and the mean-square error (MSE) is used for real images. Although some αT filters with adaptive α-values were proposed [24], [29], none of them outperforms an α-fixed αT filter averagely over the experiments. Thus, $\alpha = 0.25$ is chosen as the

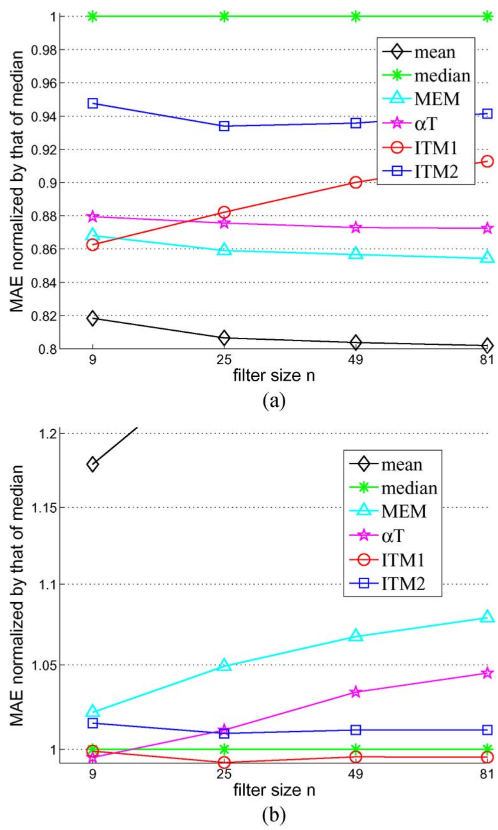

Fig. 3. Normalized MAE against filter size n for (a) Gaussian and (b) Laplacian noise. The average numbers of ITM iterations are 1.5, 3.1, 5.1, and 7.3 in (a) and 1.7, 3.5, 5.5, and 7.6 in (b), respectively, for the filter size of 9–81.

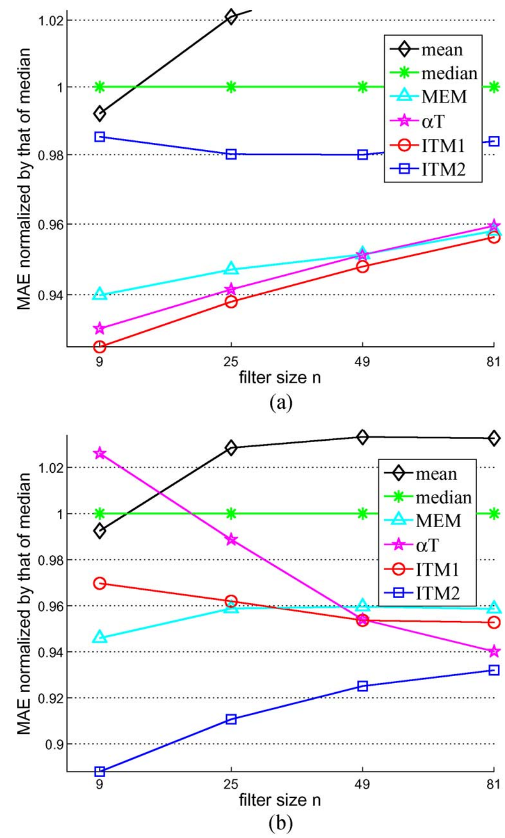

Fig. 4. Normalized MAE against filter size n for (a) Laplacian- and (b) impulsive ϵ-contaminated Gaussian noise. The average numbers of ITM iterations are 1.6, 3.3, 5.2, and 7.2 in (a) and 1.5, 3.3, 5.3, and 7.4 in (b), respectively.

αT filter approaches the median if $\alpha \to 0.5$ and approaches the mean if $\alpha \to 0$.

In the last two sets of experiments, the proposed filters are compared with three iterative-algorithm-based filters, namely, MA [23], OM [26], and mean–LogCauchy (MLC) [21] filters. Noise of the α-stable distribution is applied as the three filters were proposed to tackle the problem of this noise model. The MLC filter is a weighted sum of the mean and LogCauchy filters to tackle the ϵ-contaminated Gaussian noise [30]. It was suggested to use the weight λ equal to the prior probability of the Gaussian noise [21], [22]. $\lambda = 0.5$ is chosen in this paper and so is for the MEM filter.

A. *Single Type of Noise in a Homogeneous Region*

A constant image is contaminated by noise. Fig. 3 plots filters' MAE normalized by that of the median filter, where Gaussian noise is applied in Fig. 3(a) and Laplacian noise is applied in Fig. 3(b).

It is well known that the mean filter is optimal for Gaussian noise. It is hence not a surprise that Fig. 3(a) shows that αT, MEM, ITM1, and ITM2 filters all perform between the mean and median filters. Given the well-known fact that the median filter is optimal for Laplacian noise, Fig. 3(b) surprisingly demonstrates that the proposed ITM1 filter outperforms the median filter for all filter sizes. This probably unveils that the median may not be optimal for a finite number of samples. The culprit is probably the constraint of the median to one of the samples.

B. *Mixed Types of Noise in a Homogeneous Region*

The ϵ-contaminated normal distribution [30] is a weighted sum of two types of distributions as $\mathcal{P}_\epsilon = \{(1-\epsilon)\Phi + \epsilon H\}$, where Φ and H are Gaussian and longer-tailed distributions, respectively, $\epsilon \in [0\ 1]$. We first choose the Laplacian distribution as H with standard deviation $1.3\sigma_n$. Results for $\epsilon = 0.5$ are plotted in Fig. 4(a). Then, an impulsive distribution $H(x) = 0.5\delta(x - 3\sigma_n) + 0.5\delta(x + 3\sigma_n)$ is applied. The results for $\epsilon = 0.25$ are plotted in Fig. 4(b). While the ITM1 filter performs slightly better than the others for Laplacian ϵ-contaminated Gaussian noise, the ITM2 filter performs significantly better than the others in impulsive noise removal.

C. *Noisy Step Edge*

The gray levels of a horizontal or vertical step edge are modeled as the constant 1 on one edge side and -1 on the other side. Such an edge is contaminated by Gaussian noise of different levels. Outputs of a filter are used for computing the MAE if and only if the filter mask covers both sides of the edge. Fig. 5 plots filters' MAEs normalized by the mean absolute deviation of the noise.

The almost constant MAE of the median filter over different noise levels shows its excellent ability in edge preservation. The mean, MEM, αT, and ITM1 filters all blur the edge. Therefore,

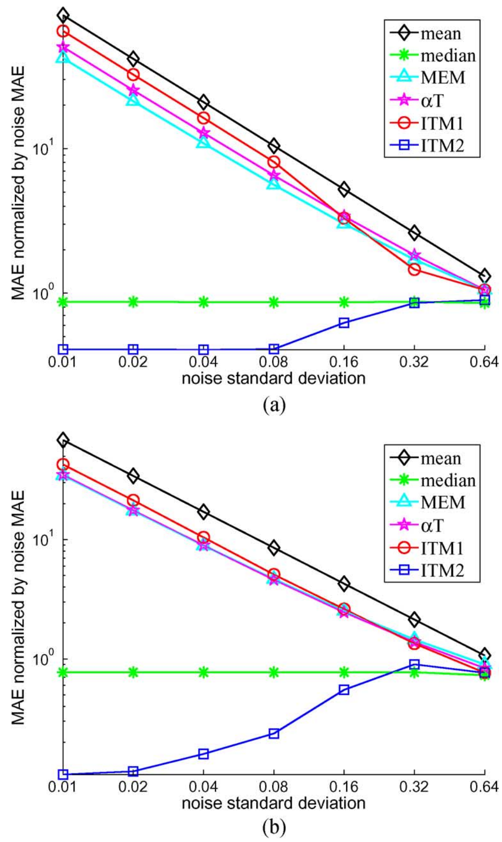

Fig. 5. Normalized MAE of filters of (a) size 3 × 3 and (b) size 11 × 11 against noise level σ_n. The average numbers of ITM iterations are 1.0 for the first four and 2.4, 3.5, and 1.9 for the last three noise levels in (a) and 1.8, 1.8, 2.0, 3.2, 3.7, 6.8, and 11.8 in (b).

their MAEs are much higher than the MAE of the median filter for low noise levels. Since they attenuate the Gaussian noise better than the median filter, they perform about the same as the median filter when σ_n reaches 0.64. The ITM2 filter significantly outperforms the others for the low and medium noise levels. At the two highest noise levels, it performs similar to the median filter. At these two noise levels, the pixel gray levels of the two edge sides are overlapped.

D. Real Images

Three real natural images, as shown in Fig. 6, that represent different image types and complexity levels are used to test the six filters of size 5 × 5. The three real natural images, named Crowd, Bank, and Girl, are of size 512 × 512, and their gray levels range from 0 to 255.

Images are first corrupted by additive Gaussian noise of different levels σ_n. For all three images, the MSE of the median filter increases from the minimum among the six filters to the maximum along with the increase of σ_n. Thus, for each image, five different noise levels are chosen so that the median filter performs best at level $\sigma_n(1)$ and worst at level $\sigma_n(5)$. The other three noise levels are determined by $\sigma_n(5)/\sigma_n(4) = \sigma_n(4)/\sigma_n(3) = \sigma_n(3)/\sigma_n(2) = \sigma_n(2)/\sigma_n(1)$. To plot the results of the three images in one graph for space saving, all MSEs are normalized by that of the median filter for a better visual comparison. The three columns in Fig. 7(a), respectively, for images Crowd, Bank, and Girl, plot the average MSE over ten runs of the six filters at five different noise levels. The results in Fig. 7(a) coincide with the theory that the median filter preserves the image structure best while attenuate the Gaussian noise worst. The proposed ITM1 filter performs best at the noise levels between these two extreme cases.

The aforementioned Gaussian-noised images are further corrupted by the exclusive impulsive noise. The probabilities of a pixel gray level unchanged, replaced by an impulse of value equal to the minimum of the Gaussian-noised image, and equal to the maximum are 0.7, 0.15, and 0.15, respectively. Fig. 7(b) plots the six filters' average MSE over ten runs at the five different Gaussian noise levels for the three real images. It demonstrates that the proposed ITM2 and the median filters perform much better than the others in the presence of the strong impulsive noise.

E. Homogeneous Images Corrupted by α-Stable Noise

One of the attractive features of the α-stable distributions is that we can adjust parameter α $(0 < \alpha \leq 2)$ to control the heaviness of the tails (degree of impulsiveness). The noise impulsiveness increases as α decreases [21]. The myriad (LogCauchy) filters are derived based on the maximum-likelihood location estimation from the samples of the α-stable distribution $(\alpha = 1)$. The "linearity parameter" k of the OM filter [26] is computed as

$$k = \sqrt{\frac{\alpha}{2-\alpha}}\gamma^{1/\alpha} \tag{43}$$

where γ is the dispersion of the α-stable distribution. The iterative algorithm presented in [31] and [32] is applied for OM and MLC filters. Another iterative-algorithm-based filter, i.e., the MA filter [23], is also included in the experiments. The fixed number of iterations, i.e., 20, is applied to both algorithms as more iterations do not lead to a visible performance gain. All filters applied are in the size of 5 × 5.

A constant image is corrupted by α-stable noise with $\gamma = 10$ and different α values. For each α value, 2.5 million inputs are generated to produce 100 000 independent filter outputs. Table I records MSEs of various filters where the results of the mean filter help show the noise impulsiveness. For each α value, the smallest MSE among all filters is underlined and in bold font, and the second smallest is in bold font. Table I shows that the OM filter performs best for an α value around 1 and the MA filter performs best for an α value approaching 2. For a high degree of impulsiveness $(\alpha = 0.5)$, the ITM2 filter performs best, and for a moderate degree of impulsiveness $(\alpha = 1.5)$, the ITM1 filter performs best. Note that the OM and MLC filters utilize the information of noise distribution α and k, whereas the MA, ITM1, and ITM2 filters do not. The average numbers of ITM iterations are 6.7, 5.1, 4.0, 3.5, and 3.2, respectively, for α values from 0.5 to 1.8.

F. Real Image Corrupted by Gaussian and α-Stable Noise

The original image "Lena" of size 512 × 512 is corrupted by ϵ-contaminated $(\epsilon = 0.5)$ Gaussian $(\sigma_n^2 = 100)$ and α-stable $(\gamma = 10)$ noise. For each of the five different α values, filters

Crowd Bank Girl

Fig. 6. Three real natural images of size 512 × 512 (Crowd, Bank, and Girl) applied in testing.

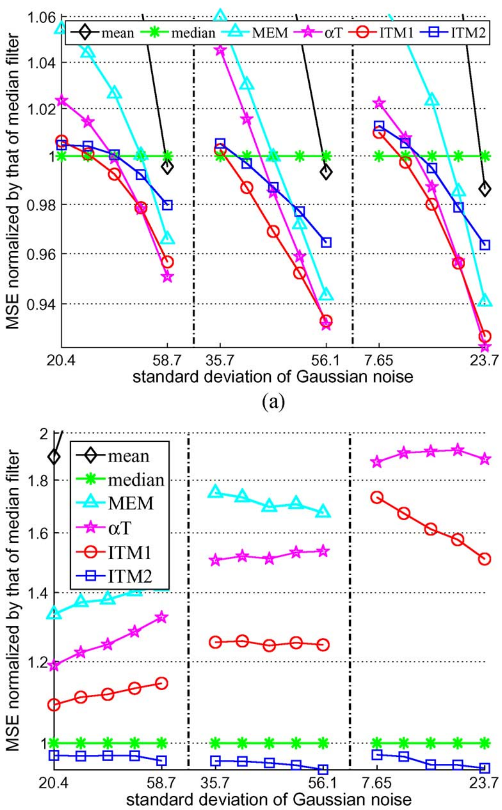

Fig. 7. Average MSE over ten runs for the three real images at five different levels of (a) additive Gaussian noise and (b) additive Gaussian noise and 30% exclusive impulses. The average numbers of ITM iterations are closely around 3.3 in (a) and closely around 4.5 in (b).

TABLE I
MSEs for a Constant Image Contaminated by α-Stable Noise

α	0.5	0.8	1.2	1.5	1.8
mean	488.62	215.22	73.074	32.059	13.962
MA	215.60	61.567	18.605	**11.324**	**9.1495**
OM	**28.988**	**8.3031**	**10.748**	12.484	11.648
MLC	165.29	61.496	25.064	15.167	10.491
ITM1	35.998	13.857	**11.470**	**10.614**	**9.9998**
ITM2	**15.149**	**12.505**	12.417	12.169	11.543

TABLE II
MSEs for a Real Image "Lena" Corrupted by ϵ-Contaminated ($\lambda = 0.5$) Gaussian ($\sigma_n^2 = 100$) and α-Stable ($\gamma = 10$) Noise

α	0.5	0.8	1.2	1.5	1.8
mean	167.37	111.51	78.860	68.240	63.217
MA	83.835	65.164	56.852	54.631	53.610
OM	62.485	**54.102**	58.666	58.917	55.587
MLC	96.343	65.409	56.177	53.434	51.795
ITM1	**58.292**	**54.655**	**51.983**	**51.246**	**50.612**
ITM2	**57.894**	55.583	**53.311**	**52.186**	**51.491**

of settings same as those in Section IV-E are applied to ten different noised versions of the image "Lena." Table II records the average MSEs over the ten noised images. The OM filter performs best only for $\alpha = 0.8$, and the ITM1 filter closely follows it. For all other four values of α, both the minimum and the second smallest MSEs are achieved by the proposed ITM filters. The average numbers of ITM iterations are 4.3, 3.8, 3.6, 3.3, and 3.1, respectively, for α values from 0.5 to 1.8. Fig. 8 shows the original image, the noise-corrupted ($\alpha = 1.2$) image, and its filter output images.

V. Conclusion

The different characteristics of the mean and median filters are well known. It is desirable to develop a filter having the merits of both the types of filters. Comparing with the arithmetic operation, data sorting required by the median-based filters is a complex process and is intractable for multivariate data. The proposed ITM filter circumvents the data-sorting process but outputs a result approaching the median. Proper termination of the proposed ITM algorithm enables the filters to own merits of the both mean and median filters and, hence, to outperform both the filters in many image denoising applications. Although it is an iterative algorithm by nature, only a few iterations are required for the ITM algorithm to achieve good results in all experiments of this paper.

This paper also shows the relation between the two very often used fundamental statistics, namely, the arithmetic mean and the order statistical median. It unveils some simple statistics

Fig. 8. Filtering results of the noise-corrupted image "Lena" of size 512 × 512. (a) Original image. (b) Corrupted image by ϵ-contaminated ($\epsilon = 0.5$) Gaussian ($\sigma_n^2 = 100$) and α-stable ($\alpha = 1.2, \gamma = 10$) noise. Outputs of (c) MA, (d) OM, (e) MLC, and (f) ITM1 filters of size 5 × 5.

of a finite data set as the upper bounds of the deviation of the median from the mean. The tightest upper bound discovered in this paper is applied as the truncation threshold, aiming at the smallest number of iterations, which, however, may not deliver the best filtering performance. Some general stopping criteria are suggested, although a better one for some specific applications is possible. Properties of the proposed ITM1 and ITM2 filters are theoretically analyzed and experimentally verified.

Comprehensive simulation results with a fixed parameter set throughout all experiments demonstrate the superiority and flexibility of the proposed filters. The significant difference of the ITM filter from the median, MEM, αT, MTM, and MA filters is that it circumvents the data-sorting operation. Different from the myriad and MLC filters, the ITM filters are not distribution-model specific and, hence, have greater application scope. One possible further development is to explore some proper criteria, based on which the filter can automatically switch between ITM1 and ITM2. Better filtering performance can be certainly expected with a proper switch mechanism because the ITM1 filter performs better in the smooth area with short-tailed and light long-tailed noise and the ITM2 filter preserves the image structure better and is more powerful in removing heavy impulsive noise.

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