OPTICAL METROLOGY

Detecting nanometric displacements with optical ruler metrology

Guang Hui Yuan[1] and Nikolay I. Zheludev[1,2]*

We introduce the optical ruler, an electromagnetic analog of a physical ruler, for nanoscale displacement metrology. The optical ruler is a complex electromagnetic field in which singularities serve as the marks on the scale. It is created by the diffraction of light on a metasurface, with singularity marks then revealed by high-magnification interferometric observation. Using a Pancharatnam-Berry phase metasurface, we demonstrate a displacement resolving power of better than 1 nanometer (λ/800, where λ is the wavelength of light) at a wavelength of 800 nanometers. We argue that a resolving power of ~λ/4000, the typical size of an atom, may be achievable. An optical ruler with dimensions of only a few tens of micrometers offers applications in nanometrology, nanomonitoring, and nanofabrication, particularly in the demanding and confined environment of future smart manufacturing tools.

Light is a perfect tool for metrology because it allows measurements of the size or position of an object against a ruler, in the same way as the Egyptians and Mesopotamians used rods divided into cubits, the earliest known unit of length, in the third millennium BCE. Displacement of an object against a ruler can be measured optically, with resolution limited by diffraction to about half of the used optical wavelength λ, typically a fraction of a micrometer. Here we demonstrate that orders of magnitude higher resolution, potentially down to the atomic level, can be achieved by using an optical ruler instead of a physical ruler: an electromagnetic field structured with singularities at the deeply subwavelength scale. This novel metrology is based on the premise that free-space singular optical fields with deeply subwavelength zones of rapid phase variations can be created by interference of multiple beams of light (*1*–*10*) and that these free-space optical fields can be imaged with no limit to resolution, far beyond the conventional diffraction limit. In contrast with superoscillatory optical microscopy using subdiffraction intensity hotspots (*8*), optical ruler metrology exploits phase singularities that are much more strongly localized.

To create an optical ruler, we used a nanostructured metasurface that, when illuminated with coherent light of wavelength λ, produces in free-space an optical field with high gradient of phase φ where modulus of the local wave vector $\boldsymbol{k} = \nabla\varphi$ considerably exceeds the free-space wave vector $|\boldsymbol{k}_0| = 2\pi/\lambda$. The interference of this field with the incident wave itself reveals the zones with high values of local wave vectors that can be as small as a fraction of one nanometer in size. We revealed these singularities interferometrically using the same metasurface as a micrometer-scale size monolithic interferometer. We imaged the singularities with high magnification and used them as the reference points (marks on the scale) for superresolution metrology in much the same way that marks on the physical ruler are used in conventional measurements (see Fig. 1).

The optical ruler is generated by a planar Pancharatnam-Berry phase metasurface (*11*, *12*) illuminated with a semiconductor laser at a wavelength of λ = 800 nm (*13*). We constructed the metasurface that creates, in the far-field from the plane of the metasurface, a superoscillatory subwavelength hotspot (*7*, *8*, *10*) with polarization orthogonal to the incident wave. Such hotspots are known to be flanked by zones of high phase gradient (*10*). We revealed the zones of high phase gradients on the optical ruler by observing the interference between the superoscillatory field and the reference wave transmitted through the metasurface with the same polarization as the incident wave.

We imaged the interference pattern of the optical ruler, without any loss of resolution, by a lens with a numerical aperture that is higher than that of the pattern generator. We used a 1300× magnification optical system, projecting the image on an image sensor with pixel size of 6.5 μm, thus achieving an effective pixel resolution of 5 nm in the observation plane (*13*). The experimental data are compared with the patterns simulated by finite-difference time-domain (FDTD) modeling. From the interferometric intensity maps (Fig. 2, A and B) taken at four different incident polarizations (+45° and –45° linear polarization, and right and left circular polarizations), using the technique described in (*10*, *13*), we retrieved the intensity (Fig. 2, C and F), phase (Fig. 2, D and G), and modulus of transverse local wave vector $k_x = \nabla_x\varphi$ (Fig. 2, E and H) profiles of the superoscillatory field. It appears that the zones of rapid variations of phase (peaks of the local wave vector) are much narrower than the intensity hotspot itself. We used these peaks as "marks" of the optical ruler.

To evaluate the practically achievable resolution of the optical ruler displacement metrology, we moved platform A, which holds the laser source and metasurface interferometer, in the lateral direction against the image sensor on platform B with nanometric steps and recorded positions of the marks of the optical ruler (see Figs. 1 and 3). The linear regression analysis shows that the dependence of the recorded position of the marks against their physical position has a unitary slope and standard deviation of less than 1 nm. This determines the displacement resolving power of the optical ruler to be better than λ/800. Here, the resolving power of ~1 nm is limited by the resolution of the piezo actuator (0.4 nm), mechanical instability in the setup, and

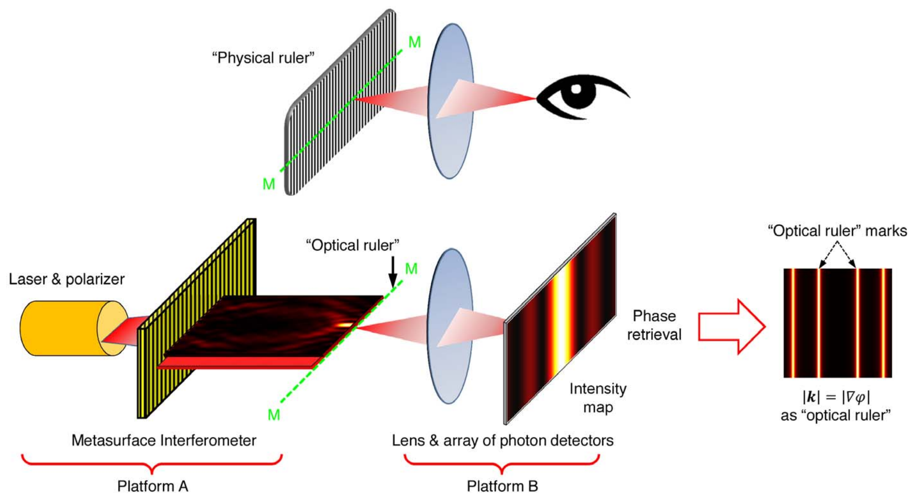

Fig. 1. Principle of the optical ruler displacement metrology. Conventional displacement metrology is based on the observation of a physical ruler moving along the MM line. The optical ruler metrology relies on observing the deeply subwavelength zones of a complex optical field with strong phase gradient $\boldsymbol{k} = \nabla\varphi$ along the MM line, which are used as marks of the optical ruler.

[1]Centre for Disruptive Photonic Technologies, The Photonics Institute, SPMS, Nanyang Technological University, Singapore 637371, Singapore. [2]Optoelectronics Research Centre and Centre for Photonic Metamaterials, University of Southampton, Highfield, Southampton SO17 1BJ, UK.
*Corresponding author. Email: niz@orc.soton.ac.uk

pixilation of the image sensor. To evaluate the potential resolution of the metrology, we calculated the autocorrelation function of the experimentally measured and simulated intensity map $G_I(\delta_x) = \int I(x+\delta_x)I(x)\mathrm{d}x$ and the wave vector map $G_k(\delta_x) = \int |k_x(x+\delta_x)k_x(x)|\mathrm{d}x$, taking into account only $|k_x(x)|/k_0 > 2$. Such autocorrelation functions allow a universal estimate for how well the original and displaced images can be distinguished, whereas the resolving power is evaluated as the width of the autocorrelation function at 80% of its height. The profile of the autocorrelation function $G_I(\delta_x)$ appears to be bell shaped, giving a resolution of 249 nm (0.31λ) from the computational data (Fig. 3, C and D) and 269 nm (0.34λ) from the empirically measured intensity map (Fig. 3, G and H). A much better resolving power can be achieved by using a *k*-vector map (Fig. 3, E and I): Its autocorrelation function $G_k(\delta_x)$ decays rapidly and is exponential at small distances, giving a resolution of about 190 pm (λ/4200), as estimated from the simulated data, and 1.7 nm (λ/470), as evaluated from the empirically measured intensity maps (Fig. 3, F and J), which is close to the value of standard deviation obtained in the displacement measurements (Fig. 3, A and B).

We have also demonstrated two-dimensional (2D) optical ruler metrology with a random 2D Pancharatnam-Berry phase metasurface (Fig. 4). The phase recovery process reveals a complex phase map with a high density of singularities (Fig. 4C) and superoscillatory wave vectors (Fig. 4D). The autocorrelation function $G_k(\delta_x;\delta_y) = \iint |k_\perp(x+\delta_x,y)k_\perp(x,y+\delta_y)\mathrm{d}x\mathrm{d}y$, where $k_\perp = \sqrt{k_x^2+k_y^2}$, decays rapidly on the nanometer scale and is exponential at small distances, allowing for a displacement resolution of about 2.2 nm (λ/360). Lower resolution of the 2D optical ruler in comparison with the 1D case is explainable by the use of a superoscillatory generator creating a pattern of singularities of diverse width.

A radical improvement of displacement resolving power from observing the intensity maps to observing the *k*-vector maps relates not only to a substantial difference in the widths of their marks; it is also helped by the different nature of the intensity and *k*-vector peak profiles that is evident from their autocorrelation functions: The smooth bell-shaped $G_I(\delta_x)$ falls much more slowly than that of the exponentially localized $G_k(\delta_x)$.

Our results show that with the optical ruler, a displacement resolving power of about λ/4000 (e.g., 100 pm at λ = 400 nm) could be potentially achievable, bringing it to the true atomic scale. Areas of large wave vectors are characteristic to many optical fields; however, narrower peaks require monochromatic light and tend to be located in areas of lower intensity (*3–5*, *14*). Moreover, high resolution will require field mapping

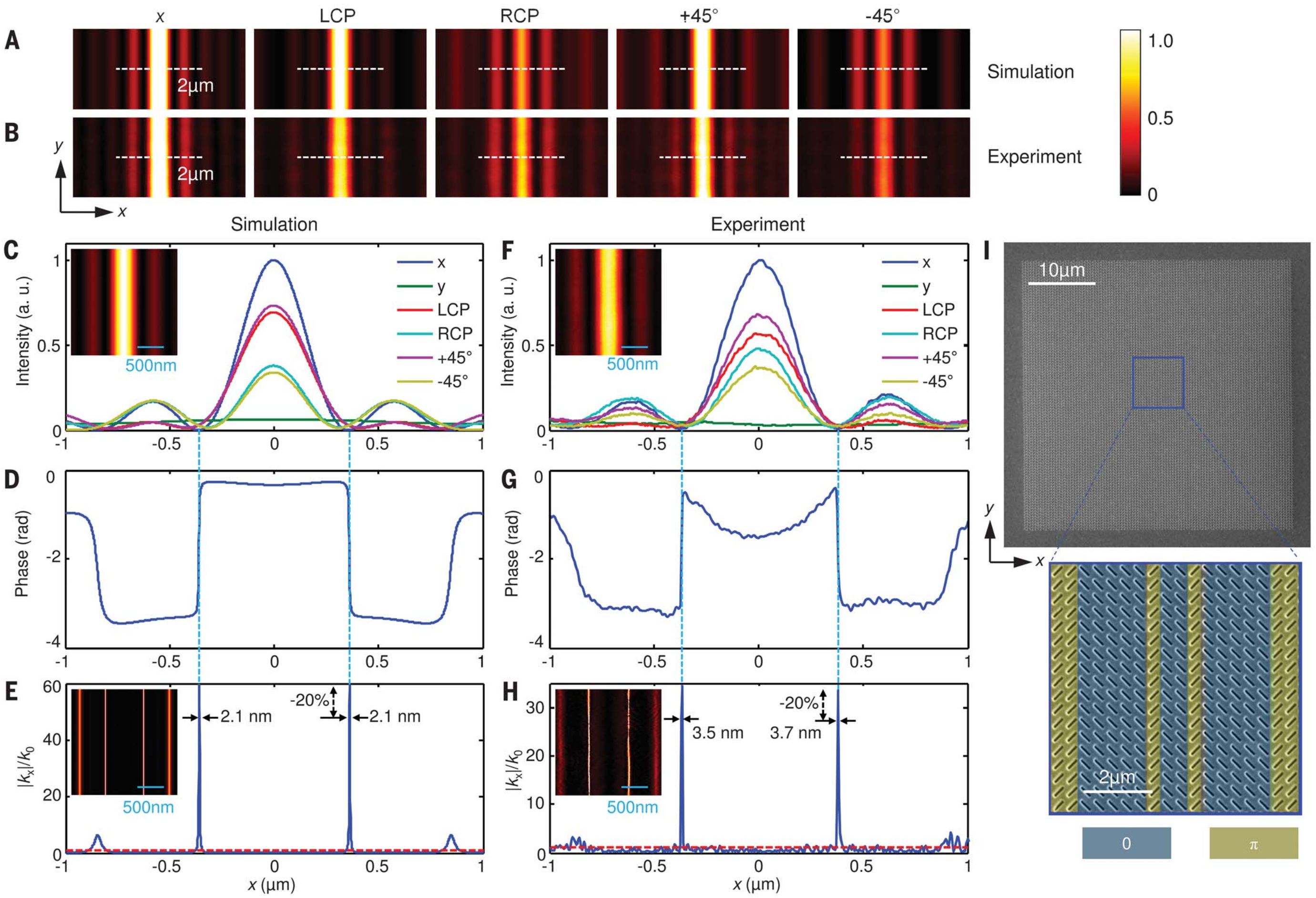

Fig. 2. Intensity, phase, and local wave vector values along the optical ruler. (**A** and **B**) The *x*-*y* cross section of intensity distribution of the *y*-polarized component of the optical field created by the metasurface at different incident polarizations [*x*, +45° and −45° linear polarizations, and right and left circular polarizations (RCP and LCP)] at a distance of ~10 μm from the metasurface. Shown are an FDTD simulation (A) and experimental data (B). The dashed horizontal bar indicates an interval of 2 μm on all images. The color scale indicates the intensity level, from 0 (black) to the maximum (white). (**C** to **H**) Intensity profiles for different input polarizations (insets show profiles as taken by the camera) [(C) and (F)]. a.u., arbitrary units. Retrieved phase profiles [(D) and (G)]. Modulus of local wave vector $k_x = \nabla_x \varphi$ exhibits peaks localized at the nanoscale (dashed red lines show $|k_x|/k_0 = 1$) [(E) and (H)]. (**I**) Scanning electron microscopy (SEM) image of the fragment of a 1D Pancharatnam-Berry phase metasurface that creates the superoscillatory field with zones of high phase gradients. Artificial colors in the magnified view indicate columns providing either 0 or π phase shift in the transmitted light.

with high magnification, which reduces light's intensity on the sensor. Therefore, the finite spectrum of the light source and noise level at the detector will limit the resolution. Our modeling shows that an increase in the noise level at the detector from 0 to 20% results in a steady decrease of resolving power from 190 pm to 1.5 nm, respectively (*13*). Above all, a demonstration of the resolving power at the level of λ/4000 will require the use of ultrastable optomechanics similar to that used in atomic-resolution scanning tunneling microscopy (STM) instruments and a further increase of optical magnification of the imaging system to reduce pixilation.

We demonstrate a novel displacement nanometrology based on the observation of optical singularities that supersedes the conventional diffraction limit of resolution for displacement measurements with a physical ruler by direct observation by several orders of magnitude. Similar to stimulated emission depletion microscopy (STED), photoactivated localization microscopy (PALM), and stochastic optical reconstruction microscopy (STORM), the proposed approach is also a far-field technique and therefore allows for noncontact operation. The subnanometer resolving power of the optical ruler demonstrated here is higher than that of the above-mentioned optical superresolution techniques. It does not require the high intensities for STED bleaching or the data accumulation required for PALM and STORM. However, the optical ruler technique does not allow full image reconstruction and is only suitable for metrology. Additionally, it does not suffer from the mechanical and thermal instabilities that affect large conventional metrological interferometric instruments. The metasurface can be manufactured at the tip of an optical fiber, allowing numerous applications where high resolution, small size, and noncontact operation are essential, including monitoring displacements of scanning stages of atomic

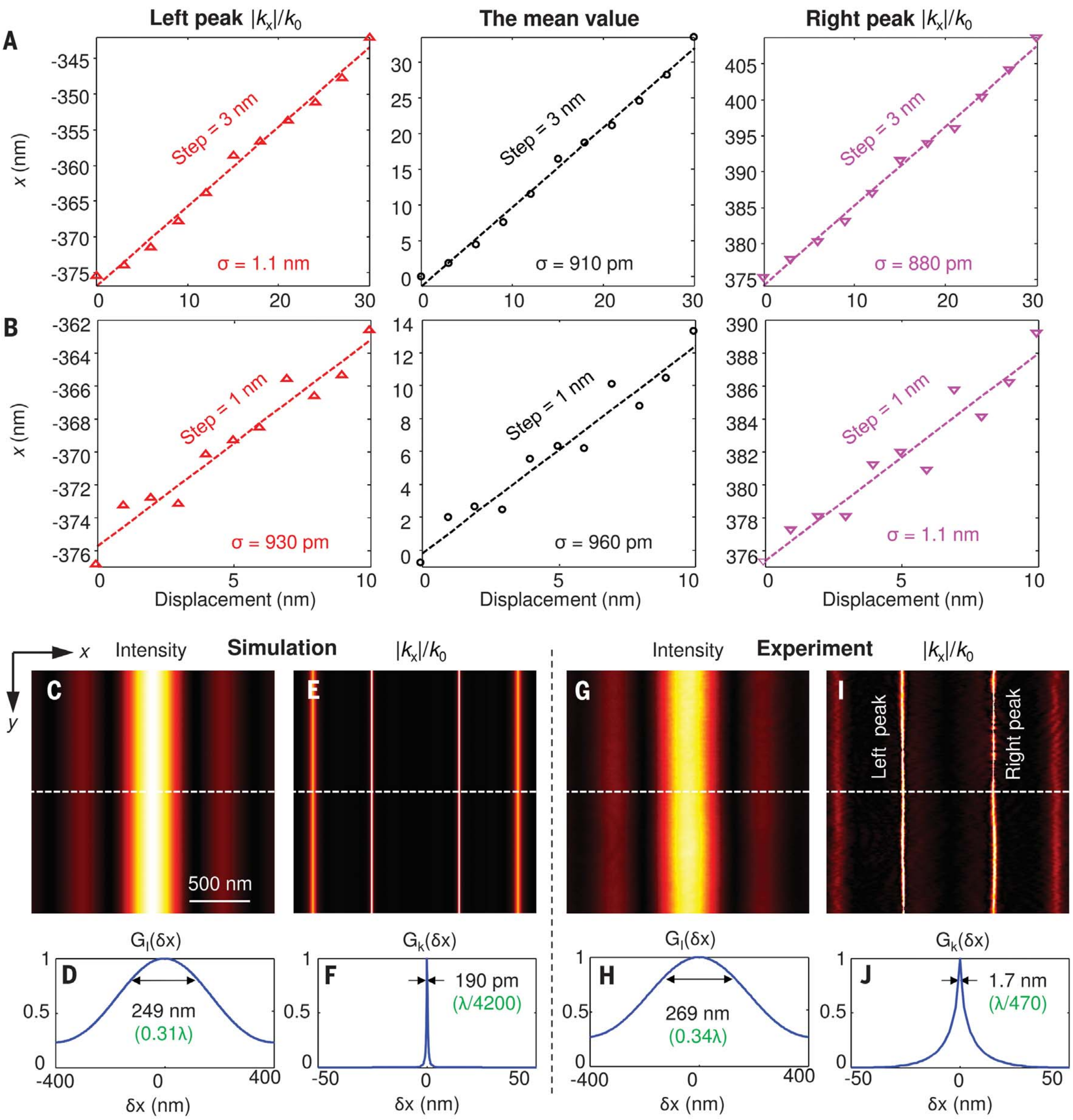

Fig. 3. Demonstration of the optical ruler displacement sensing. (**A**) Positions of the optical ruler marks (wave vector peaks) indicated by red upward triangles and magenta downward triangles and their mean values (black circles) measured at different positions of platform A with respect to platform B (see Fig. 1) with steps of 3 nm. σ, standard deviation. (**B**) Positions of the marks (wave vector peaks) measured with steps of 1 nm. (**C** to **J**) Details of the field maps of the optical ruler. The *x*-*y* cross sections of intensity distribution of the *y*-polarized complex optical field component created by the metasurface at a distance of ~10 μm from the metasurface [(C) and (G)], their autocorrelation functions [(D) and (H)], *x*-*y* maps of $|k_x|/k_0$ [(E) and (I)], and their autocorrelation functions [(F) and (J)].

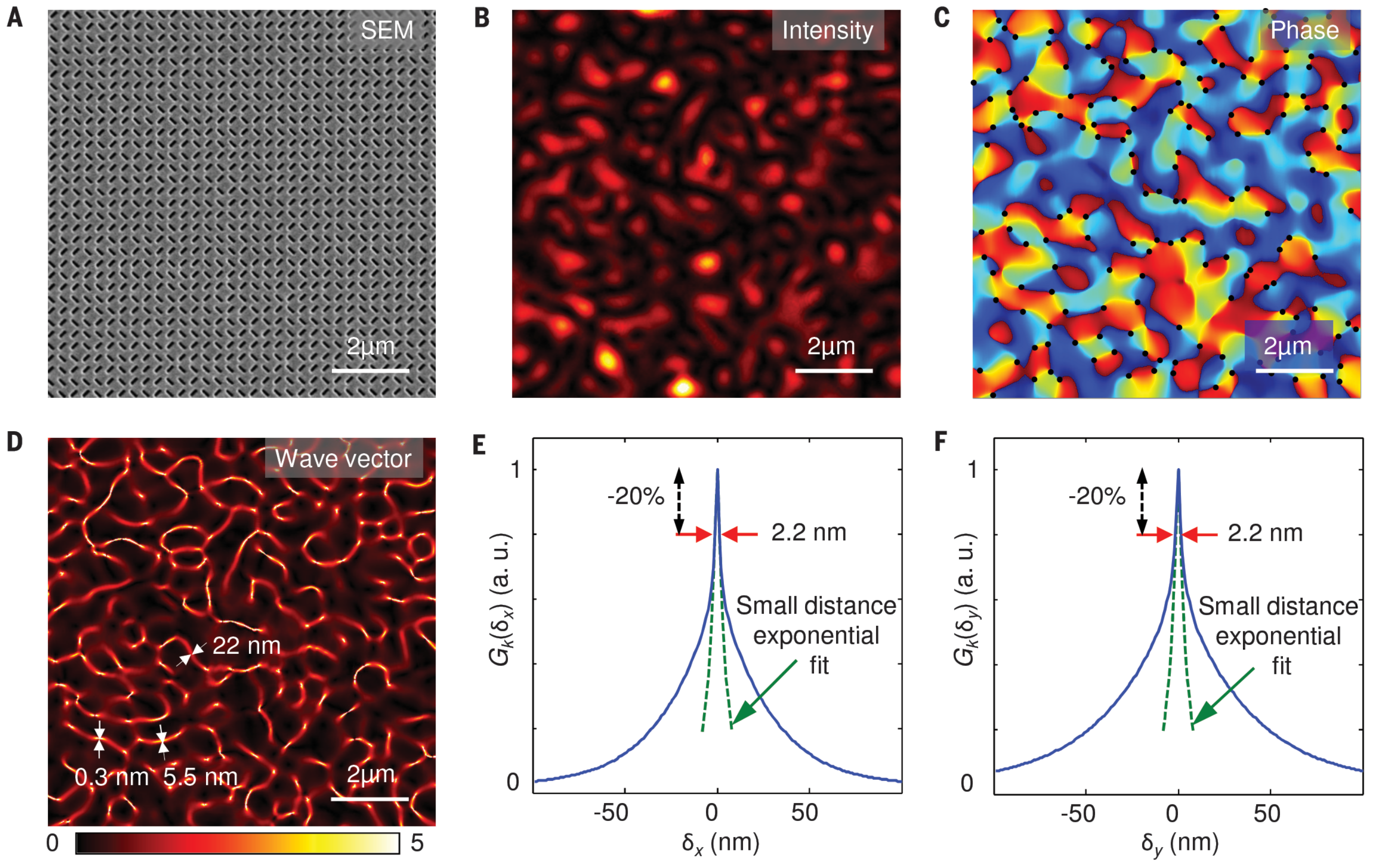

Fig. 4. Two-dimensional optical ruler. (**A**) SEM image of the fragment of a 2D random Pancharatnam-Berry phase metasurface. (**B** and **C**) *x-y* intensity (B) and phase (C) maps in free space at ~10 µm above the metasurface illuminated by coherent light source at the wavelength λ = 800 nm. Black dots show phase singularity points. In (B), the color scale indicates the intensity of random optical field, from 0 (black) to the maximum (white). In (C), the color scale indicates the phase of random optical field, from 0 (blue) to 2π (red). (**D**) Retrieved map of the modulus of the wave vector. The color scale indicates the value of $|k|/k_0$, from 0 (black) to the maximum (white). (**E** and **F**) Cross sections of the autocorrelation function along the *x* (E) and *y* (F) directions of the wave vector map.

force microscopy, STM, and superresolution optical microscopes; lithography mask alignment; and the control of motion of tools in nanoassembly. The optical ruler can also be placed on nanoindenter heads to measure the modulus of elasticity, yield stress, hardness, and wear resistance of materials. One can even envisage the optical ruler attached to a cutting tool of a smart manufacturing lathe or milling machine. Lastly, the optical ruler can be used for monitoring of mutual nanoscale displacements of parts of precision constructions, such as large optical telescopes, disc drives, microelectromechanical systems and nanoelectromechanical systems devices, and acceleration sensors, and for monitoring the deformation, fatigue, or thermal expansions of components.

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SUPPLEMENTARY MATERIALS

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Supplementary Text
Figs. S1 and S2
Reference (*16*)

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Guang Hui Yuan and Nikolay I. Zheludev

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Superoscillatory displacement metrology

The resolving power of light (or other waveforms, for that matter) is typically limited to about half the wavelength. However, multiple interference of waves gives rise to subwavelength "hotspots" in the phase owing to what is termed superoscillation of the wave field. Yuan and Zheludev used a specially designed metasurface to interfere laser light (wavelength λ = 800 nm) and created a superoscillatory ruler comprising these hotspots. They demonstrated the ability to measure displacements of around λ/800 while operating at a wavelength of 800 nm. They then showed theoretically that resolving powers of around λ/4000, i.e., atomic-scale displacements, may be possible. The technique should prove useful to metrology applications requiring precision measurements.

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