Integrating mechanism synthesis and topological optimization technique for stiffness-oriented design of a three degrees-of-freedom flexure-based parallel mechanism

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\textbf{A B S T R A C T}

This paper introduces a new design approach to synthesize multiple degrees-of-freedom (DOF) flexure-based parallel mechanism (FPM). Titled as an integrated design approach, it is a systematic design methodology, which integrates both classical mechanism synthesis and modern topology optimization technique, to deliver an optimized multi-DOF FPM. This design approach is separated into two levels. At sub-chain level, a novel topology optimization technique, which uses the classical linkage mechanisms as DNA seeds, is used to synthesize the compliant joints or limbs. At configuration level, the optimal compliant joints are used to form the parallel limbs of the multi-DOF FPM and another stage of optimization was conducted to determine the optimal space distribution between these compliant joints so as to generate a multi-DOF FPM with optimized stiffness characteristic. In this paper, the design of a 3-DOF planar motion FPM was used to demonstrate the effectiveness and accuracy of this proposed design approach. © 2014 Elsevier Inc. All rights reserved.

1. Introduction

Implementation of the compliant (a.k.a. flexure) joints on high precision positioning mechanisms has become a popular approach to deliver highly repeatable motion. Using the advantages of elastic deformation, a compliant joint overcomes the limitations of a conventional bearing-based joint such as dry friction, backlash, wear-and-tear, etc [1]. For better performance, the parallel-kinematics architectures have been widely used to develop most of the multiple degrees-of-freedom (DOF) flexure-based positioning mechanisms. This architecture plays an important role in the success of the Flexure-based Parallel Mechanism (FPM) due to its advantages of a lower inertia, programmable centers of rotations, superior dynamic behavior and less sensitive to external disturbances as compared to serial-kinematic architectures. In addition, the limited displacement of the compliant joints suits the limited motion range of the parallel-kinematic architecture. Hence, the FPMs are ideal candidates for micro/nano-scale manipulations [2–6]. The frictionless characteristic of these FPMs also suit clean vacuum environment since no particles will be generated from friction. Most importantly, utilizing compliant joints offer a simple and cost effective solution for developing macro-/micro-scale functional components such as Micro-Electro-Mechanical Systems (MEMS), and micro-scale nanopositioner [7], etc.

FPM can be classified as partially compliant or fully compliant. A partially compliant FPM mainly consists of rigid-bodies with compliant joints to deliver the desired DOF motions, while a fully compliant FPM generates motion via the deformations of the continuous compliant limbs. The most important design criteria of a FPM is to maximize the stiffness ratio, i.e., between the off-axis stiffness and natural stiffness. Here, the natural stiffness quantifies the compliance in the desired motion direction while the off-axis stiffness quantifies how stiff the joint is in all other directions. Hence, a FPM must have high stiffness ratio since it directly affects the robustness of the entire positioning system. Currently, exact constraint design approach can be used to synthesize a compliant joint of a partially compliant FPM or a compliant limb of a fully compliant FPM based on the desired DOFs that need to be constrained [8,9]. The stiffness of the compliant joint or limb can be analyzed based on existing kinetostatic modeling approaches [10–13] and...
subsequently the stiffness of the entire FPM can be predicted based on classical mechanics theory [9,14].

Among the FPMs, the stiffness of a partially compliant FPM is more predictable since each limb can be divided into several compliant joint modules where each module can be designed to deliver targeted DOF via the exact constraint design approach. To ensure high off-axis stiffness, parallel configuration separated by a distance was adopted to increase the polar moment of inertia [1,10]. On the other hand, a fully compliant FPM possesses more complex stiffness characteristic since each compliant limb is designed with the assumption that the compliance in all directions will contribute to the overall motion. In addition, synthesizing and analyzing a fully compliant FPM become increasingly challenging when the size goes towards micro-scale level since there will be limited space for proper configured compliant joints. Thus, synthesizing a microscale fully compliant FPM through traditional design approach involves iterative human intuitions [7]. However, such a synthesis method seldom leads to an optimized solution that offers high stiffness ratio. Understanding these limitations, other variations of design approaches, e.g., the freedom and constraint topologies approach [15], and the constraint design with screw theory method [16], etc, were proposed lately to synthesize the compliant joints or limbs more effectively.

This paper presents a new design approach for synthesizing and optimizing a multi-DOF FPM. Treated as an integrated design approach, it is a systematic design methodology that delivers an optimized FPM based on the desired tasks and specifications. Within this methodology, a novel topology optimization technique is used to synthesize the compliant joints or limbs at the sub-chain level. At the configuration level, the stiffness of the FPM will be optimized based on the desired workspace and size constraints. In this paper, the effectiveness of the proposed design methodology is demonstrated through the development and evaluation of a 3-DOF planar motion FPM. The remainder of the paper is organized as follows: Section 2 introduces the proposed integrated design approach and Section 3 presents the new topology optimization technique. Section 4 focuses on how optimization was conducted at the configuration level via the classical mechanics theory and objective functions. Lastly, the experimental investigations on the stiffness characteristic of the FPM were conducted and results are discussed in detail in Section 5.

2. Integrated design approach

A FPM can be treated as a continuum structure with either distributed or lumped compliance to deliver specific DOF motion. For example, a compliant gripper can be synthesized through a kinematic-based design approach via a parallel architecture with a slider as shown in Fig. 1a. On the other hand, topology optimization can also synthesize a continuum structure that not only delivers the same function but with better stiffness characteristics (Fig. 1b). In the past, topology optimization is mainly used to design continuum structures or functional parts such as the micro-grippers instead of synthesizing the compliant limbs or joints of a FPM. In this paper, an integrated design approach, which integrates classical mechanism synthesis and modern topology optimization, is introduced as a systematic design methodology for synthesizing a multi-DOF FPM with optimized stiffness characteristics. Referring to Fig. 2, the first step of this design approach is to understand the design specifications, e.g., the desired degrees-of-freedom, workspace, and size constraints, etc. Next, appropriate parallel-kinematic architecture will be selected and general kinematic analyses will be conducted. At sub-chain level, topology optimization will be used to determine the optimized topology of the compliant joint or limb. With these optimized topologies being generated, the compliant matrix of each compliant joint or limb will be used to determine the overall stiffness of the FPM via the classical mechanics theory. At configuration level, the overall stiffness of the FPM will be optimized based on the desired workspace and size constraints. Lastly, a FPM design with an optimized stiffness characteristic, which meets all the desired specifications, will be generated.

2.1. Designing a 3-DOF planar motion FPM

In this work, the design and development of a 3-DOF planar motion FPM was used to demonstrate the effectiveness of the proposed integrated design approach. This 3-DOF planar motion FPM delivers an $X-Y-\theta_2$ motion within a footprint of 300 mm$^2$ and a low profile thickness of 20 mm. Having such a large footprint ensures that any form of manufacturing uncertainty and tolerance is kept to a minimum without affecting the accuracy of the experiment data. Most importantly, performance parameters such as displacements, forces, moments, etc, were amplified to facilitate the experimental investigations. The main objective was to synthesize a FPM and optimize its stiffness ratio based on the desired specifications, and constraints.

2.2. A partially decoupled motion 3PPR parallel architecture

To realize a $X-Y-\theta_2$ planar motion, the 3-DOF FPM can be articulated based on three possible parallel-kinematic architectures formed by 1-DOF prismatic or revolute joint. These architectures include the 3-legged revolute-revolute-revolute (3RRR), the 3-legged prismatic-revolute-revolute (3PRR), and the 3-legged...
prismatic-prismatic-revolute (3PPR) [3]. In this work, the 3PPR architecture was selected over the 3PRR and 3RRR architectures because the compliant prismatic joints are generally stiffer and more deterministic than the compliant revolute joints. The schematic of the 3PPR architecture is shown in Fig. 3 whereby the end-effector, i.e., at the center of the moving platform, is connected to the fixed base by three identical parallel chains separated by 120°. Each chain comprises of a serially connected active prismatic joint (P) and a passive prismatic-revolute (PR) joint. With the active P joint being fixed to the base, the weight of the actuator will not contribute to the moving masses of the FPM. The relationship between the desired planar motion, i.e., δx, δy, and δz, and the active prismatic joints, i.e., p1, p2, and p3, is given as [3]

\[
\begin{pmatrix}
  p_1 \\
  p_2 \\
  p_3
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{2} & \frac{\sqrt{3}}{2} & r \\
  -1 & 0 & r \\
  \frac{1}{2} & -\frac{\sqrt{3}}{2} & r
\end{pmatrix} \begin{pmatrix}
  \delta x \\
  \delta y \\
  \delta z
\end{pmatrix}
\]

(1)

where \( r \) is the fixed length between the end-effector and the edge of the moving platform, which the PR joint is attached to.

3. A novel topology optimization technique: mechanism-based seeding technique

Based on the 3PPR architecture, each limb of the proposed FPM was articulated by a serially connected P and PR compliant joints. In this work, the optimized topology of each joint was determined by a new topology optimization technique. Terned as the mechanism-based seeding technique [17], it uses the traditional linkage mechanisms as the seeds of the compliant joints, which are subsequently synthesized and optimized based on the desired stiffness characteristics. Unlike existing topological optimization techniques [18–20], e.g., solid isotropic material with penalization (SIMP) and evolutionary structural optimization (ESO), the proposed approach exhibits good convergence capabilities and eliminates the non-feasible solutions such as disconnected solid elements and ambiguous “gray” elements. When compared against the morphological technique [21], the proposed technique allows the seeds to evolve during the optimization process.

The key principle of this proposed technique is to first select an appropriate linkage mechanism (LM), which has the same DOF as the targeted compliant joint, as the seed for the optimization process. For example, a 4-bar LM will be used as the initial seed for a 1-DOF compliant joint, a 5-bar LM as the initial seed for a 2-DOF compliant joint, and a 6-bar LM to represent the initial seed of a 3-DOF compliant joint. For other 3-DOF LMs, the possible variations can be enumerated by well-established classical mechanism synthesis methods and Gruebler’s criteria. Subsequently, the selected seed is superimposed onto a design domain where all its finite elements are initially void as illustrated in Fig. 4. The fixed points and the coupler point of the seed correspond to the fixed points and loading point of the compliant joint respectively. For the any optimization problem, the initial and evolved seeds stay within a fixed design domain and the loading, and fixed points can only be placed at the boundary of the design domain.

Each link of the seed will be represented by one cubic curve, one harmonic curve, and their respective reflected curves about the link. These four curves form the boundaries that will be used in the selection of the solid elements. Based on the value of \( m \) assigned to each link (\( m \in \mathbb{Z}^+ \), \( 1 \leq m \leq 3 \)), different combinations of solid elements can be generated. Here, all the elements bounded between the original curves and the link are solid when \( m = 1 \). If \( m = 2 \), all the elements bounded by the reflected curves and the link are solid. For \( m = 3 \), the solid elements will be the combined elements of the first two cases.

In this proposed technique, varying the number of variables will obtain different topologies. The first group of variables come from the initial seed, i.e., the length of the links, \( l \), and their orientations, \( \theta \), as shown in Fig. 5a. By changing both \( l \) and \( \theta \), it will vary the pose of the seed. Next, each harmonic curve is represented by the height, \( j \), and location of the peak, \( k \), as illustrated in Fig. 5b while each cubic curve is represented by the height, \( h \), the starting location, \( s \), and the ending location, \( e \), as shown in Fig. 5c. By varying these groups of variables will change the shape and topology of each limb. Throughout the optimization process, all these variables will be varied within a pre-defined range. Consequently, the different topologies were generated for the evolution during the optimization process. Examples of these topologies are illustrated in Fig. 6.

Another uniqueness of this proposed technique is that the seed can change if the length of any link approaches zero during the optimization as shown in Fig. 6. Thus, the initial seed will not constrain the evolution of the topology throughout the optimization process. In general, the number of population will determine the number of topologies to be evolved for each generation. At each generation, the stiffness characteristics of each compliant joint, which is articulated from each of
minimizes the actuating stiffness and maximizes the non-actuating stiffness, the solver Genetic Algorithm (G.A.) will eventually obtain an optimum compliant joint.

As each compliant joint is generated based on selecting the solid finite elements discretely, G.A solver was selected because it can handle discrete optimization processes as compared to other gradient methods that cannot be used to solve discrete optimization processes directly. Being an evolutionary function, G.A. allows the solution to jump out from a local solution. Hence, there are higher chances to obtain a global solution as compared to the meta-heuristic and stochastic methods. In addition, conventional optimization techniques such as ESO/BESO are solved by using heuristic rules rather than an optimization solver. Thus, these techniques are largely criticized to be non-convergent. Using the proposed mechanism-based seeding technique and GA solver, there is no convergence issue and no complexity in solving the stiffness matrices.

3.1. Synthesis of the passive PR compliant joint

The synthesis of the PR compliant joint was broken down into two stages to reduce the computational time. The first stage was performed through a coarse mesh while the second refined the design via a fine mesh, i.e., each element size of 20 mm × 0.4 mm × 10 mm (length × width × thickness). In both stages, a design domain of 50 mm × 20 mm × 10 mm was used and the Young’s Modulus and Poisson ratio were selected as 71 GPa and 0.33 respectively. The design domain is discretized into a mesh of 3-D 8-node bilinear finite elements for both stages. The passive PR compliant joint needs to deliver a translation motion along the x-axis and a rotation motion about the z-axis. Hence, it must possess low actuating stiffness along the x-axis and about the z-axis. Such stiffness characteristics were represented by achieving high values for C_{11} and C_{66} in the C. To achieve high non-actuating stiffness, all other components in C must be as low as possible. Thus, the objective function is

\[
\min \left( f_1(x) = \frac{\prod_{i=2}^{6} \prod_{j=1}^{i-1} C_{ij}}{(C_{11})^{19}(C_{66})^{19}} \right)
\]

where the 6 diagonal components and the 15 non-diagonal components are required to optimize through Eq. (6). Based on these 21 components, the C_{11} and C_{66} components are raised to the power of 19 since there are 19 remaining components that have to achieve high non-actuating stiffness. The vector \( x \) represents the variables (Fig. 5) and Eq. (6) is governed by (4). The inequality constraints ensure that the pose of the seed remains in the design domain and the equality constraint represents the FEA governing equations.

As the passive PR compliant joint has to deliver a translation motion along the x-axis and a rotation motion about the z-axis, a corresponding 2-DOF 5-bar LM with a coupler point is selected as the seed for the first stage of optimization as shown in Fig. 7a. The coupler point of the seed, which was also the loading point, was constrained to move along the top row elements while two fixed points were located at the base. Using the G.A. solver with 500 populations and 100 generations, the initial seed evolved from a 5-bar LM to a 4-bar topology and subsequently to a 3-bar topology during the first stage as shown in Fig. 7b. This is because two links of the initial seed were reduced to zero throughout the first stage making the topology of the initial 5-bar LM seed into an “inverted-Y” topology. The second stage of optimization used Eq. (6) as its objective function to refine the design. Using the G.A. solver to evolve 200 population via 50 generations, the final PR compliant joint was obtained as shown in Fig. 7c. The optimal PR joint resembles a
Fig. 7. Synthesis of the PR compliant joint via (a) a 5-bar LM seed and (b) its evolution from a 5-bar to a 3-bar topology. (c) The optimal PR compliant joint topology with fine mesh solution.

Fig. 8. Convergence plots of the topology optimization of PR compliant joint from (a) the first stage and (b) second stage.

The rest of the components, which contribute to the off-axis stiffness, must be low. With $C_{11}$ representing the actuating compliant component in $C$, the objective function is

$$
\min \left( F_2(x) = \frac{\prod_{i=2}^{n} \prod_{j=1}^{m} |C_{ij}|}{(C_{11})^{20}} \right)
$$

(7)

Considering the $C$ is a symmetrical matrix with 21 essential components, $C_{11}$ component was raised to the power of 20 since there were 20 remaining components to be accounted for. Similar to previous efforts, the vector $x$ represents the variables while Eq. (7) is governed by (4).

As the active $P$ compliant joint needs to deliver a translation motion, a 1-DOF 4-bar LM with a coupler point was selected as the seed (Fig. 10). The coupler point, which is also the loading point of the seed, was located at the top row's central element. There are two fixed points and they are located at the bottom row. The first stage of optimization was carried out by 400 population via 100 generations. From Fig. 10b, the end result still resembles a 4-bar topology but the limbs evolved to be parallel to each other. Subsequently, the second stage of optimization refined the optimal seed pose obtained from the first stage with finer mesh, i.e., $20 \text{ mm} \times 0.4 \text{ mm} \times 10 \text{ mm}$ and Eq. (7). Consequently, the optimal $P$ compliant joint

3.2. Synthesis of the active $P$ compliant joint

Similar to the synthesis of the PR joint, synthesizing the active $P$ compliant joint was split into two optimization stages to reduce computational time. In both stages, the Young’s Modulus, Poisson ratio and finite element type were similar to those of the PR compliant joint except that the width of the design domain was changed to 25 mm. As the active $P$ compliant joint needs to deliver a translation motion, the compliance along the $x$-axis must be high while

Fig. 9. Final design of the PR compliant joint.

Fig. 10. Synthesis of the $P$ compliant joint via (a) a 4-bar LM seed, (b) its evolution towards and (c) the optimal topology with fine mesh solution.

Fig. 11. Convergence plots of the topology optimization of $P$ compliant joint from (a) the first stage and (b) second stage.
was obtained after the G.A solver evolved a random initial population of 200 topologies with 50 generations. As shown in Fig. 10c, the optimal P compliant joint resembles a tapered-shape rigid-link supported by two thin beams. The convergence plots from both stages are plotted in Fig. 11.

4. Configuration level stiffness optimization

The optimal compliant joints obtained from the proposed topology optimization technique at sub-chain level were combined together in series to form each limb of the proposed 3PPR FPM as shown in Fig. 12. Note that the sharp edges, which appeared in the compliant joints have been smoothed out to prevent stress concentration. In this work, electromagnetic voice-coil (VC) actuators will be used to drive the proposed FPM due to their ability to deliver millimeters stroke range. During the design stage, it was estimated that each VC actuator needs to generate a continuous force of at least 30 N. Based on this requirement, the dimensions of such a VC actuator was estimated to be at least Ø60 mm × 60 mm. Thus, each limb of the FPM was fixed at 90 mm × 90 mm in order to encase each VC actuator within it. In this work, the proposed FPM will be monolithically cut from a SUS316 stainless steel workpiece. The Young’s Modulus and Poisson ratio of the SUS316 are 200 GPa and 0.33 respectively. Lastly, a workpiece with a standard thickness of 19 mm will be used to enhance the stiffness along the z-axis.

At configuration level, the stiffness optimization of the proposed FPM was conducted based on the targeted size constraint. This optimization is another essential step to determine the optimal space distribution between the compliant joints so as to achieve the best stiffness characteristics for the end-effector. This is because by increasing $l_3$, it will increase the non-actuating stiffness of the PR joints (refer to Fig. 12) but will also decrease the actuating compliance of the active P compliant joint. In order to retain the actuating compliance of these joints, the optimization at this level does not alter the thickness of the beams. From Section 3, the stiffness matrix of each compliant joint obtained via the proposed topology optimization technique is expressed in terms of their local chain frame. These chain frames are illustrated in Fig. 3 where (1), (2), and (3) have a z-axis rotation angle of $[\alpha_1, \alpha_2, \alpha_3] = [\pi/3, 0, -\pi/3]$ with respect to the global frame $I$ respectively. Based on the classical mechanism stiffness modeling approach [14,22], the compliance matrix of kinematic chain $i$, $C_{chain \ i}$, at the PR joint loading point can be determined by

$$[i]C_{chain \ i} = [i]C_{PR} + [i]J_i [i]C_{P} [i]J_i^T \ C \in \mathbb{R}^{6 \times 6}$$

(8)

where

$$J_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} I_{6 \times 6} \ C \in \mathbb{R}^{3 \times 3}$$

Referring to Eq. (8), $J_i$ is the Jacobian matrix with $I$, $0$ and $I$ representing the identity, zero and the skew-symmetry matrices of the position vector, $r_i$, as shown in Fig. 12. Note that $r_i$ represents the displacement vector from chain $i$’s the loading point to its P compliant joint’s loading point and $N$ represents the matrix form, e.g., $6 \times 6$ matrix or $3 \times 3$ matrix, etc. With the chain’s stiffness being identified, the stiffness matrix of the end-effector, $K_{ee}$, is determined by

$$[i]K_{ee} = \sum_{i=1}^{3} [i]C_{chain}^{-1} [i]Ad_{r,i}^{-1} \ K_{ee} \in \mathbb{R}^{6 \times 6}$$

(9)

where

$$[i]Ad_{r,i} = \begin{bmatrix} R_z(\alpha_i) & \mathbf{b} & R_z(\alpha_i) \\ 0 & R_z(\alpha_i) \end{bmatrix} R_{6 \times 6}$$

By referring to Eq. (9), $[i]Ad_{r,i}$ is the adjoint matrix with $R_z$ and $\mathbf{b}$ representing the standard z-axis rotational and the skew-symmetry matrices of the position vector, $b_i$, respectively as shown in Fig. 12. Note that $\mathbf{b}$ represents the displacement vector from the end-effector to chain $i$’s loading point. As the main objective was to maximize the stiffness ratio of the proposed FPM, i.e., maximizing non-actuating diagonal stiffness while minimizing the actuating stiffness of the end-effector, the fitness function becomes

$$\min \left( F_3(l_3) = \frac{K_{33}K_{44}K_{55}}{K_{11}K_{22}K_{66}} \right)$$

(10)

After solving Eq. (10) using the G.A. solver via an initial population of 10 with 10 generations of evolution, the optimal solution of $l_3$ was 20 mm based on the desired size constraint. Through this configuration level optimization process, the stiffness matrix of the optimized FPM, $K_{opt}^{ee}$, is given as

$$K_{opt}^{ee} = \begin{bmatrix} 2.82E4 & 8.5E-9 & 2.82E4 \\ -1.2E-14 & 5.7E-14 & 8.93E5 \\ -1.2E-7 & -2.4E-7 & -3.6E-12 & 2.46E3 \\ -2.4E-7 & -1.2E-7 & -3.7E-12 & -5.9E-9 & 2.46E3 \\ 8.5E-14 & 2.7E-14 & 5.9E-8 & 1.8E-16 & -4.5E-16 & 41.4 \end{bmatrix} \text{SYM}$$

(11)

(Note: At this stage of research, the stress concentrations are ignored during the optimization process as the goal of these optimization processes is to obtain a FPM with optimized stiffness characteristic based on the design constraints. As a result, the bending stress equation was not included as one of the governing equations in the optimization processes. However, a post process, which smoothen the jagged edges of the compliant joints to prevent any stress concentration, was conducted. Most importantly, a final numerical simulation was conducted via ANSYS10 to ensure that the maximum stress concentration of the optimized FPM do not exceed the maximum yield strength of the selected material.)

5. Evaluations and experimental investigations

5.1. Evaluating the stiffness ratio of the proposed FPM

In this work, another 3PPR FPM, which was articulated by compliant joints with traditional topologies, was used to evaluate the stiffness characteristic of the optimized FPM (Fig. 13a). Termed as the conventional-FPM (Fig. 13b), its PR compliant joint is formed by a cantilever beam with both ends being fixed to the translation portion of the P compliant joint, which is formed by a conventional parallel linear spring configuration. The design of this stage also underwent a similar configuration level optimization process to
versus R

Subsequently, C about Instead of making a physical prototype for the conventional-FPM, it was more economical to conduct the comparison of both FPMs via the FEA approach. For a fair comparison, both FPMs should have one identical actuating compliance. Consequently, the compliance about the z-axis was chosen to be identical. For the conventional-FPM, the flexure thickness of the traditional PR joints was selected as 0.6 mm to match the compliance about the z-axis of the optimized FPM.

\[
C_{ee}^{\text{opt}} = \begin{bmatrix}
3.55E - 5 \\
9.6E - 18 \\
6.71E - 24 -2.9E - 23 1.12E - 6 \\
1.67E - 15 -3.6E - 06 1.78E - 21 4.06E - 6 \\
3.61E - 06 1.67E - 15 5.18E - 22 5.63E - 16 4.06E - 6 \\
-7.5E - 20 5.94E - 21 -2.5E - 15 -9.6E - 22 1.92E - 23 2.42E - 2
\end{bmatrix} \quad \text{(12)}
\]

Based on Eq. (11), the compliance matrix of the optimized FPM at the end-effector was given in Eq. (12). Through the similar FEA solver, the compliance matrix of the conventional-FPM, \(C_{ee}^{\text{con}}\), is

\[
C_{ee}^{\text{con}} = \begin{bmatrix}
1.86E - 5 \\
-4.0E - 17 1.86E - 5 \\
9.33E - 24 -5.1E - 23 1.96E - 6 \\
-4.6E - 17 -7.1E - 06 1.90E - 21 5.41E - 6 \\
7.13E - 06 -4.6E - 17 6.84E - 22 5.72E - 17 5.41E - 6 \\
-3.8E - 20 -8.81E - 20 3.34E - 15 1.03E - 20 -3.6E - 21 2.42E - 2
\end{bmatrix} \quad \text{(13)}
\]

Subsequently, the ratio between Eqs. (12) and (13) is

\[
R^e = \frac{C_{ee}^{\text{opt}}}{C_{ee}^{\text{con}}} = \text{diag}[1.91 0.57 0.75 0.75 1] \quad \text{(14)}
\]

The ratio between \(C_{ee}^{\text{opt}}\) and \(C_{ee}^{\text{con}}\) in Eq. (14) is represented by a diagonal matrix because the values of all non-diagonal components are too small and thus omitted in this comparison. The remaining diagonal components represent the compliance of the actuating directions and the stiffness of the non-actuating directions. Here, the values of \(R_{11}^e\) and \(R_{22}^e\) are almost 2. This comparison suggests that the actuating compliance along the x- and y-axes of the optimized FPM are almost twice of the conventional-FPM. \(R_{66}^e\) is 1 because both FPMs should exhibit the same actuating compliance about the z-axis. On the other hand, \(R_{13}^e, R_{24}^e\) and \(R_{35}^e\) are all less than 1. This comparison suggests that the stiffness along the z-axis of the optimized FPM is almost twice of the conventional-FPM. It also suggest that the stiffness about the x- and y-axes of the optimized FPM are higher than the conventional-FPM. In summary, this comparison shows that the stiffness characteristic of the optimized FPM is more superior than the conventional-FPM.

5.2. Evaluating the optimized FPM prototype

A prototype of the optimized FPM was developed as shown in Fig. 14a. To validate the accuracy of the predicted compliance matrix, i.e., Eq. (12), an experimental investigation was conducted to evaluate the actual stiffness characteristic of the developed prototype. In this work, a high resolution 3-Dimensional (3D) scanner (GOM, model: ATOS Triple scan) was used to record the motion of the end-effector of the FPM generated by the picomotors as shown in Fig. 14b. External loadings produced by these picomotors were simultaneously recorded by a 6-axes Force/Torque (F/T) sensor (ATI, model: MINI40; resolution: 0.01N or Nm). Here, the F/T sensor was mounted to the end-effector and covered by a precise cut square cover, which served as a reference datum for the loadings from the picomotors and the scanning of motions by the 3D scanner. The recorded motions are images of the corresponding motions of the square cover due to the changes of external loadings. In this work, the only displacements that directly correspond to the axis of induced force or moment were considered while the parasitic displacements were discarded since the evaluations were on the diagonal components of the compliance matrix.

As the developed FPM prototype has 3-DOF, its end-effector should have three actuating compliances, i.e., the translation displacement along the x-axis due to \(F_x\) loading, the translation
displacement along the y-axis due to $F_y$ loading, and the angular displacement about the z-axis due to $M_z$ loading. Fig. 15 plots the experimental results of the compliance along the x-axis recorded from the prototype due to $F_x$ loading. From the collected data points, the least squares method was used to estimate the actual compliance. From Fig. 15, the actual compliance along the x-axis due to $F_x$ loading is $3.8 \times 10^{-5}$ m/N. As compared to the $C_{11}$ of $C_{opt}$, the deviation is 8.6%. The experimental results of the compliance along the y-axis recorded from the prototype due to $F_y$ loading are plotted in Fig. 16. Using the least squares method, the compliance along the y-axis due to $F_y$ loading is estimated as $3.48 \times 10^{-5}$ m/N. When compared to the $C_{22}$ of $C_{opt}$, the deviation is 2%. From Fig. 17, the actual compliance about the z-axis of the prototype due to $M_z$ loading was identified as $2.63 \times 10^{-5}$ rad/Nm. By comparing with $C_{66}$ of $C_{opt}$, the deviation is 8.7%. In this work, experimental investigations on the stiffness in the non-actuating directions were also conducted. Unfortunately, the changes in displacement about the x- and y-axes are too small to be recorded by the 3D scanner. Hence, only some experimental data relates to the stiffness along the z-axis are valid for conclusive evaluation. These experimental results are plotted in Fig. 17. From Fig. 18, the compliance along the z-axis due to $F_z$ loading is estimated as $1.20 \times 10^{-6}$ m/N based on the least squares method. As compared to the $C_{33}$ of $C_{opt}$, the deviation is 7.1%. Although $C_{44}$ and $C_{55}$ cannot be validated via this investigation, the collected experimental results and various comparisons with theoretical predictions are sufficient to suggest that the predicted stiffness characteristic agrees with the actual stiffness characteristic of the developed prototype.

6. Conclusion

This paper presents a novel design methodology to synthesize multi-DOF FPM. Termed as the integrated design approach, it is a systematic design methodology, which integrates both classical mechanism synthesis and modern topological optimization technique, to design a multi-DOF FPM with optimized stiffness characteristics based on the desired tasks, and specifications. This design approach is separated into two levels. At sub-chain level, a novel topology optimization technique, which uses the classical linkage mechanisms as DNA seeds, is used to synthesize the compliant joints or limbs. At configuration level, the optimal compliant joints are used to form the parallel limbs of the multi-DOF FPM and another stage of optimization was conducted to determine the optimal space distribution between these compliant joints so as to generate a multi-DOF FPM with optimized stiffness characteristic. Effectiveness of this design methodology was demonstrated through the design of a 3-DOF planar motion FPM and experimental results have shown that the deviations between the actual and predicted stiffness are less than 9%. At this stage of research, this design
methodology only concentrates on the stiffness optimization. The future work is to enhance this methodology with dynamics optimization.

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