A large deflection and high payload flexure-based parallel manipulator for UV nanoimprint lithography: Part I. Modeling and analyses

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A B S T R A C T

This paper presents a flexure-based parallel manipulator (FPM) that delivers nanometric co-planar alignment and direct-force imprinting capabilities to automate an ultra-violet nanoimprint lithography (UV-NIL) process. The FPM is articulated from a novel 3-legged prismatic-prismatic-spherical (3PPS) parallel-kinematic configuration to deliver a $\theta_x$-$\theta_y$-$Z$ motion. The developed FPM achieves a positioning and orientation resolution of $\pm 10$ nm and $0.05^\circ$ respectively, and a continuous output force of 150 N/Amp throughout a large workspace of $5 \times 5 \times 5$ mm. Part I mainly focuses on a new theoretical model that is used to analyze the stiffness characteristics of the compliant joint modules that formed the FPM, and experimental evaluations of each compliant joint module. Part II presents the stiffness modeling of the FPM, the performance evaluations of the developed prototype, and the preliminary results of the UV-NIL process.

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1. Introduction

The UV Nanoimprint Lithography (UV-NIL) process was first introduced in 1999 as a low cost approach for producing sub-100 nm circuit patterns [1]. Unlike the thermal-based NIL process, the UV-NIL is a room-temperature and low pressure process that relies on chemical and mechanical steps to transfer high resolution patterns from the templates (or molds) to the substrates [2]. Such key differences are due to a liquid etch barrier used by the UV-NIL process that eliminates high temperatures and imprinting forces, which are undesirable characteristics that cause major technical issues such as inaccuracies in multiple layers overlaying. Hence, the UV-NIL makes nano-scale duplication a highly potential process for replacing conventional optical lithography in multi-layer electronics chip fabrications with sub-100 nm electronic components. Most importantly, recent advances in optics science have demonstrated how nano-scale features can affect the light propagation [3], creating ultra-thin optical lenses [4], and even controlling the direction of propagation of surface plasmon polaritons [5]. Other than nano-scale electronic components fabrication, UV-NIL process is also a promising solution for rapid fabrication of nano-scale optical couplers and 2D/3D metamaterials.

In past literature, the UV-NIL system developed by Choi et al. [6] mainly comprised of a fine-orientation stage, a coarse-motion $Z$-stage, a $X$–$Y$ stage for step and repeat positioning, a wafer orientation stage, and other etching and UV light exposure components. The fine-orientation stage, which is driven by three Piezoelectric (PZT) actuators, is used to minimize the co-planer misalignment between the template and substrate by providing active orientation about the $x$-axis, $\theta_x$, the $y$-axis, $\theta_y$. Due to the limited stroke of each PZT actuator, the coarse-motion $Z$-stage, which carries and lifts the fine-orientation stage away from the substrate, is essential for executing other process steps such as the dispensing of UV-curable solution, de-molding, and the overlay process. Yet, the combination of coarse and fine stages not just makes the entire UV-NIL system mechanically complex and bulky, such complexities also extended to the control scheme and system. Most importantly, the backlash of the lead-screw within the coarse-motion $Z$-stage also compromises the repeatability of the overlay process. Thus, the $\theta_x$-$\theta_y$-$Z$ motion system of the entire UV-NIL system may not be the most effective solution in automating the UV-NIL process.
Only a few manipulators with $\theta_\alpha-\theta_\beta-Z$ motion were developed for high-precision applications over the past few decades [7,8]. Example include a micro-motion flexure-based Parallel Manipulator (FPM) that is articulated by a 3-limbs revolute-prismatic-spherical (3RPS) parallel-kinematic architecture [9]. Based on the same architecture, a FPM termed as Orion was developed to deliver a large workspace of $\pm2.5^\circ \times \pm2.5^\circ \times \pm5\text{mm}$ [10]. Another large workspace FPM, which is termed as the Nasmyth-Adaptive-Optics-System (NAOS), was developed for space telescope mirror alignment [11]. The main limitation of a 3RPS architecture is the actuator forms an integral part of its moving limb and thus becomes a moving mass. Large moving masses will disturb the dynamics of the entire system and affect the accuracy, and response of the system. In addition, this architecture is inefficient in transferring the force generated from the actuators to the end-effector. Another form of $\theta_\alpha-\theta_\beta-Z$ manipulator was developed based on an electromagnetically driven scheme and an air-bearing suspension system [12]. However, the air-bearing suspension system has limited stroke and is not feasible for delivering large displacement.

None of the existing parallel-kinematic architectures and solutions could be used to develop a $\theta_\alpha-\theta_\beta-Z$ motion FPM that is suitable for automating a UN-NIL process, e.g., a large workspace of few millimeters and degrees, nanometric positioning resolution, arc-second orientation resolution, large continuous output force, simple positioning control, and direct force control capability, etc. This paper presents a FPM that meets all those desirable requirements. Part I mainly focuses on a new theoretical model that is used to analyze the stiffness characteristics of the compliant joint modules that formed the FPM, and experimental evaluations of each compliant joint module. Part II presents the stiffness modeling of the proposed FPM, the performance evaluations of the developed prototype, and the preliminary results of the UV-NIL process.

2. A novel 3-legged prismatic-prismatic-spherical FPM

In this work, a novel 3-legged prismatic-prismatic-spherical (3RPS) parallel-kinematic architecture was proposed to develop the desired $\theta_\alpha-\theta_\beta-Z$ motion FPM to automate the UN-NIL process. This 3RPS architecture consists of three parallel limbs where each limb comprises of an active prismatic joint, a passive prismatic joint, and a spherical joint. As compared to the previous $\theta_\alpha-\theta_\beta-Z$ motion FPMs, the 3RPS architecture allows the high-precision linear actuators to be fixed onto the base so as to reduce the moving masses and inertias. In addition, the vertical position of each actuator allows direct transmission of the imprinting force from the actuators to the end-effector. Based on this architecture, the proposed FPM has three identical compliant limbs where each limb is articulated by an active prismatic compliant joint module and a spatial compliant joint module as shown in Fig. 1. Here, the passive prismatic and spherical joints of the proposed 3RPS architecture is articulated by the spatial compliant joint module.

The intended UN-NIL system will be mainly used to replicate one quarter of a wafer-sized template onto a substrate via each imprinting process. For proof-of-concept, a scaled-down version was developed for evaluating the proposed FPM and the entire process. Hence, the proposed FPM was designed to achieve an orientation of $\pm2.5^\circ$ about the x- and y-axis, and a displacement of 5 mm along the z-axis. In the scaled-down version, an AFM calibration grating with an array of 104 nm parallel lines over an area of $5 \times 5 \text{mm}^2$ was used as a template and the targeted imprinting force was 200 N. To obtain nanometric positioning and large force generation throughout a displacement of 5 mm, the concept of a Flexure-based Electromagnetic Linear Actuator (FELA) [13] was employed to form the active prismatic compliant joint module.

The proposed design of the prismatic and spatial compliant joint modules are illustrated in Fig. 2. In general, each compliant joint module is mainly formed by beam-based flexure joints coupled with rigid-links, which have thickness of at least 10 times thicker than those of flexure joints. In this work, $l$ represents the flexure joint length and $L$ represents the rigid-link length. From Fig. 2, it shows that all compliant joint modules are formed by different flexure configurations:

(a) the active prismatic compliant module is formed by a flexure configuration where the rigid-link length is at least 10 times more than the flexure length, i.e., $L \gg l$, to achieve displacement amplification purposes,
(b) the spatial compliant joint module is formed by a flexure configuration where the rigid-link is equal or shorter than the flexure length, i.e., $0 \leq L \leq l$, to withstand high external loading yet achieve substantial rotation motions.

3. Semi-analytic model: a generic approximation model for analyzing any form of flexure configuration

Designing each of these compliant joint modules often goes through an iterative process of changing the dimensions of the rigid-links or the beam-based flexure joints to achieve desired motion stiffness and off-axis stiffness within a given size constraint. In addition, various forms of flexure configurations will also cause the vertical end load to pose different loading conditions on the beam-based flexure joints. Although various modeling approaches [14–17] were proposed to analyze large nonlinear deflection of beam-based flexure joints, pseudo-rigid-body (PRB) model [18], which uses a torsional spring to represent the stiffness of a flexure joint, simplifies the modeling of a compliant mechanism.

Fig. 1. Proposed 3-DOF FPM articulated directly from the 3PPS parallel-kinematics configuration.

Fig. 2. Different flexure configurations for both (a) prismatic and (b) spatial compliant joint modules.
However, different flexure configurations require different forms of PRB models and coefficients. Thus, selecting a suitable PRB model and re-modeling the flexure configuration based on the selected PRB model requirements also become an iterative process during the design stage. Any misjudgment and inappropriate selection of PRB models often lead to inaccurate analysis because such an approximation technique cannot provide a simple and generic solution for all forms of flexure configurations. In this paper, a semi-analytic model [19] that offers a generic, simple and quick solution for analyzing large nonlinear deflection, and deflection stiffness of any form of flexure configuration is used to design each proposed compliant joint module. With all derivations being presented in the previous work [19], the semi-analytic model states that the deflection of a flexure configuration is given as

\[ \delta = \left( L + \frac{l}{2\sigma} \right) \sin \alpha \]  

(1)

where \( \alpha \) represents the deflection angle and \( \sigma \) is a Sine function, i.e., \( \sigma = \sin \alpha \).

The resultant parasitic motion, \( \delta_p \), normal to the deflection axis is given as

\[ \delta_p = \left( L + \frac{l}{2} \right) - \left( L + \frac{l}{2\sigma} \right) \cos \alpha \]  

(2)

The loading force, \( F \), is expressed as

\[ F = \frac{El\alpha}{\left[ L + \left( \rho l / 2 \sigma \right) \right] \sin((\pi/2) - \alpha)} \]  

(3)

where \( E \) and \( I \) represents the Young’s Modulus and second moment of area of the flexure joint respectively, while \( \rho \) is

\[ \rho = \frac{l\sqrt{1+L}}{1+L} \]  

(4)

The maximum bending stress, \( \sigma_{\text{max}} \), is given as

\[ \sigma_{\text{max}} = \frac{F(h/2)}{(l/2 + (L + l/2\sigma) \cos \alpha)(h/2)} \]  

(5)

where \( h \) represents the thickness of the flexure joint.

(Note: Termed as semi-analytic model since deflection or loading force can be solved directly with a given deflection angle while solving deflection angle based on a known deflection or loading force would require iterative root-finding procedures, e.g., Newton–Raphson method etc.)

3.1. Active prismatic compliant joint module

As mentioned in Section 2, each active compliant prismatic joint module is represented by a FELA. In general, FELA comprises of a large thrust force electromagnetic driving module (EDM) and a pair of flexure-based bearings, which guides the moving-coil translator of the EDM, to deliver large stroke with nanometric positioning resolution [13]. In this work, the active compliant prismatic joint resembles a FELA except the flexure-based bearings will be constructed based on a compound linear spring configuration as shown in Fig. 3a. Here, the moving-coil translator is attached to the moving platforms of a pair of compound linear spring compliant modules. In general, a compound linear spring configuration comprises of two linear spring configurations connected in series while each linear spring configuration is formed by a pair of parallel limbs as shown in Fig. 3b.

3.1.1. Translation stiffness along the z-axis

Translation stiffness along the z-axis is the major design consideration in designing the active prismatic compliant joint module. From Fig. 2a, each limb comprises of two beam-based flexure joints connected by a rigid-link. Hence, each limb can be represented by two identical flexure configurations where each beam-based flexure joint is represented by a revolute joint with a torsional spring attached to it (Fig. 3c). Here, the deflection stiffness for one flexure configuration is expressed as

\[ K_{z, FC} = \frac{2F_z}{\Delta z} \]  

(6)

where \( \Delta z \) represents the desired displacement and the driving force, \( F_z \), is obtained from Eq. (3) using \( l=I_{yy} \) and deflection angle, \( \alpha \), derived from Eq. (1) based on \( \delta = \Delta z/2 \) and \( L = \Delta z/2 \).

As two identical flexure configurations connected in series forms each limb, the deflection stiffness along the z-axis of each limb, \( K_{z, FC} \), is half of each flexure configuration, i.e., \( K_{z, FC} = K_{z, FC}/2 \). Due to the translation stiffness of a compound linear spring, \( K_{CLS} \), being similar to one limb, i.e., \( K_{z, FC} = K_{CLS} \), the translation stiffness along the z-axis for a pair of compound linear spring compliant modules is

\[ K_{AP, z} = 2K_{CLS} = K_{FC}^{2z} \]  

(7)

With \( \Delta z \) being achieved by two linear springs connected in series, the maximum bending stress is also shared between both linear springs. Hence, the maximum bending stress of each compound linear spring compliant module is given as

\[ \sigma_{AP, z} = \frac{\sigma_{\text{max}}}{2} \]  

(8)

where \( \sigma_{\text{max}} \) is determined from Eq. (5) based on \( F=F_z \), \( L = \Delta z/2 \), \( I=I_{yy} \), and \( \alpha \) that delivers \( \Delta z/2 \).

3.1.2. Evaluating the semi-analytic model

Over the past half a decade, a couple of analytical approaches based on the classical small-deflection (S-D) theory,
approximation approach (A–A), and PRB approach (See Appendix A–C) were introduced to model flexure-based mechanisms [20,18]. These analytical models are extremely useful during the initial design stage as varying design parameters such as the thickness, length, and width of both beam-based flexure and rigid-link will have significant influence on the stiffness and bending stress. For such an iterative and time-consuming design process, it is important that these analytical models are quick, accurate, and generic. In this section, the above mentioned analytical models including the proposed semi-analytic (S–A) model are used to analyze the stiffness and bending stress of a compound linear spring configuration. Here, its rigid-link length, L, varies from 2.5 mm, 5 mm, 10 mm, 20 mm, to 50 mm. For each variation of L, FEM analysis is also conducted through ANSYS 10 with SHELL63 element. (Note: finest meshing option is selected in the quick mesh tool provided by ANSYS.) Assuming that the FEM analysis is accurate, the numerical solution of the stiffness and bending stress are used as benchmark for evaluating the accuracy and robustness of each analytical model due to the variation of L. Except for L, all remaining parameters are fixed, i.e., the flexure length, l of 5 mm, the flexure thickness, h, of 0.3 mm, the rigid-link thickness of 6 mm, and an overall width, b, of 20 mm. The deviations in the stiffness and bending stress predictions between each analytical model and FEM analysis are listed in Tables 1 and 2, respectively.

Assuming that the FEM analysis is accurate, the S–D model fairs poorly in both stiffness and bending stress predictions when L < l. From Tables 1 and 2, deviations are well above 20% when compared against the stiffness and stress predicted by the FEM analysis. Accuracy of its predictions increases as the ratio of L/l increases and it is clear that S–D model is accurate if L > l. In contrast to S–D model, A–A model is accurate in stiffness prediction when L < l, especially when L = l, while its accuracy in stiffness prediction decreases as the ratio of L/l decreases. Above all, its stress prediction is accurate and inconsistent through the variation of L. The PRB model, which has similar characteristic as S–D model, fairs the worst among these analytical models when L < l. This is true given in literature that K = El/l only works well when L > l. Yet it is well-known that an elementary rod flexure joint has a 5-DOF motion and only by keeping the length short would allow researchers to treat it as a compliant spherical joint that delivers three rotation motions. Unfortunately, an elementary rod flexure joint runs into its elastic deformation limit when large deflections in multi-DOF is required for this proposed FPM. Although new forms of compliant joints with larger deflection or rotation motion have been presented [21–23], these compliant joints have less than 2-DOF motion and possess high driving stiffness.

In this work, a 5-DOF spatial compliant joint module, which offers larger deflection and rotation with lower driving stiffness, is introduced (Fig. 4). The design concept of this compliant joint relies very much on the basic motions of an elementary beam-based flexure joint. From past literature [20], an elementary beam-based flexure joint has 3-DOF motion, which include a deflection, an angular rotation and a torsional motion. Instead of avoiding the summary, this evaluation shows that the S–A model is a simple, accurate, and generic model for analyzing all flexure configurations.

### 3.1.3. Design parameters selection

With the EDM of the FELA capable of producing 60N/Amp, the proposed active prismatic compliant module must use less than 10N to achieve ±2.55 mm so that each FELA can deliver a continuous output force of 50N/Amp. With safety factor being considered, the compliant module is designed to achieve a maximum displacement limit of ±3 mm within the material’s maximum yield strength. In this work, the aluminum, i.e., 7075–T6 series, will be used to develop all the compliant modules. This series has a Young’s Modulus of 71 GPa and a maximum yield strength of 500 MPa. Although the selection of design parameters is an iterative and time-consuming design process, the proposed S–A model accelerated the process by offering quick and accurate analyses to determine the design parameters that define major stiffness characteristics, i.e., $K_{AP}$ and $\sigma_{AP}$, via Eqs. (7) and (8). After several iterations, the length of the rigid-link is selected as 35 mm, the length and thickness of the beam-based flexure joint are identified as 3 mm, and 0.3 mm respectively. The overall width is chosen to be 20 mm. Based on the selected design parameters, the translation stiffness of the proposed active prismatic compliant joint module is 2950 N/m, Hence, the expected driving force is 8.85 N to deliver ±3 mm within maximum bending stress of 151 MPa.

### 3.2. Passive spatial compliant joint module

Using an elementary rod flexure joint has been a popular approach to realize the rotation motions delivered from a conventional bearing-based spherical joint [9,12]. Yet it is well-known that an elementary rod flexure joint has a 5-DOF motion and only by keeping the length short would allow researchers to treat it as a compliant spherical joint that delivers three rotation motions. Unfortunately, an elementary rod flexure joint runs into its elastic deformation limit when large deflections in multi-DOF is required for this proposed FPM. Although new forms of compliant joints with larger deflection or rotation motion have been presented [21–23], these compliant joints have less than 2-DOF motion and possess high driving stiffness.

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### Table 1

Deviations between each analytical model and FEM analysis in predicting the translation stiffness of a compound linear spring with f fixed at 5 mm and a variation in L.

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>S–D model error (%)</th>
<th>A–A model error (%)</th>
<th>PRB model error (%)</th>
<th>S–A model error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>18.83</td>
<td>2.66</td>
<td>258.42</td>
<td>8.13</td>
</tr>
<tr>
<td>5</td>
<td>11.46</td>
<td>0.92</td>
<td>122.43</td>
<td>4.73</td>
</tr>
<tr>
<td>10</td>
<td>6.34</td>
<td>10.29</td>
<td>59.27</td>
<td>2.87</td>
</tr>
<tr>
<td>20</td>
<td>2.40</td>
<td>28.00</td>
<td>29.56</td>
<td>2.27</td>
</tr>
<tr>
<td>50</td>
<td>2.31</td>
<td>57.00</td>
<td>12.59</td>
<td>2.03</td>
</tr>
</tbody>
</table>

### Table 2

Deviations between each analytical model and FEM analysis in predicting the maximum bending stress of each flexure joint within a compound linear spring at displacement of 3 mm with f fixed at 5 mm and a variation in L.

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>S–D model error (%)</th>
<th>A–A model error (%)</th>
<th>PRB model error (%)</th>
<th>S–A model error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>24.60</td>
<td>32.04</td>
<td>1669.72</td>
<td>10.09</td>
</tr>
<tr>
<td>5</td>
<td>22.14</td>
<td>22.14</td>
<td>582.48</td>
<td>6.81</td>
</tr>
<tr>
<td>10</td>
<td>18.08</td>
<td>7.69</td>
<td>260.00</td>
<td>4.23</td>
</tr>
<tr>
<td>20</td>
<td>12.33</td>
<td>9.59</td>
<td>92.47</td>
<td>2.74</td>
</tr>
<tr>
<td>50</td>
<td>5.99</td>
<td>29.29</td>
<td>17.31</td>
<td>2.10</td>
</tr>
</tbody>
</table>
deflection motion, all these basic motions will be considered during the development of this spatial compliant joint module. In general, the module consists of two segments. With respect to its local coordinate frame, Segment 1 is formed by a pair of beam-based flexure joints and delivers the deflection along the x-axis, \( \Delta_x \), the rotation about the y-axis, \( \Psi_y \), and the torsional motion about the z-axis, \( \Psi_z \). Next, Segment 2 is placed orthogonally to Segment 2, which is also formed by another pair of beam-based flexure joints to deliver the deflection along the y-axis, \( \Delta_y \), the rotation about the x-axis, \( \Psi_x \), and also the rotation about the z-axis, \( \Psi_z \).

### 3.2.1. Nonlinear deflection stiffness along the x- or y-axis

Segment 1 or 2 is formed by a pair of beam-based flexure joints connected to a rigid member for mounting purposes. From Fig. 2b, the flexure configuration of Segment 1 and 2 has the rigid-link length being equal or shorter than the flexure joint length, i.e., \( 0 < L < L \). Using the proposed S-A model, the nonlinear deflection stiffness along the x- or y-axis of the spatial compliant joint module is

\[
K_{pS}^x = 2 \frac{F_n}{\Delta_n}
\]

(9)

where, \( \Delta_n \) represents the desired deflection with \( n \) denotes x- or y-axis. The driving force, \( F_n \), is obtained from Eq. (3) using deflection angle, \( \alpha \), derived from Eq. (1).

In addition, the nonlinear parasitic motion along the z-axis, \( \delta_z \), can be obtained from Eq. (2). However, the dominant deviation along the z-axis comes from the amplification, \( \Delta_z \), due to the radius of the spatial compliant joint module, \( r \) (Fig. 5b). Hence, stiffness along the z-axis is expressed as

\[
K_{pS}^z = 2 \frac{F_n}{\Delta_z} = 2 \frac{F_n}{r \sin \alpha}
\]

(10)

Subsequently, the maximum bending stress, \( \sigma_{pS}^{EMI_{k}} \), is obtained from Eq. (5). (Note: \( I = I_{xx} \) for \( \Delta_x \) and \( I = I_{yy} \) for \( \Delta_y \)).

### 3.2.2. Nonlinear angular stiffness about the x- or y-axis

The rotation about the x- or y-axis from respective Segment 1 or 2 can be obtained based on Bernoulli–Euler law (See Appendix D). Based on \( K = E/l \), the nonlinear angular stiffness about the x- or y-axis of the spatial compliant joint module is

\[
K_{pS}^{\Psi_n} = 2 \frac{M_n}{\Psi_n} = \frac{E l}{r}
\]

(11)

where \( M_n = F_n \times r \). (Note: \( I = I_{xx} \) for \( \Psi_x \) and \( I = I_{yy} \) for \( \Psi_y \)).

Subsequently, the maximum bending stress, \( \sigma_{pS}^{Mn} \), can be obtained from Eq. (30). From Fig. 5c, \( \Psi_z \), derived from Eq. (11) can be used to determine the parasitic stiffness along the z-axis due to external moment in the form of

\[
K_{pS}^{Dm_n} = 2 \frac{M_n}{\Delta_z} = 2 \frac{M_n}{r \sin \Psi_n}
\]

(12)

From Fig. 5c, the nonlinear parasitic motion along the x- or y-axis due to the external moment is written as

\[
\Delta_n = \delta + \Delta_\ell = \ell \left( \frac{1 - \cos \Psi_n}{\Psi_n} \right) + L \sin \Psi_n
\]

(13)

Here, \( \delta \) represents the nonlinear deflection of the beam-based flexure joint obtained from Eq. (28) (see Appendix C) and \( \Delta_\ell \) is the amplification due to the rigid-link.

### 3.2.3. Axial and critical loading in the z-axis

When subjected to an axial force, \( P \), the axial stiffness of Segments 1 and 2, which are connected in series, represents the overall the axial stiffness, i.e., \( K_{pS}^{Dp} = EA/l \), where \( A \) represents the cross-sectional area of a beam-based flexure joint. The stress due to the axial force, \( \sigma_{pS}^{Dp} \), is simply \( \sigma_{pS}^{Dp} = P/A \). Based on past literature [24], the critical load can be written as \( P_{cr} = \pi E l / (k^2) \). Here, \( k \) was selected as 2 with the assumptions that the beam-based flexure joints are in guided-free, guided-hinge or clamped-free conditions.

### 3.2.4. Angular stiffness about the z-axis

The angular motion caused by an external moment can be considered as force couples acting on the beam-based flexure joints (Fig. 6). In such a condition, the angular stiffness is affected by the internal torsion of each beam-based flexure and the torsional deflection resistance of a pair of flexure with a spacing in-between. Thus, the combination of the internal torsion stiffness, \( K_{pS}^{Dm_z} \), and overall torsional deflection stiffness, \( K_{pS}^{EMI_{k}} \), form the overall torsional stiffness about the z-axis of the proposed spatial compliant joint module

\[
K_{pS}^{\Psi_n} = 2K_{pS}^{Dm_z} + K_{pS}^{EMI_{k}}
\]

(14)
where the applied moment is generalized as $M_z = F \times r$ and $r = b + D/2$ (refer to Appendix E for detailed derivations).

The stress concentration in the beam-based flexure joints due to the applied moment is formed by the combination of the bending stress and the shear stress. Based on $\Psi$, derived from Eq. (14), the planar angular motion about the $z$-axis is $\Delta_{SOH} = r\Psi_z$. Hence, the bending stress, $\sigma_{\text{bending}}$, is obtained from Eq. (5) based on $F$ and $\alpha$ obtained from Eqs. (1) and (5) using $\Delta_{SOH}$ where $m$ denotes Segment 1 or 2. The overall angular motion about the $z$-axis is given as $\Delta_{PS} = \Delta_{S1} + \Delta_{S2}$. In 2D plane, the shear stress, $\sigma_{\text{shear}}$, is equivalent to the torsional shear stress, i.e., $\sigma_{\text{shear}} = \tau_{\text{max}}$. Hence, the total stress experience by the proposed spatial compliant joint module is

$$\sigma_{FS} = \sigma_{\text{bending}} + \sigma_{\text{shear}} \quad (15)$$

where $\sigma_{\text{shear}}$ is obtained from Eq. (32) with $\Omega = 2\Delta_{SOH}/b$.

### 3.2.5. Pure translation deflection stiffness along the x- or y-axis

As all three spatial compliant joint modules will be connected in parallel via the mobile platform, such a constrained condition forces each of these modules to deliver a pure translation deflection as shown in Fig. 7. To predict the pure translation stiffness, the analytical modeling based on the S-A model was conducted with the translation force, $F$, acting on the tip of the beam-based flexure joint (Fig. 7a).

To deliver a pure translation deflection, the beam-based flexure joint with a length, $l$, will deflect in "S"-like shape and can be represented as two identical cantilever beams with length, $l/2$ as shown in Fig. 7b. The translation motion, $\Delta_0$, becomes twice of the deflection of each cantilever beam, i.e., $\Delta_0/2$. Thus, the pure translation stiffness of a cantilever beam is

$$K_{\text{beam}} = \frac{2F}{\Delta} \quad (16)$$

where driving force, $F$, is obtained from Eq. (3) using $l = l_z$ and deflection angle, $\alpha$, derived from Eq. (1) based on $\delta = \Delta/2$, $l = l/2$, and $L = 0$.

With each beam-based flexure joint being formed by two identical cantilever beam connected in series, the translation stiffness of each beam-based flexure joint, $K_{l}^k$, is half of a cantilever beam, i.e., $K_{l}^k = K_{\text{beam}}/2$. Subsequently, the pure translational stiffness along the x- or y-axis of a proposed passive spatial compliant joint module is given as

$$K_{FS}^{\Delta_{FS}} = 2K_{l}^k \quad (17)$$

The pure translation stiffness along the x- or y-axis will be essential for the stiffness modeling of the proposed FPM, which will be presented in Part II. For individual spatial compliant joint module analysis, the presented analytical models in Section 3.2.1 will be used to analyze the deflection stiffness along the x- or y-axis under unconstrained condition.

### 3.2.6. Design parameters selection

All stiffness and stress analytical models presented in previously are used to determine the design parameters of the proposed spatial compliant joint module during the iterative design process. Here, the radius was selected as 17.5 mm, the rigid-link length as 15 mm and the beam-based flexure length, width, and thickness as 5 mm, 10 mm, and 0.5 mm, respectively. Based on the desired imprinting force of 200 N, each spatial compliant joint module should withstand an axial loading of at least 100 N. Based on the axial stiffness and critical loading presented in Section 3.2.3, the analytical results suggest that with 0.5 mm thickness, the critical load of these flexure joints is 730 N and 100 N sharing should deliver a deformation of 117 mm. All other parameters were selected based on a main goal of achieving lowest deflection and torsional stiffness to facilitate the proposed FPM to deliver large workspace.

Assuming FEM analysis is accurate, all predicted stiffness and maximum stress concentration of this compliant module are evaluated via ANSYS10 with SOLID93 element. (Note: finest meshing option is selected in the quick mesh tool provided by ANSYS.) For nonlinear deflection analyses, large displacement analysis option is used with sub-step of 5 over 1000 iteration steps. The nonlinear deflections predicted by the FEM were converted into the rotation motions using $\Psi = \sin^{-1}(\Delta r)$ before comparing with the rotation motions predicted from the presented analytical models. In this work, the proposed FPM would also have a safety of limit of $\pm3^\circ$ orientation motion. Hence, each spatial compliant joint module must also deliver similar magnitude of rotation motion along the x- and y-axes when subjected to a moment loading about the x- or y-axis. Table 3 lists the predicted rotation motion about the x- or y-axis and bending stress from both analytical, and FEM analyses. It shows that the deviations between the prediction on the rotation motion and bending stress from both analyses are less than 4%. Both analyses suggested that at least $\pm3^\circ$ can be achieved at maximum bending stress of 210MPa based on the selected design parameters.

The predicted rotation motions about the z-axis and bending stress when subjected to a moment loading about the z-axis are listed in Table 4. It shows that the deviations between the analytical and FEA analyses are less than 1%. Finally, the predictions on the nonlinear deflection and bending stress due to a tangential force along the x- or y-axis are listed in Table 5. For predictions on the nonlinear deflection along the x- or y-axis, the deviations between the analytical and FEM analyses are not more than 4.2%. For predictions on the parasitic deflection along the z-axis and the bending stress, the deviations between both analyses are less than 3.4% and 1.8% respectively. In summary, the conducted evaluations

<table>
<thead>
<tr>
<th>Moment (Nm)</th>
<th>Analytical predictions</th>
<th>FEM predictions</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation angle, $\psi_{\Delta z}$ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.035</td>
<td>0.678</td>
<td>0.707</td>
<td>4.17</td>
</tr>
<tr>
<td>0.070</td>
<td>1.356</td>
<td>1.411</td>
<td>3.95</td>
</tr>
<tr>
<td>0.105</td>
<td>2.034</td>
<td>2.119</td>
<td>4.02</td>
</tr>
<tr>
<td>0.140</td>
<td>2.712</td>
<td>2.823</td>
<td>3.95</td>
</tr>
<tr>
<td>0.175</td>
<td>3.391</td>
<td>3.332</td>
<td>3.99</td>
</tr>
<tr>
<td>Bending stress (MPa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.035</td>
<td>42.0</td>
<td>42.5</td>
<td>1.18</td>
</tr>
<tr>
<td>0.070</td>
<td>84.0</td>
<td>85.0</td>
<td>1.18</td>
</tr>
<tr>
<td>0.105</td>
<td>126.0</td>
<td>128.0</td>
<td>1.56</td>
</tr>
<tr>
<td>0.140</td>
<td>168.0</td>
<td>170.0</td>
<td>1.18</td>
</tr>
<tr>
<td>0.175</td>
<td>210.0</td>
<td>213.0</td>
<td>1.41</td>
</tr>
</tbody>
</table>
Table 4
Analytical and FEM predictions on the rotation angle and stress due to external moment about the z-axis.

<table>
<thead>
<tr>
<th>Moment (Nm)</th>
<th>Analytical predictions</th>
<th>FEM predictions</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation angle, $\psi_1$ (deg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.75 x 10^-2</td>
<td>0.155</td>
<td>0.155</td>
<td>0.37</td>
</tr>
<tr>
<td>1.75 x 10^-1</td>
<td>0.309</td>
<td>0.310</td>
<td>0.37</td>
</tr>
<tr>
<td>2.63 x 10^-1</td>
<td>0.464</td>
<td>0.466</td>
<td>0.37</td>
</tr>
<tr>
<td>3.50 x 10^-1</td>
<td>0.619</td>
<td>0.621</td>
<td>0.37</td>
</tr>
<tr>
<td>4.38 x 10^-1</td>
<td>0.773</td>
<td>0.776</td>
<td>0.37</td>
</tr>
<tr>
<td>Bending stress (MPa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.75 x 10^-2</td>
<td>50.0</td>
<td>50.3</td>
<td>0.60</td>
</tr>
<tr>
<td>1.75 x 10^-1</td>
<td>100</td>
<td>101</td>
<td>1.00</td>
</tr>
<tr>
<td>2.63 x 10^-1</td>
<td>150</td>
<td>151</td>
<td>0.67</td>
</tr>
<tr>
<td>3.50 x 10^-1</td>
<td>200</td>
<td>201</td>
<td>0.50</td>
</tr>
<tr>
<td>4.38 x 10^-1</td>
<td>250</td>
<td>251</td>
<td>0.40</td>
</tr>
</tbody>
</table>

suggest that the presented analytical stiffness and stress models are accurate.

4. Prototype of the proposed 3-DOF FPM

With all the predicted stiffness and stress concentrations from the presented analytical models verified by FEM analyses, a prototype of the proposed 3-DOF FPM was developed as shown in Fig. 8. Here, all the proposed compliant joint modules were monolithically-cut to minimize assembly errors. As mentioned previously, Aluminum 7075-T6 was selected for developing these modules due to lower density and Young’s Modulus as compared to other alloy materials. Wire-EDM process was used to fabricate these monolithic compliant joint modules with a controlled dimension tolerance of ±20 μm

5. Experimental investigations on compliant modules

5.1. Translation stiffness of the active prismatic compliant joint module

An evaluation was conducted on the module to determine the translation stiffness. In this evaluation, an external Voice-Coil (VC) actuator, which is attached with a 6-DOF Force/Torque (F/T) sensor (model: ATI MINI40; resolution: 0.01 N), is connected to the coil-holder as shown in Fig. 9. By energizing the VC actuator, a force is generated to drive the coil-holder while the displacement of the module is registered by an optical linear encoder (model: MicroE-Systems; resolution: 20 nm/count) and the generated force is measured by the F/T sensor. Most importantly, the moving air-core coil, which is attached to coil-holder via the F/T sensor, provides a non-contact actuation that increases the accuracy of this experimental setup.

All experimental data are plotted against the analytical results obtained from the S-A modeling in Fig. 10. Although the translation stiffness changes according to displacement for large displacement operation, the comparison shows that the active prismatic compliant joint module has an average translation stiffness of 3 kN/m and is well-predicted by the S-A modeling conducted in Section 3.1.3, which predicted a translation stiffness of 2.95 kN/m. The mean deviations between the experimental and analytical results obtained from S-A, PRB, and S-D modeling are plotted in Fig. 11. It shows that the S-A model has a maximum mean deviation of 4.60% in approximating the translation stiffness. On the other hand, the PRB model registered a maximum mean deviation of 9.86% while the S-D model has a maximum mean deviation of 4.62% in approximating the translation stiffness.

The trends of the mean deviations from all three analytical models are also very interesting. From Fig. 11, it shows that the mean deviations from the comparisons of translation stiffness between the experimental and PRB model decreases as the displacement increases. Such a trend suggests that the PRB model is more effective at larger displacement prediction for such a flexure

Fig. 8. A zero-torsion $\theta_x$-$\theta_y$-Z FPM prototype.

Fig. 9. Translation stiffness evaluation of an active compliant prismatic joint.
configuration, i.e., $l \gg l$. At a displacement of 3 mm, the PRB model predicts a driving force that is much closer to the experimental result as compared to the S-A and S-D models. On the other hand, both S-A and S-D models are effective in predicting the translation stiffness throughout the entire displacement of the active prismatic compliant joint module with a maximum mean deviation of 5%. This experimental investigation also demonstrates that either the S-A model or the S-D model is suitable for predicting the translation stiffness of the compliant joint module articulated by a double compound linear spring with a flexure configuration of $l \gg l$.

5.2. Deflection stiffness of the passive spatial compliant joint module

In this work, the experimental investigations are conducted on the deflection stiffness along the $x$- or $y$-axis of the passive spatial compliant joint module to evaluate the accuracy of the proposed S-A model. The entire experimental setup is shown in Fig. 12. In the setup, Segment 1 of the joint module was tied to a nylon string, which was used to carry the weights to generate deflections along the $x$-axis from the joint module. Based on the analytical studies conducted in Section 3.2.1, the height displacement of Segment 1 is almost proportional to the deflection along the $x$-axis. Hence, a laser displacement sensor (KEYENCE, model: LJ-G030, resolution: 1 μm) was used to record the deflection along the $x$-axis by measuring the height displacement of Segment 1. During the experiment, increasing the weight increases the height displacement of Segment 1.

All experimental and analytical results are plotted in Fig. 13. It shows that the spatial compliant joint module has a linear height displacement stiffness of 8 kN/m. The maximum mean deviation between the experimental and analytical results is $\sim 2\%$. As mentioned, the amplitude of both deflection and height displacement is similar. Hence, this investigation also shows that the prediction of the S-A model offers accurate approximations on the deflection stiffness along the $x$-axis of the developed spatial compliant joint module. As the beam-based flexure joints of Segment 1 and 2 have similar dimensions, it is reasonable to assume that the deflection stiffness along the $y$-axis (contribute from Segment 2) was also well-predicted by the S-A model.

5.3. Angular stiffness about the $z$-axis of the passive spatial compliant joint module

The angular stiffness about the $z$-axis of the spatial compliant joint module was also investigated and the entire experimental setup is shown in Fig. 14. A micro-stepping motor was connected...
The spatial compliant joint module has a linear relationship between the deflection angle and the applied torque about the z-axis. Comparing with the analytical results obtained from Section 3.2.4 shows that the angular stiffness about the z-axis is well predicted by the presented analytical model with a maximum mean deviation of 2%. Hence, the comparison between the experimental and analytical results suggest that the analytical model can be used for quick and accurate predictions on the angular stiffness about the z-axis for such kind of spatial compliant joint modules.

6. Conclusion

A $\theta_x-\theta_y-Z$ motion FPM is presented in this paper. Based on a novel 3PPS parallel-kinematic architecture, the conversion of all the kinematic joints from this architecture to the appropriate compliant joint modules were aided by the proposed S-A modeling approach. Based on this new S-A model, the analytical predictions on stiffness and stress concentration of each proposed compliant joint module are presented. The comparisons with predictions made by FEM analyses suggest that the proposed S-A model offers a simple, accurate, and generic solution for analyzing any form of flexure configuration. Subsequently, accurate predictions using the S-A model also lead to accurate approximations on parasitic motions from each compliant joint and accelerate the intuitive and time-consuming design process. Experimental investigations have shown that the stiffness characteristics of the compliant joint modules are well-predicted by the S-A model. Comparing with experimental results show that the S-A model predicted the translation stiffness of the active prismatic compliant joint module accurately with a maximum mean deviation of 4.60%. In addition, the deflection stiffness of the passive spatial compliant joint module is also accurately predicted by the S-A model with a maximum mean deviation of 2%. With the complete modeling and analysis of the proposed compliant joint modules presented in this paper, Part II presents the stiffness modeling of the proposed FPM, the experimental investigations, and the preliminary UV-NIL results.

Appendix A. Classical solution for a compound linear spring configuration based on small deflection theory

The translation stiffness of a single linear spring system can be expressed as [20]

$$K_{AF} = \frac{4K_{L,0}}{(L')^2}$$

(18)

where $L' = L + l$ as shown in Fig. 16a. As a compound linear spring is formed by a pair of linear springs connected in series and the stiffness becomes half of a single linear spring. Based on Eq. (18)
and $K_{im} = (EI/l)$, the translation stiffness of each compound linear spring is expressed as

$$K_{AF} = \frac{F}{\Delta} = \frac{2EI}{k(L)^2}$$  \hspace{1cm} (19)

The maximum bending stress of each flexure joint of the compound linear spring is

$$\sigma_{\text{max}} = \frac{Et}{2l} \max \left[ \frac{F}{2Et} \left( \frac{\Delta}{L} \right)^2 \right] = \frac{Et \Delta}{4Et}$$  \hspace{1cm} (20)

(Note: Displacement is half of the linear spring for a double compound linear spring, i.e., $\Delta/2 \sin \alpha = \alpha$ for small angles.)

**Appendix B. Classical solution for a compound linear spring configuration based on approximation approach**

Based on past literature [25,20], the pivoting point of a cantilever hinge with a rigid-body attached to the free end subjected to a tangential force is approximated at $S = (2/3)l$ (Fig. 16b). The approximated deflection stiffness is

$$K_{AF} = \frac{3EI}{(L + l)}$$  \hspace{1cm} (21)

From Fig. 16a, each limb of a compound linear spring can be represented by two cantilever hinges with one rigid-body attached to their free end. Hence, the stiffness of each limb is

$$K_{limb} = \frac{K_{AF}}{2}$$  \hspace{1cm} (22)

Subsequently, the stiffness of a compound linear spring configuration is equivalent to the stiffness of each limb. Based on Eq. (20), the maximum bending stress is expressed as

$$\sigma_{\text{max}} = \frac{Et}{2l} \left[ \frac{F}{2Et} (L + l)^2 \right]$$  \hspace{1cm} (23)

**Appendix C. Classical pseudo-rigid-body modeling on a compound linear spring configuration**

For PRB modeling, the force-deflection relation of a compound linear spring configuration (Fig. 16a), which can be obtained through static force-moment equilibrium or virtual work method [18], is expressed as

$$F_{in} = -\frac{2K(\theta - \theta_o)}{d \sin \theta}$$  \hspace{1cm} (24)

where $F_{in}$ representing the input force, $\theta$ representing the resultant deflection angle, and $\theta_o$ representing the initial reference angle, i.e., $\theta_o = (\pi/2)$. Based on “small-length flexural pivots” of PRB model, the torsional spring constant, $K$, is $EI/l$. The resultant parasitic motion normal to the deflection is

$$\delta_p = \frac{l}{2} (L + l) \cos(\theta - \theta_o)$$  \hspace{1cm} (25)

and the maximum bending stress is given as

$$\sigma_{\text{max}} = \frac{F_{in} \delta_p (h/2)}{l}$$  \hspace{1cm} (26)

To apply PRB model effectively, it is important to understand that $K$ is only suitable for a beam-based flexure joint with a rigid-body attached at free end and $L \gg l$. $K$ is represented differently for other forms of flexure configurations or loading characteristics, and the force-deflection given in Eq. (24) also differs accordingly, thus re-modeling is required.

**Appendix D. Classical nonlinear solution for a cantilever beam subjected to moment end loading**

Bernoulli–Euler law states that the bending moment at any point of the bar is proportional to the change in the curvature

$$M = \frac{d\theta}{dx}$$  \hspace{1cm} (27)

Using the chain rule on Eq. (27) with $dy, dx$, and $ds$ been infinitesimal and integrating both sides of equations gives the deflection along the $y$-axis as

$$\frac{\delta_y}{l} = \frac{1}{\theta_y}$$  \hspace{1cm} (28)

and the deflection along the $x$-axis as

$$\frac{l - \delta_x}{l} = \sin \theta_x$$  \hspace{1cm} (29)

The maximum bending stress is given as

$$\sigma = \frac{M(h/2)}{l}$$  \hspace{1cm} (30)

where $h$ represents the thickness of cantilever beam.

**Appendix E. Derivation of angular stiffness**

From past literature [24], the torsional stiffness of a beam about its longitudinal axis is given as

$$K^{GM}_{\text{rev}} = \frac{M_z}{\omega^2} = \frac{\gamma \ Gbh^3}{\Omega h^3 - 3l}$$  \hspace{1cm} (31)

where

$$\Omega = \frac{8a^2b^2}{3a + 1.8b} \gamma = ab^3 \left\{ \frac{16}{3} - \frac{3.36b(1 - (b^4/12a^4))}{a} \right\}$$

In addition, $\tau_{\text{max}}$ can be also be expressed as [20]

$$\tau_{\text{max}} = \frac{\gamma \ G2h}{\Omega h}$$  \hspace{1cm} (32)

Based on literature [20], the stiffness of the torsional deflection resistance of a pair of beam-based flexure joints with an in-between space, $D$, is given as

$$K^{GM}_{\text{re}} = \frac{M_z}{\beta} = \frac{Ebh^3}{2l^3} D^2$$  \hspace{1cm} (33)

**References**


