

**SYLLABUS FOR ENTRANCE EXAMINATION
NANYANG TECHNOLOGICAL UNIVERSITY
FOR INTERNATIONAL STUDENTS**

A-LEVEL MATHEMATICS

STRUCTURE OF EXAMINATION PAPER

1. There will be one 2-hour paper consisting of 4 questions.
2. Each question carries 25 marks.
3. Candidates will be required to answer all 4 questions.

Electronic Calculators

1. The use of common electronic scientific calculators is **allowed**.

2. Graphic calculators will **not** be permitted.

The detailed syllabus is on the next page.

DETAILED SYLLABUS

Knowledge of the content of the Ordinary level Syllabus (or an equivalent syllabus) is assumed.

MATHEMATICS

THEME OR TOPIC	CURRICULUM OBJECTIVES
<i>Candidates should be able to:</i>	
1 Functions and graphs	<ul style="list-style-type: none"> – understand the terms function, domain, range and one-one function; – find composite functions and inverses of functions, including conditions for their existence; – understand and use the relation $(fg)^{-1} = g^{-1}f^{-1}$ where appropriate; – illustrate in graphical terms the relation between a one-one function and its inverse; – understand the relationship between a graph and an associated algebraic equation, and in particular show familiarity with the forms of the graphs of <ul style="list-style-type: none"> $y = kx^n$, where n is a positive or negative integer or a simple rational number, $ax + by = c$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (knowledge of geometrical properties of conics is not required); – understand and use the relationships between the graphs of $y = f(x)$, $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$, where a is a constant, and express the transformations involved in terms of translations, reflections and scalings; – relate the equation of a graph to its symmetries; – understand, and use in simple cases, the expression of the coordinates of a point on a curve in terms of a parameter.
2 Partial fractions	<ul style="list-style-type: none"> – recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than <ul style="list-style-type: none"> $(ax + b)(cx + d)(ex + f)$, $(ax + b)(cx + d)^2$, $(ax + b)(x^2 + c^2)$, including cases where the degree of the numerator exceeds that of the denominator.
3 Inequalities; the modulus function	<ul style="list-style-type: none"> – use properties of inequalities, and in particular understand that $x > y$ and $z > 0$ imply that $xz > yz$ while $x > y$ and $z < 0$ imply $xz < yz$; – find the solution set of inequalities that are reducible to the form $f(x) > 0$, where $f(x)$ can be factorised, and illustrate such solutions graphically;

- understand the meaning of $|x|$ and sketch the graph of functions of the form $y = |ax + b|$;
- use relations such as $|x - a| < b \Leftrightarrow a - b < x < a + b$ and $|a| = |b| \Leftrightarrow a^2 = b^2$ in the course of solving equations and inequalities.

- 4 Logarithmic and exponential functions
- recall and use the laws of logarithms (including change of base) and sketch graphs of simple logarithmic and exponential functions;
 - recall and use the definition $a^x = e^{x \ln a}$;
 - use logarithms to solve equations reducible to the form $a^x = b$, and similar inequalities.

- 5 Sequences and series
- understand the idea of a sequence of terms, and use notations such as u_n to denote the n th term of a sequence;
 - recognise arithmetic and geometric progressions;
 - use formulae for the n th term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions;
 - recall the condition for convergence of a geometric series, and use the formula for the sum to infinity of a convergent geometric series;
 - use \sum notation;
 - use the binomial theorem to expand $(a + b)^n$, where n is a positive integer;
 - use the binomial theorem to expand $(1 + x)^n$, where n is rational, and recall the condition $|x| < 1$ for the validity of this expansion;
 - recognise and use the notations $n!$ (with $0! = 1$) and $\binom{n}{r}$.

- 6 Permutations and Combinations
- understand the terms 'permutation' and 'combination';
 - solve problems involving arrangements (of objects in a line or in a circle), including those involving
 - repetition (e.g. the number of ways of arranging the letters of the word NEEDLESS),
 - restriction (e.g. the number of ways several people can stand in a line if 2 particular people must – or must not – stand next to each other).

- 7 Trigonometry
- use the sine and cosine formulae;
 - calculate the angle between a line and a plane, the angle between two planes, and the angle between two skew lines in simple cases.

- 8 Trigonometrical functions
- understand the definition of the six trigonometrical functions for angles of any magnitude;
 - recall and use the exact values of trigonometrical functions of 30° , 45° and 60° , e.g. $\cos 30^\circ = \frac{1}{2}\sqrt{3}$;
 - use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometrical relations;
 - relate the periodicity and symmetries of the sine, cosine and tangent functions to the form of their graphs, and use the concepts of periodicity and/or symmetry in relation to these functions and their inverses;
 - use trigonometrical identities for the simplification and exact evaluation of expressions, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \text{ and } \frac{\cos \theta}{\sin \theta} = \cot \theta,$$

$$\sin^2 \theta + \cos^2 \theta = 1 \text{ and equivalent statements,}$$
 the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$,
 the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$,
 the formulae for $\sin A \pm \sin B$ and $\cos A \pm \cos B$,
 the expression of $a \cos \theta + b \sin \theta$ in the forms $R \cos(\theta \pm \alpha)$ and $R \sin(\theta \pm \alpha)$;
 - find the general solution of simple trigonometrical equations, including graphical interpretation;
 - use the small-angle approximations $\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$.

- 9 Differentiation
- understand the idea of a limit and the derivative defined as a limit, including geometrical interpretation in terms of the gradient of a curve at a point as the limit of the gradient of a suitable sequence of chords;
 - use the standard notations $f'(x)$, $f''(x)$ etc., and $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ etc., for derived functions;
 - use the derivatives of x^n (for any rational n), $\sin x$, $\cos x$, $\tan x$, e^x , a^x , $\ln x$, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$; together with constant multiples, sums, differences, products, quotients and composites;
 - find and use the first derivative of a function which is defined implicitly or parametrically;
 - locate stationary points, and distinguish between maxima, minima and stationary points of inflexion (knowledge of conditions for general points of inflexion is not required);
 - find equations of tangents and normals to curves, and use information about gradients for sketching graphs;
 - solve problems involving maxima and minima, connected rates of change, small increments and approximations;
 - derive and use the first few terms of the Maclaurin series for a function.

10 Integration

- understand indefinite integration as the reverse process of differentiation;
- integrate x^n (including the case where $n = -1$), e^x , $\sin x$, $\cos x$, $\sec^2 x$, together with sums, differences and constant multiples of these, expressions involving a linear substitution (e.g. e^{2x-1}), applications involving the use of partial fractions, applications involving the use of trigonometrical identities (e.g. $\int \cos^2 x \, dx$);
- recognise an integrand of the form $\frac{kf'(x)}{f(x)}$ and integrate, e.g. $\frac{x}{x^2 + 1}$ or $\tan x$;
- integrate $\frac{1}{a^2 + x^2}$ and $\frac{1}{\sqrt{a^2 - x^2}}$;
- recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, e.g. $x \sin 2x$, $x^2 e^x$, $\ln x$;
- use the method of integration by substitution to simplify and evaluate either a definite or an indefinite integral (including simple cases in which the candidates have to select the substitution themselves, e.g. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$);
- evaluate definite integrals (including e.g. $\int_0^1 x^{-\frac{1}{2}} \, dx$ and $\int_0^\infty e^{-x} \, dx$);
- understand the idea of the area under a curve as the limit of a sum of the areas of rectangles and use simple applications of this idea;
- use integration to find plane areas and volumes of revolution in simple cases;
- use the trapezium rule to estimate the values of definite integrals, and identify the sign of the error in simple cases by graphical considerations.

11 Vectors

- use rectangular cartesian coordinates to locate points in three dimensions, and use standard notations for vectors, i.e.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \quad \vec{AB}, \quad \mathbf{a};$$

- carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms;
- use unit vectors, position vectors and displacement vectors;
- recall the definition of and calculate the magnitude of a vector and the scalar product of two vectors;
- use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors;

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- understand the significance of all the symbols used when the equation of a straight line is expressed in either of the forms

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \text{ and } \frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}, \text{ and convert equations of lines}$$

from vector to cartesian form and *vice versa*;

- solve simple problems involving finding and using either form of the equation of a line;
- use equations of lines to solve problems concerning distances, angles and intersections, and in particular
 - determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists,
 - find the perpendicular distance from a point to a line,
 - find the angle between two lines;
- use the ratio theorem in geometrical applications.

12 Mathematical induction

- understand the steps needed to carry out a proof by the method of induction;
- use the method of mathematical induction to establish a given result e.g. the sum of a finite series, or the form of an n th derivative.

13 Complex numbers

- understand the idea of a complex number, recall the meaning of the terms 'real part', 'imaginary part', 'modulus', 'argument', 'conjugate', and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;
- carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $(x + iy)$;
- recall and use the relation $zz^* = |z|^2$;
- use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;
- represent complex numbers geometrically by means of an Argand diagram;
- carry out operations of multiplication and division of two complex numbers expressed in polar form $(r(\cos \theta + i \sin \theta) = re^{i\theta})$;
- understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying, dividing two complex numbers;
- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram
(e.g. $|z - a| < k$, $|z - a| = |z - b|$, $\arg(z - a) = \alpha$, but excluding $\arg(z - a) - \arg(z - b) = \gamma$).

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- 14 Curve sketching
- understand the relationships between the graphs of $y = f(x)$, $y^2 = f(x)$ and $y = |f(x)|$;
 - determine, in simple cases, the equations of asymptotes parallel to the axes;
 - use the equation of a curve, in simple cases, to make deductions concerning symmetry or concerning any restrictions on the possible values of x and/or y that there may be;
 - sketch curves of the form $y = f(x)$, $y^2 = f(x)$ and $y = |f(x)|$; (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes).
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- 15 First order differential equations
- formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;
 - find by integration a general form of solution for a first order differential equation in which the variables are separable;
 - find the general solution of a first order linear differential equation by means of an integrating factor;
 - reduce a given first order differential equation to one in which the variables are separable or to one which is linear by means of a given simple substitution;
 - understand that the general solution of a differential equation is represented in graphical terms by a family of curves, and sketch typical members of a family in simple cases;
 - use an initial condition to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.
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- 16 Numerical methods
- locate approximately a root of an equation by means of graphical considerations and/or searching for a sign change;
 - use the method of linear interpolation to find an approximation to a root of an equation;
 - understand the idea of, and use the notation for, a sequence of approximations which converges to the root of an equation;
 - understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration to determine a root to a prescribed degree of accuracy (conditions for convergence are not included);
 - understand, in geometrical terms, the working of the Newton-Raphson method, and derive and use iterations based on this method;
 - appreciate that an iterative method may fail to converge to the required root.
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- 17 Probability
- use addition and multiplication of probabilities, as appropriate, in simple cases, and understand the representation of events by means of tree diagrams;
 - understand the meaning of mutually exclusive and independent events, and calculate and use conditional probabilities in simple cases;
 - understand and use the notations $P(A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A | B)$ and the equations $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cap B) = P(A) P(B | A) = P(B) P(A | B)$ (Bayes' Theorem).
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MATHEMATICAL NOTATION

The list which follows summarises the notation used in NTU's Entrance Examinations in Mathematics. Although primarily directed towards A level, the list also applies, where relevant, to examinations at AO-level.

1. Set Notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x: \dots\}$	the set of all x such that
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	universal set
A'	the complement of the set A
\mathbb{N}	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Z}_n	the set of integers modulo n , $\{0, 1, 2, \dots, n - 1\}$
\mathbb{Q}	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geq 0\}$
\mathbb{R}^n	the real n tuples
\mathbb{C}	the set of complex numbers
\subseteq	is a subset of
\subset	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
yRx	y is related to x by the relation R

2. Miscellaneous Symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\cong	is isomorphic to
\propto	is proportional to
$<; \ll$	is less than; is much less than
$\leq; \nlessgtr$	is less than or equal to; is not greater than
$>; \gg$	is greater than; is much greater than
$\geq; \ngtr$	is greater than or equal to; is not less than
∞	infinity

3. Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
\sqrt{a}	the positive square root of the real number a
$ a $	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$ ($0! = 1$)
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leq r \leq n$ $\frac{n(n-1)\dots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$f(x)$	the value of the function f at x
$f: A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$g \circ f, gf$	the composite function of f and g which is defined by $(g \circ f)(x)$ OR $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x; \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, n th derivatives of $f(x)$ with respect to x
$\int y dx$	indefinite integral of y with respect to x
$\int_a^b y dx$	the definite integral of y with respect to x for values of x between a and b
$\frac{\partial y}{\partial x}$	the partial derivative of y with respect to x
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

$\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$	} the circular functions
$\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	} the inverse circular functions

7. *Complex Numbers*

i	square root of -1
z	a complex number, $z = x + iy$ $= r(\cos \theta + i \sin \theta)$, $r \in \mathbb{R}_0^+$ $= re^{i\theta}$, $r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re}(x+iy) = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im}(x + iy) = y$
$ z $	the modulus of z , $ x + iy = \sqrt{(x^2 + y^2)}$, $ r(\cos \theta + i \sin \theta) = r$
$\arg z$	the argument of z , $\arg(r(\cos \theta + i \sin \theta)) = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $(x + iy)^* = x - iy$

8. *Matrices*

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the square matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$	the determinant of the square matrix \mathbf{M}

9. *Vectors*

\mathbf{a}	the vector \mathbf{a}
\vec{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} $	the magnitude of \mathbf{a}
$ \vec{AB} $	the magnitude of \vec{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10. *Probability and Statistics*

A, B, C , etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A , the event 'not A '
$P(A B)$	probability of the event A given the event B
X, Y, R , etc.	random variables
x, y, r , etc.	value of the random variables X, Y, R , etc.
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations, $x_1, x_2 \dots$ occur

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$p(x)$	the value of the probability function $P(X = x)$ of the discrete random variable X
$p_1, p_2 \dots$	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x) \dots$	the value of the probability density function of the continuous random variable X
$F(x), G(x) \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$B(n, p)$	binominal distribution, parameters n and p
$N(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}	sample mean
s^2	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample
$\text{Cov}(X, Y)$	covariance of X and Y