

**ENTRANCE EXAMINATION**

**Mathematics at A-Level (Sample)**

Time Allowed : **2 hours**

**INSTRUCTIONS**

1. This paper consists of **FIVE (5)** questions and comprises **TWO (2)** pages.
  2. Answer any **FOUR (4)** questions only.
  3. The marks are allocated at the end of each part/question.
  4. Answers will be graded for content and appropriate presentation.
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Question 1

(a) Sketch the graph of the function  $y = f(x) = \frac{x+2}{x-2}$ . Determine and show clearly on your diagram, the asymptotes, intercepts with the coordinate axes, and the coordinates of the stationary point(s), if any. (12 marks)

(b) Explain how you can obtain, from the graph of  $f$ , a sketch of the graph of the function  $y = g(x) = \left(\frac{x+2}{x-2}\right)^2$ . Show this sketch of  $g$  on the same diagram as  $f$ . In addition, sketch  $y = \frac{1}{g(x)}$ . (13 marks)

Question 2

(a) If  $f(x+x^{-1}) = x^3 + x^{-3}$ , determine the function  $f$ . (7 marks)

(b) Let  $f(x) = (m^2 - 1)x^2 + (m - 1)x + n + 2$ , ( $m \neq 1$ ), be an odd function and  $m$  and  $n$  are constants. Determine whether  $g(x) = x^m + x^n$  is an even or an odd function, or neither. (8 marks)

(c) Find the range of  $x$  for which

$$\frac{4x^2}{(1 - \sqrt{1 + 2x})^2} < 2x + 9.$$

(10 marks)

### Question 3

- (a) Ten persons,  $A, B, C, \dots, J$ , sit round a circular table. The chairs are all numbered. What is the probability that  $A$  and  $B$  sit next to each other?

(6 marks)

- (b) Use the relationship  $e^{i\theta} = \cos\theta + i\sin\theta$  to express  $\cos 5\theta$  in terms of  $\cos\theta$ .

Hence show that  $x = \cos\left(\frac{1}{10}\pi\right)$  is a root of the equation  $16x^4 - 20x^2 + 5 = 0$ .

(10 marks)

- (c) Points  $A$  and  $B$  are specified by the position vectors  $\mathbf{a}$  and  $\mathbf{b}$ . Prove that the equation of the plane bisecting the segment  $AB$  perpendicularly is

$$\mathbf{r} \cdot (\mathbf{a} - \mathbf{b}) = \frac{1}{2}(|\mathbf{a}|^2 - |\mathbf{b}|^2)$$

(9 marks)

### Question 4

- (a) By solving the equation  $\sin 2\theta = \cos 3\theta$ , find the value of  $\sin 18^\circ$ . Show your answer in the form of  $\frac{\sqrt{a+b}}{c}$ , where  $a, b$  and  $c$  are natural numbers. A numerical value from your calculator or mathematical table is NOT acceptable.

(12 marks)

- (b) Given  $(x^2 - 6x + 9)^{x^2 - 4} = 1$ , solve for all real values of  $x$ .

(7 marks)

- (c) Determine the sum  $\sum_{k=1}^{\infty} \frac{4^k + 3^{k-1} - 2^{k-2}}{5^{k+1}}$ .

(6 marks)

### Question 5

- (a) Find the volume of the solid generated by revolving the area between the curve  $y = \frac{\cos x}{x}$  and the  $x$ -axis for  $\pi/6 \leq x \leq \pi/2$  about the  $y$ -axis.

(7 marks)

- (b) Evaluate  $\int x^2 e^x dx$ .

(6 marks)

- (c) Show that  $\int_0^1 x^m (1-x)^n dx = \frac{n}{m+1} \int_0^1 x^{m+1} (1-x)^{n-1} dx$  for  $m > 0$  and  $n > 0$ . Hence, or

otherwise, show that  $\int_0^1 x^m (1-x)^n dx = \frac{m!n!}{(m+n+1)!}$

(12 marks)

**- END OF PAPER -**