NANYANG TECHNOLOGICAL UNIVERSITY
SYLLABUS FOR ENTRANCE EXAMINATION
FOR INTERNATIONAL STUDENTS

AO-LEVEL MATHEMATICS

STRUCTURE OF EXAMINATION PAPER

1. There will be one 2-hour paper consisting of 4 questions.
2. Each question carries 25 marks.
3. Candidates will be required to answer all 4 questions.

The detailed syllabus is on the next page.
Knowledge of the content of the O Level Mathematics syllabus is assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics.

<table>
<thead>
<tr>
<th>Topic / Sub-topics</th>
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<tbody>
<tr>
<td><strong>PURE MATHEMATICS</strong></td>
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<tr>
<td>1 Functions and graphs</td>
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</tbody>
</table>
| 1.1 Exponential and logarithmic functions and Graphing techniques | Include:  
- concept of function  
- use of notation such as $f(x) = x^2 + 5$  
- functions $e^x$ and $\ln x$ and their graphs  
- laws of logarithms  
- equivalence of $y = e^x$ and $x = \ln y$  
- use of a graphic calculator to graph a given function  
- characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes  
Exclude:  
- concepts of domain and range  
- the use of notation $f: x \mapsto x^2 + 5$ |
| 1.2 Equations and inequalities | Include:  
- solving simultaneous equations, one linear and one quadratic, by substitution  
- conditions for a quadratic equation to have real or equal roots  
- solving quadratic inequalities  
- conditions for $ax^2 + bx + c$ to be always positive (or always negative)  
- solving inequalities by graphical methods  
- formulating an equation from a problem situation  
- finding the numerical solution of an equation using a graphic calculator |
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| 2.1 Differentiation | Include:  
- derivative of \( f(x) \) as the gradient of the tangent to the graph of \( y = f(x) \) at a point  
- use of standard notations \( f'(x) \) and \( \frac{dy}{dx} \)  
- derivatives of \( x^n \) for any rational \( n \), \( e^x \), \( \ln x \), together with constant multiples, sums and differences  
- use of chain rule  
- graphical interpretation of \( f'(x) > 0 \), \( f'(x) = 0 \) and \( f'(x) < 0 \)  
- stationary points (local maximum and minimum points and points of inflexion)  
- finding the numerical value of a derivative at a given point using a graphic calculator  
- finding equations of tangents and normals to curves  
- solving practical problems involving differentiation  
Exclude:  
- differentiation from first principles  
- derivatives of products and quotients of functions  
- use of \( \frac{dy}{dx} = \frac{1}{dx} \frac{dy}{dx} \)  
- differentiation of functions defined implicitly or parametrically  
- finding non-stationary points of inflexion  
- problems involving small increments and approximation  
- relating the graph of \( y = f'(x) \) to the graph of \( y = f(x) \) |
| 2.2 Integration | Include:  
- integration as the reverse of differentiation  
- integration of \( x^n \), for any rational \( n \), and \( e^x \), together with constant multiples, sums and differences  
- integration of \( (ax + b)^n \), for any rational \( n \), and \( e^{(ax + b)} \)  
- definite integral as the area under a curve  
- evaluation of definite integrals  
- finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves  
- finding the numerical value of a definite integral using a graphic calculator  
Exclude:  
- definite integral as a limit of sum  
- approximation of area under a curve using the trapezium rule  
- area below the \( x \)-axis |
<table>
<thead>
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<tr>
<td><strong>STATISTICS</strong></td>
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<tr>
<td><strong>3 Probability</strong></td>
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<tr>
<td>3.1 Probability</td>
<td>Include:</td>
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<tr>
<td></td>
<td>• addition and multiplication of probabilities</td>
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<td>• mutually exclusive events and independent events</td>
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<td>• use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities</td>
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<td>• calculation of conditional probabilities in simple cases</td>
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<td>• use of $P(A') = 1 - P(A)$</td>
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<td></td>
<td>$P(A \cup B) = P(A) + P(B) - P(A \cap B)$</td>
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<td>$P(A</td>
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<td><strong>4 Binomial and normal distributions</strong></td>
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<tr>
<td>4.1 Binomial distribution</td>
<td>Include:</td>
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<td></td>
<td>• knowledge of the binomial expansion of $(a+b)^n$ for positive integer $n$</td>
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<td>• use of the notations $n!$ and $\binom{n}{r}$</td>
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<td></td>
<td>• concept of binomial distribution $B(n, p)$ and use of $B(n, p)$ as a probability model</td>
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<td></td>
<td>• use of mean and variance of a binomial distribution (without proof)</td>
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<td>• solving problems involving binomial variables</td>
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<td></td>
<td>Exclude calculation of mean and variance for other probability distributions</td>
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<td>4.2 Normal distribution</td>
<td>Include:</td>
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<td></td>
<td>• concept of a normal distribution and its mean and variance; use of $\mathcal{N}(\mu, \sigma^2)$ as a probability model</td>
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<td>• standard normal distribution</td>
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<td>• finding the value of $P(X &lt; x_i)$ given the values of $x_i$, $\mu$, $\sigma$</td>
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<td>• use of the symmetry of the normal distribution</td>
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<td>• finding a relationship between $x_i$, $\mu$, $\sigma$ given the value of $P(X &lt; x_i)$</td>
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<td>• solving problems involving normal variables</td>
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<td>• solving problems involving the use of $E(aX + b)$ and $\text{Var}(aX + b)$</td>
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<td>• solving problems involving the use of $E(aX + bY)$ and $\text{Var}(aX + bY)$, where $X$ and $Y$ are independent</td>
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<td>• normal approximation to binomial</td>
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<td>Exclude:</td>
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<td></td>
<td>• finding probability density functions and distribution functions</td>
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<td>• calculation of $E(X)$ and $\text{Var}(X)$ from other probability density functions</td>
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| 5 Sampling and hypothesis testing | Include:  
- concepts of population and sample  
- random, stratified, systematic and quota samples  
- advantages and disadvantages of the various sampling methods  
- distribution of sample means from a normal population  
- use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large  
- calculation of unbiased estimates of the population mean and variance from a sample  
- solving problems involving the sampling distribution |
| 5.1 Sampling |  
- concepts of population and sample  
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- advantages and disadvantages of the various sampling methods  
- distribution of sample means from a normal population  
- use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large  
- calculation of unbiased estimates of the population mean and variance from a sample  
- solving problems involving the sampling distribution |
| 5.2 Hypothesis testing | Include:  
- concepts of null and alternative hypotheses, test statistic, level of significance and p-value  
- tests for a population mean based on:  
  - a sample from a normal population of known variance  
  - a large sample from any population  
- 1-tail and 2-tail tests  
Exclude testing the difference between two population means |
| 6 Correlation and Regression | Include:  
- concepts of scatter diagram, correlation coefficient and linear regression  
- calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line  
- concepts of interpolation and extrapolation  
Exclude:  
- derivation of formulae  
- hypothesis tests  
- use of a square, reciprocal or logarithmic transformation to achieve linearity |
| 6.1 Correlation coefficient and linear regression | Include:  
- concepts of scatter diagram, correlation coefficient and linear regression  
- calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line  
- concepts of interpolation and extrapolation  
Exclude:  
- derivation of formulae  
- hypothesis tests  
- use of a square, reciprocal or logarithmic transformation to achieve linearity |
MATHEMATICAL NOTATION

1. Set Notation

\(\in\) is an element of
\(\notin\) is not an element of
\(\{x_1, x_2, \ldots\}\) the set with elements \(x_1, x_2, \ldots\)
\(\{x: \ldots\}\) the set of all \(x\) such that
\(n(A)\) the number of elements in set \(A\)
\(\emptyset\) the empty set
\(\mathbb{C}\) universal set
\(A'\) the complement of the set \(A\)
\(\mathbb{Z}\) the set of integers, \(\{0, \pm1, \pm2, \pm3, \ldots\}\)
\(\mathbb{Z}^+\) the set of positive integers, \(\{1, 2, 3, \ldots\}\)
\(\mathbb{Q}\) the set of rational numbers
\(\mathbb{Q}^+\) the set of positive rational numbers, \(\{x \in \mathbb{Q}: x > 0\}\)
\(\mathbb{Q}_0^+\) the set of positive rational numbers and zero, \(\{x \in \mathbb{Q}: x \geq 0\}\)
\(\mathbb{R}\) the set of real numbers
\(\mathbb{R}^+\) the set of positive real numbers, \(\{x \in \mathbb{R}: x > 0\}\)
\(\mathbb{R}_0^+\) the set of positive real numbers and zero, \(\{x \in \mathbb{R}: x \geq 0\}\)
\(\mathbb{R}^n\) the real \(n\) tuples
\(\mathbb{C}\) the set of complex numbers
\(\subseteq\) is a subset of
\(\subset\) is a proper subset of
\(\not\subset\) is not a subset of
\(\not\subset\) is not a proper subset of
\(\cup\) union
\(\cap\) intersection
\([a, b]\) the closed interval \(\{x \in \mathbb{R}: a \leq x \leq b\}\)
\([a, b)\) the interval \(\{x \in \mathbb{R}: a \leq x < b\}\)
\((a, b]\) the interval \(\{x \in \mathbb{R}: a < x \leq b\}\)
\((a, b)\) the open interval \(\{x \in \mathbb{R}: a < x < b\}\)
2. Miscellaneous Symbols

= is equal to
≠ is not equal to
≡ is identical to or is congruent to
≈ is approximately equal to
∝ is proportional to
< is less than
≤; ≥ is less than or equal to; is not greater than
> is greater than
≥; ≧ is greater than or equal to; is not less than
∞ infinity

3. Operations

\( a + b \) \( a \) plus \( b \)
\( a \)− \( b \) \( a \) minus \( b \)
\( a \times b, ab, a.b \) \( a \) multiplied by \( b \)
\( a \div b, \frac{a}{b}, a/b \) \( a \) divided by \( b \)
\( a : b \) the ratio of \( a \) to \( b \)
\( \sum_{i=1}^{n} a_i \) \( a_1 + a_2 + \ldots + a_n \)
\( \sqrt{a} \) the positive square root of the real number \( a \)
\( |a| \) the modulus of the real number \( a \)
\( n! \) \( n \) factorial for \( n \in \mathbb{Z}^+ \cup \{0\} \), \((0! = 1)\)
\( \binom{n}{r} \) the binomial coefficient \( \frac{n!}{r!(n-r)!} \), for \( n, r \in \mathbb{Z}^+ \cup \{0\}, 0 \leq r \leq n \)
\( \frac{n(n-1)...(n-r+1)}{r!} \), for \( n \in \mathbb{Q}, r \in \mathbb{Z}^+ \cup \{0\} \)
4. Functions

\[ f \]  
function \( f \)

\[ f(x) \]  
the value of the function \( f \) at \( x \)

\[ f: A \rightarrow B \]  
\( f \) is a function under which each element of set \( A \) has an image in set \( B \)

\[ f: x \rightarrow y \]  
the function \( f \) maps the element \( x \) to the element \( y \)

\[ f^{-1} \]  
the inverse of the function \( f \)

\( g \circ f, gf \)  
the composite function of \( f \) and \( g \) which is defined by \( (g \circ f)(x) \) or \( gf(x) = g(f(x)) \)

\[ \lim_{x \to a} f(x) \]  
the limit of \( f(x) \) as \( x \) tends to \( a \)

\( \Delta x; \ \delta x \)  
an increment of \( x \)

\[ \frac{dy}{dx} \]  
the derivative of \( y \) with respect to \( x \)

\[ \frac{d^n y}{dx^n} \]  
the \( n \)th derivative of \( y \) with respect to \( x \)

\( f'(x), f''(x), \ldots, f^{(n)}(x) \)  
the first, second, \( \ldots \) \( n \)th derivatives of \( f(x) \) with respect to \( x \)

\[ \int y \, dx \]  
indefinite integral of \( y \) with respect to \( x \)

\[ \int_a^b y \, dx \]  
the definite integral of \( y \) with respect to \( x \) for values of \( x \) between \( a \) and \( b \)

\( \dot{x}, \ddot{x}, \ldots \)  
the first, second, \( \ldots \) derivatives of \( x \) with respect to time

5. Exponential and Logarithmic Functions

\( e \)  
base of natural logarithms

\( e^x, \exp x \)  
exponential function of \( x \)

\( \log_a x \)  
logarithm to the base \( a \) of \( x \)

\( \ln x \)  
natural logarithm of \( x \)

\( \lg x \)  
logarithm of \( x \) to base 10

6. Circular Functions and Relations

\( \sin, \cos, \tan, \)  
the circular functions

\( \cosec, \sec, \cot \)  
the circular functions

\( \sin^{-1}, \cos^{-1}, \tan^{-1} \)  
the inverse circular functions

\( \cosec^{-1}, \sec^{-1}, \cot^{-1} \)  
the inverse circular functions
7. Complex Numbers

\( i \) square root of \(-1\)

\( z \) a complex number, \( z = x + iy \)

\[ r(\cos \theta + i \sin \theta), r \in \mathbb{R}^+ \]

\( = re^{i\theta}, r \in \mathbb{R}^+ \)

\( \text{Re } z \) the real part of \( z \), \( \text{Re} (x + iy) = x \)

\( \text{Im } z \) the imaginary part of \( z \), \( \text{Im} (x + iy) = y \)

\( |z| \) the modulus of \( z \), \( |x + iy| = \sqrt{x^2 + y^2}, |r(\cos \theta + i \sin \theta)| = r \)

\( \arg z \) the argument of \( z \), \( \arg(r(\cos \theta + i \sin \theta)) = \theta, \pi < \theta \leq \pi \)

\( z^* \) the complex conjugate of \( z \), \( (x + iy)^* = x - iy \)

8. Matrices

\( M \) a matrix \( M \)

\( M^{-1} \) the inverse of the square matrix \( M \)

\( M^T \) the transpose of the matrix \( M \)

\( \det M \) the determinant of the square matrix \( M \)

9. Vectors

\( \mathbf{a} \) the vector \( \mathbf{a} \)

\( \overrightarrow{AB} \) the vector represented in magnitude and direction by the directed line segment \( AB \)

\( \mathbf{a} \) a unit vector in the direction of the vector \( \mathbf{a} \)

\( \mathbf{i}, \mathbf{j}, \mathbf{k} \) unit vectors in the directions of the cartesian coordinate axes

\( |\mathbf{a}| \) the magnitude of \( \mathbf{a} \)

\( |\overrightarrow{AB}| \) the magnitude of \( \overrightarrow{AB} \)

\( \mathbf{a} \cdot \mathbf{b} \) the scalar product of \( \mathbf{a} \) and \( \mathbf{b} \)

\( \mathbf{a} \times \mathbf{b} \) the vector product of \( \mathbf{a} \) and \( \mathbf{b} \)

10. Probability and Statistics

\( A, B, C, \text{ etc.} \) events

\( A \cup B \) union of events \( A \) and \( B \)

\( A \cap B \) intersection of the events \( A \) and \( B \)

\( P(A) \) probability of the event \( A \)

\( A' \) complement of the event \( A \), the event 'not \( A \)'

\( P(A \mid B) \) probability of the event \( A \) given the event \( B \)

\( X, Y, R, \text{ etc.} \) random variables
value of the random variables $X$, $Y$, $R$, etc.

observations

frequencies with which the observations, $x_1$, $x_2$ ... occur

the value of the probability function $P(X = x)$ of the discrete random variable $X$

probabilities of the values $x_1$, $x_2$, ... of the discrete random variable $X$

the value of the probability density function of the continuous random variable $X$

the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable $X$

expectation of the random variable $X$

expectation of $g(X)$

variance of the random variable $X$

binominal distribution, parameters $n$ and $p$

Poisson distribution, mean $\mu$

normal distribution, mean $\mu$ and variance $\sigma^2$

population mean

population variance

population standard deviation

sample mean

unbiased estimate of population variance from a sample,

$$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

probability density function of the standardised normal variable with distribution $N(0, 1)$

corresponding cumulative distribution function

linear product-moment correlation coefficient for a population

linear product-moment correlation coefficient for a sample