# NANYANG TECHNOLOGICAL UNIVERSITY
# SYLLABUS FOR ENTRANCE EXAMINATION
# FOR INTERNATIONAL STUDENTS

# AO-LEVEL MATHEMATICS

## STRUCTURE OF EXAMINATION PAPER

1. There will be one 2-hour paper consisting of 4 questions.

2. Each question carries 25 marks.

3. Candidates will be required to answer all 4 questions.

**Electronic Calculators**

1. The use of common electronic scientific calculators is **allowed.**

The detailed syllabus is on the next page.

Nanyang Technological University
September 2010

# CONTENT OUTLINE

Knowledge of the content of the Ordinary Level Syllabus (or an equivalent syllabus) is assumed. The use of a scientific calculator, where appropriate, is expected.

| | Topic / Sub-topics | Content |
|---|---|---|
| **PURE MATHEMATICS** | | |
| **1** | **Functions and graphs** | |
| 1.1 | Exponential and logarithmic functions and Graphing techniques | Include:<br>• concept of function<br>• use of notation such as $f(x) = x^2 + 5$<br>• functions $e^x$ and $\ln x$ and their graphs<br>• laws of logarithms<br>• equivalence of $y = e^x$ and $x = \ln y$<br>• use of a graphic calculator to graph a given function<br>• characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes<br><br>Exclude:<br>• concepts of domain and range<br>• the use of notation $f: x \mapsto x^2 + 5$ |
| 1.2 | Equations and inequalities | Include:<br>• solving simultaneous equations, one linear and one quadratic, by substitution<br>• conditions for a quadratic equation to have real or equal roots<br>• solving quadratic inequalities<br>• conditions for $ax^2 + bx + c$ to be always positive (or always negative)<br>• solving inequalities by graphical methods<br>• formulating an equation from a problem situation<br>• finding the numerical solution of an equation using a graphic calculator |

| | Topic / Sub-topics | Content |
|---|---|---|
| **2** | **Calculus** | |
| 2.1 | Differentiation | Include:<br>• derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point<br>• use of standard notations $f'(x)$ and $\frac{dy}{dx}$<br>• derivatives of $x^n$ for any rational $n$, $e^x$, ln $x$, together with constant multiples, sums and differences<br>• use of chain rule<br>• graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$<br>• stationary points (local maximum and minimum points and points of inflexion)<br>• finding the numerical value of a derivative at a given point using a graphic calculator<br>• finding equations of tangents and normals to curves<br>• solving practical problems involving differentiation<br><br>Exclude:<br>• differentiation from first principles<br>• derivatives of products and quotients of functions<br>• use of $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$<br>• differentiation of functions defined implicitly or parametrically<br>• finding non-stationary points of inflexion<br>• problems involving small increments and approximation<br>• relating the graph of $y = f'(x)$ to the graph of $y = f(x)$ |
| 2.2 | Integration | Include:<br>• integration as the reverse of differentiation<br>• integration of $x^n$, for any rational $n$, and $e^x$, together with constant multiples, sums and differences<br>• integration of $(ax+b)^n$, for any rational $n$, and $e^{(ax+b)}$<br>• definite integral as the area under a curve<br>• evaluation of definite integrals<br>• finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves<br>• finding the numerical value of a definite integral using a graphic calculator<br><br>Exclude:<br>• definite integral as a limit of sum<br>• approximation of area under a curve using the trapezium rule<br>• area below the $x$-axis |

| | Topic / Sub-topics | Content |
|---|---|---|
| **STATISTICS** | | |
| **3** | **Probability** | |
| 3.1 | Probability | Include:<br>• addition and multiplication of probabilities<br>• mutually exclusive events and independent events<br>• use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities<br>• calculation of conditional probabilities in simple cases<br>• use of<br>$P(A') = 1 - P(A)$<br>$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$<br>$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ |
| **4** | **Binomial and normal distributions** | |
| 4.1 | Binomial distribution | Include:<br>• knowledge of the binomial expansion of $(a+b)^n$ for positive integer $n$<br>• use of the notations $n!$ and $\binom{n}{r}$<br>• concept of binomial distribution $\mathrm{B}(n,p)$ and use of $\mathrm{B}(n,p)$ as a probability model<br>• use of mean and variance of a binomial distribution (without proof)<br>• solving problems involving binomial variables<br><br>Exclude calculation of mean and variance for other probability distributions |
| 4.2 | Normal distribution | Include:<br>• concept of a normal distribution and its mean and variance; use of $\mathrm{N}(\mu, \sigma^2)$ as a probability model<br>• standard normal distribution<br>• finding the value of $\mathrm{P}(X < x_1)$ given the values of $x_1$, $\mu$, $\sigma$<br>• use of the symmetry of the normal distribution<br>• finding a relationship between $x_1$, $\mu$, $\sigma$ given the value of $\mathrm{P}(X < x_1)$<br>• solving problems involving normal variables<br>• solving problems involving the use of $\mathrm{E}(aX + b)$ and $\mathrm{Var}(aX + b)$<br>• solving problems involving the use of $\mathrm{E}(aX + bY)$ and $\mathrm{Var}(aX + bY)$, where $X$ and $Y$ are independent<br>• normal approximation to binomial<br><br>Exclude:<br>• finding probability density functions and distribution functions<br>• calculation of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ from other probability density functions |

| | Topic / Sub-topics | Content |
|---|---|---|
| **5** | **Sampling and hypothesis testing** | |
| 5.1 | Sampling | Include:<br>• concepts of population and sample<br>• random, stratified, systematic and quota samples<br>• advantages and disadvantages of the various sampling methods<br>• distribution of sample means from a normal population<br>• use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large<br>• calculation of unbiased estimates of the population mean and variance from a sample<br>• solving problems involving the sampling distribution |
| 5.2 | Hypothesis testing | Include:<br>• concepts of null and alternative hypotheses, test statistic, level of significance and *p*-value<br>• tests for a population mean based on:<br>- a sample from a normal population of known variance<br>- a large sample from any population<br>• 1-tail and 2-tail tests<br><br>Exclude testing the difference between two population means |
| **6** | **Correlation and Regression** | |
| 6.1 | Correlation coefficient and linear regression | Include:<br>• concepts of scatter diagram, correlation coefficient and linear regression<br>• calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line<br>• concepts of interpolation and extrapolation<br><br>Exclude:<br>• derivation of formulae<br>• hypothesis tests<br>• use of a square, reciprocal or logarithmic transformation to achieve linearity |

# MATHEMATICAL NOTATION

The list which follows summarises the notation used in NTU Entrance Examinations in Mathematics. Although primarily directed towards A-Level, the list also applies, where relevant, at AO-Level.

1. *Set Notation*

| | |
|---|---|
| $\in$ | is an element of |
| $\notin$ | is not an element of |
| $\{x_1, x_2, \ldots\}$ | the set with elements $x_1, x_2, \ldots$ |
| $\{x: \ldots\}$ | the set of all $x$ such that |
| $\mathrm{n}(A)$ | the number of elements in set $A$ |
| $\varnothing$ | the empty set |
| $\mathscr{E}$ | universal set |
| $A'$ | the complement of the set $A$ |
| $\mathbb{Z}$ | the set of integers, {0, ±1, ±2, ±3, ...} |
| $\mathbb{Z}^+$ | the set of positive integers, {1, 2, 3, ...} |
| $\mathbb{Q}$ | the set of rational numbers |
| $\mathbb{Q}^+$ | the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$ |
| $\mathbb{Q}_0^+$ | the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$ |
| $\mathbb{R}$ | the set of real numbers |
| $\mathbb{R}^+$ | the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$ |
| $\mathbb{R}_0^+$ | the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$ |
| $\mathbb{R}^n$ | the real $n$ tuples |
| $\mathbb{C}$ | the set of complex numbers |
| $\subseteq$ | is a subset of |
| $\subset$ | is a proper subset of |
| $\nsubseteq$ | is not a subset of |
| $\not\subset$ | is not a proper subset of |
| $\cup$ | union |
| $\cap$ | intersection |
| $[a, b]$ | the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$ |
| $[a, b)$ | the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$ |
| $(a, b]$ | the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$ |
| $(a, b)$ | the open interval $\{x \in \mathbb{R}: a < x < b\}$ |

2. *Miscellaneous Symbols*

| | |
|---|---|
| $=$ | is equal to |
| $\neq$ | is not equal to |
| $\equiv$ | is identical to or is congruent to |
| $\approx$ | is approximately equal to |
| $\propto$ | is proportional to |
| $<$ | is less than |
| $\leqslant$; $\not>$ | is less than or equal to; is not greater than |
| $>$ | is greater than |
| $\geqslant$; $\not<$ | is greater than or equal to; is not less than |
| $\infty$ | infinity |

3. *Operations*

| | |
|---|---|
| $a + b$ | $a$ plus $b$ |
| $a - b$ | $a$ minus $b$ |
| $a \times b$, $ab$, $a.b$ | $a$ multiplied by $b$ |
| $a \div b$, $\frac{a}{b}$, $a/b$ | $a$ divided by $b$ |
| $a : b$ | the ratio of $a$ to $b$ |
| $\sum_{i=1}^{n} a_i$ | $a_1 + a_2 + \ldots + a_n$ |
| $\sqrt{a}$ | the positive square root of the real number $a$ |
| $\lvert a \rvert$ | the modulus of the real number $a$ |
| $n!$ | $n$ factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$ |
| $\binom{n}{r}$ | the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n$, $r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ |
| | $\frac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$ |

4. *Functions*

| | |
|---|---|
| $\mathrm{f}$ | function $\mathrm{f}$ |
| $\mathrm{f}(x)$ | the value of the function $\mathrm{f}$ at $x$ |
| $\mathrm{f}: A \to B$ | $\mathrm{f}$ is a function under which each element of set $A$ has an image in set $B$ |
| $\mathrm{f}: x \mapsto y$ | the function $\mathrm{f}$ maps the element $x$ to the element $y$ |
| $\mathrm{f}^{-1}$ | the inverse of the function $\mathrm{f}$ |
| $\mathrm{g} \circ \mathrm{f}$, $\mathrm{gf}$ | the composite function of $\mathrm{f}$ and $\mathrm{g}$ which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$ |
| $\lim_{x \to a} \mathrm{f}(x)$ | the limit of $\mathrm{f}(x)$ as $x$ tends to $a$ |
| $\Delta x$; $\delta x$ | an increment of $x$ |
| $\frac{\mathrm{d}y}{\mathrm{d}x}$ | the derivative of $y$ with respect to $x$ |
| $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$ | the $n$th derivative of $y$ with respect to $x$ |
| $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$ | the first, second, … $n$th derivatives of $\mathrm{f}(x)$ with respect to $x$ |
| $\int y \mathrm{d}x$ | indefinite integral of $y$ with respect to $x$ |
| $\int_a^b y \mathrm{d}x$ | the definite integral of $y$ with respect to $x$ for values of $x$ between $a$ and $b$ |
| $\dot{x}, \ddot{x}, \ldots$ | the first, second, …derivatives of $x$ with respect to time |

5. *Exponential and Logarithmic Functions*

| | |
|---|---|
| $\mathrm{e}$ | base of natural logarithms |
| $\mathrm{e}^x$, $\exp x$ | exponential function of $x$ |
| $\log_a x$ | logarithm to the base $a$ of $x$ |
| $\ln x$ | natural logarithm of $x$ |
| $\lg x$ | logarithm of $x$ to base 10 |

6. *Circular Functions and Relations*

| | |
|---|---|
| $\sin$, $\cos$, $\tan$, $\mathrm{cosec}$, $\sec$, $\cot$ | } the circular functions |
| $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ $\mathrm{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$ | } the inverse circular functions |

## 7. *Complex Numbers*

| | |
|---|---|
| i | square root of –1 |
| $z$ | a complex number, $z = x + \mathrm{i}y$<br>$= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$<br>$= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$ |
| $\mathrm{Re}\ z$ | the real part of $z$, $\mathrm{Re}\ (x + \mathrm{i}y) = x$ |
| $\mathrm{Im}\ z$ | the imaginary part of $z$, $\mathrm{Im}\ (x + \mathrm{i}y) = y$ |
| $\lvert z \rvert$ | the modulus of $z$, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$ |
| $\arg z$ | the argument of $z$, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta, -\pi < \theta \leqslant \pi$ |
| $z^*$ | the complex conjugate of $z$, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$ |

## 8. *Matrices*

| | |
|---|---|
| $\mathbf{M}$ | a matrix $\mathbf{M}$ |
| $\mathbf{M}^{-1}$ | the inverse of the square matrix $\mathbf{M}$ |
| $\mathbf{M}^{\mathrm{T}}$ | the transpose of the matrix $\mathbf{M}$ |
| $\det \mathbf{M}$ | the determinant of the square matrix $\mathbf{M}$ |

## 9. *Vectors*

| | |
|---|---|
| $\mathbf{a}$ | the vector $\mathbf{a}$ |
| $\overrightarrow{AB}$ | the vector represented in magnitude and direction by the directed line segment $AB$ |
| $\hat{\mathbf{a}}$ | a unit vector in the direction of the vector $\mathbf{a}$ |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | unit vectors in the directions of the cartesian coordinate axes |
| $\lvert \mathbf{a} \rvert$ | the magnitude of $\mathbf{a}$ |
| $\lvert \overrightarrow{AB} \rvert$ | the magnitude of $\overrightarrow{AB}$ |
| $\mathbf{a}.\mathbf{b}$ | the scalar product of $\mathbf{a}$ and $\mathbf{b}$ |
| $\mathbf{a} \times \mathbf{b}$ | the vector product of $\mathbf{a}$ and $\mathbf{b}$ |

## 10. *Probability and Statistics*

| | |
|---|---|
| $A, B, C$, etc. | events |
| $A \cup B$ | union of events $A$ and $B$ |
| $A \cap B$ | intersection of the events $A$ and $B$ |
| $\mathrm{P}(A)$ | probability of the event $A$ |
| $A'$ | complement of the event $A$, the event 'not $A$' |
| $\mathrm{P}(A \mid B)$ | probability of the event $A$ given the event $B$ |
| $X, Y, R$, etc. | random variables |

| | |
|---|---|
| $x$, $y$, $r$, etc. | value of the random variables $X$, $Y$, $R$, etc. |
| $x_1$, $x_2$, ... | observations |
| $f_1$, $f_2$, ... | frequencies with which the observations, $x_1$, $x_2$ ...occur |
| p($x$) | the value of the probability function P($X = x$) of the discrete random variable $X$ |
| $p_1$, $p_2$ ... | probabilities of the values $x_1$, $x_2$, ...of the discrete random variable $X$ |
| f($x$), g($x$)... | the value of the probability density function of the continuous random variable $X$ |
| F($x$), G($x$)... | the value of the (cumulative) distribution function P($X \leqslant x$) of the random variable $X$ |
| E($X$) | expectation of the random variable $X$ |
| E[g($X$)] | expectation of g($X$) |
| Var($X$) | variance of the random variable $X$ |
| B($n$, $p$) | binominal distribution, parameters $n$ and $p$ |
| Po($\mu$) | Poisson distribution, mean $\mu$ |
| N($\mu$, $\sigma^2$) | normal distribution, mean $\mu$ and variance $\sigma^2$ |
| $\mu$ | population mean |
| $\sigma^2$ | population variance |
| $\sigma$ | population standard deviation |
| $\bar{x}$ | sample mean |
| $s^2$ | unbiased estimate of population variance from a sample, $s^2 \quad \frac{1}{n \quad 1}\sum(x \quad \bar{x})^2$ |
| $\phi$ | probability density function of the standardised normal variable with distribution N (0, 1) |
| $\Phi$ | corresponding cumulative distribution function |
| $\rho$ | linear product-moment correlation coefficient for a population |
| $r$ | linear product-moment correlation coefficient for a sample |