

**NANYANG TECHNOLOGICAL UNIVERSITY
SYLLABUS FOR ENTRANCE EXAMINATION
FOR INTERNATIONAL STUDENTS**

AO-LEVEL MATHEMATICS

STRUCTURE OF EXAMINATION PAPER

1. There will be one 2-hour paper consisting of 4 questions.
2. Each question carries 25 marks.
3. Candidates will be required to answer all 4 questions.

Electronic Calculators

1. The use of common electronic scientific calculators is **allowed**.

The detailed syllabus is on the next page.

CONTENT OUTLINE

Knowledge of the content of the Ordinary Level Syllabus (or an equivalent syllabus) is assumed. The use of a scientific calculator, where appropriate, is expected.

	Topic / Sub-topics	Content
PURE MATHEMATICS		
1	Functions and graphs	
1.1	Exponential and logarithmic functions and Graphing techniques	Include: <ul style="list-style-type: none"> • concept of function • use of notation such as $f(x) = x^2 + 5$ • functions e^x and $\ln x$ and their graphs • laws of logarithms • equivalence of $y = e^x$ and $x = \ln y$ • use of a graphic calculator to graph a given function • characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes Exclude: <ul style="list-style-type: none"> • concepts of domain and range • the use of notation $f: x \mapsto x^2 + 5$
1.2	Equations and inequalities	Include: <ul style="list-style-type: none"> • solving simultaneous equations, one linear and one quadratic, by substitution • conditions for a quadratic equation to have real or equal roots • solving quadratic inequalities • conditions for $ax^2 + bx + c$ to be always positive (or always negative) • solving inequalities by graphical methods • formulating an equation from a problem situation • finding the numerical solution of an equation using a graphic calculator

	Topic / Sub-topics	Content
2	Calculus	
2.1	Differentiation	<p>Include:</p> <ul style="list-style-type: none"> derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point use of standard notations $f'(x)$ and $\frac{dy}{dx}$ derivatives of x^n for any rational n, e^x, $\ln x$, together with constant multiples, sums and differences use of chain rule graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$ stationary points (local maximum and minimum points and points of inflexion) finding the numerical value of a derivative at a given point using a graphic calculator finding equations of tangents and normals to curves solving practical problems involving differentiation <p>Exclude:</p> <ul style="list-style-type: none"> differentiation from first principles derivatives of products and quotients of functions use of $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ differentiation of functions defined implicitly or parametrically finding non-stationary points of inflexion problems involving small increments and approximation relating the graph of $y = f'(x)$ to the graph of $y = f(x)$
2.2	Integration	<p>Include:</p> <ul style="list-style-type: none"> integration as the reverse of differentiation integration of x^n, for any rational n, and e^x, together with constant multiples, sums and differences integration of $(ax+b)^n$, for any rational n, and $e^{(ax+b)}$ definite integral as the area under a curve evaluation of definite integrals finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves finding the numerical value of a definite integral using a graphic calculator <p>Exclude:</p> <ul style="list-style-type: none"> definite integral as a limit of sum approximation of area under a curve using the trapezium rule area below the x-axis

	Topic / Sub-topics	Content
STATISTICS		
3	Probability	
3.1	Probability	Include: <ul style="list-style-type: none"> • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities • calculation of conditional probabilities in simple cases • use of $P(A') = 1 - P(A)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A B) = \frac{P(A \cap B)}{P(B)}$
4	Binomial and normal distributions	
4.1	Binomial distribution	Include: <ul style="list-style-type: none"> • knowledge of the binomial expansion of $(a+b)^n$ for positive integer n • use of the notations $n!$ and $\binom{n}{r}$ • concept of binomial distribution $B(n, p)$ and use of $B(n, p)$ as a probability model • use of mean and variance of a binomial distribution (without proof) • solving problems involving binomial variables Exclude calculation of mean and variance for other probability distributions
4.2	Normal distribution	Include: <ul style="list-style-type: none"> • concept of a normal distribution and its mean and variance; use of $N(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $P(X < x_1)$ given the values of x_1, μ, σ • use of the symmetry of the normal distribution • finding a relationship between x_1, μ, σ given the value of $P(X < x_1)$ • solving problems involving normal variables • solving problems involving the use of $E(aX + b)$ and $\text{Var}(aX + b)$ • solving problems involving the use of $E(aX + bY)$ and $\text{Var}(aX + bY)$, where X and Y are independent • normal approximation to binomial Exclude: <ul style="list-style-type: none"> • finding probability density functions and distribution functions • calculation of $E(X)$ and $\text{Var}(X)$ from other probability density functions

	Topic / Sub-topics	Content
5	Sampling and hypothesis testing	
5.1	Sampling	<p>Include:</p> <ul style="list-style-type: none"> • concepts of population and sample • random, stratified, systematic and quota samples • advantages and disadvantages of the various sampling methods • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large • calculation of unbiased estimates of the population mean and variance from a sample • solving problems involving the sampling distribution
5.2	Hypothesis testing	<p>Include:</p> <ul style="list-style-type: none"> • concepts of null and alternative hypotheses, test statistic, level of significance and p-value • tests for a population mean based on: <ul style="list-style-type: none"> - a sample from a normal population of known variance - a large sample from any population • 1-tail and 2-tail tests <p>Exclude testing the difference between two population means</p>
6	Correlation and Regression	
6.1	Correlation coefficient and linear regression	<p>Include:</p> <ul style="list-style-type: none"> • concepts of scatter diagram, correlation coefficient and linear regression • calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line • concepts of interpolation and extrapolation <p>Exclude:</p> <ul style="list-style-type: none"> • derivation of formulae • hypothesis tests • use of a square, reciprocal or logarithmic transformation to achieve linearity

MATHEMATICAL NOTATION

The list which follows summarises the notation used in NTU Entrance Examinations in Mathematics. Although primarily directed towards A-Level, the list also applies, where relevant, at AO-Level.

1. Set Notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x: \dots\}$	the set of all x such that
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	universal set
A'	the complement of the set A
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Q}	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geq 0\}$
\mathbb{R}^n	the real n tuples
\mathbb{C}	the set of complex numbers
\subseteq	is a subset of
\subset	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. Miscellaneous Symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\propto	is proportional to
$<$	is less than
$\leq; \preceq$	is less than or equal to; is not greater than
$>$	is greater than
$\geq; \succeq$	is greater than or equal to; is not less than
∞	infinity

3. Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
\sqrt{a}	the positive square root of the real number a
$ a $	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, ($0! = 1$)
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leq r \leq n$ $\frac{n(n-1)\dots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. Functions

f	function f
$f(x)$	the value of the function f at x
$f: A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$g \circ f, gf$	the composite function of f and g which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x; \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... n th derivatives of $f(x)$ with respect to x
$\int y dx$	indefinite integral of y with respect to x
$\int_a^b y dx$	the definite integral of y with respect to x for values of x between a and b
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to time

5. Exponential and Logarithmic Functions

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. Circular Functions and Relations

$\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$	} the circular functions
$\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	} the inverse circular functions

7. Complex Numbers

i	square root of -1
z	a complex number, $z = x + iy$ $r(\cos \theta + i \sin \theta)$, $r \in \mathbb{R}_0^+$ $= re^{i\theta}$, $r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re}(x + iy) = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im}(x + iy) = y$
$ z $	the modulus of z , $ x + iy = \sqrt{x^2 + y^2}$, $ r(\cos \theta + i \sin \theta) = r$
$\arg z$	the argument of z , $\arg(r(\cos \theta + i \sin \theta)) = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $(x + iy)^* = x - iy$

8. Matrices

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the square matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$	the determinant of the square matrix \mathbf{M}

9. Vectors

\mathbf{a}	the vector \mathbf{a}
\overrightarrow{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} $	the magnitude of \mathbf{a}
$ \overrightarrow{AB} $	the magnitude of \overrightarrow{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10. Probability and Statistics

A, B, C , etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A , the event 'not A '
$P(A B)$	probability of the event A given the event B
X, Y, R , etc.	random variables

$x, y, r, \text{ etc.}$	value of the random variables $X, Y, R, \text{ etc.}$
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations, $x_1, x_2 \dots$ occur
$p(x)$	the value of the probability function $P(X = x)$ of the discrete random variable X
$p_1, p_2 \dots$	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x) \dots$	the value of the probability density function of the continuous random variable X
$F(x), G(x) \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$B(n, p)$	binominal distribution, parameters n and p
$\text{Po}(\mu)$	Poisson distribution, mean μ
$N(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}	sample mean
s^2	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample