ENTRANCE EXAMINATION
Mathematics at AO-Level (Sample)

Time Allowed: 2 hours

INSTRUCTIONS

1. This paper consists of FIVE (5) questions and comprises THREE (3) pages.
2. Answer any FOUR (4) questions only.
3. The marks are allocated at the end of each part/question.
4. Answers will be graded for content and appropriate presentation.

Question 1

(a) Express \(8 - 6x - x^2\) in the form \(a - (x + b)^2\) and hence, or otherwise, find the range of the function \(f(x) = 8 - 6x - x^2\) for real \(x\).

(b) Solve the simultaneous equations

\[
\begin{align*}
3x + 7y &= 1 \\
2x^2 + 4y &= 3
\end{align*}
\]

(c) It is known that the variables \(x\) and \(y\) satisfy an equation of the form

\[
\frac{x + y}{xy} = a
\]

where \(a\) is a constant. The table below shows approximate experimental values of \(x\) and \(y\):

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>3.0</td>
<td>2.5</td>
<td>1.8</td>
<td>1.6</td>
<td>1.5</td>
</tr>
</tbody>
</table>

However, one of the values of \(y\) has been wrongly recorded. Redefine the dependent and independent variables so that there is a linear relationship between them. Plot this straight-line graph, identify the incorrect value and estimate the value of \(a\).

(12 marks)
Question 2
(a) Given that \( \sin(A + B) = 2\sin(A - B) \), show that \( \tan A = 3\tan B \). Hence find all the solutions of the equation \( \sin(A + 30^\circ) = 2\sin(A - 30^\circ) \) for \( A \) in \( (-\pi, \pi) \).

(9 marks)

(b) In a certain geometric series, the sum of the first \( n \) terms is 48, and the sum of the first \( 2n \) terms is 60. Find the sum of the first \( 3n \) terms.

(8 marks)

(c) By means of the substitution \( y = 8^x \), find the exact values of \( x \) which satisfy the equation

\[
64^x - 5(8^x) + 4 = 0
\]

(8 marks)

Question 3
(a) Convert the parametric equations \( x = \sec t \) and \( y = \tan t \) into a Cartesian equation. Plot the curve.

(6 marks)

(b) How many licence plates can be made by using 2 English letters in uppercase followed by a 3-digit number? The first digit of a licence plate should not be a zero. How many of those licence plates have 2 vowels followed by 3 identical digits?

(7 marks)

(c) Three unit vectors \( \hat{a}, \hat{b} \) and \( \hat{c} \) have the property that the angle between any two is a fixed angle \( \theta \).

(i) Find in terms of \( \theta \) the length of the vector \( \mathbf{v} = \hat{a} + \hat{b} + \hat{c} \).

(ii) Find the largest possible value of \( \theta \).

(iii) Find the cosine of the angle \( \beta \) between \( \hat{a} \) and \( \mathbf{v} \).

(12 marks)

Question 4
(a) Chord \( AB \) intersects diameter \( CD \) at right angles as shown in Figure 4.1. Let the area of the circle be \( 36\pi \) cm\(^2\) and the length of chord \( AB \) be \( 6\sqrt{3} \) cm. Determine the area of the shaded region.

Figure 4.1

(10 marks)
(b) Find \( \frac{dy}{dx} \) if \( y = e^{\sin x} \).

(5 marks)

(c) A body moves along a horizontal line according to \( s = f(t) = t^3 - 9t^2 + 2At \) where

\( s \) is the displacement and \( t \) is the time.

(i) When is \( s \) increasing and when is it decreasing?

(ii) When is the velocity \( v \) increasing, and when is it decreasing?

(iii) Find the total distance travelled in the first 5 seconds of motion.

(10 marks)

Question 5

(a) Find \( \frac{d^2y}{dx^2} \) if \( y = \frac{u-1}{u+1} \) and \( u = \sqrt{x} \).

(8 marks)

(b) Find \( \int \sin^4 x \cos^4 x \, dx \).

(9 marks)

(c) Find the area bounded by the curves \( y = x^2 - 4 \) and \( y = 8 - 2x^2 \).

(8 marks)

- END OF PAPER -