NANYANG TECHNOLOGICAL UNIVERSITY

ENTRANCE EXAMINATION

Mathematics at AO-Level (Sample)

Time Allowed : **2 hours**

<u>**INSTRUCTIONS**</u>

1. This paper consists of **FIVE (5)** questions and comprises **THREE (3)** pages.
2. Answer any **FOUR (4)** questions only.
3. The marks are allocated at the end of each part/question.
4. Answers will be graded for content and appropriate presentation.

<u>Question 1</u>

(a) Express $8-6x-x^2$ in the form $a-(x+b)^2$ and hence, or otherwise, find the range of the function $f(x)=8-6x-x^2$ for real x.

(7 marks)

(b) Solve the simultaneous equations

$$3x+7y=1$$
$$2x^2+4y=3$$

(6 marks)

(c) It is known that the variables x and y satisfy an equation of the form

$$\frac{x+y}{xy}=a$$

where a is a constant. The table below shows approximate experimental values of x and y:

x	2	3	4	5	6
y	3.0	2.5	1.8	1.6	1.5

However, one of the values of y has been wrongly recorded. Redefine the dependent and independent variables so that there is a linear relationship between them. Plot this *straight-line* graph, identify the incorrect value and estimate the value of a.

(12 marks)

Question 2

(a) Given that $\sin(A+B)=2\sin(A-B)$, show that $\tan A = 3\tan B$. Hence find all the solutions of the equation $\sin(A+30^\circ)=2\sin(A-30^\circ)$ for A in $(-\pi,\pi)$.

(9 marks)

(b) In a certain geometric series, the sum of the first n terms is 48, and the sum of the first $2n$ terms is 60. Find the sum of the first 3n terms.

(8 marks)

(c) By means of the substitution $y=8^x$, find the exact values of x which satisfy the equation

$$64^x - 5(8^x) + 4 = 0$$

(8 marks)

Question 3

(a) Convert the parametric equations $x = \sec t$ and $y = \tan t$ into a Cartesian equation. Plot the curve.

(6 marks)

(b) How many licence plates can be made by using 2 English letters in uppercase followed by a 3-digit number? The first digit of a licence plate should not be a zero. How many of those licence plates have 2 vowels followed by 3 identical digits?

(7 marks)

(c) Three *unit* vectors $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ have the property that the angle between any two is a *fixed* angle θ.

(i) Find in terms of θ the length of the vector $\mathbf{v} = \hat{\mathbf{a}} + \hat{\mathbf{b}} + \hat{\mathbf{c}}$.

(ii) Find the largest possible value of θ.

(iii) Find the cosine of the angle β between $\hat{\mathbf{a}}$ and $\mathbf{v}$.

(12 marks)

Question 4

(a) Chord *AB* intersects diameter *CD* at right angles as shown in Figure 4.1. Let the area of the circle be 36π cm^2 and the length of chord *AB* be $6\sqrt{3}$ cm. Determine the area of the shaded region.

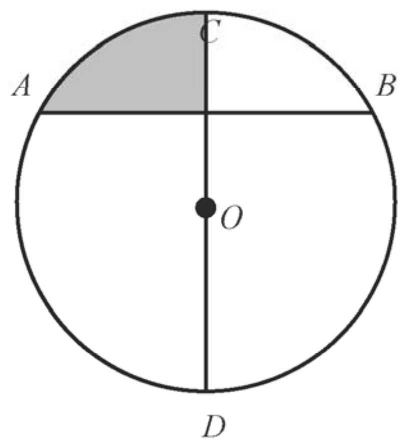

Figure 4.1

(10 marks)

(b) Find $\frac{dy}{dx}$ if $y = e^{\sin x}$.

(5 marks)

(c) A body moves along a horizontal line according to $s = f(t) = t^3 - 9t^2 + 24t$ where s is the displacement and t is the time.

(i) When is s increasing and when is it decreasing?

(ii) When is the velocity v increasing, and when is it decreasing?

(iii) Find the total distance travelled in the first 5 seconds of motion.

(10 marks)

Question 5

(a) Find $\frac{d^2y}{dx^2}$ if $y = \frac{u-1}{u+1}$ and $u = \sqrt{x}$.

(8 marks)

(b) Find $\int \sin^4 x \cos^5 x \, dx$.

(9 marks)

(c) Find the area bounded by the curves $y = x^2 - 4$ and $y = 8 - 2x^2$.

(8 marks)

- END OF PAPER -