ON THE CROWDING IN OF PRIVATE CAPITAL BY PUBLIC CAPITAL

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Abstract

This paper provides a more complete perspective to the crowding-in issue than is currently available in the literature. In addition to the channel identified in the literature, in which public capital crowds in private capital through gross complementarity between public and private capital, the paper shows that public capital can crowd in private capital by raising the marginal productivity of labor and, in turn, savings. Empirical studies on the crowding-in issue that ignore this channel have focused on the less important of the two channels and made an empirically-untenable assumption about the slope of the savings function.

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1. Introduction

In an interesting empirical paper "Does public capital crowd out private capital?" Aschauer (1989b) adopts a theoretical framework in which public capital crowds out or crowds in private capital, depending on the relative strength of two opposing forces: (1) as a substitute in production for private capital, public capital tends to crowd out private capital, and (2) by raising the return to private capital, public capital tends to crowd in private capital. This framework yields the result that, on balance, public capital will crowd out or crowd in private capital,[1] depending on whether public and private capital are gross substitutes or gross complements.

Aschauer's (1989b) adoption of the aforementioned theoretical framework leaves the impression that crowding out or crowding in takes place solely through gross substitutability or gross complementarity between public and private capital, respectively.[2] However, this framework is incomplete as it does not capture fully the channels through which crowding out or crowding in takes place. The purpose of this paper is to provide a more complete theoretical framework that clarifies the conditions under which public capital crowds in private capital and points out the deficiencies of Aschauer's study of the crowding-in issue.

The rest of the paper is organized as follows. To provide a theoretical framework for analyzing the issue, Section 2 considers the formulation of an overlapping-generations (OG) model that incorporates public capital, in addition to private inputs, in the production function.[3] Section 3 discusses the results of the paper. It is shown that public capital crowds in private capital through two channels. In addition to the channel identified in Aschauer (1989b), public capital crowds in private capital by raising the marginal productivity of labor and, in turn, savings. Consequently, in addition to the degree of gross substitutability/complementarity between public and private capital, the magnitude of the effect of public capital on the marginal productivity of labor and savings also influences whether public capital will crowd in private capital. Section 4 discusses the implications of the findings of the paper for empirical studies of the crowding-in issue. Section 5 contains concluding remarks.

2. Model Formulation

2.1. Consumers

Consumers live for two periods and form overlapping generations, growing at the rate n. In period i, generation i supply one unit of labor each, pay unit-labor income taxes τ_i and earn a post-tax wage rate $\omega_i \equiv w_i - \tau_i$, where w_i is the pre-tax wage rate. They consume c_i^i of a private good and save s^i each, governed by $c_i^i + s^i \leq \omega_i$. They save by purchasing private capital, and earn an interest rate of r_{i+1} from periods i to i+1. In period i+1, they retire and consume their entire savings. Their consumption is c_{i+i}^i each, governed by $c_{i+i}^i \leq (1+r_{i+1})s^i$.

Preferences of consumers are represented by a utility function $u(c^i) \equiv u(c_i^i,c_{i+i}^i)$. $u(c^i)$ defined for defined for $c^i \geq 0$ is continuous, strictly quasi-concave and increasing.

Under the consumer regularity condition assumed, consumption of the private good is uniquely determined: $c^i = c(r_{i+1},\omega_i)$. So is saving: $s^i = s(r_{i+1},\omega_i)$. Assuming that c_i^i satisfies the normality assumption, $0 < c_{1\omega} \equiv \partial c_i^i/\partial\omega_i < 1$, the post-tax wage derivative of saving satisfies $0 < s_\omega \equiv \partial s^i/\partial\omega_i < 1$. It is well known that the interest-rate derivative of saving is ambiguous in sign.

2.2. Producers

Producers sell a single good produced with labor, private capital and public capital. Capital is simply prior production of the good that is not consumed. On a per-capita basis, producers hire one unit of labor, rent k_i units of private capital, use g_i units of public capital and sell y_i units of output in period i. Production is subject to constant returns to scale in the private inputs.[4] The per-capita production function $f(k_i,g_i)$ is continuous, strictly concave in k_i for given g_i and increasing in (k_i,g_i).[5]

Following Tan (1995a), define a unit-labor pre-wage profit function:[6]

$$\pi(r_i;g_i) \equiv \max\{y_i - r_ik_i: f(k_i;g_i) \geq y_i; (y_i,k_i;g_i) \geq 0\}. \qquad (1)$$

$\pi(r_i;g_i)$ yields the maximum profits before deducting for wages when producers employ unit labor given $(r_i;g_i)$.

Under constant returns to scale in the private inputs, profits are assumed to be zero. Hence, the wage rate is:

$$w_i = \pi(r_i;g_i). \qquad (2)$$

By Hotelling's (1932) Lemma, the per-capita demand for private capital in period i+1 is

$$k_{i+1} = - \pi_r(r_{i+1};g_{i+1}). \tag{3}$$

Under the monotonicity assumption of the production function, it is well known that $k_r < 0$. It is clear that the effects of public capital on private capital satisfy $k_g > 0$ or < 0 according as public and private capital are gross complements or gross substitutes.

Since $\pi(r;g) = y(r;g) - rk(r;g)$, it follows that $\pi_g = y_g - rk_g$. If π_g is to be positive, then the marginal productivity of public capital must exceed the marginal returns to private capital arising from an additional unit of public capital.

2.3. Government

The government has, at the beginning of period i, a stock $N^i g_i$ of public capital, where N^i is the size of generation i. During the period, it collects labor-income taxes of $N^i\tau_i$ to finance the accumulation of public capital, which will grow to $N^{i+1}g_{i+1}$ by the end of the period. Given the growth rate of population n, the per-capita government-budget constraint is

$$\tau_i = (1+n)g_{i+1} - g_i. \tag{4}$$

In the steady state, the government-budget constraint becomes

$$\tau = ng. \tag{4'}$$

2.4. Excess supply of savings

Define a per-capita excess-supply-of-savings function at the end of period i as:[7]

$$\alpha_{i+1}(r_{i+1},r_i;g^i) \equiv (1+n)^{-1}s[(r_{i+1},w(r_i;g_i) - \tau_i(g^i)] - k(r_{i+1};g_{i+1}). \quad (5)$$

By differentiating (5) and using (2)-(4), the derivatives of $\alpha_{i+1}(r_{i+1},r_i;g^i)$ are:

$$\alpha_1 \equiv \partial\alpha_{i+1}/\partial r_{i+1} \equiv (1+n)^{-1}[s_r-(1+n)k_r],$$

$$\alpha_2 \equiv \partial\alpha_{i+1}/\partial r_i \equiv - (1+n)^{-1}s_\omega k_i,$$

$$\alpha_3 \equiv \partial\alpha_{i+1}/\partial g_i \equiv (1+n)^{-1}(s_\omega\pi_g+s_\omega),$$

$$\alpha_4 \equiv \partial\alpha_{i+1}/\partial g_{i+1} \equiv - (s_\omega+k_g). \quad (6)$$

In the steady state, $\alpha_{i+1}(.)$ reduces to $\alpha(r;g)$. Its derivatives are:

$$\alpha_r \equiv (1+n)^{-1}[s_r - (1+n)k_r - s_\omega k],$$

$$\alpha_g \equiv (1+n)^{-1}[s_\omega(\pi_g-n) - (1+n)k_g]. \quad (6')$$

The signs of $\alpha_3 + \alpha_4$ and α_g are summarized, respectively, in:

Lemma 1a: Under consumption normality, $\alpha_3 + \alpha_4 <$ or > 0 according as $s_\omega(\pi_g-n) - (1+n)k_g <$ or > 0.

Lemma 2a: Under consumption normality, $\alpha_g <$ or > 0 according as $s_\omega(\pi_g-n) - (1+n)k_g <$ or > 0.

2.5. The Model

The model comprises the following equations:

$$\alpha_{i+1}(r_{i+1},r_i;g^i) = 0, \quad (7a)$$

$$r_1 = \bar{r}_1, \quad (7b)$$

$$k_{i+1} = k_{i+1}(r_{i+1};g_{i+1}), \quad (8a)$$

$$k_1 = \bar{k}_1. \quad (8b)$$

Equation (7a) is the capital-market equilibrium condition, showing the evolution of the economy from the initial condition (7b). (7a) and (7b) determine the time path of interest rates forward in time from the initial interest rate given the time paths of public capital. With interest rates determined and given the time paths of public capital, (8a) determines the time paths of the stock of private capital from the initial condition (8b).

A temporary equilibrium in period i is an $r_{i+1} > 0$ satisfying the capital-market equilibrium condition (7a) for given $(r_i;g^i)$. A steady-state equilibrium is an $r > 0$ satisfying the steady-state counterpart to (7a) for given g. Assume that a temporary equilibrium and a steady-state equilibrium exist. Then we have:

Lemma 1b: $\alpha_1 > 0$ if, and only if, the capital market is locally Walrasian stable at a temporary equilibrium.[8]

Lemma 2b: Under consumption normality and local Walrasian stability of the capital market at a temporary equilibrium, the steady-state equilibrium is locally dynamically stable only if $\alpha_r > 0$.[9]

Suppose the government increases public capital by a small amount dg from period 1 such that $dg_i = dg$ for $i \geq 1$. The model comprising (7a)-(8b) can be differentiated at the initial equilibrium to yield:

$$\alpha_1 dr_{i+1} = - \alpha_2 dr_i - (\alpha_3+\alpha_4)dg, \quad (9a)$$

$$dr_1 = 0, \quad (9b)$$

$$dk_{i+1} = k_r dr_{i+1} + k_g dg, \quad (10a)$$

$$dk_1 = 0. \quad (10b)$$

In the steady state, the differential system of equations (9a)-(10b) becomes

$$\alpha_r dr = - \alpha_g dg, \quad (9')$$

$$dk = k_r dr + k_g dg. \quad (10')$$

The systems of equations given by (9a)-(10b) and (9')-(10') will be used to determine, respectively, the short-run and long-run crowding-in effects of private capital by public capital.

3. Results

Using equations (9a) and (9b), as a result of public capital increasing by $dg_i = dg$ for $i \geq 1$, the interest rate in period 2 will change by

$$dr_2 = -(\alpha_1)^{-1}(\alpha_3+\alpha_4)dg. \qquad (9c)$$

Using (9c) to eliminate dr_2 from (10a), the stock of private capital in period 2 will change by

$$dk_2 = [-(\alpha_1)^{-1}(\alpha_3+\alpha_4)k_r + k_g]dg. \qquad (10c)$$

Since $\alpha_1 > 0$ by Lemma 1b, $dg > 0$ by assumption, we have $dk_2 > 0$ if, and only if, $\alpha_1 k_g - (\alpha_3+\alpha_4)k_r > 0$. Substituting the identities (6) for α_1 and $\alpha_3+\alpha_4$ in the condition just obtained and rearranging yields $s_\omega(\pi_g-n) - (s_r/k_r)k_g > 0$. Hence, we have:

Proposition 1: Let public capital be increased by a small amount $dg_i = dg$ for $i \geq 1$. Then, subject to the assumptions of the model, $dk_2 > 0$ if, and only if, $s_\omega(\pi_g-n) - (s_r/k_r)k_g > 0$.

The interpretation of the condition $s_\omega(\pi_g-n) - (s_r/k_r)k_g > 0$ is straightforward. The first term on the left-hand side represents the effect of an increase in public capital on savings via its impact on the marginal productivity of labor. Recall from Section 2.2 that an increase in public capital has a positive effect on the marginal productivity of labor if the marginal productivity of public capital exceeds the marginal returns to private capital arising from an additional unit of public capital. This positive impact tends to increase savings. However, the financing of the increase in public capital by labor-income taxes tends to decrease savings. Whether an increase in public capital will increase savings, on balance, via this channel and, hence, crowd in private capital depends on whether π_g exceeds n.

The second term on the left-hand side of the inequality represents the effect of an increase in public capital on savings via gross complementarity/substitutability between public and private capital. An increase in public capital, by increasing or decreasing private capital for a given interest rate according as public and private capital are gross complements or gross substitutes, will increase or decrease the amount of savings and, hence, crowd in or crowd out private capital, respectively, if the savings function is positively sloped, an assumption implicit in Aschauer (1989b). If the savings function is negatively sloped and the capital market is Walrasian stable, an increase in public capital will increase or decrease the amount of savings and, hence, crowd in or crowd out private capital according as public and private capital are gross substitutes or gross complements.

Thus, Proposition 1 merely says that, to crowd in private capital in the short run, an increase in public capital must increase savings, overall, through two channels: (a) via its impact on the marginal productivity of labor, and (b) via gross substitutability/complementarity between public and private capital. Aschauer (1989b) considers only the second channel but neglects the first. It is clear that, for given s_ω, $s_r > 0$ ($s_r < 0$) and k_r, the stronger the effect of public capital on the marginal productivity of labor and, in turn, on savings and the greater the degree of gross complementarity (substitutability) between public and private capital, the greater will be the crowding-in effect.

The short-run effect of the increase in public capital on the interest rate is easily determined. If public and private capital are gross complements, then, under the conditions in Proposition 1, the interest rate changes ambiguously in the short run. If, however, public and private capital are gross substitutes, the interest rate falls in the short run.

Suppose public and private capital are gross complements. Bearing in mind $\alpha_1 \equiv (1+n)^{-1}[s_r - (1+n)k_r] > 0$ by Lemma 1b, it is easy to show that $\alpha_3+\alpha_4 \equiv (1+n)^{-1}[s_\omega(\pi_g\text{-}n) - (1+n)k_g)] > 0$ implies $s_\omega(\pi_g\text{-}n) - (s_r/k_r)k_g > 0$. Then, using Proposition 1, we have:

Proposition 2: Suppose $k_g > 0$. Let public capital be increased by a small amount $dg_i = dg$ for $i \geq 1$. Then, subject to the assumptions of the model, $dk_2 > 0$ if $\alpha_3+\alpha_4 \equiv (1+n)^{-1}[s_\omega(\pi_g\text{-}n) - (1+n)k_g)] > 0$.

In this case, it is easy to see from equation (9c) that $dr_2 < 0$. This means that, when public capital is increased, the increase in savings exceeds the increase in private capital. Hence, in crowding in private capital under the conditions of Proposition 2, the channel through which public capital raises the marginal productivity of labor and, in turn, savings is more effective than the channel through which public capital raises private capital through complementarity.

Following a similar argument in determining the short-run crowding-in effect, we have the following proposition on the long-run effect:

Proposition 3: Let public capital be increased by a small amount dg. Then, subject to the assumptions of the model, $dk > 0$ if, and only if, $s_\omega(\pi_g - n) - (s_r - s_\omega k)(k_g/k_r) > 0$.

We see from Propositions 1 and 3 that the condition for the long-run crowding-in effect differs from that for the short-run crowding-in effect. Ignoring the first term in both conditions, public capital will crowd in private capital in the long run, depending on whether public and private capital are gross complements if the savings function satisfies $s_r > s_\omega k$, or depending on whether public and private capital are gross substitutes if $s_r < s_\omega k$, while public capital will crowd in private capital in the short run, depending on whether public and private capital are gross complements if $s_r > 0$, or depending on whether public and private capital are gross substitutes if $s_r < 0$.

Using a similar argument in deriving Proposition 2, the steady-state version of the proposition is obtained as:

Proposition 4: Suppose $k_g > 0$. Let public capital be increased by a small amount $dg_i = dg$ for $i \geq 1$. Then, subject to the assumptions of the model, $dk > 0$ if $\alpha_g \equiv (1+n)^{-1}[s_\omega(\pi_g - n) - (1+n)k_g] > 0$.

4. Implications

Empirically, s_r is close to 0.[10] Using this empirical estimate, we can deduce from Propositions 1 and 3 that public capital will crowd in private capital in the short run provided $s_\omega(\pi_g - n) > 0$ and in the long run provided $s_\omega(\pi_g - n) - (s_r - s_\omega k)(k_g/k_r) > 0$. These conditions imply that: (a) complementarity between public and private capital is irrelevant for the crowding-in effect in the short run, (b) the higher the degree of complementarity between public and private capital, the more it detracts from the crowding-in effect in the long run and, therefore, (c) the effect of public capital on the marginal productivity of labor plays a more important role than complementarity between public and private capital in the crowding-in process.[11]

Implication (c) above suggests that Aschauer (1989b), by ignoring the channel embodied by the term $s_\omega(\pi_g\text{-}n)$, has focused on the less important of the two channels in the crowding-in process. There is yet another criticism that can be leveled against Aschauer's study. If the channel embodied by $s_\omega(\pi_g\text{-}n)$ is ignored and $k_g > 0$ assumed, as in Aschauer (1989b), the conditions deduced in the previous paragraph to achieve both short-run and long-run crowding in will never be satisfied and, accordingly, crowding in cannot take place. To obtain the crowding-in result in the absence of the aforementioned channel and under the assumption of $k_g > 0$, Aschauer would have to assume, additionally, that the savings function is positively sloped, (in fact, satisfying $s_r > s_\omega k$ in the long run), an assumption that is inconsistent with the empirical evidence. To be consistent with the empirical evidence, he cannot choose to ignore the channel embodied by $s_\omega(\pi_g\text{-}n)$. [12]

5. Conclusion

This paper has shown that public capital crowds in private capital through two channels, namely, via its impact on the marginal productivity of labor and savings, and via gross complementarity/substitutability between public and private capital. Empirical studies that ignore the first channel can be criticized on the ground that they would have to assume a positively-sloped savings function, an assumption that is untenable empirically. To be consistent with the empirical evidence on the slope of the savings function, these studies must incorporate the channel through which public capital raises the marginal productivity of labor and savings, a channel that has, surprisingly, received inadequate attention in the literature on the crowding-in issue, despite being a more important channel than the one identified in the literature.

Notes

1 Two related issues are the crowding out of private consumption by public spending, and the crowding out of private capital by public debt. Feldstein (1982), Kormendi (1983) and Aschauer (1985), among others, study these issues empirically. See also the references in Barro (1989).

2 This is not to say that Aschauer (1989b) is unaware that public capital can affect private capital by effects other than through gross substitutability/complementarity between public and private capital. See Aschauer (1989b, p.183).

3 OG models are due to Samuelson (1958) and Diamond (1965). An OG model incorporating public capital in the production function can be found in Pestieau (1974).

4 An alternative assumption is that of constant returns to scale in the private and public inputs, as in Pestieau (1974) and Aschauer (1988).

5 On whether public capital is productive, see Aschauer (1989a).

6 Unlike most OG models, which adopt a semi-dual approach, the model here adopts a fully-dual approach.

7 $\alpha_{i+1}(.)$ embodies utility maximization, profit maximization and compliance with the government-budget constraint.

8,9 See Tan (1995a) for a proof of $\alpha_1 > 0$ and Tan (1995b) for a proof of $\alpha_r > 0$.

10 See Deaton (1992, chap.2).

11 Empirically, π_g is likely to be significant for public infrastructure, especially for developing economies. See the World Bank (1994, p.1).

12 Ignoring the channel embodied by $s_\omega(\pi_g\text{-}n)$ would be correct for a small open economy. In that case, public capital will crowd in private capital so long as public and private capital are gross complements.

REFERENCES

Aschauer, D. (1985), "Fiscal Policy and Aggregate Demand," *American Economic Review* 75, 117-127.

Aschauer, D. (1988), "The Equilibrium Approach to Fiscal Policy," *Journal of Money, Credit and Banking* 20, 41-62.

Aschauer, D. (1989a), "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23, 177-200.

Aschauer, D. (1989b), "Does Public Capital Crowd Out Private Capital?" *Journal of Monetary Economics* 24, 171-188.

Barro, R.J. (1989), "The Neoclassical Approach to Fiscal Policy," R.J. Barro (ed), *Modern Business Cycle Theory*, Basil Blackwell.

Deaton, A. (1992), *Understanding Consumption*, Clarendon Press, Oxford.

Diamond, P.A. (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review* 55, 1126-50.

Feldstein, M. (1982), "Government Deficits and Aggregate Demand," *Journal of Monetary Economics* 9, 1-20.

Hotelling, H. (1932), "Edgeworth Taxation Paradox and the Nature of Demand and Supply Functions," *Journal of Political Economy* 40, 577-616.

Kormendi R.C. (1983), "Government Debt, Government Spending and Private Sector Behavior," *American Economic Review* 73, 994-1010.

Pestieau, P. (1974), "Optimal Taxation and Discount Rate for Public Investment in a Growth Setting," *Journal of Public Economics* 3, 217-235.

Samuelson, P.A. (1958), "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy* 66, 467-482.

Tan, K.H. (1995a), "The Intergenerational Incidence of Government Spending," *Economic Record* 71, 54-65.

Tan, K.H. (1995b), "Strictly Pareto-Improving Bilateral Reforms of Public Debts," *Journal of Economics* 62, 141-156

The World Bank (1994), *World Development Report 1994: Infrastructure for Development*, Oxford University Press.