INTRODUCTION

A prominent identifying feature of a landscape is called a landmark. Anatomical landmarks are those points on the body surface with pronounced characteristics compared with other points in the near vicinity. Such points have raised people’s great interests because sometimes they help to get an accurate difference between a healthy person and a patient. Doctors or orthopedists can easily infer or know what happened to their patients. The vertebra prominens landmark on the back surface is a typical example with a convex surface region surrounded by a hyperbolic (saddle shaped) surface region [1]. Since the 1980’s to recently, methods based on the analysis of body surfaces have been proposed with different purposes. With the development of laser technology, the 3D non-contact scanner becomes an important and fast way of collecting the data on the body surface. It is desirable to develop a method suitable for evaluation of the data collected from the 3D non-contact scanner based on those methods for stereophotogrammetry. The method should allow the extraction of curvatures containing the information for local shape analysis.

CURVATURE CALCULATION

A two-parametric position vector can denote any point on a two-dimensional surface

\[ \mathbf{x} = \mathbf{x}(u, v) = (x(u, v), y(u, v), z(u, v)) \]

whose components are functions of two parameters of \( u \) and \( v \).

Taylor’s theorem gives

\[ \mathbf{x}(u + du, v + dv) \approx \mathbf{x}_0 + \mathbf{x}_u du + \mathbf{x}_v dv + \frac{1}{2} \left( \mathbf{x}_{uu} dudu + \mathbf{x}_{uv} du dv + \mathbf{x}_{vv} dv dv \right) \]

This can be used to approximate the surface in the neighborhood of \( \mathbf{x} \). The coefficient \( \mathbf{x}_u, \mathbf{x}_v \) etc. is the first and second derivatives of \( \mathbf{x} \) with respect to \( u, v \) at surface point \( \mathbf{x}_0 = \mathbf{x}(u, v) \). With these first and second derivatives, we can calculate the two principle curvatures \( \kappa_1 \) and \( \kappa_2 \). Meanwhile we can get Gaussian curvature and mean curvature (refer to [3] for details).

These quantities are well suited for an invariant shape analysis [2]. The sign of Gaussian curvature enables elliptic areas to be distinguished from hyperbolic areas. Similarly in the case of parabolic and elliptic areas one can easily discriminate convexity from concavity by the sign of mean curvature. It is easy to calculate the Gaussian curvature and mean curvature once the analytical surface in form of (1) has been given. But from the experiment, we could only get some discrete data, \( (x, y, z) \). Now we consider how to derive those curvatures from randomly distributed data. We consider \( z = f(x, y) \). Because the surface is only described at a finite number of collected data, interpolation is impossible. For this reason and for further analysis, a surface represented by a set of regular grid data is desirable. To extract a grid point from the scattered data, we need to consider fitting a small second polynomial surface patch for this grid point and other points in the neighborhood. A set of regular grid points has the following forms:

\[ x_k = g \cdot i, y_k = g \cdot j, z_k = z(x_k, y_k), \]

where \( i \) and \( k \) are grid indices and \( g \) is grid constant. We will transform our randomly distributed data into regular grid data by introducing a polynomial

\[ z = c_1 u + c_2 v + c_3 u^2 + c_4 uv + c_5 v^2 \]

Here

\[ u = x - x_k, v = y - y_k \]

We aim to get those coefficients \( c_1, c_2, c_3, c_4, c_5 \) and \( c_6 \) in such a way that the discrepancy between the fitting surface patch and those discrete data is minimum. This is obtained by least squares fitting of polynomial surface. In our case, the surface is to be determined by (3). At each data point, the difference between the surface elevation and data’s \( z \) coordinates is \( z - z_n \). We then adjust \( z \) by choice of those coefficients to minimize

\[ E(z) = \sum_{n=1}^{N} (z(u_n, v_n) - z_n) \]

The function \( E \) of \( c_1, c_2, c_3, c_4, c_5, c_6 \) will have a minimum only when \( \partial E / \partial c_i = 0 \) for \( i = 1, 2, \ldots 6 \). These conditions yield six linear equations in the unknowns \( c_1, c_2, c_3, c_4, c_5 \) and \( c_6 \) (the normal equations). The technique of normal equations can be performed conveniently in matrix-vector form

\[ \mathbf{z} + \mathbf{r} = \mathbf{AC} \]

where

\[ \mathbf{z} = (z_1, z_2, \ldots, z_n)^T \]

\[ \mathbf{r} = (r_1, r_2, \ldots, r_n)^T \]

\[ \mathbf{C} = (c_1, c_2, c_3, c_4, c_5, c_6)^T \]

\[ \mathbf{A} = \begin{bmatrix} 1 & u_1 & v_1 & u_1^2 & u_1 v_1 & v_1^2 \\ 1 & u_2 & v_2 & u_2^2 & u_2 v_2 & v_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & u_n & v_n & u_n^2 & u_n v_n & v_n^2 \end{bmatrix} \]

are the vectors of the measured \( z \) coordinates of data, the residuals, the polynomial coefficients and the matrix containing the \( x, y \) differences between the grid point and its neighborhood points. A weighting function is used to put weightings on those points with respect to their distances to the grid point (the center point) in such a way that the weightings drop off with increasing distance from center point. We introduce a diagonal weighting matrix \( \mathbf{G} \) to meet the requirement.
\[ G = \text{Diag}(g_1, g_2, \cdots, g_n) \]  
\[ g_i = \exp(-(u_i^2 + v_i^2)/d^2) \quad i=1,2,\ldots,n \]
d is the grid spacing constant.

We can optimize the coefficients by minimizing the weighted residuals
\[ r^T G r = \text{minimum} \]

From equation (13), we arrive at the normal equations
\[ NC = A^T G z \]
with N being the normal equation matrix
\[ N = A^T GA \]

Solving (15) by multiply the inverse matrix \( N^{-1} \) on both sides, we get
\[ C = N^{-1} A^T G z \]

Now we have fitted the second order polynomial surface patch in a small area around the center point by way of least squares method. At the same time the irregular data points have been transformed into points on the intersection of grid. The first component of \( C \) is the \( z \) value of the grid point. The other components of \( C \) can be interpreted as the coefficients \( x_{uv}, x_u, x_v \) etc. which are the first and second derivatives of \( x \) with respect to \( u, v \) at surface point \( x_m = x(u, v) \).

**RESULTS**

A set of photogrammetric measurement data with a nominal accuracy of less than 0.05 mm (ATOS system, Braunschweig, Germany) of a right foot from a 14 year old girl has been analyzed by calculating curvatures. From Fig.1., one can see the elliptic shape of outer malleoli and the hyperbolic shape immediately below it. From Fig.2., the area of convexity and concavity could be differentiated. Besides the outer malleoli, the pronounced triangle (red in Fig.2.) can also be taken as a landmark on the foot surface. The standard deviation is calculated in the 2x2-mm² square around the center of the grid point. Most standard deviations being smaller than 1 mm indicate that our method of surface patch fitting is accurate enough for our application. Further efforts should be concentrated on locating the landmarks accurately and building a coordinate system with additional anatomical landmarks of the foot.

**Related Publication:**


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