Product Portfolio Identification Based on Association Rule Mining

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Abstract: It has been well recognized that product portfolio planning has far-reaching impact on the company's business success in competition. In general, product portfolio planning involves two main stages, namely portfolio identification and portfolio evaluation and selection. The former aims to capture and understand customer needs effectively and accordingly to transform them into specifications of product offerings. The latter concerns how to determine an optimal configuration of these identified offerings with the objective of achieving best profit performance. Current research and industrial practice have mainly focused on the economic justification of a given product portfolio, whereas the portfolio identification issue has been received only limited attention. This paper intends to develop explicit decision support to improve product portfolio identification by efficient knowledge discovery from past sales and product records. As one of the important applications of data mining, association rule mining lends itself to the discovery of useful patterns associated with requirement analysis enacted among customers, marketing folks, and designers. An association rule mining system (ARMS) is proposed for effective product portfolio identification. Based on a scrutiny into the product definition process, the paper studies the fundamental issues underlying product portfolio identification. The ARMS differentiates the customer needs from functional requirements involved in the respective customer and functional domains. Product portfolio identification entails the identification of functional requirement clusters in conjunction with the mappings from customer needs to these clusters. While clusters of functional requirements are identified based on fuzzy clustering analysis, the mapping mechanism between the customer and functional domains is incarnated in association rules. The ARMS architecture and implementation issues are discussed in detail. An application of the proposed methodology and system in a consumer electronics company to generate a vibration motor portfolio for mobile phones is also presented.

Keywords: Data mining, mass customization, product portfolio, association rule, variety, requirement management, customer satisfaction, product definition.

1. Introduction

Understanding and fulfilling each individual customer need has been recognized as an enormous challenge for companies across industries. Rather than offering market-focused products, which corresponds to an average satisfaction of several customer needs, companies are pursuing a strategy of mass customization (Pine 1993), which strives to offer customer-focused products with a large degree of individuality. To compete in the marketplace, manufacturers have been seeking for expansion of their product lines and differentiation of their product offerings with the intuitively-appealing belief that high product variety may stimulate sales and thus conduce to revenue (Ho and Tang, 1998). While a high variety strategy may offer an effective means for companies to differentiate themselves from their competitors, it unavoidably leads to high complexity and costs in product fulfillment (Child et al., 1991). Moreover, making wide variety of products available and letting customers vote on the shelf may cause customer to be overwhelmed by the huge assortment offered or frustrated by the complexity involved with making a choice (Huffman and Kahn, 1998). Therefore, it becomes imperative for the manufacturer to determine how to offer “right” product variety to the target market.

This type of decisions adheres to the general wisdom as suggested in the Boston Consulting Group's notion of product portfolio strategy, one of the most popular contemporary approaches to strategic planning (Henderson, 1970). While representing the spectrum of a company’s product offerings, the product portfolio must be carefully set up, planned and managed so as to match those customer needs in the target market (Warren, 1983). The product portfolio strategy has far-reaching impact on the company’s business success such as to achieve financial goals in maximizing return and R&D productivity, to maintain the competitive edge of the business by increasing sales and market share, to allocate scare resources properly and efficiently, to forge the link between project selection and business strategies, to better communicate priorities within the organization both vertically and horizontally, and so on (Cooper et al., 2001).

In general, product portfolio planning involves two main stages (Li and Azarm, 2002). The first is called product portfolio identification. The goal is to capture and understand customer needs effectively and accordingly to transform them into specifications of product offerings (e.g., functional features). The second is called product portfolio evaluation and selection. The key issue is to determine an optimal setup or configuration of these planned offerings (e.g., the go/kill decision of an offering) with the objective of achieving best profit performance. Current researchers and industrial practitioners in this field involve themselves mostly in the economic justification of product portfolio (e.g., product line design), *viz.*, the latter stage of product portfolio planning. They usually imply the specification of offerings in a product portfolio is given. However, the first issue – how to identify customer needs and generate product portfolio specifications - has received only limited attention. During this phase, many factors are to be considered including any combination of customer needs, corporate objectives, product ideas and related technological capabilities, etc. Usually, product offerings are represented as a list of functional features and target values. This information is often a mix of quantitative values and qualitative descriptions of product functionality. In most cases, the company may produce a formal document that requires to undergo routinely many amendments along with scrutiny, or to be signed off by many individuals (Prasad, 1996). Even though product portfolio identification is of paramount importance, past research has not addressed it well, nor has actual practice availed to formulate effective means. This may stem from the complications inherent in the product portfolio identification process, as discussed below.

1.1 Fundamentals of Product Portfolio Identification

To leverage the market benefits of customization and the costs of providing variety, it is reasonable to fulfill mass customization within a company’s capabilities in design and production. In practice, this is often achieved by developing product and process platforms (Simpson, 2004; Jiao et al., 2003). A product platform performs as a base product from which product families can variegate designs to satisfy individual customer requirements (Meyer and Lehnerd, 1997). Corresponding to a product platform, production processes can be organized as a process platform in the form of a bill-of-operations (e.g., standard routings), hence facilitating build or configure-to-order production for given customer orders (Jiao et al., 2000). Both product and process platforms originate from, and are thus supposed to conform to, a planned product portfolio.

As shown in Figure 1, a holistic view on product portfolio can be illustrated along the entire spectrum of product development according to the domain framework in axiomatic design (Suh, 2001). Product development in general encompasses three consecutive stages: (1) product definition – mapping of customer needs (CNs) in the customer domain to functional requirements (FRs) in the functional domain; (2) product design – mapping of FRs in the functional domain to design parameters (DPs) in the physical domain; and (3) process design – mapping of DPs in the physical domain to process variables (PVs) in the process domain. Accordingly, the customer, functional, physical and process domains address the customer satisfaction, functionality, technical feasibility, and manufacturability/cost issues associated with the products, respectively (Jiao and Tseng, 1999). Within the context of mass customization, product design and process design are embodied in the respective product and process platforms. Product definition is characterized by the product portfolio representing the target of mass customization (i.e., the "right" product offerings), which in turn becomes the input to the downstream design activities and is propagated to product and process platforms in a coherent fashion. In this sense, a product portfolio represents the functional specification of product families, i.e., the functional view of product and process platforms (Jiao and Tseng, 2004).

Figure 1 Product portfolio within the spectrum of product development

Consistent with the product definition process, product portfolio identification involves a tedious elaboration process enacted among customers, marketing folks, and designers, as shown in Figure 2. Tseng and Jiao (1998) point out the difficulties associated with product definition. Their observations are also supported in the study by Tarasewich and Nair (2001).

First, the customer requirements are normally qualitative and tend to be imprecise and ambiguous due to their linguistic origins. In most cases, requirements are negotiable and conflict with one another, and thus tradeoffs are often necessary. Frequently, customers, marketing folks and designers employ different sets of context to express the requirements. Differences in semantics and terminology always impair the ability to convey requirement information effectively from customers to designers due to their different positions. The differentiation of requirements in terms of CNs and FRs is of practical significance. An organization should put considerable efforts in capturing the genuine or "real" needs of the customers (CNs), rather than too much focus on the technological issues (FRs) during early stage of product development (Yan et al., 2002).

Second, there rarely exists any definite structure of requirement information. Variables used to describe requirements are often poorly understood and are usually expressed in abstract, fuzzy, or conceptual terms, leading to work on the basis of vague assumptions and implicit inference. A few researchers have enforced a hierarchical structure or an AND/OR tree structure for the articulation of customer requirements, for example, the requirement taxonomy (Hauge and Stauffer, 1993), the customer attribute hierarchy (Yan et al., 2001), and the FR topology (Tseng and Jiao, 1998). Nevertheless, the non-structure nature of requirement information itself coincides with those findings in nature language processing (Shaw and Gaines, 1996).

Third, the interrelationships (i.e., mapping) between CNs and FRs are often not clearly available in an early stage of design. Customers are often not aware of the underlying coupling and interrelationships among various requirements with regard to product performance. It is difficult, if not impossible, to estimate the consequences (in particular, in terms of economic, scheduling and quality concerns) of specifying different requirements. Christopher *et al.* (1980) discern customer needs and product specifications and point out the mapping problem between them is the key issue in "design for customers".

Fourth, the specification of requirements results from not only the transformation of customer requirements from those end-users, but also considerations of many engineering concerns, involving any internal customer from downstream of the design team along the product realization process (Du et al., 2003). In practice, product development teams must keep track of a myriad of requirement information derived from different perspectives on the product life-cycle, such as product technologies, manufacturability, reliability, maintainability, and environmental safety, to name but a few (Prudhomme et al., 2003).

Therefore, the process of product portfolio identification can be described as: $L \leftarrow G(CNs, Eng.)$, where L represents a specification of product offerings, CNs indicate the customer needs of end-users, $Eng.$ means engineering considerations associated with CNs, and G denotes the mapping relationship from CNs and $Eng.$ to a particular product portfolio, L.

Figure 2 Product definition process inherent in product portfolio identification

1.2 Strategy for Solution

Due to the difficulties inherent in the portfolio identification process, reusing knowledge from historical data suggests itself as a natural technique to facilitate the handling of requirement information and tradeoffs among many customer, marketing and engineering concerns. Tseng and Jiao (1998) propose to identify FR patterns from previous product designs for addressing a broad spectrum of domain-specific customer requirements and to organize requirement information during design. In their model, various FRs are grouped according to the similarity among customers (i.e., market segments). The focus is on the functional domain. Du et al. (2003) extend the idea to study the patterns of CNs for better customization and personalization. Chen et al. (2002) apply neural network techniques to construct a customer attribute hierarchy (CAH) in order to improve customer requirement elicitation. Both ideas emphasize on the customer domain. While these proposed solutions emphasize on the identification of either CN or FR patterns, the mapping relationship between CNs and FRs has not been taken into account. We assert that FR patterns should not be identified in isolation from those patterns of CNs, and vice versa. The patterns of CN-FR mappings play an important role in bringing engineering concerns into product portfolio identification as well as in determining CN and FR patterns within a cohesive context.

To this end, this paper proposes to apply data mining techniques to improve the product portfolio identification process. Data mining has been well recognized for decision support by efficient knowledge discovery of previously unknown and potentially useful patterns of information from past data (Chen et al., 1996). As one of the important applications of data mining, association rule mining lends itself to the discovery of knowledge associated with mappings from CNs to FRs. Based on association rule mining, this research develops an inference system for effective product portfolio identification.

In the next section, the background research leading to product portfolio identification is presented. In Section 3, the methodology of product portfolio identification based on association rule mining is described. An association rule mining system (ARMS) architecture and its implementation issues are discussed in Section 4. In Section 5, an application of the proposed methodology and system to generate a vibration motor portfolio for mobile phones is presented. Observations from the case study and managerial implications are derived in Section 6. An outline of future work and conclusions are drawn in Section 7.

2. Related Work

Approaches to defining product specifications by capturing, analyzing, understanding, and projecting customer requirements, sometimes called the Voice of the Customer (VoC), have received a significant amount of interests in recent years (McKay et al., 2001). A method used for transforming the VoC to product specifications is developed by Shoji *et al.* (1993), in which semantics methods, such as the Kawakita Jiro (KJ) method (i.e., affinity diagram) and multi-pickup method (MPM), are applied as the basis for discovering underlying facts from affective language. Kano *et al.* (1984) propose a diagram to categorize different types of customer requirements for product definition.

In this regard, market researchers have emphasized customer profiling by applying regression analysis to compare customer characteristics and to determine their overall ranking in contribution towards profitability (Jenkins, 1995). Traditionally, market analysis techniques are adopted for investigating customers' responses to design options. For example, conjoint analysis is widely used to measure preferences for different product profiles and to build market simulation models (Green and DeSarbo, 1978). Louviere *et al.* (1990) use discrete choice experiments to predict customer choices pertaining to design options. Turksen and Willson (1993) employ fuzzy systems to interpret the linguistic meaning regarding customer preferences as an alternative to conjoint analysis. Others have taken a qualitative approach and used focus groups to provide a reality check on the usefulness of a new product design (LaChance-Porter, 1993). Similar techniques include one-on-one interviews and similarity-dissimilarity attribute rankings (Griffin and Hauser, 1992). While these types of methods are helpful for discovering the VoC, it is still difficult to obtain design requirement information because marketing folks do not know what engineers need to know. It is difficult to apply the VoC alone to achieve a synergy of marketing and engineering concerns in developing product specifications (Veryzer, 1993).

A number of complex customer behaviors such as perceptions, motivations, attitudes and personality can be grouped under psychological factors for making rational decisions (Louder and Bitta, 1988). These factors influence the way in which customers select, organize and interpret a company and its product offerings. Kansei engineering (Nagamachi, 1989) is a technique for the translation of consumers' psychological feeling about a product into perceptual design elements (JSKE, 2003). As a structured questioning methodology built upon Kelly's repertory grid technique (Kelly, 1955), the laddering technique has been widely used to transform customers' psychological factors into useful inputs for design applications (Rugg and McGeorge, 1995). Many methods and tools in the field of knowledge acquisition, such as observation, self-report (Cortazzi and Roote, 1975), interview, protocol, ethnographic methods (Mead, 1928), and sorting techniques (Shaw, 1980), have some applicability in requirement elicitation for product development (Shaw and Gaines, 1996). Maiden and Rugg (1996) propose a framework called acquisition of requirements (ACRE) to assist practitioners in understanding the strengths and weaknesses of each of the methods for requirement elicitation. Chen and his co-authors propose an integrated approach to the elicitation of customer requirements by combining picture sorts, fuzzy evaluation, laddering, and neural network techniques (Chen and Occeña, 1995; Chen et al., 2000, 2002; Yan et al., 2001, 2002).

From an engineering design perspective, Hauge and Stauffer (1993) develop a taxonomy of product requirements to assist in traditional qualitative market research. To elicit knowledge from customers (ELK), the taxonomy of customer requirements is deployed as an initial concept graph structure in the methodology for question probe – a method used in the development of expert systems. While ELK aims at making customer information more useful to the designer, the taxonomy developed for ELK is too general to be a domain independent framework (Tseng and Jiao, 1998). A key component of Quality Function Deployment (QFD; Clausing, 1994) is the customer requirements frame to aid the designer's view in defining product specifications. While QFD excels in converting customer information to design requirements, it is limited as a means of actually discovering the VoC (Hauge and Stauffer, 1993). To empower QFD with market aspects, Fung and Popplewell (1995) propose to pre-process the VoC prior to its being entered as customer attributes into the House of Quality (HoQ). In this process, the VoC is categorized using an affinity diagram (KJ method). Fung et al. (1998) further adopt the Analytic Hierarchy Process (AHP; Saaty, 1980) to analyze and prioritize customer requirements. Fung et al. (2002) extend their QFD-based customer requirement analysis method to a non-linear fuzzy inference model. Fukuda and Matsuura (1993) also propose to prioritize the customer's requirements by AHP for concurrent design. Researchers at IBM have applied structured brainstorming techniques to build customer requirements into the QFD process (Byrne and Barlow, 1993). McAdams et al. (1999) propose a matrix approach to the identification of relationships between product functions and customer needs.

In summary, most approaches assume product development starts from a clean sheet of paper. In practice, most new products evolve from existing products, i.e., so-called variant design. Historical data, product evolution paths, and feedback from customers on current products are often considered only implicitly, if not ignored. As a result, product design seldom has the opportunity to take advantage of the wealth of customer requirement information accumulated in existing products. In addition, these methods do not explicitly differentiate the customer preference from the designer's preference of requirement information (Tarasewich and Nair, 2001), nor exists any approach to handling effectively the mapping from the customer domain to the functional domain. Furthermore, new product development in mass customization is facing the challenge of maintaining the continuity of manufacturing and service operations. Therefore, product definition should effectively preserve the strength of product families to obtain significant cost savings in tooling, learning curves, inventory, maintenance, and so on. This demands a structured approach to product definition and to the capturing of gestalt requirement information from previous designs as well as existing product and process platforms.

3. Methodology of Product Portfolio Identification

3.1 Problem Formulation

Figure 3 illustrates the principle of product portfolio identification based on association rule mining. In general, customer needs can be described as a set of features or attributes, $A \equiv \{a_1, a_2, \ldots, a_M\}$. Each feature, $a_i \mid \forall i \in [1, \ldots, M]$, may take on one out of a finite set of options, $A_i^* \equiv \{a_{i1}^*, a_{i2}^*, \ldots, a_{in_i}^*\}$. That is, $a_i =:: a_{ij}^* \mid \exists a_{ij}^* \in A_i^*$, where $j = 1, \ldots, n_i$, denotes the *j*-th option of a_i. Suppose all customers comprise a set, $C \equiv \{c_1, c_2, \ldots, c_S\}$, where S denotes the total number of customers. In the customer domain, requirement information of a particular customer, $c_s \in C \mid \exists s \in [1, \ldots, S]$, can be depicted by a vector of certain options of these features, for example, $\overline{a_s^*} \equiv [a_{13}^*, a_{22}^*, \ldots, a_{M1}^*]$, where a_{13}^* refers to the 3-rd option of feature a_1 as desired by customer c_s,

a_{22}^* the 2-nd option of feature a_2, and a_{M1}^* the 1-st option of feature a_M. All population of customers' needs become a set, $A^* \equiv \{\overline{a_1^*}, \overline{a_2^*}, \cdots, \overline{a_S^*}\}$, which characterizes the customer domain.

In the functional domain, the functionality of each product is characterized by a set of FRs, $V \equiv \{v_1, v_2, \cdots, v_N\}$. Each FR, $v_q \mid \forall q \in [1, \cdots, N]$, possesses a few possible values, $V_q^* \equiv \{v_{q1}^*, v_{q2}^*, \cdots, v_{qn_q}^*\}$. That is, $v_q =:: v_{qr}^* \mid \exists v_{qr}^* \in V_q^*$, where $r = 1, \cdots, n_q$, denotes the *r*-th possible value of v_q. Suppose all existing products comprise a set, $P \equiv \{p_1, p_2, \cdots, p_T\}$, where T refers to the total number of products. The requirement specification of a particular product, $p_t \in P \mid \exists t \in [1, \cdots, T]$, can be represented as a vector of certain FR values of those FRs, for example, $\overline{v_t^*} \equiv [v_{12}^*, v_{21}^*, \cdots, v_{N5}^*]$, where v_{12}^* means product p_t involves the 2-nd value of FR v_1, v_{21}^* the 1-st value of FR v_2, and v_{N5}^* the 5-th value of FR v_N. All the instances of FRs (i.e., FR values) in the functional domain constitute a set, $V^* \equiv \{\overline{v_1^*}, \overline{v_2^*}, \cdots, \overline{v_T^*}\}$.

Based on the company's sales records and product documentation, we can extract transaction data related to which customer was met with which product. Therefore, transaction data can be summarized as CN-FR pairs in the form of $\langle \overline{a_s^*}, \overline{v_t^*} \rangle$, where s and t stand for customer ID and product ID, respectively. Each pair of such transaction data not only indicates a specific case of requirement information from both the customer and manufacturer viewpoints, but also implies a particular instance of mapping relationship between the customer and functional domains.

The difference between the customer and functional domains suggests that what a customer *de facto* perceives is the CNs, rather than FRs. While providing customer-perceived diversity in CNs, the manufacturer must seek for economy of scale in product fulfillment, which is meant by FRs. In addition, mass customization is by no means to provide whatever customers may want, as excessive variety results in a dramatic increase of costs (Huffman and Kahn, 1998). As postulated in the classic Hotelling-Lancaster model (Hotelling, 1929), some products close together on the spectrum are better substitutes than those further apart. This implies that customers are willing to choose from those products with functional values closest to their desired values if they can't find any product on the market that exactly matches their desired values. Consumer behavior study also suggests that the consumers falling into the same cluster usually hold the same purchase trend and thus the customer can be met by providing such a product that the total variations of functionality from what the customer prefers to are the smallest. This implies that individual customers within a cluster can most probably be satisfied with a product whose functional values assume the mean values of different expectations by all customers in the same cluster (namely, the centroid of the cluster).

Therefore, in order to take advantage of commonality in product family design, existing instances of FRs, V^*, should be analyzed and clustered according to the similarity among them (Tseng and Jiao, 1996). This process is called FR clustering. The result is a few FR clusters, noted as $X = \{c_1, c_2, \ldots, c_L\}$, where $c_l \in X \mid \forall l \in [1, \ldots, L]$, meaning the *l*-th FR cluster. As a result, all FR instances related to a FR cluster, i.e., $c_l \sim V^{*l} \subset V^*$, can be grouped and represented by the characteristics of c_l – the mean value of these FR instances, $m_l \equiv [x_1^l, x_2^l, \ldots, x_N^l]$, and the variation range of these FR instances within c_l, $D_l \equiv [d_1^l, d_2^l, \ldots, d_N^l]$. Therefore, each FR cluster can be described as a tuple: $c_l = (m_l, D_l)$.

Subsequently, these identified FR clusters become the functional specification of product offerings that can be derived from common product platforms and are supposed to be able to accommodate all the customer needs (Du et al., 2001). In other words, the specification of a product portfolio should cover a group of existing and latent CNs by mapping these needs to the identified FR clusters. At this stage, data mining techniques are applied to figure out the mapping relationship between CNs and FR clusters, noted as $A^* \Rightarrow X$, where an association rule, $\Rightarrow$, indicates an inference from the precedent (A^*) to the consequence (X). As a result, a product portfolio specification, L, consists of two elements: FR clusters and mappings from CNs to FR clusters, namely, $L = \langle X, \Rightarrow \rangle$.

Figure 3 Product portfolio identification based on association rule mining

3.2 FR Clustering

Clustering analysis refers to a process of grouping a set of physical or abstract objects into classes of similar objects. A cluster is a collection of objects that are similar to one another within the same cluster yet dissimilar to the objects in other clusters (Han and Kamber, 2001). The specification of FRs usually presents in the form of numerical, binary or nominal variables. To handle both quantitative and qualitative variables, this research adopts a fuzzy clustering approach to FR clustering. Fuzzy equivalence relations excel in revealing the similarity between any two objects involving subjectiveness and imprecision (Zimmermann, 1985). Fuzzy clustering is to create a hierarchical decomposition of the given set of objects, in which each object forms a separate group and successively the objects or groups close to one another are merged at different similarity levels. In our case, historical data about FR instances contained in the platform can be used to measure the similarity degree based on the compatibility of FR value ranges. In comparison with the k-means method, fuzzy clustering partitions FR instances based on the similarity degree that is derived from the real data of FR values, rather than based on subjectively pre-defined clusters.

Given a collection of objects (i.e., FR instances), $Z = V^* = \{v_t^* \mid \forall t = 1, \ldots, T\}$, a fuzzy set F in Z is defined as a set of ordered pairs: $F = \{(z, \mu_F(z)) \mid z \in Z\}$, where $\mu_F(z)$ is called the membership function of z in F that maps Z to $[0,1]$. The membership function is also referred to as the degree of compatibility or degree of truth. A certain set of objects that belong to the fuzzy set F at least to the degree λ is called the λ-cut.

Assume Z is a finite, non-empty set called the universe. Let R be a fuzzy relation in $Z \times Z$, that is, $R = \{(x,y) \mid \forall (x,y) \in Z \times Z\}$, then (Lin and Lee, 1996):

(1) R is reflexive if $\mu_R(z,z) = 1 \mid \forall z \in Z$;

(2) R is symmetric if $\mu_R(z,x) = \mu_R(x,z) \mid \forall x, z \in Z$; and

(3) R is max-min-transitive if $\mu_R(z,x) \geq max_{y \in Y}\{min\{\mu_R(z,y), \mu_R(y,x)\}\}$, i.e., $R \circ R \subseteq R$.

If R is reflexive and symmetric, R is said to be a fuzzy compatible relation. If R is reflexive, symmetric, and transitive, R is said to be a fuzzy equivalence relation. Fuzzy clustering becomes a set of T objects of Z to be clustered, given a fuzzy compatible relation R defined on Z. Assume R^t denotes the *t*-th power of fuzzy relation R, i.e., $R^t = R^{t-1} \circ R$, where $\circ$ is max-min composition. Then the max-min-transitive closure of R, denoted as R^*, can be defined as $R^* = \bigcup_{i=1}^{T} R^i$. Therefore, R^* is a fuzzy equivalence relation. Assume $0 \leq \lambda \leq 1$ and let $R_\lambda^* = \{(z,x) \mid \mu_{R^*}(z,x) \geq \lambda, \forall x, z \in Z\}$. Then we know (Wang and McCauley-Bell, 1996):

(1) R_λ^* is an equivalence relation on Z; and

(2) Let $G_{R_\lambda^*}$ denote the partition on Z induced according to R_λ^*. Then for each $B \in G_{R_\lambda^*}$, there exists $E \in G_{R_{\lambda'}^*}$, so that $B \subseteq E$, as long as $\lambda' \leq \lambda$.

As a result, the λ-cut of fuzzy equivalence relation R^*, R_λ^*, becomes an equivalence relation. As λ increased, a finer partition can be achieved. With a hierarchy of partitions of objects, k-clusters of objects can be identified. Figure 4 illustrates the nested partitions corresponding to a fuzzy equivalence relation defined based on the FR instances. Given different values of similarity threshold, l, different clustering results can be obtained.

Figure 4 Fuzzy clustering of FR instances

3.3 Association Rule Mining

FR clustering can separate data items into clusters of items, but cannot explain the clustering results specifically. It needs other methods to figure out the underlying mechanisms of CN-FR mapping between the customer and functional domains. Knowledge is usually represented in the form of rules. Rules are used for deducing the degree of association among variables, mapping data into predefined classes, identifying a finite set of categories or clusters to describe the data, etc. Therefore, this research employs association rules to explain the meaning of each FR cluster as well as the mapping of CNs to each cluster. Association rule mining is one of the major forms of data mining and is perhaps the most common form of knowledge discovery in unsupervised learning systems (Chen et al., 1996). Association rules are produced by finding the interesting associations or correlation relationships among a large set of data items. The flexibility of association rule induction lies in its capability to deal with those qualitative data that can't be treated by traditional operations research methods.

The basic problem of mining association rules is introduced by Agrawal et al. (1993). Let $I=\{i_1,i_2,\mathbf{L},i_m\}$ be a set of literals, called items. Let DB be a database of transactions, where each transaction, T, is a set of items such that $T \subseteq I$, and each transaction is associated with an identifier, called TID. Given $Z \subseteq I$, a transaction T contains Z if and only if $Z \subseteq T$. An association rule is an implication of the form $X \Rightarrow Y$, where $X \subseteq I$, $Y \subseteq I$, and $X \cap Y = \emptyset$. The association rule $X \Rightarrow Y$ holds in DB with confidence c if $c\%$ of the transactions in DB that contain X also contain Y. This is taken to be a conditional probability, $P(y/x \mid \forall x \in X, \forall y \in Y)$. The association rule $X \Rightarrow Y$ has support s in DB if $s\%$ of the transactions in DB contain X and Y. The support is taken to be a probability, $P(x \wedge y \mid \forall x \in X, \forall y \in Y)$.

While the confidence denotes the strength of implication, the support indicates the frequencies of the occurring patterns in the rule. Given a minimum confidence threshold, min_conf, and a minimum support threshold, min_sup, the problem of mining association rules becomes a searching for all the association rules whose confidence and support are larger than the respective thresholds. Based on if can meet the thresholds (min_conf and min_sup) or not, association rules are distinguished between strong rules and weak ones. A set of items is referred to as an itemset. An itemset that contains k items is called a k-itemset. Given a minimum support threshold, min_sup, an itemset is called large if its support is no less than min_sup. Association rule mining involves a two-step process (Agrawal et al., 1993):

(1) Discover all large itemsets whose support is larger than the predetermined minimum support threshold. Itemsets with minimum support are called frequent itemsets; and

(2) Generate strong association rules from the large itemsets.

The most crucial factor affecting the performance of mining association rules lies in the first step. After the large itemsets are identified, the corresponding association rules can be derived in a straightforward manner. Efficient counting of large itemsets is hence the focus of most prior studies on algorithms for mining association rules.

4. ARMS Architecture and Implementation

Knowledge discovery for CN-FR mapping mechanisms is an interactive and iterative process. Based on association rule mining, an inference system can be constructed for effective product portfolio identification. Figure 5 illustrates the architecture of such an association rule mining system (ARMS). The system involves four consecutive stages interacted one and another to achieve the goals, namely the data preprocessing, FR clustering, association rule mining and rule evaluation and presentation modules. First, historical data are selected and transformed to proper target data sets, which are further analyzed and preprocessed for subsequent mining procedures. The data mining procedure then starts to search for interesting patterns by the clustering module and rule mining module. After mining of association rules, the work of rule evaluation is performed to eliminate any weak rules under the initial criteria predefined by the system. The useful rules are stored with different presentation styles in the knowledge base that may be in the forms of case bases, rule bases, and others. Equipped with such knowledge about the patterns of CNs, FRs and their mappings, the system can provide better recommendations and high-degree predictions to improve portfolio identification.

Figure 5 ARMS system architecture

4.1 Data Preprocessing Module

Before proceeding to rule mining of data sets, raw data must be preprocessed in order to be useful for knowledge discovery. Three tasks are involved at this stage, as described below.

(1) Target data transformation. Generally, there are lots of data records in a company’s databases. Only those records that correlate closely with the mining purpose are taken into account. Based on

raw data stored in the company, target data sets should be identified, involving such data cleaning and filtering tasks as integration of multiple databases, removal of noises, handling of missing data files, etc.

All target data should be organized into a proper transaction database. This involves understanding of variables, selection of attributes and metrics, and identification of entity relationships among data. Within the ARMS, sales records and product documentation are transformed into transaction data (TID). Transaction data consists of customer records (C) and their ordered products (P). Each customer is described by his choices of certain options (A^*) for some functional features (A). The product ordered by this customer is described by specific values (V^*) of related FRs (V). The results of CN-FR mappings, i.e., $\left\langle \overline{a_s^*}, \overline{v_t^*} \right\rangle$, are embodied in the transaction records ($\left\langle C, P \right\rangle$). Figure 6 shows the entity relationships among these target data sets.

Figure 6 Entity relationships of target data sets

(2) Prioritization of FR variables. The specification of FRs involves multiple variables, i.e., $V = \{v_q \mid \forall q = 1, \ldots, N\}$. These FR variables contribute to the overall functionality of a product differently – some may play more roles than others. Hence, FR variables should be prioritized to differentiate their different effects, in particular those important ones. The relative importance of FR variables is usually quantified by assigning different weights. That is, each v_q is associated with a weight, w_q, subjective to $\sum_{q=1}^{N} w_q = 1$. For the ARMS, the AHP (Saaty, 1980) is adopted for the prioritization of FR variables, owing to its advantages in maintaining consistence among a large number of variables through pair-wise comparisons.

(3) Standardization of FR values. Prior to clustering analysis of FR instances, all V^* data need to be transformed into standard forms because FR variables may involve different metrics and ranges of values. In general, expressing a variable in smaller units will lead to a larger range for that variable, and thus a larger impact on the clustering structure. To avoid dependence on the choice of different metrics or dominance of certain variables over others, those FR instances that are of numerical type should be standardized to become dimensionless. This is achieved by normalization. Many methods are available such as the z-score method, the max-min normalization method (Han and Kamber, 2001). The ARMS adopts the latter method. Assume some of the FR variables, $v_k \in V \mid \forall k = 1, \ldots, Q \le N$, are of numerical type. It means that their values, $v_{kr}^* \in V_k^* \mid \forall r = 1, \ldots, n_k$, are numerical, where n_k

refers to the number of values that v_k can assume. Applying the max-min method, each individual value of v_k, v_{kr}^*, can be normalized to become a dimensionless number ranged between 0 and 1, that is,

$$N\_v_{kr}^* = \frac{v_{kr}^* - min\{v_{kj}^* \mid \forall j = 1, \ldots, n_k\}}{max\{v_{kj}^* \mid \forall j = 1, \ldots, n_k\} - min\{v_{kj}^* \mid \forall j = 1, \ldots, n_k\}}, \tag{1}$$

where $N\_v_{kr}^*$ denotes the normalized value for the *r*-th value of FR v_k, v_{kr}^* is the original value of v_k, and $max\{v_{kj}^* \mid \forall j = 1, \ldots, n_k\}$ and $min\{v_{kj}^* \mid \forall j = 1, \ldots, n_k\}$ are the maximum and minimum values among all values of v_k with size-n_k, respectively.

In some cases, those non-numerical FR instances, such as nominal FRs, should be transformed into normalized numerical values. For instance, the data type of FR "coating material" is originally of nominal type (i.e., character strings). A scaling transformation can be applied such that, for example, "Au coating" is supplanted by 0.2, "Alloy coating" becomes 0.4, and so on. When all FR instances possess the same measurements and ranges, we can proceed to the FR clustering process.

4.2 FR Clustering Module

Within the ARMS, FR clustering includes two steps: distance measure and fuzzy clustering. As a preparatory stage for fuzzy clustering, the distance measure module measures the dissimilarity between FR instances in order to define the fuzzy compatible relations among such data objects.

(1) Distance measure. In general, each FR instance, $\overline{v_t^*} = [v_{1t}^*, v_{2t}^*, \ldots, v_{qt}^*, \ldots, v_{Nt}^*] \in V^*$, where $\forall v_{qt}^* \equiv v_{qr}^*$, $\exists v_{qr}^* \in V_q^*$, $\forall r = 1, \ldots, n_q$, may involve three types of FR variables: numerical, binary, and nominal FRs. For example, v_{1t}^* may be a numerical value whilst v_{2t}^* may be a binary or nominal value. The distance between any two FR instances means the dissimilarity of them and thus is measured as a composite distance of three distance components corresponding to these three types of FR variables.

Numerical FRs — A number of methods of distance measure have been proposed for purpose of numerical clustering, including the Euclidean distance, Manhattan distance, Minkowski distance and weighted Euclidean distance measure (Han and Kamber, 2001). The ARMS employs the weighted Euclidean distance. It is computed as the following,

$$d_{numerical}\left(\overline{v_i^*}, \overline{v_j^*}\right) = \sqrt{\sum_{q=1}^{Q} \left(w_q \left(N\_v_{qi}^* - N\_v_{qj}^*\right)\right)^2}, \tag{2}$$

where $d_{numerical}\left(\overline{v_i^*},\overline{v_j^*}\right)$ indicates the numerical distance between two FR instances, $\overline{v_i^*}$ and $\overline{v_j^*}$, $\forall\overline{v_i^*},\overline{v_j^*}\in V^*$, w_q means the relative importance of the q-th numerical FR variable, $v_q\in V^{numerical}\subseteq V$, Q represents the total number of numerical FR variables among the total size-N FR variables ($Q\le N$), and $N\_v_{qi}^*$ and $N\_v_{qj}^*$ denote the normalized values of original v_{qi}^* and v_{qj}^* according to Eq.(1), respectively,

Binary FRs — A binary variable assumes only two states: 0 or 1, where 0 means the variable is absent and 1 means it is present. The ARMS uses a well-accepted coefficient for assessing the distance between symmetric binary variables, called the simple matching coefficient (Han and Kamber, 2001). It is calculated as the following,

$$d_{binary}\left(\overline{v_i^*},\overline{v_j^*}\right)=\frac{a_2+a_3}{a_1+a_2+a_3+a_4}, \tag{3}$$

where $d_{binary}\left(\overline{v_i^*},\overline{v_j^*}\right)$ indicates the binary distance between two FR instances, $\overline{v_i^*}$ and $\overline{v_j^*}$, $\forall\overline{v_i^*},\overline{v_j^*}\in V^*$, a_1 is the total number of binary FR variables in V (i.e., $v_q\in V^{binary}\subseteq V$) that equal to 1 for both $\overline{v_i^*}$ and $\overline{v_j^*}$, a_2 is the total number of binary FR variables that equal to 1 for $\overline{v_i^*}$ but 0 for $\overline{v_j^*}$, a_3 is the total number of binary FR variables that equal to 0 for $\overline{v_i^*}$ but 1 for $\overline{v_j^*}$, and a_4 is the total number of binary FR variables that equal to 0 for both $\overline{v_i^*}$ and $\overline{v_j^*}$.

Nominal FRs — A nominal variable can be regarded as a generalization of a binary variable in that it can take on more than two states. This type of variables can not be expressed by numerical values but by qualitative expressions with more than one option. Therefore, the simple matching coefficient can also be used here to measure the nominal distance between two FR instances containing nominal FR variables (Han and Kamber, 2001):

$$d_{nominal}\left(\overline{v_i^*},\overline{v_j^*}\right)=\frac{b-g}{b}, \tag{4}$$

where $d_{nomical}\left(\overline{v_i^*},\overline{v_j^*}\right)$ indicates the nominal distance between two FR instances, $\overline{v_i^*}$ and $\overline{v_j^*}$, $\forall\overline{v_i^*},\overline{v_j^*}\in V^*$, g means the total number of nominal FR variables in V (i.e., $v_q\in V^{nomical}\subseteq V$) that assume the same states for $\overline{v_i^*}$ and $\overline{v_j^*}$; and b is the total number of nominal variables among total size-N FR variables ($b\le N$).

Given a set of FR variables, $V\equiv\{v_1,v_2,\mathbf{L},v_N\}$, every FR instance assumes a certain value for each of the FR variable, and thus consists of a combination of numerical, binary and/or nominal FR values, that is, $V^{numerical}\cup V^{binary}\cup V^{nominal}=V$. As a result, the overall distance between $\overline{v_i^*}$ and $\overline{v_j^*}$ comprises three components: the numerical, binary and nominal distances. A composite distance can thus be obtained by the weighted sum:

$$d\left(\overline{v_i^*},\overline{v_j^*}\right)=W_{numerical}d_{numerical}\left(\overline{v_i^*},\overline{v_j^*}\right)+W_{binary}d_{binary}\left(\overline{v_i^*},\overline{v_j^*}\right)+W_{nominal}d_{nominal}\left(\overline{v_i^*},\overline{v_j^*}\right), \tag{5}$$

$$\sum\left(W_{numerical}+W_{binary}+W_{nominal}\right)=1 , \tag{6}$$

where $W_{numerical}$, W_{binary} and $W_{nominal}$ refer to the relative importance of numerical, binary and nominal distances, respectively. These weights can be determined in the similar way as that of FR variables – applying the AHP.

(2) Fuzzy clustering. The first step of fuzzy clustering is to define a fuzzy compatible relation, R, for a given set of FR instances, $V^*=\left\{\overline{v_1^*},\overline{v_2^*},\ldots,\overline{v_T^*}\right\}$. The R is constructed in a matrix form, that is, $R=\left[r\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{T\times T}\mid\forall\left(\overline{v_i^*},\overline{v_j^*}\right)\in V^*\times V^*$, where $\left(\overline{v_i^*},\overline{v_j^*}\right)$ suggests pair-wise relationships among FR instances. Within the context of FR clustering, R is called the compatible matrix. A matrix element $r\left(\overline{v_i^*},\overline{v_j^*}\right)$ indicates the similarity grade between any two FR instances, $\overline{v_i^*}$ and $\overline{v_j^*}$. As a measure of similarity, it can be derived from the aforementioned dissimilarity measure that is determined by the distance between FR instances. Then we have the following:

(a) Normalize the distance measure between $\overline{v_i^*}$ and $\overline{v_j^*}$ based on Eqs. (1) and (5), i.e.,

$$N\_d\left(\overline{v_i^*},\overline{v_j^*}\right)=\frac{d\left(\overline{v_i^*},\overline{v_j^*}\right)-min\left\{d\left(\overline{v_x^*},\overline{v_y^*}\right)\mid\forall x,y=1,\ldots,T\right\}}{max\left\{d\left(\overline{v_x^*},\overline{v_y^*}\right)\mid\forall x,y=1,\ldots,T\right\}-min\left\{d\left(\overline{v_x^*},\overline{v_y^*}\right)\mid\forall x,y=1,\ldots,T\right\}}, \tag{7}$$

where $N\_d\left(\overline{v_i^*},\overline{v_j^*}\right)\in[0,1]$ is the normalized value of original distance $d\left(\overline{v_i^*},\overline{v_j^*}\right)$, and $d\left(\overline{v_x^*},\overline{v_y^*}\right)\mid\forall\overline{v_x^*},\overline{v_y^*}\in V^*$ stands for a distance measure between any two FR instance based on pair-wise comparisons, $(x,y)\in T\times T$; and

(b) Derive the similarity grade $r\left(\overline{v_i^*},\overline{v_j^*}\right)$ from normalized distance measure $N\_d\left(\overline{v_i^*},\overline{v_j^*}\right)$, since it indicates the dissimilarity, i.e.,

$$r\left(\overline{v_i^*},\overline{v_j^*}\right)=1-N\_d\left(\overline{v_i^*},\overline{v_j^*}\right). \tag{8}$$

Hence, we have $0\leq r\left(\overline{v_i^*},\overline{v_j^*}\right)\leq 1$. In addition, we can infer that $r\left(\overline{v_i^*},\overline{v_i^*}\right)=1\mid\forall i=1,\ldots,T$, suggesting that R is reflexive, and $r\left(\overline{v_i^*},\overline{v_j^*}\right)=r\left(\overline{v_j^*},\overline{v_i^*}\right)\mid\forall i,j=1,\ldots,T$, suggesting R is symmetrical. As a result, matrix $R = \left[r\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{T\times T} \mid r\left(\overline{v_i^*},\overline{v_j^*}\right) \in [0,1]$ becomes a fuzzy compatible relation defined on V^*. Representing a subset of Cartesian product $V^* \times V^*$, matrix R is called a fuzzy compatible matrix.

The second step is to construct a fuzzy equivalence relation for V^* with transitive closure of the fuzzy compatible relation defined above. The fuzzy compatible matrix R is a fuzzy equivalence matrix if and only if the transitive condition can be met, i.e.,

$$r\left(\overline{v_i^*},\overline{v_j^*}\right) \geq max\left\{min\left\{r\left(\overline{v_i^*},\overline{v_z^*}\right), r\left(\overline{v_z^*},\overline{v_j^*}\right) \mid \forall \overline{v_i^*},\overline{v_z^*},\overline{v_j^*} \in V^*\right\}\right\}. \tag{9}$$

To convert a compatible matrix to an equivalence matrix, the "continuous multiplication" method is often used. Multiplication in fuzzy relations is also known as max-min composition (Lin and Lee, 1996). Let $R\left(\overline{v_i^*},\overline{v_z^*}\right)$ and $R\left(\overline{v_z^*},\overline{v_j^*}\right)$ be two fuzzy compatible relations, then $R\,\mathbf{o}\,R = \left[\left(\overline{v_i^*},\overline{v_j^*}\right), max\left\{min\left\{r\left(\overline{v_i^*},\overline{v_z^*}\right), r\left(\overline{v_z^*},\overline{v_j^*}\right)\right\}\right\}\right]$ is also a fuzzy compatible relation. To achieve the max-min-transitive closure of R, the flowchart of max-min composition is shown in Figure 7.

Figure 7 The flowchart of converting a compatible matrix to an equivalent matrix

The third step is to determine l -cut of the equivalence matrix. The l -cut is a crisp set, R_l, that contains all the elements of the universe, V^*, such that the similarity grade of R is no less than l, that is,

$$R_l = \left[t\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{T\times T}, \tag{10}$$

where
$$t\left(\overline{v_i^*},\overline{v_j^*}\right) = \begin{cases} 1 & if\ r\left(\overline{v_i^*},\overline{v_j^*}\right) \geq l \\ 0 & if\ r\left(\overline{v_i^*},\overline{v_j^*}\right) < l \end{cases}, \quad r\left(\overline{v_i^*},\overline{v_j^*}\right) \in [0,1]. \tag{11}$$

Then each l -cut, R_l, is an equivalence relation representing the presence of similarity among FR instances to the degree l. For this equivalence matrix, there exists a partition on V^*, $y\left(R_l\right)$, such that each compatible matrix is associated with a set, $y\left(R\right) = \left\{y\left(R_l\right)\right\}$. The ARMS applies a netting method (Yang and Gao, 1996) to identify partitions of FR instances with respect to a given equivalence matrix. The procedure of generating a fuzzy netting graph is summarized as the following,

(a) Fill the signals of the elements in the diagonal;

(b) Replace element 1 as signal * and element 0 as blank;

(c) Connect longitude and latitude to the nodes where the signals * are located; and

(d) Assign the elements that are connected through the nodes into the same cluster.

The value of $\lambda \in [0,1]$ indicates the similarity threshold of a λ -cut. Given an equivalence matrix, different clustering results can be obtained according to individual similarity thresholds, as shown in Figure 4(c). In practice, the value of λ is often determined by domain experts with many practical considerations (Lin and Lee, 1996). Furthermore, latent and future customer needs, trends of product and process technologies, repeatability in design and manufacturing, ease of configuration, core competencies, and many others, are also important dimensions of decision making for the threshold.

Finally, with the hierarchy of partitions of objects, k-clusters of objects can be identified. The ARMS adopts a straightforward algorithm introduced by Wang and McCauley-Bell (1996) for hierarchical clustering. Each FR cluster, $c_l = (m_l, D_l) \mid \forall l = 1, \ldots, L$, is described by a vector of its mean, $m_l = [x_q^l]_N$, and a vector of its variation range, $D_l = [d_q^l]_N$.

For a numerical FR value (i.e., $v_{qt}^* \sim v_q \in V^{numerical}$), the mean value and the variation range are calculated as the following,

$$x_q^l = \sum_{t=1}^{n_l} v_{qt}^* \Big/ n_l \ , \tag{12}$$

$$d_q^l = max\left\{\left|v_{qt}^* - x_q^l\right| \mid \forall t = 1, \ldots, n_l\right\}, \tag{13}$$

where $\forall q \in [1, \ldots, N]$, $\forall \overline{v_t^*} = [v_{qt}^*]_N \in V^*$, and n_l refers to the number of FR instances associated with the *l*-th FR cluster, i.e., $\forall \overline{v_t^*} \sim c_l \mid \forall t = 1, \ldots, n_l \le T$.

For a binary FR value (i.e., $v_{qt}^* \sim v_q \in V^{binary}$), the mean value and the variation range are determined as the following,

$$x_q^l = \begin{cases} 1 & if\ a_Y \ge a_N \\ 0 & if\ a_Y < a_N \end{cases}, \tag{14}$$

$$d_q^l = 0, \tag{15}$$

where $\forall q \in [1, \ldots, N]$, $\forall \overline{v_t^*} = [v_{qt}^*]_N \in V^*$, $a_Y + a_N = n_l$, n_l refers to the number of FR instances associated with the *l*-th FR cluster, i.e., $\forall \overline{v_t^*} \sim c_l \mid \forall t = 1, \ldots, n_l \le T$, a_Y is the total number of FR instances that assume a 1-state for v_q, and a_N is the total number of FR instances that assume a 0-state for v_q.

For a nominal FR value (i.e., $v_{qt}^* \sim v_q \in V^{nominal}$), the mean value and the variation range are determined as the following,

$$x_q^l = v_{qr}^* \mid r = max(a_r), \tag{16}$$

$$d_q^l = 0, \tag{17}$$

where $\forall q \in [1,\ldots,N]$, $\forall \overline{v_t^*} = [v_{qt}^*]_N \in V^*$, v_{qr}^* represents the *r*-th state of v_q that possesses n_q possible states, i.e., $\exists r \in [1,n_q]$, and a_r is the total number of FR instances that assume a v_{qr}^*-state for v_q.

4.3 Association Rule Mining Module

As reviewed in Section 3.3, traditional association rule mining ($Z \Rightarrow Y$) conforms to the general model of market basket analysis, where all items are assumed to belong to one itemset of transaction data ($Z \subseteq I$ and $Y \subseteq I$). In the ARMS scenario, rule mining involves two different itemsets, that is, $Z \subseteq A^*$ and $Y \subseteq V^*$, corresponding to the customer and functional domains, respectively. Based on the clustered FR instances, association rules regarding the mappings between individual A^* and V^* turn out to be the association rules mapping A^* to FR clusters, X, that is, $A^* \Rightarrow X$. Therefore, the ARMS's transaction data comprises these two itemsets, i.e., $DB \sim \langle A^*, X \rangle$, where $A^* = \{\overline{a_s^*} \mid \forall s = 1,\ldots,S\}$ and $X = \{c_l \mid \forall l = 1,\ldots,L\}$. Itemset A^* consists of a number of sales records of CNs embodied in various combinations of customer choices for diverse options of features, i.e., $\{a_{ij}^* \mid \forall i = 1,\ldots,M, \forall j = 1,\ldots,n_i\}$, where a_{ij}^* corresponds to the *j*-th option of feature a_i, which possesses n_i possible options. Each customer's order indicates a particular combination of these options, i.e., $\overline{a_s^*} = [a_{ij}^*]_M$. Itemset X comprises a set of FR clusters in the form of mean-variation tuples, i.e., $\{(m_l, D_l) = ([x_q^l]_N, [d_q^l]_N) \mid \forall l = 1,\ldots,L\}$. As a result, the general form of an association rule in the ARMS is given as the following,

$$a_1 \wedge a_2 \ldots \wedge a_e \ldots \wedge a_E \Rightarrow b_1 \wedge b_2 \ldots \wedge b_f \ldots \wedge b_F \quad [Support = s\%; Confidence = c\%], \tag{18}$$

where $\exists a_e \in \{a_{ij}^*\}_{\sum_{i=1}^{M} n_i} \mid \forall e = 1,\ldots,E \le M$, $\exists b_f \in \{(x_q^l, d_q^l)\}_{N \times L} \mid \forall f = 1,\ldots,F \le N$, and $s\%$ and $c\%$ refer to the support and confidence levels for this rule, respectively. They are calculated based on the following,

$$s\% = \frac{count(a_1 \wedge a_2 \ldots \wedge a_E \wedge b_1 \wedge b_2 \ldots \wedge b_F)}{count(DB)} \times 100\%, \tag{19}$$

$$c\% = \frac{count(a_1 \wedge a_2 \ldots \wedge a_E \wedge b_1 \wedge b_2 \ldots \wedge b_F)}{count(a_1 \wedge a_2 \ldots \wedge a_E)} \times 100\%, \tag{20}$$

where $count(a_1 \wedge a_2 \cdots \wedge a_E \wedge b_1 \wedge b_2 \cdots \wedge b_F)$ is the number of transaction records in DB containing all items a_1, a_2,…, and a_E as well as b_1, b_2,…, and b_F, $count(DB)$ is the total number of data records contained in DB, and $count(a_1 \wedge a_2 \cdots \wedge a_E)$ is the number of transaction records in DB containing all items a_1, a_2,…, and a_E. In general, $count(DB) = S$, because each TID corresponds to a $s-t$ pair. In addition, the set $\{a_1, a_2, \cdots, a_e, \cdots, a_E\}$ embodies a non-empty subset of $\{a_{ij}^* \mid \forall i \in [1,M], \exists j \in [1,n_i]\}$, whereas the set $\{b_1, b_2, \cdots, b_f, \cdots, b_F\}$ exhibits a non-empty subset of $\{(x_q^l, d_q^l) \mid \forall q \in [1,N], \exists l \in [1,L]\}$. The association rule in Eq. (18) means that the data occurrence of a_1, a_2,…, and a_E will most likely (at a $s\%$-support and with a $c\%$- confidence) associate with the data occurrence of b_1, b_2,…, and b_F.

A large number of efficient algorithms for mining association rules have been proposed (Chen et al., 1996). The ARMS adopts a well-known algorithm, called Apriori algorithm (Agrawal and Srikant, 1994) to determine frequent itemsets. Once the frequent itemsets are identified from DB, it is straightforward to generate strong association rules from them. For a large volume of source relations, the performance of rule generation may be slow. Rather than updating the association rule base continuously, the ARMS derives association rules incrementally by storing the record counts of previous computing data into the existing rule set and adding the new record counts during the new data computing process. Table 1 shows the procedure of such an incremental strategy for rule mining.

Table 1 Algorithm of incremental mining of association rules in the ARMS

4.4 Rule Evaluation and Presentation Module

Based on all the association rules created, the evaluation and presentation module comes into play to refine these rules in order to keep the most relevant and valuable rules in the knowledge base in the form of either case bases or rule bases. The characteristics of each FR cluster should also be explored based on the rules and the related support and confidence levels. Moreover, the causality of original association rules are defined for single feature options, as the precedent of each rule is a subset of $\{a_{ij}^*\}$ and the consequence of each rule is a subset of $\{(x_q^l, d_q^l)\}$ per se. Nevertheless, inference relationships do exist in various combinations of more feature options. This means a need for generating combinatorial rules. To solve such a rule refinement problem, the ARMS adopts an equivalence class method proposed by ChangChien and Lu (2001). Finally, users can retrieve all the rules stored in the knowledge base to understand the mappings of CNs to FRs clearly, to gain insights into the consequences of diverse customer preferences on the product fulfillment, and thus to justify the proper specification of product offerings in a portfolio.

5. Case Study

The potential of ARMS has been tested in an electronics company that produces a large variety of vibration motors for major world-leading mobile phone manufacturers. The company had conducted extensive market studies and derived data of customer expressions of various functionality related to mobile phones. These data have been collected from market surveys and analyzed based on natural language processing. As far as the "Alarm" function is concerned, the related features and their options are summarized in Table 2. Those CNs listed in Table 2 provide the ground for diverse specifications of the "Alarm" function as perceived by different mobile phone users. A variety of the "Alarm" functions correspond to different vibration motor designs. In other words, the "Alarm"-related CNs of mobile phones are fulfilled by the FRs of vibration motors. Based on existing product documentation and consultation with design engineers, we know that the functional specification of vibration motors is described by a set of FRs and their values, as shown in Table 3. Among these 9 FRs, the "Pbfree" is of binary type and the "Coating" is of nominal type, while all the rest are numerical variables.

It is interesting to observe the difference between CNs and FRs in this case. What customers really perceive is how they feel about the "Alarm" function of mobile phones. Customers have no idea of the implications of this functionality in engineering – vibration motors. From the company's viewpoint, CNs refer to mobile phones, whereas FRs are related to vibration motors. When the company makes decisions about its vibration motor portfolio, it has to understand the mapping mechanisms between the customer and functional domains, as well as the tradeoffs of requirement specification between mobile phones and vibration motors.

Table 2 List of CNs

Table 3 List of FRs

Based on the sales records, target data are identified and organized into a transaction database, as shown in Table 4. For illustrative simplicity, only 30 out of hundreds of transaction records are used in the case study here. As shown in Table 4, each customer order indicates the customer's choice of certain feature options related to the "Alarm" function of mobile phones, which is presented as a specific instance of a subset of $A = \{a_i\}_M$. Corresponding to the 30 customers (end-users of mobile phones), there are 30 vibration motors provided, whose requirement information are described as particular instances of FR vector, $\left[v^*_{qr}\right]_N$.

Table 4 Transaction database

To prioritize 9 FR variables, the AHP is applied. A 9-scale rating system is used to provide subjective judgments of preference, as shown in Table 5. The result of each weight associated with each FR variable is given in Table 6.

Table 5 Scale for subjective judgment

Table 6 Relative importance among FR variables

Due to different metrics used for FR variables, all FR instances in Table 4 need to be standardized based on the max-min normalization method. After that, the distances between every two FR instances are calculated to suggest the dissimilarity among them. The SPSS software package (SPSS 12.0 for Windows, http://www.spss.com/) is used to obtain the weighted Euclidean distance measures. The 30 records of product specifications are input into the SPSS software for processing, in which the original data are normalized automatically and then the distances are calculated. The pair-wise measures of distances are presented as a 30×30 matrix. Figure 8 shows the raw data for distance measures of numerical FR instances before the normalization. The normalized distance measures of numerical FR instances are presented in a matrix form, $\left[N\_d_{numerical}\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{30\times 30}$, as shown in Figure 9. The results of distance measures for binary and nominal FR instances, $\left[N\_d_{binary}\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{30\times 30}$ and $\left[N\_d_{nominal}\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{30\times 30}$, are shown in Figures 10 and 11, respectively. Based on these three distance components, the composite distances are calculated and presented as a dissimilarity matrix, $\left[d\left(\overline{v_i^*},\overline{v_j^*}\right)\right]_{30\times 30}$, for all FR instances, as shown in Figure 12. Based on the relative importance of FR variables, the weights associated with numerical, binary and nominal distance components are determined as $W_{numerical} = w_1 + w_3 + w_4 + w_6 + w_7 + w_8 + w_9 = 0.677$, $W_{binary} = w_2 = 0.304$ and $W_{nominal} = w_5 = 0.019$, respectively.

Figure 8 Raw data for distance measures of numerical FR instances

Figure 9 Result of distance measures for numerical FR instances

Figure 10 Result of distance measures for binary FR instances

Figure 11 Result of distance measures for nominal FR instances

Figure 12 Dissimilarity matrix based on distance measures for all FR instances

Based on the dissimilarity matrix, a fuzzy compatible matrix, R, is determined, as shown in Figure 13. Obviously, R meets both the reflexive and symmetric characteristics. To obtain a fuzzy equivalence matrix, the max-min composition is applied. The result of $R^2 = R \,\mathbf{o}\, R$ is shown in Figure 14. As $R^2 \neq R$, we know R^2 is not a fuzzy equivalence matrix yet. Continuing to apply the max-min composition, $R^4 = (R \,\mathbf{o}\, R) \,\mathbf{o}\, (R \,\mathbf{o}\, R)$ is obtained, which equals to R^2. The result of R^4 is also shown in Figure 14. As a result, R^4 turns out to be a fuzzy equivalence matrix. Based on R^4, the l -cut is derived with a similarity threshold setting at 0.84. The result of the l -cut is shown in Figure 15.

Figure 13 Result of R

Figure 14 Result of R^2 and R^4

Figure 15 Result of a l -cut with $l = 0.84$

With the obtained l -cut, a fuzzy netting graph is constructed, as shown in Figure 16. Based on the partitions derived from the fuzzy netting graph, 3-clusters of FR instances are identified. The mean value and variation range for each FR cluster are calculated based on those FR instances that are grouped into this cluster. The result of FR clustering is given in Table 7, in which, for example, FR cluster, c_1, is associated with its mean, $\mu_l = [100, Y, 9.2, 4.5, Au, 44.5, 67, 2.4, 49]$, and variation range, $\delta_l = [0, 0, 1.2, 0.5, 0, 10.5, 27, 0.6, 21]$, and contains 10 FR instances, including $\overline{v_1^*}$, $\overline{v_2^*}$, $\overline{v_7^*}$, $\overline{v_8^*}$, $\overline{v_{11}^*}$, $\overline{v_{12}^*}$, $\overline{v_{14}^*}$, $\overline{v_{15}^*}$, $\overline{v_{24}^*}$, and $\overline{v_{29}^*}$.

Figure 16 Fuzzy netting graph

Table 7 Result of FR clustering

The resulted FR clusters comprise an itemset, $X = \{(x_q^l, d_q^l) \mid \forall q \in [1,9]; \exists l \in [1,3]\}$, as shown in Table 8. The characteristics of each FR cluster entail the specification of a product platform – a set of base values together with the related variation ranges, and therefore can be used to suggest standard settings for vibration motor portfolio. These items are added to the transaction database. The link of each customer order to a FR instance is then replaced with the link to the items of the FR cluster that this FR instance belongs to. To mining rules between itemsets A^* and X, a data mining tool, called Magnum Opus (Version 2.0, http://www.rulequest.com/), is employed. All data are extracted from the transaction database and input as a text file to the Magnum Opus. The system allows data to be input as identifier-item files that list customers to be analyzed in the identifier-item format. Each customer has a unique identifier consisting of two columns: one for the identifier and one for the item. The Magnum Opus provides five association metrics: leverage, lift, strength, coverage, and support, each of which is supported by a search mode. The case study only uses the support and strength modes for the handling of support and confidence measures, respectively. This is because the coverage, lift and leverage criteria are not considered in the Apriori algorithm. Under either search mode, the Magnum Opus finds a number of association rules specified by the user. The search guarantees that only those rules with the highest values on the specified metric are found according to user specified search settings.

The Magnum Opus will find fewer than the specified number of association rules if the search is terminated by the user or there are fewer than the specified number of associations that satisfy user specified search settings. In our case, the maximum number of associations is set to be 10000 to make sure that the association rules can be derived completely. The minimum leverage, minimum lift, minimum strength, minimum coverage, and minimum support are set as 0, 1.0 (default value required by the system), 0.6, 0, and 0.5, respectively. Figure 17 shows the setting of search modes and their metrics as well as the rule induction process in the Magnum Opus.

Table 8 Specification of vibration motor portfolio based on FR clusters

Figure 17 Association rule induction in the Magnum Opus

At the end of mining, the system generates 37 association rules, as shown in Table 9. These rules serve as the basis of knowledge discovery. Some rules, for example, Rules 31, 32 and 33, are coupled and should be aggregated into one. The possibility of some rule combinations is also considered to discover more implicit rules. For example, Rules 15, 16 and 17 together with Rules 23, 24 and 25 can give more hints to optimize the size of motors. In addition to such rule refinement, the characteristics of each FR cluster and implicit relationships among them are explored to gain more understanding of vibration motor design specifications, so as to identify prominent settings of particular FR variables, to analyze the tradeoffs between different customer perceptions on mobile phones and the relevant FR values of vibration motors, and so on. All the identified patterns of CNs, FRs and the mapping are built into the knowledge base and are utilized to assist users in portfolio decision making based on the generated portfolio (Table 8).

Table 9 Result of association rule mining

6. Sensitivity Analysis

To evaluate the performance of the ARMS, the sensitivity of the identified product portfolio is studied with respect to varying values of data mining parameters, including the similarity threshold, and the minimum support and confidence levels. These parameters involve two modules of the ARMS: FR clustering and association rule mining, respectively.

The FR clustering module entails the specification of an optimal value of similarity threshold of l-cut. Essentially, it gives rise to a tradeoff issue of FR granularity inherent in mass customization (Tseng and Jiao, 1996). With a large (small) value of l-cut, more (less) FR clusters will be identified. These FR clusters affect the downstream planning of the product and process platforms. In terms of the economic latitude, the cost of introducing more FRs (i.e., finer FR clustering) and its contribution to customer-perceived values should reach a balance at the right level of aggregation of the product and process platforms. If the differentiation of FRs is too spread or too low a level of aggregation, such as at the nuts and bolts level, then the number of DPs and PVs may be too many and the product fulfillment becomes difficult to leverage the investments. To the contrary, if the FR aggregation is at a very high level, such as complete subassemblies, then the repetition may not be sufficient to take advantage of mass production efficiency.

An optimal granularity can normally be determined by assessing the performance of the product and process platforms in accordance with the resulted FR clusters. Jiao et al. (2004) apply the real option theory to the valuation of flexibility enabled by the product and process platforms. On the other hand, the construction of the product and process platforms embodies a type of fixed costs (Meyer and Lehnerd, 1997; Du et al., 2001). Therefore, we introduce a performance measure of l-cut, Y^{l}, as the following,

$$Y^{l} = \frac{E[V]}{C^{F}}, \tag{21}$$

where $E[V]$ denotes the expected value of the product and process platforms, which is determined based on a real option framework (Jiao et al., 2004; Gonzalez-Zugasti et al., 2001), and C^{F} stands for the fix cost of the product and process platforms. Furthermore, Jiao and Tseng (2004) posit the rationale of justifying cost implications of the product and process platforms based on process variations. Following Jiao and Tseng (2004) and Jiao et al. (2004b), we employ a process capability index to measure the above fixed cost, as the following,

$$C^{F} = b^{F} e^{\frac{1}{PCI}} = b^{F} e^{\frac{6s}{USL-LSL}}, \tag{22}$$

where b^{F} is a constant indicating the average dollar cost per variation of process capabilities, USL, LSL and s are the upper specification limit, lower specification limit and standard deviation of part-worth cost estimates corresponding to individual FR clusters, respectively. The part-worth cost estimates are determined using a pragmatic approach based on standard time estimation (Jiao et al., 2004b).

To analyze the sensitivity of product portfolio identification, a total number of 17 runs of FR clustering are generated by changing l value from 0.1 to 0.95 with an increment of 0.05. Using process data of vibration motors in Jiao et al. (2003) and flexibility valuation data of vibration motors in Jiao et al. (2004a), the result of sensitivity analysis is obtained. As shown in Figure 18, the performance measure in Eq. (21) is presented as a normalized comparison. The result clearly shows that a l value of 0.84 yields the best performance of FR clustering for product portfolio identification.

Figure 18 Sensitivity analysis of product portfolio identification with respect to similarity threshold

The difficulty in association rule mining originates from the need for determining appropriate thresholds for the support and confidence levels. If the support and confidence thresholds are planned with low values, useful information may be overwhelmed in excessive rules. To the contrary, certain relationship patterns that are of interest may be ignored if the support and confidence criteria are specified very strict.

Association rules basically suggest the mapping relationships between CNs and FRs. To meet the required CNs, the associated FRs must be fulfilled through configuration of DPs and PVs within the existing product and process platforms – a process of product variant derivation (Du et al., 2001). Such a variant derivation exhibits the accounting of a type of variable costs (Meyer and Lehnerd, 1997). Jiao et al. (2004b) review the implications of customer-perceived value per unit cost in regard to the measure of profitability. Therefore, we introduce a performance measure of association rule mining, y^{AR}, based on the ratio of utility and the variable cost, as the following,

$$Y^{AR} = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{U_{ij}}{C_j^V}, \tag{23}$$

where the resulted product portfolio comprises $j = 1, \mathbf{L}, J$ products that are offered to meet a target market segment with $i = 1, \mathbf{L}, I$ customers, U_{ij} denotes the utility of the *i*-th customer with respect to the *j*-th product, and C_j^V is the related variable cost of producing this product variant. As suggested in Jiao et al. (2004b), product level utilities, $\{U_{ij}\}_{I \cdot J}$, are derived from part-worth utilities of individual CNs based on conjoint analysis (Green and Krieger, 1978). Likewise, product costs, $\{C_j^V\}_J$, are determined by the regression of part-worth cost estimates of individual FRs. The association rules indicate what FRs are to be used to satisfy what CNs. Such customer choice and product instantiation can be implemented by introducing binary variables to the part-worth regressions (Jiao et al., 2004b).

To analyze the sensitivity of association rule mining, a total number of 18×18=324 runs of ARMS are set up by enumerating all combinations of the *min_sup* and *min_conf* values, where both the *min_sup* and *min_conf* values are changed from 0.05 to 0.95 with an increment of 0.05. Using utility data of vibration motors in Jiao et al. (2004a) and process data of vibration motors in Jiao et al. (2003), the result of sensitivity analysis is obtained. As shown in Figure 19, the performance measure in Eq. (23) is presented as a normalized comparison. The result of sensitivity analysis suggests that the optimal criteria of association rule mining are given as the support and confidence thresholds of 0.5 and 0.6, respectively.

Figure 19 Sensitivity analysis of product portfolio identification with respect to minimum support and confidence levels

7. Discussions

As witnessed in the case study, it is profound to discern CNs from FRs in the respective customer and functional domains. Such a contextual difference in requirement information, as a matter of fact, constitutes the major tradeoffs inherent in the product definition process. While customers concern about the “Alarm” function of a mobile phone, designers have to interpret the implications of these CNs in terms of the functional specification of a vibration motor. During this process, engineering concerns play different roles in analyzing CNs and FRs. In accordance, product portfolio identification should seek for a synergy of these two sets of requirement information so as to achieve the desired “dynamic” functional variety while keeping “stability” in technical variety (Du et al., 2001). Therefore, the ARMS specifies a portfolio in terms of clusters of FRs while bearing correspondence to CNs. We believe this is more reasonable than most models in market research and requirement management, in which customer groups, market segments, or requirement patterns are all built upon the assumption that CNs and FRs connote the same semantic set of requirements. In this sense, the ARMS is more applicable to those consumer products than capital products (industrial products, e.g., power supplies). Consumer products usually involve more explicit interfaces between customers and engineering, whereas capital products involve less explicit customer involvement in engineering. In addition, knowledge recovery by data mining should be more useful for variant designs rather than new designs. Moreover, we advocate the importance of reusing knowledge from past data in order to deliver mass customization within the existing capabilities. In this regard, the portfolio identification has to conform to the product and process platforms that have been installed in the company. So the specification of product offerings in a portfolio indeed represents the functional view of the product and process platforms.

In terms of requirement pattern recognition, association rule mining is advantageous over the tradition method based on decision trees. The key difference between the two techniques lies in that the decision tree method can only produce rules that are mutually exclusive, whilst association rule mining can produce rules that may not be mutually exclusive (Berson et al., 1999). The reason behind this originates from the way they operate. Association rule mining seeks to go from the bottom up and collect all possible patterns that are of interest, and then use these patterns for some prediction targets. Decision trees, on the other hand, work from a prediction target downward in a manner known as "greedy" search. They look for the best possible split at the next step. Furthermore, decision trees deal with data records that belong to the same category, whereas association rule mining can handle data records from different itemsets.

Nevertheless, the applicability of ARMS requires intensive collaboration with domain experts and considerations of particular problem contexts. Decisions on the proper similarity threshold and reasonable support and confidence levels may be too complex, and tricky as well, for enterprise managers. In practice, this can be alleviated through iterative interactions between portfolio identification and portfolio evaluation, as what we have done in the sensitivity analysis. Usually, a few scenarios with different settings of these parameters are identified and then input to the ARMS. Based on the running results, the performances of them are evaluated against a few pre-defined business objectives. Then the best setup is determined and the portfolio specification is refined. Hence, portfolio identification and its evaluation are iterative in implementation and thereby should be integrated within a unified framework of product portfolio planning.

While data mining techniques excels in identifying hidden patterns of mapping relationships between CNs and FRs, a practical data mining application is often complex, involving a number of interactive and iterative steps (Han and kamber, 2001). The processing of data throughout the data mining process deserves a particular attention for the achievement of good results. This is, however, often neglected and difficult to implement in practice. Pyle (1999) provides a comprehensive coverage of existing data preparation techniques, including discretization, dimensionality reduction, normalization, etc. Treatment of missing values and data cleaning are important exercises for the implementation of data mining. The post-processing of discovered patterns is also important. This may involve interpreting association rules, analyzing the patterns automatically or semi-automatically, or identifying those truly interesting and useful patterns for the user. Also important is to extract target data sets from transaction records based on thorough understanding of the application domain and the application goals.

As for association rule mining, the support-confidence framework has been the subject of several criticisms. The confidence measure does not adequately capture the intuitive and natural semantics of direct associations, in which the associations are obvious (Adamo, 2001). To improve this, Brin et al. (1997) propose an alternative measure, called conviction, to account for the strength of direct associations. In addition, the support-confidence framework tends to favor those rules with dense consequent. As a result, the rule generation process inclines to overstress those rules with a high consequent support. For instance, certain biased rules involving negated attributes are likely to appear in the outcome, making it contain many spurious rules (Aggarwal and Yu, 1998). Towards this end, a number of improvements have been proposed, including improvement-based rule pruning, collective strength, correlated attribute-set enumeration, intensity measure, and so on (Adamo, 2001). Moreover, traditional association rule mining adopts only a single minimum support in rule generation. However, classification data often contains a huge number of rules, which may cause combinatorial exploration. To tackle such an unbalanced data class distribution, Liu et al. (1998) introduce the use of multiple class minimum supports to rule generation by assigning a different minimum support for each class. By incorporating appropriate measures into the association rule mining process, the quality of rules could be improved dramatically. For example, the Magnum Opus data mining tool employed in this study provides five instruments: coverage, support, strength, lift, and leverage. In the current mining process, we have only used two of them: support and strength. Conjoint use of all these five measures could improve the predictive accuracy of association rule mining substantially (http://www.rulequest.com/MOnew.html). However, the challenge lies in how to apply appropriate measures in accordance with the specific problem context of domain applications.

Another weakness inherent in data mining is that most rule induction methods perform a local, greedy search in the space of candidate rules. Intuitively, a global search can discover interesting rules and patterns that would be missed by the greedy search. Along this line, a number of efforts have been added into the research agenda. For example, evolutionary algorithms are applied to data mining to enable a robust search method that performs a global search in the solution space (Freitas, 2002). The amalgamation of statistical and data mining techniques has also attracted much attention (Mani et al., 2001).

8. Conclusions

This paper presents a domain independent inference system for analyzing and organizing requirement information to support product portfolio identification. The methodology is based on the mining of association rules so as to provide an integration of requirement information from both customer and design viewpoints within a coherent framework.

Product portfolio identification entails a mapping process from customer needs in the customer domain to functional requirements in the functional domain. The specification of product offerings in a portfolio is embodied in a set of functional requirement clusters in conjunction with a set of associations of customer needs and the clusters. Each functional requirement cluster performs as a functional platform to satisfy a group of customers by enabling a certain range of variation with respect to a base value. For most variant product designs, where market segments have been established and product platforms have been installed, the association rule mining methodology can improve the efficiency and quality of portfolio identification by alleviating the tedious, ambiguous and error-prone process of requirement analysis enacted among customers, marketing folks, and designers. Generating the portfolio based on knowledge discovery from past data avails to maintain the integrity of existing product and process platforms, as well as the continuity of the infrastructure and core competencies, hence leveraging existing design and manufacturing investments. The application of data mining opens opportunities for incorporating experts' experiences into the projection of portfolio patterns from historical data, thereby enhancing the ability to explore and utilize domain knowledge more effectively.

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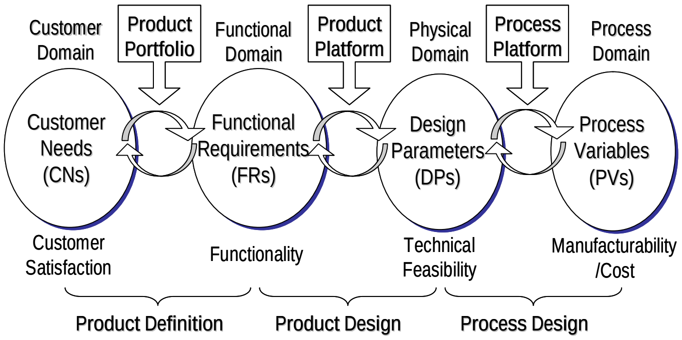


Figure 1 Product portfolio within the spectrum of product development

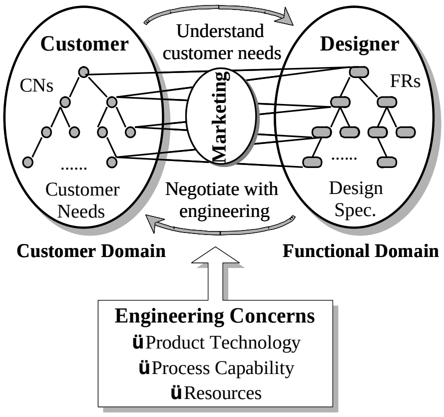


Figure 2 Product definition process inherent in product portfolio identification

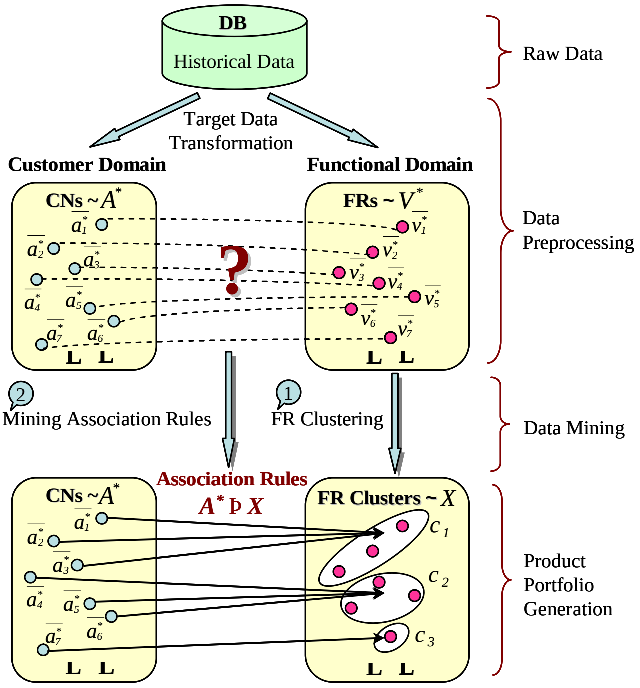
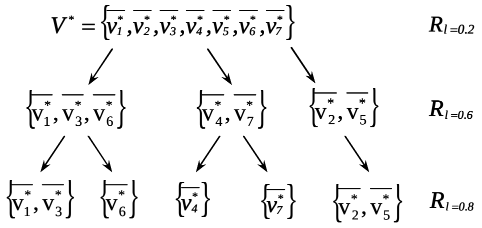


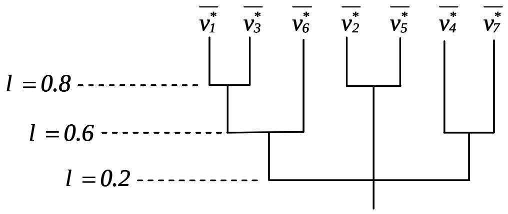
Figure 3 Product portfolio identification based on association rule mining

$$R = \begin{array}{c|ccccccc|} V^* & \overline{v_1^*} & \overline{v_2^*} & \overline{v_3^*} & \overline{v_4^*} & \overline{v_5^*} & \overline{v_6^*} & \overline{v_7^*} \\ \overline{v_1^*} & 1 & 0.2 & 1 & 0.4 & 0.3 & 0.7 & 0.5 \\ \overline{v_2^*} & 0.2 & 1 & 0.2 & 0.3 & 0.8 & 0.3 & 0.4 \\ \overline{v_3^*} & 1 & 0.2 & 1 & 0.4 & 0.1 & 0.6 & 0.3 \\ \overline{v_4^*} & 0.4 & 0.3 & 0.4 & 1 & 0.1 & 0.5 & 0.7 \\ \overline{v_5^*} & 0.3 & 0.8 & 0.1 & 0.1 & 1 & 0.2 & 0.5 \\ \overline{v_6^*} & 0.7 & 0.3 & 0.6 & 0.5 & 0.2 & 1 & 0.3 \\ \overline{v_7^*} & 0.5 & 0.4 & 0.3 & 0.7 & 0.5 & 0.3 & 1 \end{array}$$

(a) A fuzzy equivalence relation defined on V^*



(b) Nested partitions of V^* induced according to R_l



(c) Different FR clusters resulted from different values of similarity threshold

Figure 4 Fuzzy clustering of FR instances

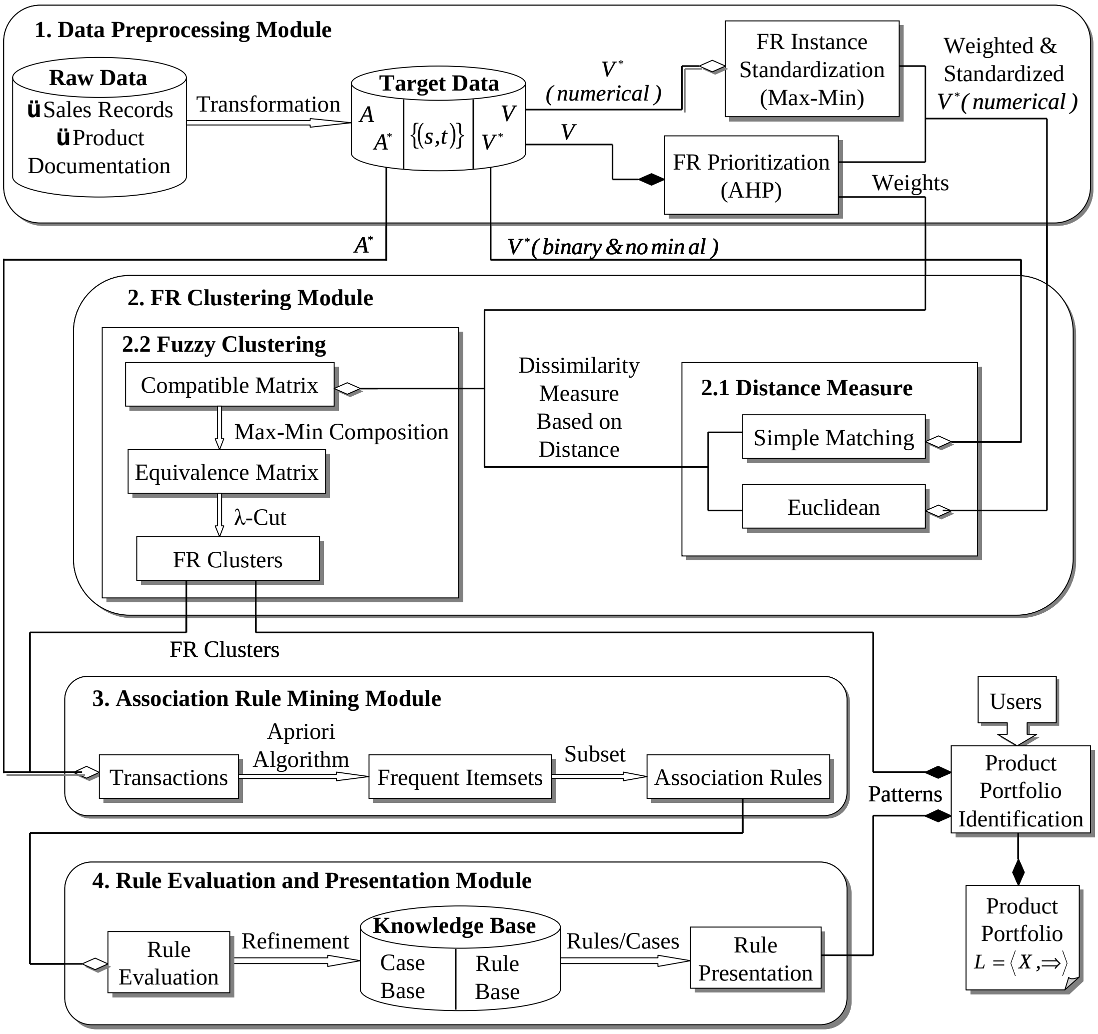


Figure 5 ARMS architecture

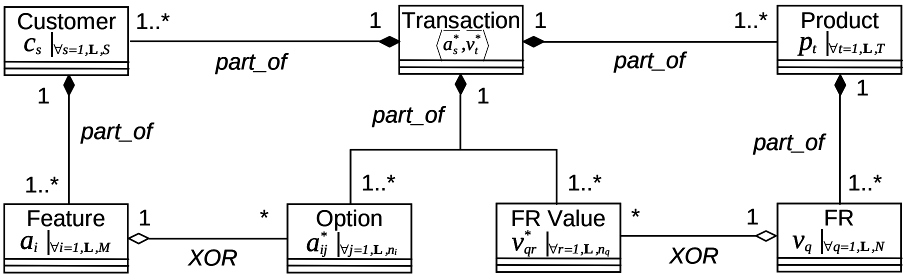


Figure 6 Entity relationships of target data sets

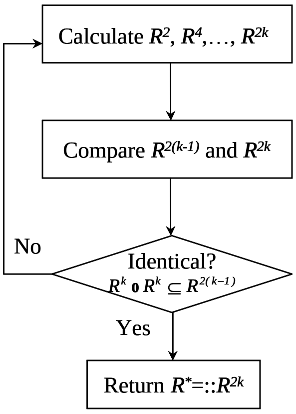


Figure 7 The flowchart of converting a compatible matrix to an equivalent matrix

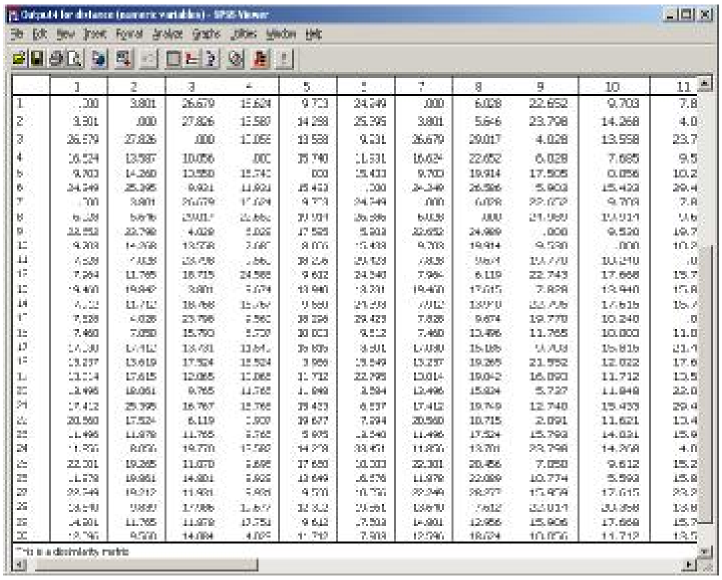


Figure 8 Raw data for distance measures of numerical FR instances

$$\begin{bmatrix}
0 \\
.076 & 0 \\
.534 & .557 & 0 \\
.332 & .272 & .2 & 0 \\
.194 & .285 & .271 & .315 & 0 \\
.485 & .508 & .2 & .24 & .309 & 0 \\
0 & .076 & .534 & .332 & .194 & .485 & 0 \\
.121 & .113 & .58 & .453 & .398 & .532 & .12 & 0 \\
.453 & .476 & .08 & .121 & .352 & .118 & .453 & .5 & 0 \\
.194 & .285 & .271 & .154 & .161 & .309 & .194 & .398 & .19 & 0 \\
.157 & .08 & .476 & .191 & .366 & .588 & .157 & .193 & .4 & .2 & 0 \\
.16 & .235 & .374 & .492 & .192 & .487 & .159 & .122 & .455 & .353 & .316 & 0 \\
.39 & .397 & .076 & .193 & .279 & .275 & .39 & .352 & .157 & .279 & .316 & .23 & 0 \\
.158 & .234 & .375 & .335 & .191 & .488 & .158 & .279 & .456 & .352 & .315 & .156 & .23 & 0 \\
.157 & .081 & .476 & .191 & .366 & .588 & .157 & .193 & .396 & .2 & 0 & .316 & .316 & .315 & 0 \\
.149 & .157 & .316 & .115 & .2 & .192 & .149 & .27 & .235 & .2 & .238 & .309 & .24 & .152 & .238 & 0 \\
.341 & .348 & .275 & .231 & .316 & .076 & .341 & .304 & .194 & .316 & .429 & .342 & .2 & .343 & .43 & .116 & 0 \\
.265 & .272 & .35 & .31 & .079 & .313 & .265 & .385 & .431 & .24 & .353 & .263 & .274 & .262 & .353 & .196 & .237 & 0 \\
.277 & .352 & .257 & .217 & .234 & .456 & .276 & .397 & .338 & .234 & .272 & .274 & .113 & .118 & .272 & .195 & .312 & .23 & 0 \\
.27 & .361 & .195 & .235 & .237 & .072 & .27 & .317 & .115 & .237 & .442 & .272 & .2 & .273 & .442 & .12 & .08 & .316 & .316 & 0 \\
.348 & .508 & .335 & .375 & .309 & .137 & .348 & .395 & .255 & .309 & .588 & .27 & .275 & .35 & .588 & .192 & .076 & .313 & .319 & .07 & 0 \\
.411 & .35 & .122 & .079 & .394 & .16 & .411 & .374 & .042 & .232 & .27 & .413 & .115 & .414 & .27 & .913 & .152 & .389 & .296 & .157 & .297 & 0 \\
.23 & .238 & .235 & .195 & .12 & .273 & .23 & .35 & .316 & .28 & .318 & .228 & .16 & .072 & .318 & .08 & .197 & .115 & .115 & .2 & .273 & .274 & 0 \\
.237 & .161 & .395 & .272 & .285 & .669 & .237 & .274 & .476 & .285 & .08 & .235 & .236 & .234 & .08 & .318 & .509 & .272 & .191 & .522 & .669 & .35 & ..238 & 0 \\
.446 & .385 & .238 & .194 & .353 & .2 & .446 & .409 & .157 & .192 & .305 & .448 & .23 & .604 & .305 & .309 & .192 & .274 & .411 & .272 & .337 & .115 & .389 & .385 & 0 \\
.238 & .397 & .296 & .179 & .273 & .334 & .238 & .442 & .215 & .112 & .317 & .4 & .235 & .24 & .371 & .157 & .273 & .352 & .122 & .193 & .197 & .257 & .237 & .397 & .372 & 0 \\
.445 & .384 & .239 & .2 & .191 & .2 & .445 & .566 & .319 & .352 & .465 & .443 & .23 & .287 & .465 & .152 & .193 & .112 & .255 & .273 & .338 & .277 & .07 & .384 & .318 & .377 & 0 \\
.273 & .197 & .36 & .394 & .246 & .397 & .273 & .152 & .44 & .407 & .277 & .114 & .2 & .27 & .277 & .279 & .238 & .158 & .313 & .326 & .397 & .315 & .198 & .197 & .275 & .519 & .27 & 0 \\
.296 & .235 & .238 & .355 & .192 & .35 & .296 & .259 & .318 & .353 & .316 & .137 & .23 & .293 & .316 & .309 & .342 & .263 & .411 & .272 & .487 & .276 & .228 & .235 & .311 & .534 & .306 & .114 & 0 \\
.252 & .191 & .282 & .081 & .234 & .158 & .252 & .372 & .2 & .234 & .272 & .411 & .274 & .255 & .272 & .03 & .15 & .23 & .298 & .155 & .295 & .159 & .115 & .352 & .274 & .259 & .118 & .313 & .274 & 0
\end{bmatrix}_{30\times 30}$$

Figure 9 Result of distance measures for numerical FR instances

$$\begin{bmatrix}
0 \\
0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}_{30\times 30}$$

Figure 10   Result of distance measures for binary FR instances

$$\begin{bmatrix}
0 \\
0 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}_{30\times 30}$$

Figure 11   Result of distance measures for nominal FR instances

$$\left[\begin{array}{*{30}{c}}
0 \\
.076 & 0 \\
.554 & .58 & 0 \\
.352 & .29 & .2 & 0 \\
.52 & .6 & .57 & .62 & 0 \\
.51 & .53 & .22 & .26 & .53 & 0 \\
.3 & .38 & .85 & .65 & .22 & .81 & 0 \\
.12 & .11 & .6 & .47 & .82 & .55 & .42 & 0 \\
.47 & .5 & .08 & .12 & .65 & .14 & .77 & .52 & 0 \\
.22 & .31 & .29 & .17 & .48 & .31 & .52 & .42 & .21 & 0 \\
.48 & .4 & .78 & .49 & .37 & .91 & .18 & .51 & .7 & .52 & 0 \\
.16 & .24 & .39 & .51 & .51 & .51 & .46 & .12 & .48 & .37 & .64 & 0 \\
.41 & .42 & .08 & .19 & .58 & .3 & .71 & .37 & .16 & .3 & .62 & .25 & 0 \\
.16 & .24 & .4 & .34 & .51 & .51 & .46 & .28 & .48 & .37 & .64 & .26 & .25 & 0 \\
.46 & .38 & .74 & .51 & .39 & .91 & .16 & .49 & .72 & .52 & .02 & .62 & .64 & .62 & 0 \\
.17 & .18 & .32 & .12 & .5 & .21 & .47 & .29 & .24 & .22 & .54 & .33 & .24 & .17 & .44 & 0 \\
.36 & .37 & .3 & .25 & .64 & .08 & .66 & .32 & .21 & .32 & .75 & .36 & .22 & .36 & .75 & .14 & 0 \\
.59 & .6 & .67 & .63 & .01 & .62 & .29 & .71 & .75 & .54 & .37 & .58 & .6 & .58 & .37 & .52 & .54 & 0 \\
.3 & .32 & .26 & .22 & .53 & .48 & .6 & .42 & .34 & .25 & .57 & .29 & .11 & .14 & .6 & .2 & .33 & .55 & 0 \\
.29 & .38 & .2 & .24 & .54 & .09 & .59 & .34 & .12 & .26 & .74 & .29 & .2 & .29 & .76 & .12 & .1 & .64 & .32 & 0 \\
.27 & .53 & .36 & .4 & .53 & .14 & .67 & .42 & .28 & .31 & .91 & .29 & .3 & .37 & .91 & .21 & .01 & .63 & .34 & .09 & 0 \\
.43 & .37 & .12 & .08 & .69 & .18 & .73 & .39 & .04 & .25 & .57 & .43 & .12 & .43 & .59 & .93 & .15 & .69 & .3 & .16 & .32 & 0 \\
.25 & .26 & .24 & .2 & .42 & .25 & .55 & .37 & .32 & .3 & .62 & .24 & .16 & .09 & .64 & .08 & .22 & .44 & .12 & .2 & .29 & .27 & 0 \\
.24 & .16 & .42 & .29 & .61 & .69 & .44 & .27 & .5 & .31 & .4 & .24 & .26 & .23 & .38 & .34 & .53 & .59 & .21 & .54 & .69 & .37 & .27 & 0 \\
.77 & .7 & .56 & .51 & .37 & .5 & .47 & .73 & .48 & .49 & .23 & .77 & .55 & .62 & .23 & .63 & .49 & .27 & .73 & .6 & .64 & .44 & .71 & .71 & 0 \\
.56 & .72 & .62 & .5 & .29 & .54 & .26 & .76 & .54 & .41 & .34 & .72 & .56 & .56 & .34 & .48 & .57 & .35 & .44 & .51 & .5 & .58 & .56 & .72 & .69 & 0 \\
.47 & .4 & .26 & .22 & .51 & .2 & .77 & .59 & .34 & .35 & .79 & .46 & .25 & .25 & .79 & .17 & .19 & .41 & .28 & .29 & .34 & .3 & .09 & .4 & .34 & .68 & 0 \\
.29 & .22 & .36 & .39 & .55 & .42 & .59 & .17 & .44 & .43 & .58 & .13 & .2 & .2 & .6 & .28 & .26 & .48 & .31 & .33 & .42 & .32 & .2 & .22 & .3 & .84 & .29 & 0 \\
.3 & .24 & .26 & .38 & .51 & .37 & .6 & .26 & .34 & .37 & .54 & .14 & .25 & .25 & .62 & .33 & .36 & .58 & .43 & .29 & .51 & .3 & .25 & .24 & .31 & .85 & .23 & .13 & 1 \\
.27 & .21 & .28 & .08 & .53 & .16 & .57 & .39 & .2 & .25 & .57 & .43 & .27 & .27 & .59 & .03 & .17 & .55 & .3 & .16 & .32 & .16 & .18 & .37 & .29 & .58 & .14 & .31 & .29 & 0
\end{array}\right]_{30\times 30}$$

Figure 12 Dissimilarity matrix based on distance measures for all FR instances

$$\left[\begin{array}{*{30}{c}}
1 \\
.924 & 1 \\
.446 & .42 & 1 \\
.648 & .71 & .8 & 1 \\
.48 & .4 & .43 & .38 & 1 \\
.49 & .47 & .78 & .74 & .47 & 1 \\
.7 & .62 & .15 & .35 & .78 & .19 & 1 \\
.88 & .89 & .4 & .53 & .18 & .45 & .58 & 1 \\
.53 & .5 & .92 & .88 & .35 & .86 & .23 & .48 & 1 \\
.78 & .69 & .71 & .83 & .52 & .69 & .48 & .58 & .79 & 1 \\
.52 & .6 & .22 & .51 & .63 & .09 & .82 & .49 & .3 & .48 & 1 \\
.84 & .76 & .61 & .49 & .49 & .49 & .54 & .88 & .52 & .63 & .36 & 1 \\
.59 & .58 & .92 & .81 & .42 & .7 & .29 & .63 & .84 & .7 & .38 & .75 & 1 \\
.84 & .76 & .6 & .66 & .49 & .49 & .54 & .72 & .52 & .63 & .36 & .74 & .75 & 1 \\
.54 & .62 & .26 & .49 & .61 & .09 & .84 & .51 & .28 & .48 & .98 & .38 & .36 & .38 & 1 \\
.83 & .82 & .68 & .88 & .5 & .79 & .53 & .71 & .76 & .78 & .46 & .67 & .76 & .83 & .56 & 1 \\
.64 & .63 & .7 & .75 & .36 & .92 & .34 & .68 & .79 & .68 & .25 & .64 & .78 & .64 & .25 & .86 & 1 \\
.41 & .4 & .33 & .37 & .99 & .38 & .71 & .29 & .25 & .46 & .63 & .42 & .4 & .42 & .63 & .48 & .46 & 1 \\
.7 & .68 & .74 & .78 & .47 & .52 & .4 & .58 & .66 & .75 & .43 & .71 & .89 & .86 & .4 & .8 & .67 & .45 & 1 \\
.71 & .62 & .8 & .76 & .46 & .91 & .41 & .66 & .88 & .74 & .26 & .71 & .8 & .71 & .24 & .88 & .9 & .36 & .68 & 1 \\
.73 & .47 & .64 & .6 & .47 & .86 & .33 & .58 & .72 & .69 & .09 & .71 & .7 & .63 & .09 & .79 & .99 & .37 & .66 & .91 & 1 \\
.57 & .63 & .88 & .92 & .31 & .82 & .27 & .61 & .96 & .75 & .43 & .57 & .88 & .57 & .41 & .07 & .85 & .31 & .7 & .84 & .68 & 1 \\
.75 & .74 & .76 & .8 & .58 & .75 & .45 & .63 & .68 & .7 & .38 & .76 & .84 & .91 & .36 & .92 & .78 & .56 & .88 & .8 & .71 & .73 & 1 \\
.76 & .84 & .58 & .71 & .39 & .31 & .56 & .73 & .5 & .69 & .6 & .76 & .74 & .77 & .62 & .66 & .47 & .41 & .79 & .46 & .31 & .63 & .73 & 1 \\
.23 & .3 & .44 & .49 & .63 & .5 & .53 & .27 & .52 & .51 & .77 & .23 & .45 & .38 & .77 & .37 & .51 & .73 & .27 & .4 & .36 & .56 & .29 & .29 & 1 \\
.44 & .28 & .38 & .5 & .71 & .46 & .74 & .24 & .46 & .59 & .66 & .28 & .44 & .44 & .66 & .52 & .43 & .65 & .56 & .49 & .5 & .42 & .44 & .28 & .31 & 1 \\
.53 & .6 & .74 & .78 & .49 & .8 & .23 & .41 & .66 & .65 & .21 & .54 & .75 & .75 & .21 & .83 & .81 & .59 & .72 & .71 & .66 & .7 & .91 & .6 & .66 & .32 & 1 \\
.71 & .78 & .64 & .61 & .45 & .58 & .41 & .83 & .56 & .57 & .42 & .87 & .8 & .8 & .4 & .72 & .74 & .52 & .69 & .67 & .58 & .68 & .8 & .78 & .7 & .16 & .71 & 1 \\
.7 & .76 & .74 & .62 & .49 & .63 & .4 & .74 & .66 & .63 & .46 & .86 & .75 & .75 & .38 & .67 & .64 & .42 & .57 & .71 & .49 & .7 & .75 & .76 & .69 & .15 & .77 & .87 & 1 \\
.73 & .79 & .72 & .92 & .47 & .84 & .43 & .61 & .8 & .75 & .43 & .57 & .73 & .73 & .41 & .97 & .83 & .45 & .7 & .84 & .68 & .84 & .82 & .63 & .71 & .42 & .86 & .69 & .71 & 1
\end{array}\right]_{30\times 30}$$

Figure 13 Result of R

$$
\left[\begin{array}{*{30}{c}}
1 \\
.7&1 \\
.64&.74&1 \\
.52&.82&.8&1 \\
.7&.62&.85&.85&1 \\
.75&.71&.76&.76&.78&1 \\
.7&.7&.56&.648&.78&.79&1 \\
.88&.88&.73&.71&.58&.71&.9&1 \\
.76&.76&.92&.88&.58&.76&.53&.71&1 \\
.78&.78&.8&.83&.58&.79&.7&.78&.85&1 \\
.7&.62&.58&.6&.78&.51&.82&.6&.52&.6&1 \\
.84&.84&.82&.75&.58&.75&.7&.78&.75&.78&.6&1 \\
.76&.75&.92&.8&.52&.78&.59&.75&.82&.81&.8&.75&1 \\
.84&.74&.8&.75&.58&.79&.7&.84&.76&.78&.6&.84&.76&1 \\
.7&.62&.58&.62&.78&.56&.84&.62&.53&.78&.82&.62&.62&.62&1 \\
.83&.83&.8&.78&.58&.89&.7&.83&.78&.83&.83&.83&.81&.83&.62&1 \\
.68&.82&.79&.79&.58&.72&.64&.71&.86&.79&.81&.76&.75&.83&.62&.79&1 \\
.7&.62&.56&.49&.79&.89&.78&.58&.59&.56&.71&.65&.59&.59&.73&.59&.59&1 \\
.74&.8&.89&.81&.58&.79&.7&.73&.84&.78&.72&.76&.78&.75&.62&.83&.8&.59&1 \\
.83&.82&.8&.8&.58&.81&.7&.71&.8&.76&.6&.75&.8&.75&.62&.79&.91&.59&.8&1 \\
.79&.79&.78&.79&.58&.86&.7&.73&.82&.78&.68&.73&.78&.79&.56&.86&.76&.59&.71&.76&1 \\
.77&.71&.78&.8&.63&.86&.62&.73&.78&.83&.7&.75&.78&.75&.62&.88&.82&.59&.78&.78&.82&1 \\
.83&.82&.8&.8&.58&.79&.7&.76&.8&.8&.71&.76&.8&.81&.62&.83&.78&.58&.78&.88&.79&.8&1 \\
.84&.84&.74&.71&.61&.73&.7&.74&.74&.76&.88&.76&.75&.76&.62&.82&.74&.62&.79&.74&.73&.74&.79&1 \\
.71&.71&.71&.71&.63&.56&.77&.7&.74&.74&.76&.88&.76&.75&.76&.62&.82&.74&.62&.79&.74&.73&.74&.79&1 \\
.7&.62&.59&.56&.71&.59&.74&.58&.89&.59&.74&.59&.56&.59&.74&.59&.59&.71&.59&.59&.59&.59&.59&.62&.66&1 \\
.83&.75&.78&.78&.6&.8&.6&.71&.8&.78&.6&.75&.78&.75&.6&.79&.8&.89&.78&.8&.8&.8&.83&.74&.59&.59&1 \\
.78&.83&.8&.81&.58&.75&.7&.78&.8&.72&.7&.76&.8&.76&.62&.88&.78&.73&.8&.8&.71&.8&.8&.78&.7&.57&.75&1 \\
.84&.76&.75&.75&.58&.74&.7&.76&.74&.71&.6&.76&.75&.76&.62&.76&.75&.69&.75&.75&.71&.75&.76&.76&.71&.59&.75&.86&1 \\
.83&.82&.85&.92&.58&.8&.7&.79&.88&.83&.71&.76&.81&.76&.62&.78&.84&.71&.78&.84&.82&.82&.8&.79&.71&.59&.8&.78&.76&1
\end{array}\right]_{30\times30}
$$

Figure 14 Result of R^2 and R^4

$$
\left[\begin{array}{*{30}{c}}
1 \\
0&1 \\
0&0&1 \\
0&0&0&1 \\
0&0&1&1&1 \\
0&0&0&0&0&1 \\
0&0&0&0&0&0&1 \\
1&1&0&0&0&0&1&1 \\
0&0&1&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&1&1 \\
0&0&0&0&0&0&0&0&0&0&1 \\
1&1&0&0&0&0&0&0&0&0&0&1 \\
0&0&1&0&0&0&0&0&0&0&0&0&1 \\
1&0&0&0&0&0&0&1&0&0&0&1&0&1 \\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1 \\
1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1 \\
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1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1 \\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1 \\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1 \\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1 \\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1 \\
0&0&1&1&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1
\end{array}\right]_{30\times30}
$$

Figure 15 Result of a l -cut with $l = 0.84$

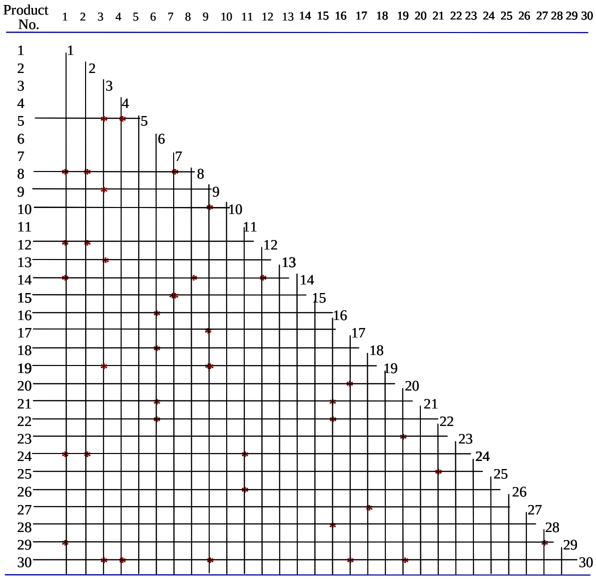
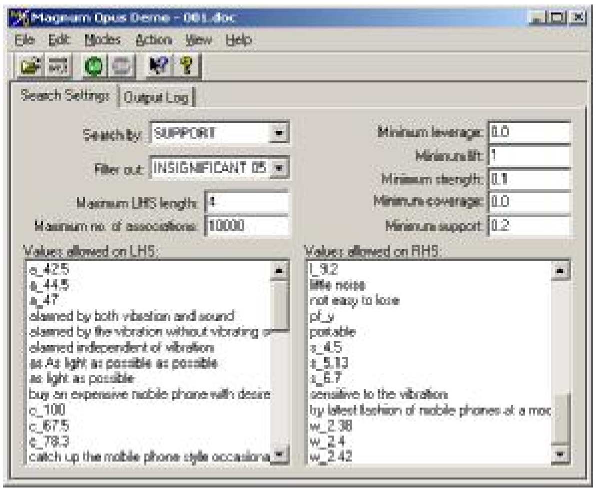


Figure 16 Fuzzy netting graph



Search by: SUPPORT
Filter out: INSIGNIFICANT 05
Maximum LHS length: 4
Maximum no. of associations: 10000
Minimum leverage: 0.0
Minimum lift: 1
Minimum strength: 0.1
Minimum coverage: 0.0
Minimum support: 0.2

Values allowed on LHS:
a_42.5
a_44.5
a_47
alarmed by both vibration and sound
alarmed by the vibration without vibrating o
alarmed independent of vibration
as As light as possible as possible
as light as possible
buy an expensive mobile phone with desire
c_100
c_67.5
c_78.3
catch up the mobile phone style occasiona

Values allowed on RHS:
l_9.2
little noise
not easy to lose
pf_y
portable
s_4.5
s_5.13
s_6.7
sensitive to the vibration
try latest fashion of mobile phones at a moc
w_2.38
w_2.4
w_2.42

Figure 17 Association rule induction in the Magnum Opus

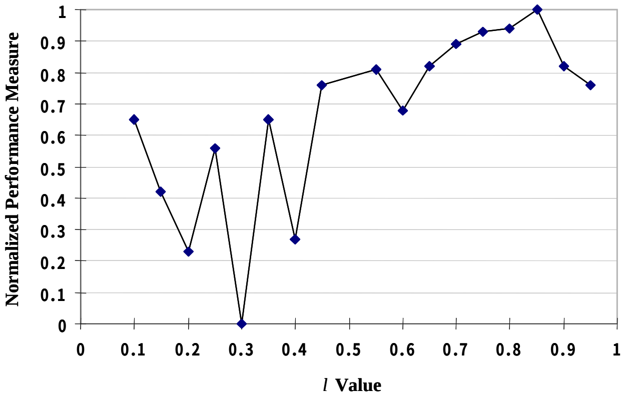


Figure 18 Sensitivity analysis of product portfolio identification with respect to similarity threshold

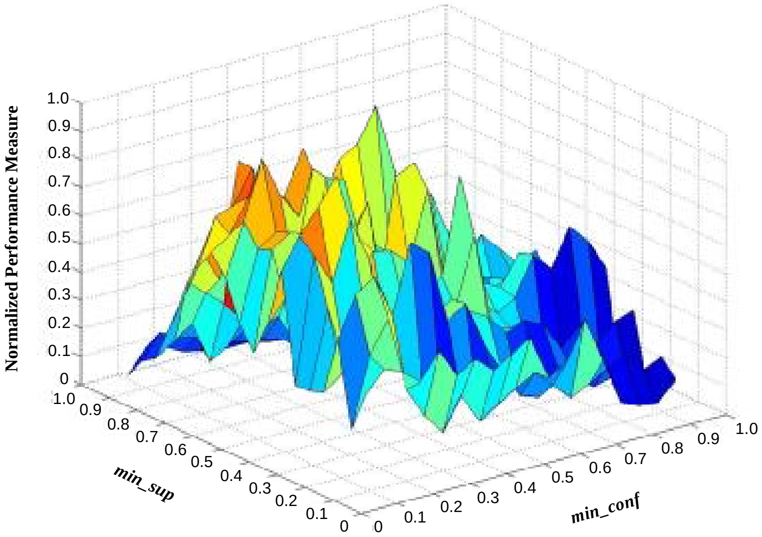


Figure 19 Sensitivity analysis of product portfolio identification with respect to minimum support and confidence levels

Table 1 Algorithm of incremental mining of association rules in the ARMS

```
Begin
Let $N = count(DB)$; /* Total data record count */
Let $S_m = min\_sup$; /* Minimum support threshold specified by the user */
Let $C_m = min\_conf$; /* Minimum confidence threshold specified by the user */
    For i = 1 to N do
        Begin
        Let $S = count(a_1 \wedge a_2 \cdots \wedge a_E \wedge b_1 \wedge b_2 \cdots \wedge b_F)$; /* Call the Apriori algorithm */
        Let $C = count(a_1 \wedge a_2 \cdots \wedge a_E)$; /* Call the Apriori algorithm */
        Let $s = (S/N) \times 100\%$;
        Let $c = (S/C) \times 100\%$;
        If $s \geq S_m$ and $c \geq C_m$
            Then $Rule_i$ is derived;
        End if;
    End;
End;
```

Table 2 List of CNs

Feature		**Option**		
$a_i \mid \forall i = 1, \cdots, M$	**Description**	$a^*_{ij} \mid \forall j = 1, \cdots, n_i$	**Code**	**Description**
a_1	Feel of vibration	a^*_{11}	A11	Feel the vibration very strongly
		a^*_{12}	A12	Alarmed by vibration without vibrating suddenly
		a^*_{13}	A13	Sensitive to the vibration
a_2	Price	a^*_{21}	A21	Buy an expensive mobile phone with desire for a long time use
		a^*_{22}	A22	Catch up the mobile phone style occasionally at a low price
		a^*_{23}	A23	Try latest fashion of mobile phones at a moderate price
a_3	Size	a^*_{31}	A31	Portable
		a^*_{32}	A32	Comfortable to hold
		a^*_{33}	A33	Not easy to lose
a_4	Volume of sound	a^*_{41}	A41	Little noise
		a^*_{42}	A42	Alarmed independent of vibration
		a^*_{43}	A43	Alarmed by both vibration and sound
a_5	Material	a^*_{51}	A51	Green material for environment friendliness
a_6	Weight	a^*_{61}	A61	As light as possible

Table 3 List of FRs

FR			**FR Value**		
$v_q \mid \forall q = 1, \ldots, N$	**Description**	**Type**	$v^*_{qr} \mid \forall r = 1, \ldots, n_q$	**Code**	**Description**
v_1	Current	Numerical	v^*_{11}	V11	100 mA
			v^*_{12}	V12	80 mA
			v^*_{13}	V13	60 mA
v_2	Pbfree	Binary	v^*_{21}	V21	1 (Yes)
			v^*_{22}	V22	0 (No)
v_3	Length	Numerical	v^*_{31}	V31	8 mm
			v^*_{32}	V32	12 mm
			v^*_{33}	V33	10 mm
v_4	Diameter	Numerical	v^*_{41}	V41	5 mm
			v^*_{42}	V42	4 mm
			v^*_{43}	V43	6 mm
v_5	Coating	Nominal	v^*_{51}	V51	Au
			v^*_{52}	V52	Alloy
			v^*_{53}	V53	None
v_6	Angle	Numerical	v^*_{61}	V61	40°
			v^*_{62}	V62	55°
v_7	Strength	Numerical	v^*_{71}	V71	7 Kg
			v^*_{72}	V72	4 Kg
v_8	Weight	Numerical	v^*_{81}	V81	2 g
			v^*_{82}	V82	3 g
v_9	Hardness	Numerical	v^*_{91}	V91	40 HB
			v^*_{92}	V92	70 HB

Table 4 Transaction database

Record (TID)	**CNs** ($\overline{a^*_s} \mid \forall s = 1, \ldots, S$)	**FRs** ($\overline{v^*_t} \mid \forall t = 1, \ldots, T$)
T001	A11, A21, A31, A43, A51, A61	V11, V21, V31, V42, V53, V62, V71, V82, V92
T002	A11, A21, A43, A51	V11, V21, V31, V41, V51, V61, V71, V81, V92
T003	A12, A22, A33, A61	V12, V21, V33, V43, V51, V61, V72, V82, V91
…	…	…
T028	A13, A22, A33, A41, A61	V13, V22, V31, V42, V52, V61, V72, V81, V91
T029	A11, A21, A31, A43, A51, A61	V12, V22, V33, V43, V52, V62, V72, V81, V92
T030	A12, A22, A33, A42, A61	V11, V22, V33, V42, V53, V61, V72, V82, V91

Table 5 Scale for subjective judgment

Verbal judgment of preference	**Numerical rating**
Extremely preferred	1
Very strong to extremely	2
Very strongly preferred	3
Strongly to very strongly	4
Strongly preferred	5
Moderately to strongly	6
Moderately preferred	7
Equally to moderately	8
Equally preferred	9

Table 6 Relative importance among FR variables

FR (v_q)	**Weight (w_q)**
v_1	0.219
v_2	0.304
v_3	0.046
v_4	0.031
v_5	0.019
v_6	0.066
v_7	0.157
v_8	0.095
v_9	0.083
	$\sum w_q = 1$

Table 7 Result of FR clustering

FR Cluster			**Clustered FR Instances** ($\{\overline{v_t^*} \sim c_l \mid \forall t = 1, \ldots, n_l \le T\}$)
c_l	**Mean Value (**m_l**)**	**Variation Range (**D_l**)**	
c_1	[100, Y, 9.2, 4.5, Au, 44.5, 6.7, 2.4, 49]	[0, 0, 1.2, 0.5, 0, 10.5, 2.7, 0.6, 21]	$\{\overline{v_1^*}, \overline{v_2^*}, \overline{v_7^*}, \overline{v_8^*}, \overline{v_{11}^*}, \overline{v_{12}^*}, \overline{v_{14}^*}, \overline{v_{15}^*}, \overline{v_{24}^*}, \overline{v_{29}^*}\}$
c_2	[78.3, Y, 11.17, 5.5, Alloy, 47, 4.5, 2.42, 57.5]	[21.7, 0, 1.17, 0.5, 0, 8, 2.5, 0.58, 17.5]	$\{\overline{v_3^*}, \overline{v_4^*}, \overline{v_5^*}, \overline{v_9^*}, \overline{v_{10}^*}, \overline{v_{13}^*}, \overline{v_{17}^*}, \overline{v_{19}^*}, \overline{v_{20}^*}, \overline{v_{23}^*}, \overline{v_{26}^*}, \overline{v_{30}^*}\}$
c_3	[67.5, Y, 10.75, 5.13, None, 42.5, 5.13, 2.38, 47.5]	[12.5, 0, 1.25, 0.87, 0, 12.5, 1.87, 0.62, 22.5]	$\{\overline{v_6^*}, \overline{v_{16}^*}, \overline{v_{18}^*}, \overline{v_{21}^*}, \overline{v_{22}^*}, \overline{v_{25}^*}, \overline{v_{27}^*}, \overline{v_{28}^*}\}$

Table 8 Specification of vibration motor portfolio based on FR clusters

FR Variable	**FR Value**	
	Base Value	**Variation Range**
Current (mA)	100	±0
	78.3	±21.7
	67.5	±12.5
Pbfree	1 (Yes)	±0
Length (mm)	9.2	±1.2
	11.17	±1.17
	10.75	±1.25
Diameter (mm)	4.5	±0.5
	5.5	±0.5
	5.13	±0.87
Coating	Au	±0
	Alloy	±0
	None	±0
Angle (°)	44.5	±10.5
	47	±8
	42.5	±12.5
Strength (Kg)	6.7	±2.7
	4.5	±2.5
	5.13	±1.87
Weight (g)	2.4	±0.6
	2.42	±0.58
	2.38	±0.62
Hardness (HB)	49	±21
	57.5	±17.5
	47.5	±22.5

Table 9 Result of association rule mining

Rule 1: Green material for environment friendliness\=>pf_y\[Support=0.882; Strength=1.000];

Rule 2: Alarmed independent of vibration\&Not easy to lose\&Catch up the mobile phone style occasionally at a low price\=>h_57.5[±17.5]\[Support=0.265; Strength=0.900];

Rule 3: Alarmed independent of vibration\&Try latest fashion of mobile phones at a moderate price\&Not easy to lose\=> c_78.3[±21.7]\[Support=0.265; Strength=0.900];

Rule 4: Alarmed independent of vibration\&Buy an expensive mobile phone with desire for a long time use \=> l_11.17[±1.17]\[Support=0.265; Strength=0.900];

Rule 5: Alarmed independent of vibration\=>h_57.5[±17.5]\[Support=0.294; Strength=0.833];

Rule 6: Not easy to lose\&Alarmed independent of vibration\=> a_47[±8]\[Support=0.265; Strength=0.900];

Rule 7: Not easy to lose\&Comfortable to hold\&Catch up the mobile phone style occasionally at a low price\=>w_2.42[±0.58]\[Support=0.265; Strength=0.750];

Rule 8: Not easy to lose\=> w_2.42[±0.58]\&h_57.5[±17.5]\[Support=0.265; Strength=0.900];

Rule 9: Catch up the mobile phone style occasionally at a low price\=>co_None\[Support=0.324; Strength=0.688];

Rule 10: Buy an expensive mobile phone with desire for a long time use\&Feel the vibration very strongly\=>h_49[±21]\[Support=0.206; Strength=1.000];

Rule 11: Buy an expensive mobile phone with desire for a long time use\=>s_6.7[±2.7]\[Support=0.206; Strength=1.000];

Rule 12: Buy an expensive mobile phone with desire for a long time use\&Alarmed by both vibration and sound\=>a_44.5[±10.5]\[Support=0.206; Strength=1.000];

Rule 13: Buy an expensive mobile phone with desire for a long time use\&Portable\=>a_44.5[±10.5\[Support=0.265; Strength=0.818];

Rule 14: Buy an expensive mobile phone with desire for a long time use\=>co_Au\[Support=0.206; Strength=1.000];

Rule 15: Feel the vibration very strongly\&Portable\=> l_9.2[±1.2]\[Support=0.206; Strength=0.875];

Rule 16: Feel the vibration very strongly\=>c_100[±0]\[Support=0.206; Strength=0.875];

Rule 17: Feel the vibration very strongly\&As light as possible\=>d_4.5[±0.5]\[Support=0.265; Strength=0.750];

Rule 18: As light as possible\=>a_42.5[±12.5]\[Support=0.206; Strength=0.875];

Rule 19: As light as possible\&Little noise=>w_2.38[±0.62]\[Support=0.206; Strength=0.875];

Rule 20: As light as possible\=>co_None\[Support=0.206 ; Strength=0.875]

Rule 21: Alarmed by the vibration without vibrating suddenly\=>s_4.5[±2.5]\[Support=0.294; Strength=0.833];

Rule 22: Alarmed by the vibration without vibrating suddenly\=>l_11.17[±1.17]\[Support=0.294; Strength=0.833];

Rule 23: Portable\&As light as possible\=>d_4.5[±0.5]\[Support=0.265; Strength=0.818];

Rule 24: Portable\&Feel the vibration very strongly\=>l_9.2[±1.2]\[Support=0.265; Strength=0.818];

Rule 25: Portable\=>a_44.5[±10.5]\[Support=0.294; Strength=0.833];

Rule 26: Sensitive to the vibration\=>d_5.13[±0.87]\[Support=0.235; Strength=0.800];

Rule 27: Sensitive to the vibration\&Little noise\=>c_67.5[±12.5]\[Support=0.235; Strength=0.800];

Rule 28: Sensitive to the vibration\&Little noise\&As light as possible\=>h_47.5[±22.5]\[Support=0.235; Strength=0.727];

Rule 29: Little noise\&As light as possible\=>s_5.13[±1.87]\[Support=0.206; Strength=0.700];

Rule 30: Little noise\=>c_67.5[±12.5]\[Support=0.206; Strength=0.700];

Rule 31: Alarmed by both vibration and sound\=>a_44.5[±10.5]\[Support=0.206; Strength=0.700];

Rule 32: Alarmed by both vibration and sound\=>d_4.5[±0.5]\[Support=0.206; Strength=0.700];

Rule 33: Alarmed by both vibration and sound\=>l_10.75[±1.25]\[Support=0.206; Strength=0.700];

Rule 34: Comfortable to hold\=>w_2.40[±0.6]\[Support=0.206; Strength=0.700];

Rule 35: Try latest fashion of mobile phones at a moderate price\&Alarmed by both vibration and sound\=>c_78.3[±21.7]\[Support=0.206; Strength=0.700];

Rule 36: Try latest fashion of mobile phones at a moderate price\=>d_5.5[±0.5]\[Support=0.206; Strength=0.700];

Rule 37: Try latest fashion of mobile phones at a moderate price\=>co_Alloy\[Support=0.294; Strength=0.833];

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