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Propagation of Financial Crises:
A Heterogenous Agents Approach

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31 July 2013

EGC Report No: 2013/09

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Version: July, 2013

We propose a two-market heterogeneous agents model with coupling mechanism to study financial crisis with contagion effect. It manages to calibrate sudden crash behavior of US and UK stock markets during "Black Monday" of 1987 besides smooth crisis and disturbing crisis categorized in literature. It is implied that financial crisis and its contagion could be endogenous and supports scenario of over-valuation causing financial crisis. In addition, the model shows that financial system could be fragile in which small shock(s) hitting individual market's fundamental could cause financial crisis which spreads to other market. It also supports scenario of external shock triggering financial crisis. Lastly, the model manages to match typical stylized facts, especially cross-correlation which is exclusive to multi-market case.

Keywords: Financial crash, Contagion, Heterogeneous agents, Financial multi-market interactions

JEL Classifications: C61 D84, G15

1. INTRODUCTION

Along the history of financial market development, financial crisis is one of the perennial phenomena, in which large decline of asset price is observed. Usually, financial crisis is not isolated within one market. Instead, it has propagation effect: financial crisis originating from one market spreads to other markets causing simultaneous or sequential crises. Kindleberger and Aliber (2005) devote two chapters to document this propagation effect in their book. One of the example of financial crisis propagation is the "Black Monday" of US stock market in October 19, 1987. In that day, other national stock markets such as UK experienced nearly simultaneous sharp decline. Manconi et al. (2012) argue that investors with liquidity constraint play a role in propagating crisis from securitized to corporate bonds during subprime crisis. Allen and Moessner (2012) identify flight to liquidity and safety as a common features in propagation of financial crises in 1931 and 2008. Nevertheless, working mechanisms of financial crisis and its propagation is not fully understood yet. Given development of financial market integration and globalization, impact and depth of financial crisis propagation, if any, should become even more severe and deserve more attention as markets become more closely linked.

The development of heterogenous agents models (HAM) has provided a tool to investigate financial crises. Following the financial crisis grouping of Rosser (2000), we have sudden crisis, smooth crisis and disturbing crisis, respectively. In sudden crisis, price falls precipitately from peak to bottom in a short period. In smooth crisis, price decreases smoothly from peak to bottom with persistent trend. In between sudden crisis and smooth crisis is disturbing crisis, in which price fluctuates disturbingly with declining trend. Day and Huang (1990) setup the stylized framework of market maker and generates randomly switching bear and bull market episodes. The transition from bull to bear market episode mimics a sudden financial crisis. He and Westerhoff (2005) also investigate sudden crisis and evaluate policy of price limiters. Chiarella et al. (2003) investigate smooth crisis. Huang et al. (2010) manage to simulate all three patterns of financial crises. Their model indicates that both fundamentalist and chartist could contribute to financial crises and hence financial crises could be endogenous. Huang and Zheng (2012) generalize regime-dependent beliefs and regime-switching dynamics to examine the triggering mechanisms for all the three financial crisis patterns. For more literature study of HAM model, we cite, in particular, Brock and LeBaron (1996), Brock and Hommes (1998), Lux (1995), Lux and Marchesi (2000), and Farmer and Joshi (2002).

All the above literatures are related to one single market. To investigate the propagation behavior of financial crisis, a multi-market model is required. Academia has pointed out the direction of multi-market model. As pioneers, Brock et al. (2009) develop multi-asset model by introducing additional Arrow securities into the stylized evolutionary equilibrium model of Brock and Hommes (1998) and demonstrate that more hedging instruments may destabilized markets. Dieci and Westerhoff (2010) build up a three-market model in which two stock markets are linked via foreign exchange market. The two stock markets have only fundamentalists while the foreign exchange market is populated with chartists and fundamentalists. It is concluded that upon market interactions, stock markets may be destabilized while foreign exchange market and the whole market system can be stabilized relatively.

This paper extends Huang and Zheng (2012)'s model to multi-market framework and propose a coupling market maker mechanism accordingly. From the point of view of endogeneity, we manage to capture the simultaneous crash behavior of US and UK stock markets during "Black Monday" of 1987 as well as other financial crisis patterns. On the other hand, from the point of view of exogeneity, further simulations demonstrate that, upon impact of permanent or temporary shock(s) on one market, financial crisis can arise and spread to other market. Factors such as magnitude of shock and duration of temporary shock also affect patterns of financial crisis. Lastly, numerical evaluation verifies capability of our model to match stylized facts of financial markets.

The rest of paper is organized as follows. Section 2 describes the dynamic two-market model and its stability. Section 3 focuses on crisis behavior matching and evaluating hypothesis of financial crisis causes. Section 4 matches stylized facts. Lastly, Section 5 concludes the paper.

2. THEORETICAL MODEL

This section develops a two-market model with coupling mechanism market maker framework. Besides that, steady state and its stability conditions are derived.

2.1. Model Setup

We develop a two-market nonlinear model in this section. There are two markets A and B. $p_{i,t}$ is price per share of risky asset of market i ($= A$ or B) at period t. Each market is populated with two kinds of investors (agents): fundamentalists (f) and chartists (c).

For a fundamentalist from j ($= A$ or B), her demand for asset i, $D^f_{ij,t}$[1], is simply defined as

$$D^f_{ij,t} = v_{i,t} \left(F_i - p_{i,t} \right) \tag{1}$$

where F_i is fundamental value of market i and $v_{i,t} > 0$ is convergence speed following the definition of Day and Huang (1990). $v_{i,t}$ is a bimodal function with modes near or at

[1] In the sequel, we shall adopt the same notation convention with the first subscript i standing for the market asset demanded, the second subscript j for market investors originated from, and the superscript k for type of investors.

bottoming price $u_1 F_i$ $(u_1 < 1)$ and topping price $u_2 F_i$ $(u_2 > 1)$. The bimodal implies that convergence speed becomes high when price deviates too much from the fundamental value. Without loss of generality, we assume

$$v_{i,t} = (p_{i,t} - u_1 F_i)^d \, (u_2 F_i - p_{i,t})^d$$

with $d < 0$. With this setting, fundamentalists buy in asset when its price is below F_i and sell it out vice versa.

For a chartist, she applies technical analysis and divides price domain P_i into n regimes

$$P_i = \cup_{l=1}^{n} P_{i,l} = [\overline{p}_{i,0}, \overline{p}_{i,1}) \cup [\overline{p}_{i,1}, \overline{p}_{i,2}) \cup \cdots \cup [\overline{p}_{i,n-1}, \overline{p}_{i,n}]$$

where $\overline{p}_{i,l}$ $(l = 1, 2, \cdots n)$ represents thresholds of price regime l. It can also be interpreted as different support/resistance in technical analysis. By conducting technical analysis, a reference price $p_{i,t}^c$ is derived by averaging the top and bottom thresholds of a price regime, in which current price $p_{i,t}$ falls in. That is

$$p_{i,t}^c = \frac{\overline{p}_{i,l-1} + \overline{p}_{i,l}}{2}, \text{ given } p_{i,t} \in [\overline{p}_{i,l-1}, \overline{p}_{i,l})$$

Given the reference price, a chartist from market j has demand for asset i, $D_{ij,t}^c$, as determined by

$$D_{ij,t}^c = \tau \left(p_{i,t} - p_{i,t}^c\right) \tag{2}$$

where τ is demand parameter of a chartist. This asset demand function captures the behavior of chartists that they buy in asset when its price is above chartist reference price and sell out vice versa.

In this two-market model, we assume composition of each type of investor w_{ij}^k $(k = c$ or $f)$ is fixed. Then, excess demand in market i, $D_{i,t}$, is derived

$$D_{i,t} = w_{iA}^f D_{iA,t}^f + w_{iA}^c D_{iA,t}^c + w_{iB}^f D_{iB,t}^f + w_{iB}^c D_{iB,t}^c$$

In classic single market framework, price change is a function of excess demand.

$$\Delta p_{i,t+1} = \beta_i D_{i,t}$$

$$\text{where } \Delta p_{i,t+1} = p_{i,t+1} - p_{i,t}$$

Empirical studies have found the existence of prices comovement among different markets. For example, Egert and Kocenda (2011) report strong correlation among returns of Germany, France and UK stock markets. The correlation can be even up to 0.9. These empirical phenomena indicate that price change of one market $\Delta p_{i,t+1}$ is correlated to another market $\Delta p_{-i,t+1}$. This cross-correlation implies the general form of price change $\Delta p_{i,t+1}$ in multi-market framework as:

$$\Delta p_{i,t+1} = \alpha_i D_{i,t} + \beta_i \Delta p_{-i,t+1}$$
$$\Delta p_{-i,t+1} = \alpha_{-i} D_{-i,t} + \beta_{-i} \Delta p_{i,t+1}$$

By solving $\Delta p_{i,t+1}$ and $\Delta p_{-i,t+1}$, we get

$$\Delta p_{i,t+1} = \frac{1}{1-\beta_i\beta_{-i}} \left(\alpha_i D_{i,t} + \alpha_{-i}\beta_i D_{B,t}\right)$$
$$\Delta p_{-i,t+1} = \frac{1}{1-\beta_i\beta_{-i}} \left(\alpha_{-i} D_{B,t} + \alpha_i\beta_{-i} D_{A,t}\right)$$

without loss of generality, this adaptive price updating can be expressed in a coupling form:

$$p_{i,t+1} = p_{i,t} + \gamma \left[\left(1-g_i\right) D_{i,t} + g_i \cdot D_{-i,t}\right] \tag{3}$$

where $D_{-i,t}$ is excess demand of market other than i, γ is adjustment speed of price, and g_i is coupling factor weighting foreign market excess demand. This coupling market maker approach is due to the fact of interrelated market connections. Aware of commonality of macroeconomics factors underlying markets and market correlation, market maker i applies g_i to take account of these factors. Weighting factor g_i shows the importance of foreign market excess demand to price updating of domestic market. A larger g_i implies a more influencing foreign market. In our model, we set $0 \leq g_i < 0.5$ to reflect that domestic factors have dominant effect in determining individual market price.

Based on Eq.3, we have a two-dimension deterministic market system:

$$p_{A,t+1} = p_{A,t} + \gamma \left[\left(1-g_A\right) D_{A,t} + g_A \cdot D_{B,t}\right]$$
$$p_{B,t+1} = p_{B,t} + \gamma \left[\left(1-g_B\right) D_{B,t} + g_B \cdot D_{A,t}\right]$$

2.2. Steady State and Stability

For theoretical analysis, steady state and its corresponding stability conditions are derived in this subsection. Steady state of the two-market system is denoted by $(\widehat{p}_A, \widehat{p}_B)$.

PROPOSITION 1. *(a). Individual steady state $\widehat{p}_A$ and $\widehat{p}_B$ can be implicitly determined separately by $D_{A,t}=0$ and $D_{B,t}=0$, where*

$$D_{A,t} = \left(w^f_{AA}+w^f_{AB}\right) v_{A,t}\left(F_A - p_{A,t}\right) + \left(w^c_{AA}+w^c_{AB}\right)\tau\left(p_{A,t}-p^c_{A,t}\right)$$
$$D_{B,t} = \left(w^f_{BA}+w^f_{BB}\right) v_{B,t}\left(F_B - p_{B,t}\right) + \left(w^c_{BA}+w^c_{BB}\right)\tau\left(p_{B,t}-p^c_{B,t}\right)$$

(b). If steady state $(\widehat{p}_A,\ \widehat{p}_B)$ exists for a particular chartist price window, it is unstable.

Remark 1. For $i=A$ and B, convergence speed $v_{i,t}$ is a function of price $p_{i,t}$; chartist reference price $p^c_{i,t}$ is pre-specified; hence, excess demand in individual market $D_{i,t}$ is a function of its corresponding price $p_{i,t}$. If condition $D_{i,t}=0$ exists, it can implicitly determine the value of $p_{i,t}$. However, this steady state is unstable.

Detailed proof is provided in Appendix A.

3. FINANCIAL CRISIS SIMULATIONS

In this section, we will apply the two-market model to simulate simultaneous crash behavior from two points of view: endogeneity and exogeneity with external shocks. Each view corresponds to different hypothesis of causes of financial crisis. Investigating from the point of view of endogeneity, we simulate different patterns of financial crises, including the sudden crisis with empirical reference to "Black Monday" of US and UK stock markets in 1987. On the other hand, from the point of view of exogeneity, permanent and temporary shocks are employed to evaluate their contribution to financial crisis.

3.1. Crisis from Endogeneity

In the morning of October 19, 1987, crash began in Far Eastern markets and then spread to Europe and US. During that day, DJIA dropped by 22.6%, the largest one-day percentage drop in history. And that day is called as "Black Monday". There are various versions of explanation for this crash, such as programming trading, over-valuation and market psychology. Our intension is to simulate a two-market crisis with propagation phenomena. If a model manages to simulate the propagation phenomena, at least it can provide a tool to understand partly, if not all, the crisis. Two markets DJIA (US) and FTSE 100 (UK) are used as reference. Sample period is from 08-01-1987 to 12-29-1987. A common set of parameters are defined for the subsequent simulations unless the parameters are specified. This common set of parameters and conditions for subsequent simulations are provided in Appendix B. With initial prices condition "endo-sudden", our deterministic two-market model manages to mimic market trend of the two indexes during the crisis – a simultaneous sudden drop of asset prices (Fig. 1).

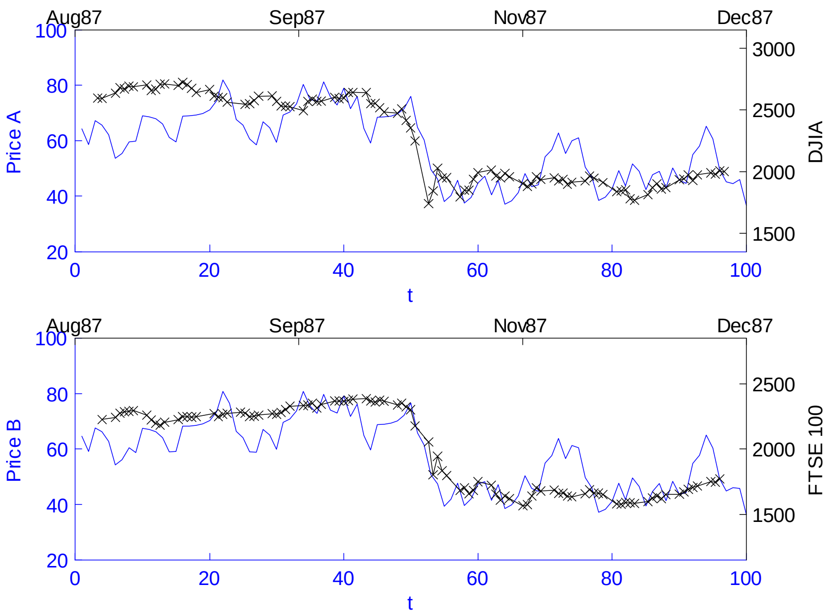
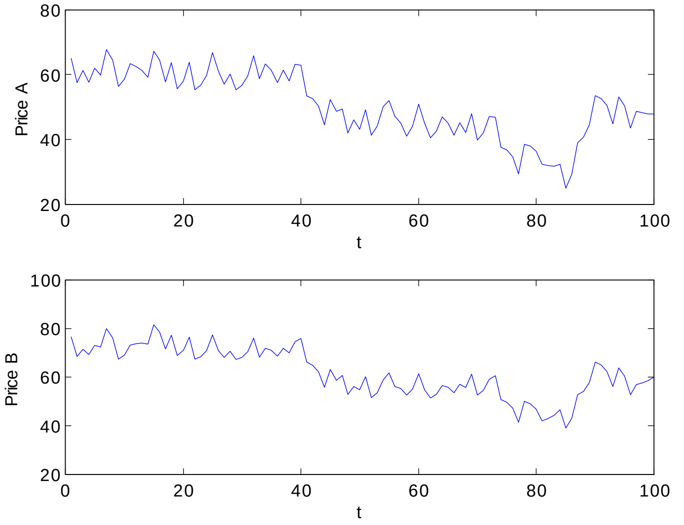
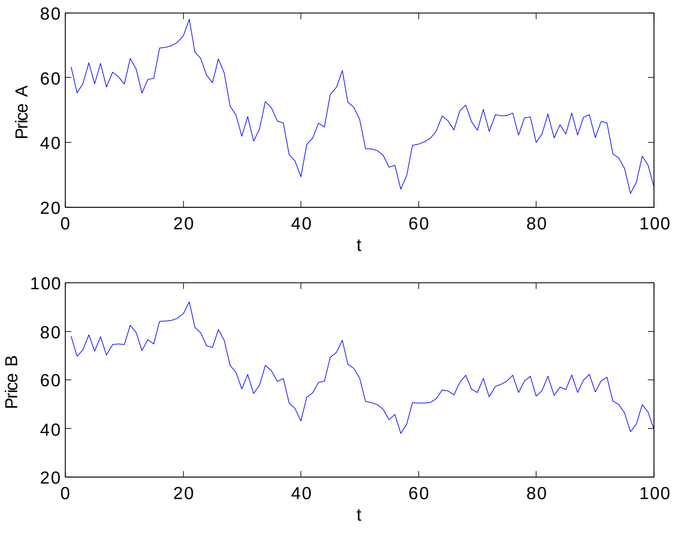


Fig. 1. Crisis matching. Blue color is for simulation while the black color with makers is for real indexes DJIA and FTSE 100.

Besides sudden crisis, just by changing initial prices, our model also manages to produce patterns of smooth crisis and disturbing crisis with propagation behavior: both markets have similar trends although individual market values are not the same. Smooth crisis is produced with "endo-smooth". Both markets evolve without large fluctuations till time step 40 and then decline gradually to bottom around time step 85. During this declining process, both markets lose around 50% of their initial market values. (Fig. 2.a). In contrast, given another set of initial prices "endo-disturbing", disturbing crisis emerges in both markets. From initial prices, both markets climb and reach their peaks at time step 21. After that, prices drop dramatically till time step 40 and then rebound. However, the rebound is temporary and prices drop to even lower bottoms at time step 58. From peaks to bottoms, both markets lose around 60% of their values (Fig. 2.b).



(a)



(b)

Fig. 2. Crisis with contagion behavior: (a) smooth crisis (b) disturbing crisis.

Simulations of this subsection for three types of financial crisis have a common feature that peaks of both markets' prices before crisis are well above fundamental values $F_i = 50$. Over-valuation causes dramatic adjustment of market without external force, which supports scenario that over-valuation causes financial crisis. These simulations demonstrate the capability of the model in certain extent to explain financial crisis and its propagation behavior. Similar to the conclusion of Huang et al. (2010) that financial crisis can be endogenous, in this case, financial crisis and its propagation effect occur without external force and are endogenous.

3.2. Crisis from Exogeneity

In real world, innovations continue to emerge and financial markets always encounter shocks affecting market fundamental. Such kinds of shocks can be technological innovation, macro-economics fluctuation and so on. In this subsection, simulations are conducted to evaluate the possibility of financial crisis induction by shocks to market fundamental. Market fundamental values F_i are no longer constant as in previous subsection. A shock can change F_i permanently or temporarily. Time frame of financial crisis usually is short in unit of days or months. The time window for this paper's study is 100 time steps. When a shock is in effect for a period longer than the time window, it is treated as permanent. Similarly, when a shock lasts less than the time window, it is treated as temporary. The purpose of the simulation is to verify whether a small shock to market fundamental value can cause dramatic fluctuation as well as whether crisis propagation is possible.

3.2.1. Permanent shock

A permanent shock can arise from sources like changes in fiscal policy, such as decrease in government expenditure, as well as some critical market events. Calvo (2012) argues that the collapse of Lehman Brothers triggers sub-prime crisis of 2008 as market conjectures that other large financial institutions might not be bailed out from then on and falls into panic. This subsection demonstrates that a small permanent shock in the fundamental value of one market can induce drastic drop in asset price – financial crisis. Also, this financial crisis can spread to the other market. Depending on the magnitude of the shock, different patterns of crisis can be induced.

With condition "permanent-shock", reference price trajectories of markets A and B are created: fundamental values F_A and F_B do not change their values 50 and $p_{A,t}$ and $p_{B,t}$ fluctuate within range 25 $(= 65 - 40)$[2] and 30 $(= 80 - 50)$, respectively. At time step 30, a permanent shock hits F_B and F_B reduces its value by 1 and changes to 49, i.e. a 2%

[2] peak of price - bottom of price

reduction in fundamental value of market B while F_A is not affected. This shock does not cause immediate effect on both markets. However, after around 15 time steps, both markets experience price drop. In terms of magnitude, $p_{A,t}$ still fluctuate approximately within the same price range with the reference price. In contrast, although $p_{B,t}$ experiences similar up and down trends with $p_{A,t}$, its adjustment is severe with lower bottom value to 40. Range of price fluctuation for market B has been increased by 30%, from 30 ($= 80 - 50$) to 40 ($= 80 - 40$) (Fig. 3.a).

To confirm the result, the same setting is utilized except the magnitude of shock changed to 2, a 4% reduction in F_B. This time, a more severe drop in asset price of market B occurs. Disturbing crisis occurs in market B. $p_{B,t}$ drops from 80 to 30 such that price fluctuation range increases by 60%. Meanwhile, similar disturbing crisis is also observed in market A with fluctuation range increased more than 100%, from 25 ($= 65 - 40$) to 55 ($= 65 - 10$) (Fig. 3.b). Here, a 4% small shock in one market's fundamental value can trigger financial crisis spreading to both market with price fluctuation range increased more than 60%.

Shocks are not always negative. What will over-valued markets response to a positive shock? A permanent 0.6% increase in F_B from 50 to 50.3 causes both markets to adjust at first. After that, both markets are pushed up and reach new peaks, followed by smooth crisis. During the smooth crisis, $p_{B,t}$ drops from 82 to 22, with price fluctuation range increased by 100%, from 30 to 60 ($= 82 - 22$); $p_{A,t}$ drops from 80 to 20, with price range increased more than 100%, from 25 to 60 (Fig. 3.c). Here, even a positive shock can trigger crisis by booming up larger assets bubbles which collapse eventually. As a comparison, magnitude of positive shock on F_B is increased to 1%, i.e., F_B increases from 50 to 50.5. Surprisingly, no crisis is observed this time. Price of both markets fluctuate within the reference range (Fig. 3.d). In an over-valued market, depending on magnitude, a positive shock on market fundamental has different possible consequences. It may pushes market to a higher peak upon which market self-correction is triggered and crisis occurs. On the other hand, it can be absorbed within normal market fluctuations.

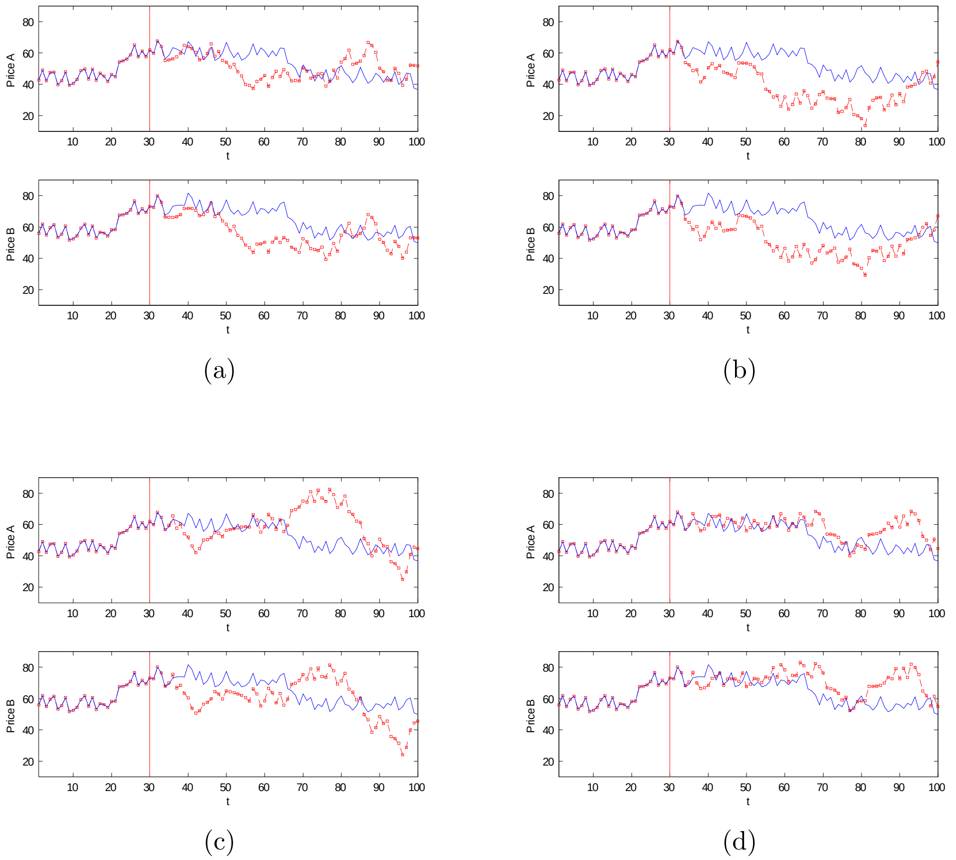
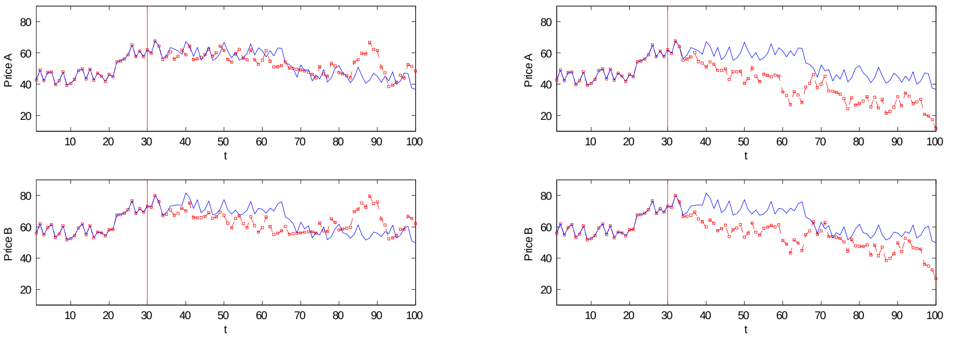


Fig. 3. Impact of permanent shock. Blue color is reference trajectory while red marker represents situation in which F_B is hit by a permanent shock at step 30, highlighted by a vertical line. (a) F_B reduces by 2%. (b) F_B reduces by 4%. (c) F_B increases by 0.6%. (d) F_B increases by 1%.

In a market system, each market can encounter shocks simultaneously. It has been shown that it is possible for a shock in one market to cause financial crisis across markets. What will happen if market members encounter shocks simultaneously? Can shocks in different markets cancel out each other? At step 30, F_A decreases by 1.8% from 50 to 49.1 while F_B increases by 0.6% from 50 to 50.3. Contrasting to Fig. 3.c for a 0.6% increase in F_B, the smooth crisis disappear. Instead, both markets fluctuate within the reference range and no crisis is triggered (Fig. 4.a). In this case, shocks hitting individual market cancel out each other. If we increase magnitude of shock in F_A to 3%, that is, F_A decreases from 50 to 48.5, smooth crisis occurs in both markets. $p_{A,t}$ decreases from 65 to 10, with fluctuation range increased more than 100%, from 25 to 55 ($= 65 - 10$). Similarly, fluctuation range of $p_{B,t}$ also increased by 100%, from 30 to 60 ($= 80 - 20$) (Fig. 4.b).



(a) (b)

Fig. 4. Impact of simultaneous permanent shocks. F_B increases by 0.6%. (a) F_A decreases by 1.8%. (b) F_A decreases by 3%.

These demonstrations show that in a closely correlated financial world, a small shock, either positive or negative, in one market can create a financial crisis spreading to other market. Shocks with different magnitudes have different impacts. Simultaneous shocks in market members can cause financial crisis or cancel out each other without causing dramatic market reactions.

3.2.2. Temporary shock

Temporary shocks can be due to short term policy changes or psychological fluctuations caused by rumors. Individual company is the common entity hit by rumors. In September 8, 2008, United Airline's stock price plummeted more than 75%, from prior day's close \$12.3 to a low of \$3. The crash was caused by a rumor of an erroneous report claiming bankruptcy of the company. Even to bigger scale of whole market, rumors could trigger financial crises. Kindleberger and Aliber (2005) discuss several cases of rumor triggering crisis. One of the examples is "Black Friday" of May 11, 1866 due to rumors of Prussian-Austrian war. We demonstrate that financial crisis could be triggered by temporary shock in this subsection.

Condition "temp-shock" is applied to create reference trajectories. Similar to the cases of permanent shock, $p_{A,t}$ and $p_{B,t}$ fluctuate in between of 25 $(= 65-40)$ and 30 $(= 80-50)$, respectively. At time step 30, F_B is hit by a shock and changes its value from 50 to 47, a 6% reduction. Since the shock is temporary, F_B recovers to its previous level 50 at time step 34. The duration of shock is 4 time steps. Meanwhile, market A is free of shock. There is no much change in the new price trajectories of both markets compared to the reference ones (Fig 5.a). However, if duration of the temporary shock to F_B is extended to a longer time, i.e. 6 time steps such that F_B recovers to its previous level 50 at time step 36, disturbing financial crisis occurs in both markets. Fluctuation range of market A increases by 40% while market B by 30% (Fig 5.b). As a comparison, we switch to positive shock and evaluate its effect. At time step 30, a positive shock impacts F_A so that F_A increases from 50 to 51.4, a 2.8% increment. If F_A recovers to 50 at time step 34, no major effect is found on new price trajectories (Fig 5.c). However, if F_A recovers at time step 36, disturbing crisis arises in market A with price range increased by 80%, from 25 to 45 ($= 65 - 20$). At the same time, market B also experiences similar crisis, with price range increased by 100%, from 30 to 60 ($= 80 - 20$) (Fig 5.d). Hence, even for a temporary shock in one market's fundamental value, depending on its duration, it is possible to cause financial crises in two markets.

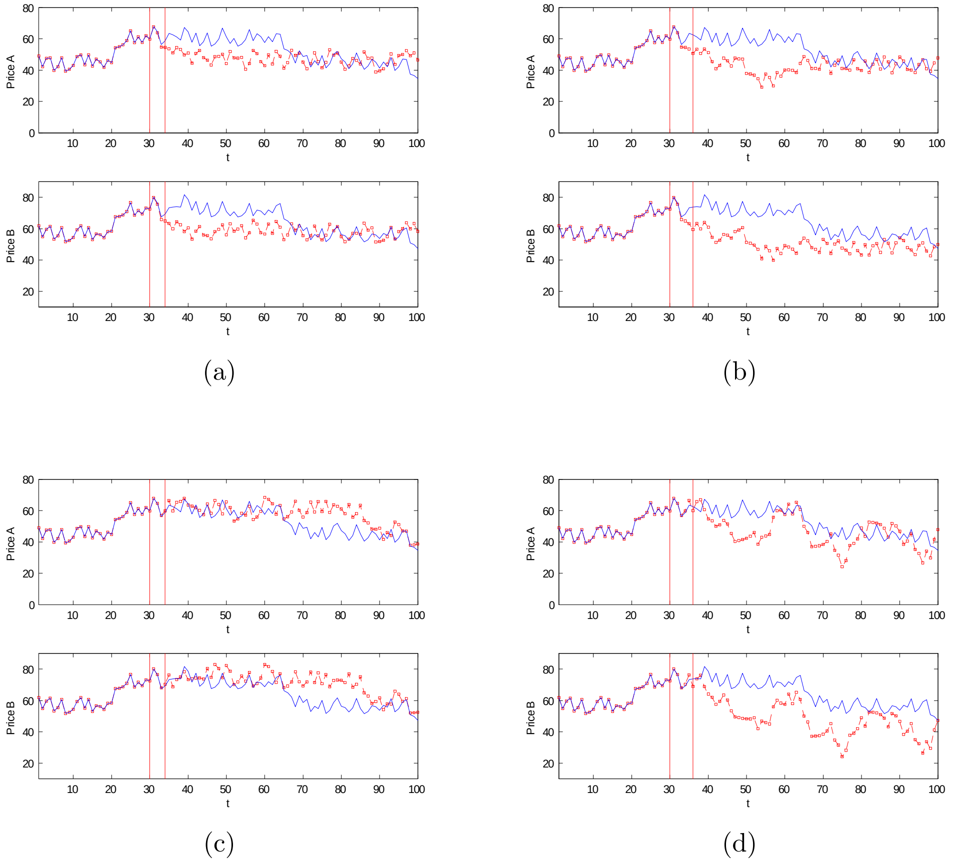
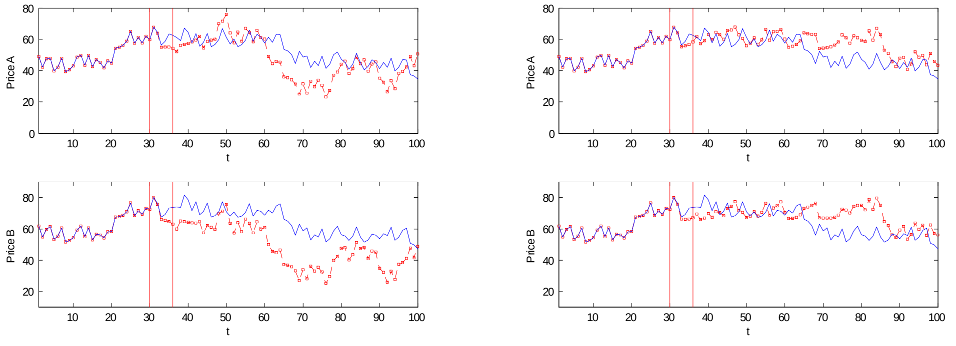


Fig. 5. Impact of temporary shock on one market's fundamental value. In between two vertical lines is effective period of shock. In (a) and (b), F_B reduces by 6% at time step 30. (a). F_B recovers to original value 50 at time step 34. (b). F_B recovers at time step 36. In (c) and (d), F_A increases by 2.8% at time step 30. (c) F_A recovers to original value 50 at time step 34. (b). F_A recovers at time step 36.

Extending to case of simultaneous temporary shocks, both F_A and F_B encounter shocks simultaneously at step 30 and both recover to original value 50 at step 36. During the shock effective period, F_B decreases by 6% from 50 to 47. If F_A increases by 1.4% from 50 to 50.7 temporarily, disturbing financial crisis develops in both markets. Fluctuation range of $p_{A,t}$ increases by more than 100%, from 25 to 60 ($= 80 - 20$). Similarly, fluctuation range of $p_{B,t}$ increases by 100%, from 30 to 60 ($= 80 - 20$) (Fig. 6.a). If magnitude of positive shock in F_A is further increased to 3% so that F_A increases from 50 to 51.5, both markets fluctuate comparably with the reference trajectories and no financial crisis is observed (Fig. 6.b). These results show that in an over-valued market system, simultaneous temporary shocks hitting individual market can produce different results, depending on magnitude of the shocks.



(a) (b)

Fig. 6. Impact of simultaneous temporary shocks. F_B reduces by 6% and effective in between step 30 and 36. (a) F_A increases by 1.4%. (b) F_A increases by 3%.

In above simulations of permanent and temporary shocks, at the time fundamental value of individual market is affected by a shock, price of both markets are above their market fundamental values. Financial crisis in both markets is triggered by shock(s). It is implied that over-valuation does not always cause financial crisis. This supports scenario that financial crisis is attributed to market shock provided asset is over-valued. In addition, financial crisis triggered by shock in one market causes similar change in the other market. This is resemblance to domino effect that a change causes similar change nearby. These simulations might capture mechanism of financial crisis partly. In a world of over-valued and closely linked financial markets, certain macro-economic changes or market shocks in market member(s) can lead to over-adjustment and even disasters to all markets. These changes or shocks can be permanent or temporary. In this sense, financial markets are fragile. Policy implication is that policy to remove asset price bubbles must be designed with deliberation. Otherwise, adverse consequence might be caused.

4. STYLIZED FACTS

In this section we calibrate our two-market model to match the stylized facts. According to Cont (2001), T. Lux (2002) and Westerhoff and Dieci (2006), real world speculative markets have following characteristics: (1) volatility cluster phenomena in which high-volatility events tend to cluster in time; (2) distribution of returns with fat tails; (3) insignificant autocorrelation for daily return; (4) strong autocorrelation for absolute daily returns. Besides the above stylized fact for single market, empirical studies already show the existence of correlation between markets. For example, Egert and Kocenda (2011) find strong correlation among returns of Germany, France and UK, up to 0.9.

We denote $r_{i,t}$ for return of market i at time step t. $r_{i,t}$ is defined as

$$r_{i,t} = \ln p_{i,t} - \ln p_{i,t-1}$$

To calibrate the stylized facts, condition "stylized-facts" are applied to generate 10,000 periods price trajectories and the corresponding returns are calculated. It is shown that individual market manages to match the typical stylized facts. Both markets exhibit volatility cluster in their return trajectories. Besides that, both market experience the same large and small volatilities most of the time (Fig. 7); distributions of return have fat tails and kurtosis of markets A and B are 3.602 and 3.600, respectively (Fig. 8); autocorrelation of return is insignificant across lags except lag 1 while autocorrelation of absolute returns is significant (Fig. 9); besides that, at 95% confidence interval, there are significant cross-correlation at different lags. Especially at lag zero, the cross-correlation is up to 0.7, which implies prices comovement of the two markets and explains the similar volatility patterns of both markets (Fig. 10).

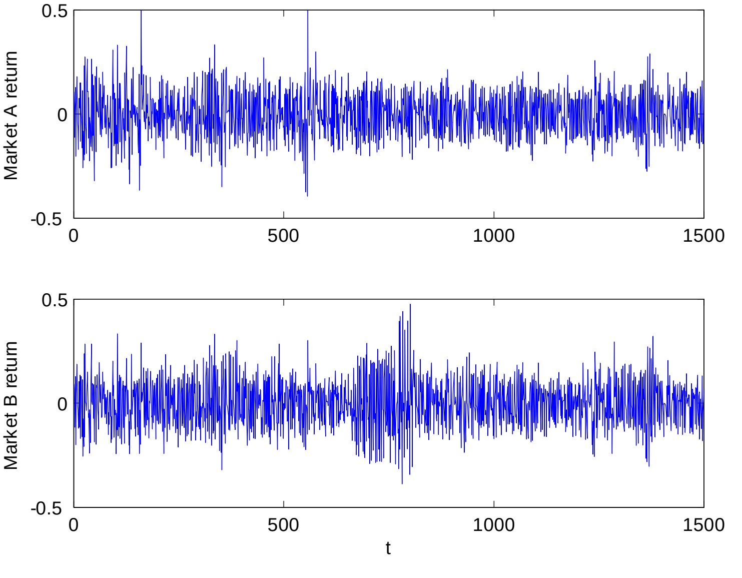
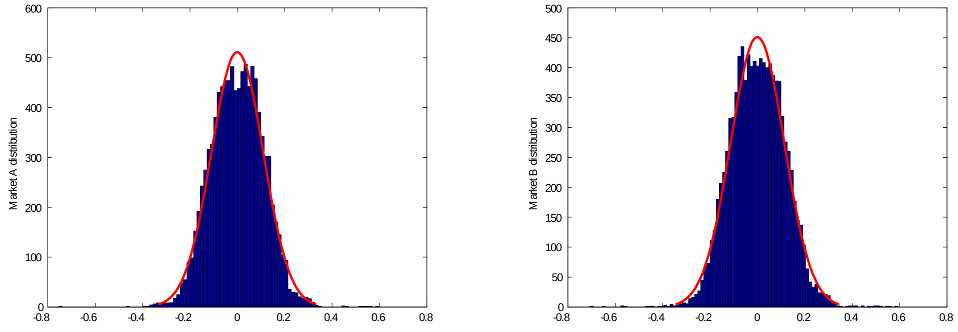
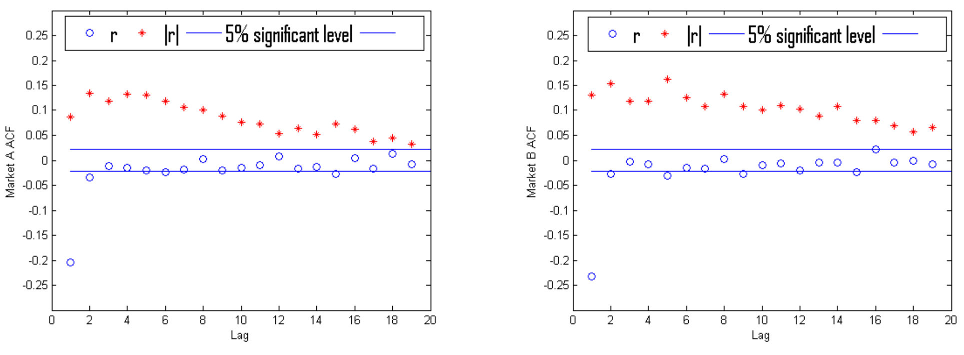


Fig. 7. Volatility clustering



(a) (b)

Fig. 8. Distribution of returns. (a) market A. (b) market B.



(a) (b)

Fig. 9. Autocorrelation of returns and absolute returns. (a) market A. (b) market B.

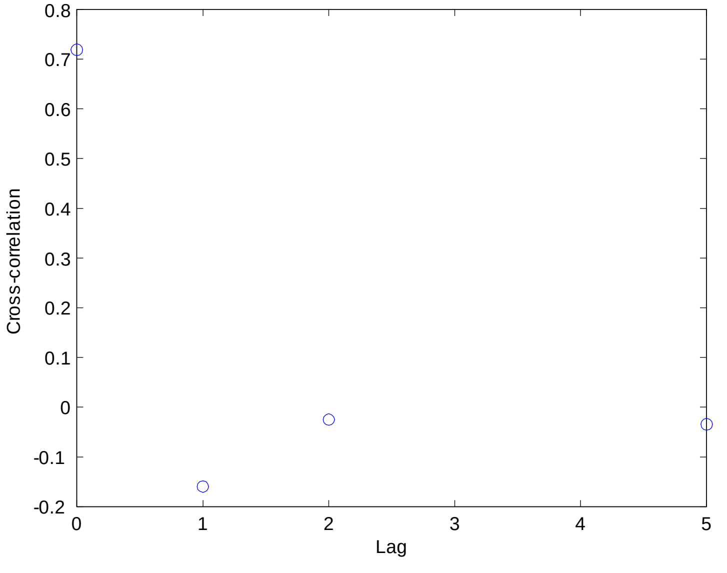


Fig. 10. Significant cross-correlation in 95% confidence interval.

5. CONCLUSION

This paper proposes a heterogeneous agents two-market model. Different agents with fixed compositions are active in both markets and are allowed to invest in both markets. Market makers of individual market adopt a coupled price updating function to reflect common factors underlying the two markets. The main purpose of this paper is to simulate financial crisis within two-market framework from points of view of endogeneity and exogeneity so that different scenarios could be tested to understand causes of financial crisis.

In terms of endogeneity, we manage to simulate different patterns of financial crisis across two markets endogenously, especially sudden crisis with empirical reference to "Black Monday" of US and UK stock markets in 1987. These simulation implies that financial crisis and its propagation could occur endogenously. As all our simulated financial crises occur at price level above market fundamental levels, they provide support to scenario of market over-valuation causing financial crisis. In terms of exogeneity, shocks are introduced to fundamental value of individual market. Depending on the magnitude, sign and duration of shocks, different patterns of financial crisis could be triggered. Similar to simulations of endogeneity, all financial crises have over-valued prices when shock(s) hit individual market. This supports scenario of financial crisis triggered by shock. In addition, the fact that financial crisis in one market caused by shock causes similar financial crisis in the other market is analogous to domino effect.

In matching stylized facts, in addition to volatility clustering, fat tails, insignificant autocorrelation of return and significant autocorrelation of absolute return, we also manage to calibrate cross-correlation, which is exclusive to multi-market model. Cross-correlation can match to empirical phenomena of prices comovement among markets, especially propagation effect of financial crisis.

Financial crisis involves a lot of aspects such as macro-economics and financial markets. Although a single model might not fully capture all the factors underlying financial crisis, a model which is more closed to realistic world and provides more intuition should be more robust in understanding financial crisis. The current model is limited by the simplicity of fixed investor composition. Features such as varying investor composition based on some evolutionary fitness should be included into the model for future research!

ACKNOWLEDGEMENTS

We thank J. Barkley Rosser, Jr. and Marcel Ausloos as well as the NED (2013) conference participants for the valuable comments and suggestions. Any remaining errors are our own.

APPENDIX A

For a two-dimension system:

$$p_{A,t+1} = p_{A,t} + \gamma\left[(1-g_A)\, D_{A,t} + g_A \cdot D_{B,t}\right]$$
$$p_{B,t+1} = p_{B,t} + \gamma\left[(1-g_B)\, D_{B,t} + g_B \cdot D_{A,t}\right]$$

where

$$D_{A,t} = \left(w^f_{AA} + w^f_{AB}\right) v_{A,t}\left(F_A - p_{A,t}\right) + \left(w^c_{AA} + w^c_{AB}\right)\tau\left(p_{A,t} - p^c_{A,t}\right)$$
$$D_{B,t} = \left(w^f_{BA} + w^f_{BB}\right) v_{B,t}\left(F_B - p_{B,t}\right) + \left(w^c_{BA} + w^c_{BB}\right)\tau\left(p_{B,t} - p^c_{B,t}\right)$$

Since $0 \le g_A,\, g_B < 0.5$, the equilibrium prices $(\widehat{p}_A,\, \widehat{p}_B)$ can be determined implicitly from:

$$D_{A,t} = 0$$
$$D_{B,t} = 0$$

Evaluated at equilibrium point $(\widehat{p}_A, \widehat{p}_B)$, denote $\frac{dD_{A,t}}{dp_{A,t}} = d_A$ and $\frac{dD_{B,t}}{dp_{B,t}} = d_B$, Jacobian matrix of this two-dimension system is expressed as:

$$\begin{matrix} 1+\gamma\left(1-g_A\right)d_A & \gamma d_B \\ \gamma d_A & 1+\gamma\left(1-g_B\right)d_B \end{matrix}$$

The corresponding eigenvalues λ_1 and λ_2 can be derived

$$\lambda_1 = 1 + \frac{1}{2}\gamma\left(A + \sqrt{B}\right)$$
$$\lambda_2 = 1 + \frac{1}{2}\gamma\left(A - \sqrt{B}\right)$$
$$\text{where, } A = d_A + d_B - g_A d_A - g_B d_B$$
$$B = \left(d_A - d_B + g_A d_A - g_B d_B\right)^2 + 4 d_A d_B$$

To have real roots, we have $B \geq 0$. Since $0 \leq g_A, g_B < 0.5$, it can be proved that $-\sqrt{B} \leq A \leq \sqrt{B}$ by

$$\begin{aligned} A^2 - B &= -4 g_A d_A^2 + 4 g_A g_B d_A d_B - 4 g_B d_B^2 \\ &\leq -2 g_A d_A^2 - 2 g_B d_B^2 \\ &\leq 0 \end{aligned}$$

Hence, $\lambda_1 \geq 1$, violating the stability condition. It is concluded that the equilibrium point $(\widehat{p}_A, \widehat{p}_B)$ is unstable.

APPENDIX B

Parameters setting for all the simulations are listed here. A common set of parameters are used for each simulation scenario unless the individual parameter is specified.

common set of parameters	$d = 0.25$, $g_A = g_B = 0.45$, $u_1 = -0.2$, $u_2 = 2$, $\gamma = 0.25$, $\lambda = 15.02$, $w^f_{AA} = w^f_{BA} = 1$, $w^f_{AB} = w^f_{BB} = 1$, $w^c_{AA}\tau = w^c_{BA}\tau = 2.9$, and $w^c_{AB}\tau = w^c_{BB}\tau = 2.9$.
endo-sudden	$p_{A,0} = 64.2691$ and $p_{B,0} = 64.6191$
endo-smooth	$p_{A,0} = 64.9957$ and $p_{B,0} = 76.5895$
endo-disturbing	$p_{A,0} = 63.2693$ and $p_{B,0} = 77.9186$,
permanent-shock	$p_{A,0} = 42.6906$ and $p_{B,0} = 55.9131$
temp-shock	$p_{A,0} = 49.0616$ and $p_{B,0} = 61.9338$
stylized-facts	$g_A = g_B = 0.3$, $p_{A,0} = 54.1100$ and $p_{B,0} = 57.0915$