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Heterogeneous Agents in Multi-markets: A Coupled Map Lattices Approach

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Abstract

In this paper we examine assets price deviation in a multi-market system with heterogeneous investors in each market. Coupled map lattices (CML) is introduced to the market maker framework. It results in market cluster sharing the same sign of deviation in the chaotic interval. Distribution plots are applied to understand the deviation persistence enhancement from the coupling effect. Besides that, external disturbance is employed to the system to examine the market pattern stability and the propagation of the disturbance. The goal of the paper is to introduce coupling effect as a bridge for multi-market interactions with heterogeneous agents.

Key words: Coupled map lattices; heterogeneous agents; multi-market interaction.

JEL classification: C61, G12, G15

1 Introduction

In 1981, Kaneko discovered the spatiotemporal pattern of coupled map lattices (CML) when he started a simulation in which a chain of logistic maps is utilized. In the chaotic regions, each logistic map couples to nearby ones. The discrete time evolvements display spatiotemporal pattern in which values of logistic maps are either greater or less than fixed point value. CML has been expanded into fields of spatiotemporal chaos and pattern formation, biology, mathematics, engineering and so on. For further understanding of CML, we cite here in particular, Kaneko [10-14] and Ouchi and Kaneko [15].

Heterogeneous agents models have managed to replicate some of the stylized facts of financial markets, such as bubbles and crashes, randomly switching bear and bull market episodes, excess volatility, volatility clustering and fat tails for returns distribution [1-4, 6,7]. Huang and Chen [9] further contribute to stylized facts matching in terms of cross-correlation. Based on the market maker framework of Day and Huang [5] and two-market model of Westerhoff and Dieci [16], Huang and Chen [9] model a two-market system with free movement of capital and apply coupling in market maker's price updating by weighting excess demand of different markets. The market system displays market pooling phenomena and strong cross-correlation.

This paper extends the framework of Huang and Chen [9] to multiple markets through CML effect. To segregate and investigate the effect from coupling, investors can only invest in their home market while coupling of market makers is still adopted. Market makers update price based on the weighted excess demand of domestic and adjacent markets. With this setup, market cluster or enhancement on persistence of asset price deviation is observed. The effect becomes prominent in the chaotic interval, where market cluster with the same sign of deviation is formed in the spatio-temporal diagram. The market clusters can regroup if market member is hit by shock in asset price. During this regrouping process, coupling effect can stabilize market member with small fluctuation compared to the isolated counterpart.

This paper is structured as follows. Section 2 describes the details of model setup and proposes an coupling market maker framework in a multi-market system. In section 3, with numerical bifurcation study as reference, deviation spatio-temporal diagrams are plotted to demonstrate the deviation enhancement effect. To understand this enhancement phenomena, phase diagram and Lorenz plot are utilized. Also, external shocks are employed to investigate the market cluster pattern stability. Lastly, section 4 concludes the paper and suggests possible future research direction.

2 Model Setup

For the isolated market i, asset price at time t is denoted by $P_{i,t}$. The fundamental value is treated as constant and is denoted by F_i. For convenience, we define the price deviation $x_{i,t} = P_{i,t} - F_i$. For a chartist (c) or a fundamentalist (f) in market i, their excess demand for the asset i is $D_{ic,t}$ and $D_{if,t}$, respectively. These excess demands are linear functions of price deviation, for simplicity.

$$D_{ic,t} = b_c x_{i,t}, D_{if,t} = b_f(-x_{i,t})$$

where b_c and b_f are the strength of demand of chartist and fundamentalist. Chartists purchase over-valued asset and sell under-value one while fundamentalists behave in an opposite way. The distribution composites of chartists and fundamentalists in market i are $W_{ic,t}$ and $W_{if,t}$, respectively. When the price deviation is larger, more investors will become fundamentalists and the proportion of fundamentalists is larger. Investors distribution composites are determined according to:

$$W_{ic,t} = \frac{h\exp\left(-\left|x_{i,t}\right|\right)}{\exp\left(\left|x_{i,t}\right|\right) + h\exp\left(-\left|x_{i,t}\right|\right)}, W_{if,t} = \frac{\exp\left(\left|x_{i,t}\right|\right)}{\exp\left(\left|x_{i,t}\right|\right) + h\exp\left(-\left|x_{i,t}\right|\right)}$$

where $h \geq 1$ is chartist distribution parameter, proportional to the confidence level of chartist. A large h indicates that chartists are more confident.

With the individual excess demand and investor compositions, the excess demand to market i, $D_{i,t}$, can be derived:

$$D_{i,t} = W_{if,t}D_{if,t} + W_{ic,t}D_{ic,t}$$

In response to the excess demand, market maker of market i updates next period's price.

$$P_{i,t+1} = P_{i,t} + aD_{i,t}$$

where a is the price adjustment coefficient.

Expressed in the price deviation form, the price updating process can be expressed as:

$$
\begin{aligned}
x_{i,t+1} &= x_{i,t} + aD_{i,t} \qquad (1)\\
&= x_{i,t} + \frac{ah\exp(-|x_{i,t}|)b_c x_{i,t}}{\exp(|x_{i,t}|) + h\exp(-|x_{i,t}|)} - \frac{a\exp(|x_{i,t}|)b_f x_{i,t}}{\exp(|x_{i,t}|) + h\exp(-|x_{i,t}|)}
\end{aligned}
$$

Above is the price dynamics for individual market i in isolation. To model dynamics of multi-market system, we concentrate on the behavior of market makers. In a connected market system, price movements of market members are correlated to each other. Price dynamics of market i is influenced by other markets. Realizing the existence of market correlation, each market maker adopts the coupling parameter g so that activities of two most adjacent markets are taken into account. The coupled market model for price updating is introduced:

$$
x_{i,t+1} = x_{i,t} + a\left[(1-g)\,D_{i,t} + g(\frac{D_{i+1,t} + D_{i-1,t}}{2})\right] \qquad (2)
$$

where $0 \leq g \leq 0.5$. The market maker i puts weight $(1-g)$ on the excess demand from market i and g on the average excess demand of adjacent markets.

3 Result

If we ignore deviation magnitude and focus on the sign of deviation, the deviations can be categorized into four types: persistent positive deviation, persistent negative deviation, alternate deviations, and diminishing to zero (the fundamental value). If the deviations fluctuate within the positive value domain or the negative value domain, the signs of the deviations do not change.

They are defined as persistent positive deviations or persistent negative deviations, respectively. Once the deviations fluctuate alternately between the positive and negative domains, they are not persistent and are defined as alternate deviations. In contrast, there are cases where deviation can diminish to zero–the fundamental value state. We are interested with the deviation persistence under cases of isolation and coupling effect–market correlation. The persistence of the deviation is related to the price deviation dynamics, which involves dynamical stability and bifurcation study such as switching between positive and negative regions. Hence, stability of the isolated market is investigated, followed by bifurcation illustration and other numerical demonstrations for multi-market cases.

3.1 Isolated market

For the isolated stock market i (1),

$$x_{i,t+1} = x_{i,t} + \frac{ah\exp(-|x_{i,t}|)b_c x_{i,t}}{\exp(|x_{i,t}|) + h\exp(-|x_{i,t}|)} - \frac{a\exp(|x_{i,t}|)b_f x_{i,t}}{\exp(|x_{i,t}|) + h\exp(-|x_{i,t}|)}$$

the steady state and stabilities properties is derived:

Proposition 1 *(a) There exists a unique fundamental steady state* $\bar{x}_i = 0$*, which is stable if and only if* $\max\{1 - 2(h+1)/(ab_f), 0\} < b_c \cdot h/b_f < 1$*;*

(b) There are two nonfundamental steady states: $\bar{x}_i = \pm\ln\sqrt{b_c \cdot h/b_f}$ *if* $b_c \cdot h/b_f > 1$*, and* $b_c \cdot h/b_f < \gamma$*, where* $\gamma = \exp\left(\frac{2(b_c+b_f)}{a\cdot b_c\cdot b_f}\right)$.

We concentrate on the bifurcation diagram as it relates the sign of deviation to parameter region. For all the numerical simulations, a common set of parameters is defined: $a = 2.131$, $b_f = 1$, and $h = 1$. Fig. 1 reports the bifurcation diagram, in which b_c is the bifurcation parameter. The diagrams report the attractors corresponding to two different initial conditions, one below and one above the fundamental value (Fig. 1.a and 1.b). When $b_c < 1$, fundamental steady state is stable. By increasing b_c, the attractors experience nonfundamental steady state and a sequence of period-doubling bifurcations to chaos states. Depending on the initial conditions, the chaotic intervals locate either above or below the fundamental state. In other words, When $b_c < 5.051$, deviations take place either in the positive or negative region–persistent deviations in our definition. After that, the deviations start to wander across both positive and negative regions. These regions are characterized by intrinsic fluctuations and erratic switching between positive and negative regions–alternate deviations. These alternate deviation regions demonstrate the market clusters under coupling effect in the sequel. In addition, the attractors experience a transition from chaotic state to 4-orbit state across positive and negative regions, followed by chaotic fluctuation again. The interval of this 4-orbit is $5.97 < b_c < 6.225$. In a short summary, an increase of the chartist strength destabilizes the market.

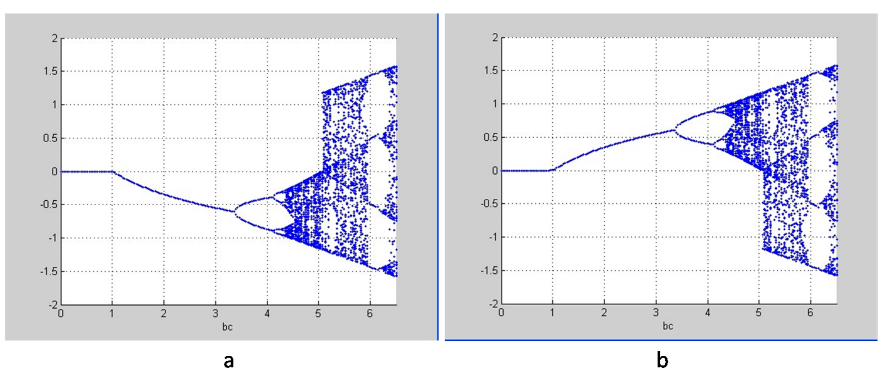

Fig. 1. Bifurcation curves of isolated market deviation x_i based on b_c.
(a). initial value of $x_i < 0$. (b). initial value of $x_i > 0$.

3.2 *Multi-markets*

To investigate the effect of coupling on deviation persistence, we study a system with 200 markets with price dynamics 2. The market system is close such that markets 2 and 200 are adjacent to market 1. After randomization, deviation evolvements of each market are plotted in the deviation sign spatio-temporal plots. The y axis is market i up to 200 and the x axis is the time step of the evolvement. Colors of green, violet and red represent positive, zero and negative deviation, respectively, regardless of the magnitudes. In this numerical demonstration, b_c and g are changed to study their effect on the deviation persistence. The demonstration is divided into two portions by the region-crossing point $b_c = 5.051$. Fig. 2 shows the deviation persistence for cases where $b_c < 5.051$. In this range, deviations converge to zero or are persistent in either the disjointed deviation regions. When $b_c < 1$, all the markets eventually converge to the fundamental steady states. The introduction of coupling effect, $g = 0.4$, prolongs the time needed to return to the fundamental states. The time steps taken to converge to fundamental steady state under isolation and coupling effect are around 130 and 650, respectively (Fig. 2.a and 2.b). In case of $1 < b_c < 5.051$, deviations locate in the persistent regions. The signs of deviations follow the ones determined by initial randomization. It seems the spatio-temporal plots under coupling effect are not significantly different from the isolated counterparts as persistence is observed visually for both cases (Fig. 2.c-2.f). However, if the same initial randomized values are evaluated with different values of g, both of the signs and magnitudes of deviations can be changed. That will be discussed in the subsequent subsection of deviation distribution study.

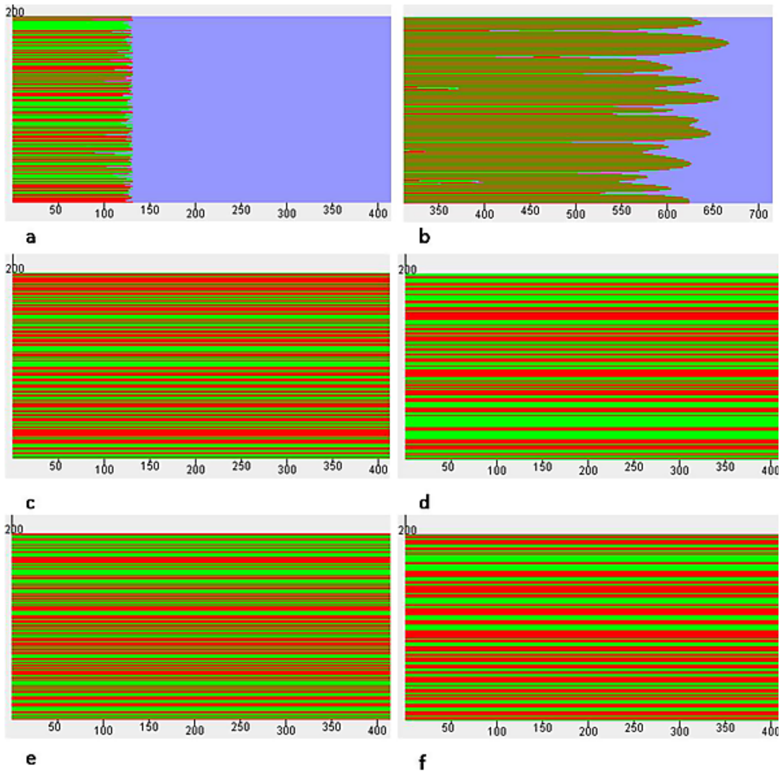

Fig. 2. Coupling effect before the region-crossing point. (a) $b_c = 0.9$, $g = 0$, deviations diminish to zero at time step 130. (b) $b_c = 0.9$, $g = 0.4$, deviations diminish to zero with a prolonged time at step 650. (c) $b_c = 3.33$, $g = 0$, deviations converge to the non-fundamental steady states. (d) $b_c = 3.33$, $g = 0.4$, deviations converge to the non-fundamental steady states, the sign and magnitude of the deviation might be changed. (e) $b_c = 3.4$, $g = 0$, deviations converge to two-period orbit in either the disjointed regions. (f) $b_c = 3.4$, $g = 0.4$, deviations converge to two-period.

The coupling effect becomes apparent when b_c is larger than the region-crosing point 5.051(Fig. 3). Before b_c exceeds 5.97 , the isolated markets are in turbulent states with varying deviation signs. In these markets, the duration that deviation remains in the same domain decreases with b_c (Fig. 3.a, 3.c and

3.e). The introduction of coupling effect enhances the deviation persistence. Market clusters with the same deviation sign are formed. Each cluster is deviation persistent. The coupling effect is not always dominant as some of the clusters become unstable with "defect" emerging when b_c increases (Fig. 3.b, 3.d and 3.f). When $5.97 < b_c < 6.225$, the isolated markets enter the state of 4-period orbit; the spatio-temporal plot shows a regular sign-switching pattern. The coupling effect still boosts the deviation persistence by increasing the duration of deviation in the same region. It disturbs the regular pattern into a turbulent state, with some intermittent market clusters growing and diminishing (Fig. 3.g and 3.h).

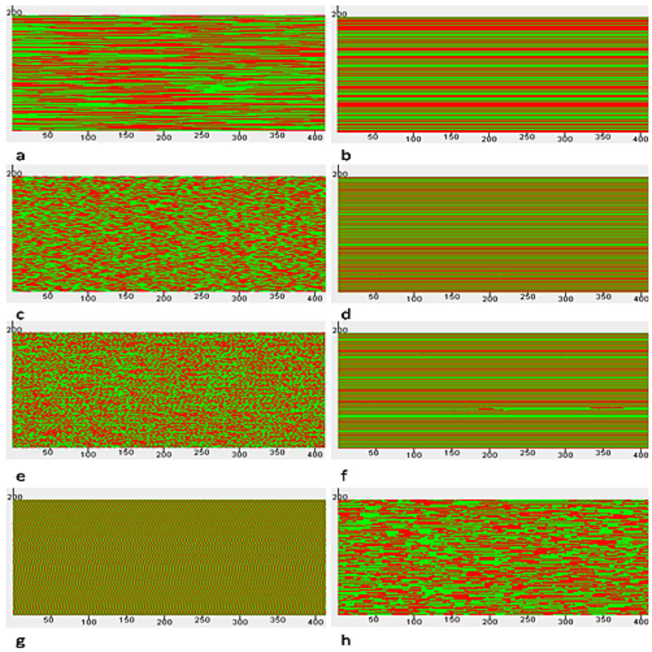

Fig. 3. Coupling effect after the region-crossing point. (a) $b_c = 5.0533333$, $g = 0$, market system is in turbulent state. (b) $b_c = 5.0533333$, $g = 0.4$, market clusters are formed with deviation persistence. (c) $b_c = 5.2$, $g = 0$, market system is in turbulent state with medium duration in the same deviation region. (d) $b_c = 5.2$, $g = 0.4$, market clusters are formed with deviation persistence. (e) $b_c = 5.3$, $g = 0$, market system is in turbulent state with short duration. (f) $b_c = 5.3$, $g = 0.4$, market system forms cluster patten with some "defect". (g) $b_c = 6.0$, $g = 0$, each isolated market is in the state of regular 4-period orbit. (h) $b_c = 6.0$, $g = 0.4$, duration of deviation in the same region is enhanced; market system shows turbulent state with intermittent market clusters.

After demonstrating the deviation persistence enhancement effect from the coupling parameter g, the coupling effect is further investigated by varying g in the region-crossing chaotic state, given $b_c = 5.0533333$. Without any coupling effect, $g = 0$, the isolated markets are in turbulent state (Fig. 4.a). The turbulent state transforms into a less turbulent one after a small coupling effect $g = 0.005$ is introduced into the system. Under the influence of weak coupling effect, there is a continual process in which clusters are destroyed by growing "defect" and new clusters emerge. (Fig. 4.b). By increasing g to 0.04825, more stable clusters are formed. The size of each cluster changes slightly with time (fig. 4.c). Further increasing g to 0.4 and eventually 0.5, the clusters are stable with small fixed sizes (Fig. 4.d and 4.e). By varying the strength of coupling effect, different spatio-temporal patterns appear. When the coupling effect is weak, market clusters are unstable. The unstable clusters can display a dynamic process of "defect" creation or cluster size change. In case the coupling effect is strong, coupling effect dominates the regions switching chaotic process; stable market clusters with persistent deviation signs appear.

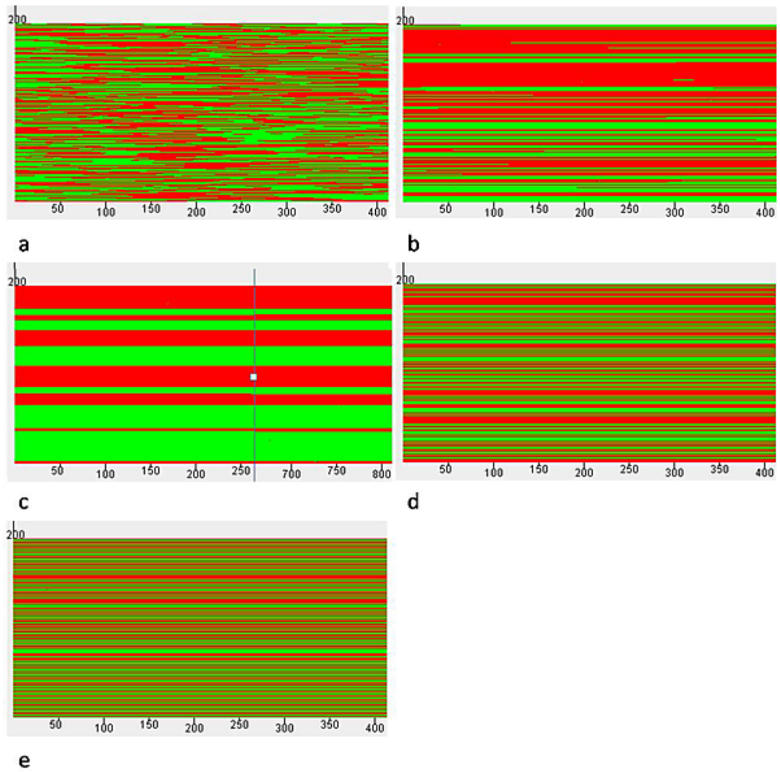

Fig. 4. Effect of g given $b_c = 5.0533333$. (a) $g = 0$, the market systen with isolated markets is in turbulent state. (b) $g = 0.005$, the market system is less turbulent, with "defect" growing in the clusters and new clusters emerging. (c)$g = 0.04825$, more stable clusters are formed; but the size of each cluster change with time. Contrast between periods 270 and 670 shows changes in the size of clusters. (d) $g = 0.4$, small fixed size clusters are formed. (e) $g = 0.5$, small fixed size clusters are formed.

3.3 Deviation Lorenz plot study

It has been shown that coupling effect tends to form clusters with same sign of deviation. Enquiries about the magnitude of deviation arise. Will the magnitude of deviation be constant? If not, is there any distribution pattern? To answer these questions, we plot Lorenz plots: $x_{i,t+1}$ vs $x_{i,t}$. One common set of initial random numbers is created to generate the 200 markets data. 50,000 iterations of evolvement are computed. The last 1000 iterations data from markets 4, 5, and 6 are used to plot the Lorenz distribution diagrams, where blue, red and green colors represent markets 4, 5, and 6, respectively. In Fig. 5, sub-figures of column one are phase diagrams of isolated market for different chartist strengths b_c. Consistent to column one, column two is the corresponding Lorenz plots of isolated markets 4, 5, and 6. Loci of the Lorenz plots match to the orbits in the corresponding phase diagrams. In contrast, column three plots the same Lorenz plots under coupling effect. When $b_c = 4.2$, each isolated market experiences 4-period orbit dynamics within the disjointed deviation regions. The introduction of coupling effect $g = 0.4$ does not alter the periods of the orbit dynamics. However, the distribution loci have been changed slightly, besides the reversal of deviation sign of green-colored market 6 (Fig. 5.a.2 and 5.a.3). When $b_c = 5.2$, in the region where the spatio-temporal plot shows turbulent state for isolated market as deviations wander across positive and negative regions, distributions of the isolated markets converge to strange attractors. Once coupling effect $g = 0.4$ is applied, the shapes of the strange attractors are changed and data points of individual markets are segregated within either disjointed regions. This explains why the coupling effect can enhance the deviation persistence (Fig. 5.b.2 and 5.b.3). In case b_c is increased to 6.0, the isolated markets converge to the 4-period orbit distribution across positive and negative regions. This 4-period circulation translates into a regular sign-switching pattern in the deviation spatio-temporal plot Fig. 3.g in the above demonstration. The application of coupling effect $g = 0.4$ destroys the 4-period pattern distribution and creates strange attractors covering all the four quadrants in the distribution diagram. (Fig. 5.c.2 and 5.c.3). These strange attractors justify the turbulent defects pattern in the spatio-temporal plot Fig. 3.h.

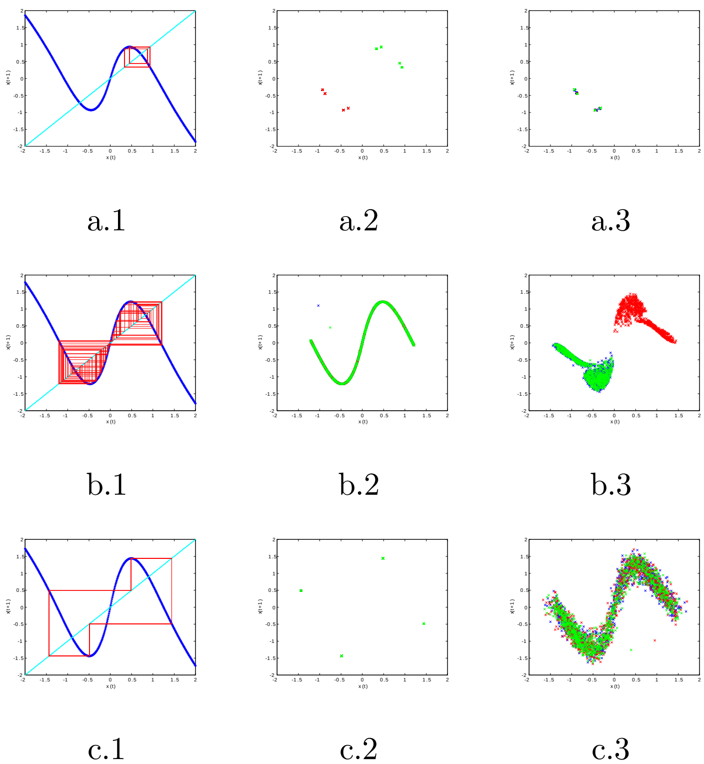

Fig. 5. Isolated market phase diagrams and Lorenz plots for deviations distribution: x_{t+1} vs x_t. Column one to three are phase diagrams, isolated market distributions, and distributions with coupling effect. For the distribution plots, Blue, red and green colors represent markets 4, 5, and 6, respectively. (a.1) $b_c = 4.2$, 4-period orbit phase diagram. (a.2) $b_c = 4.2$, $g = 0$, 4-point attractors locate in either quadrant I or III. (a.3) $b_c = 4.2$, $g = 0.4$, the green market attractor appears in quadrant III, instead of quadrant I. Besides that, the loci of distribution are also changed. (b.1) $b_c = 5.2$, phase diagram with region-crossing. (b.2) $b_c = 5.2$, $g = 0$, strange attractors covering four quadrants. (b.3) $b_c = 5.2, g = 0.4$, the shape of strange attractors are changed and loci are segregated into either quadrant I or III. (c.1) $b_c = 6.0$, phase diagram of 4-period orbit circulating across positive and negative regions. (c.2) $b_c = 6.0, g = 0$, 4-period circulation on four quadrants. (c.3) $b_c = 6.0, g = 0.4$, loci of 4-period orbit change to strange attractors covering four quadrants.

3.4 Single disturbance

Coupling effect enhances the formation of market clusters especially in chaotic intervals where deviation wanders across positive and negative regions. The stability of the resulted market structure is still ambiguous. To address this concern, we simulate by introducing a shock s to one of the markets and check the clusters activities. Given market conditions $b_c = 5.08$ and $g = 0.4$, a single shock s hits the 100^{th} market site at time step t such that $x_{100,t} = s$. The corresponding deviation spatio-temporal diagrams are plotted in Fig. 6. If the shock s is not large enough, it seems only the adjacent markets are affected. The affected markets may change the sign of deviation or result in new clusters. After adjustment for some periods, the whole market system again shows stable cluster pattern. With a larger shock, more markets are involved in adjustment with a longer adjusting time required (Fig. 6.a-6.c). Once s reaches certain strength, coupling effect no longer stabilize the cluster pattern. Instead, market-collapse in which prices diverge and are out of bound spreads from the impacted market to the whole market system through coupling effect and eventually all markets collapse. An avalanche appears. (Fig. 6.d).

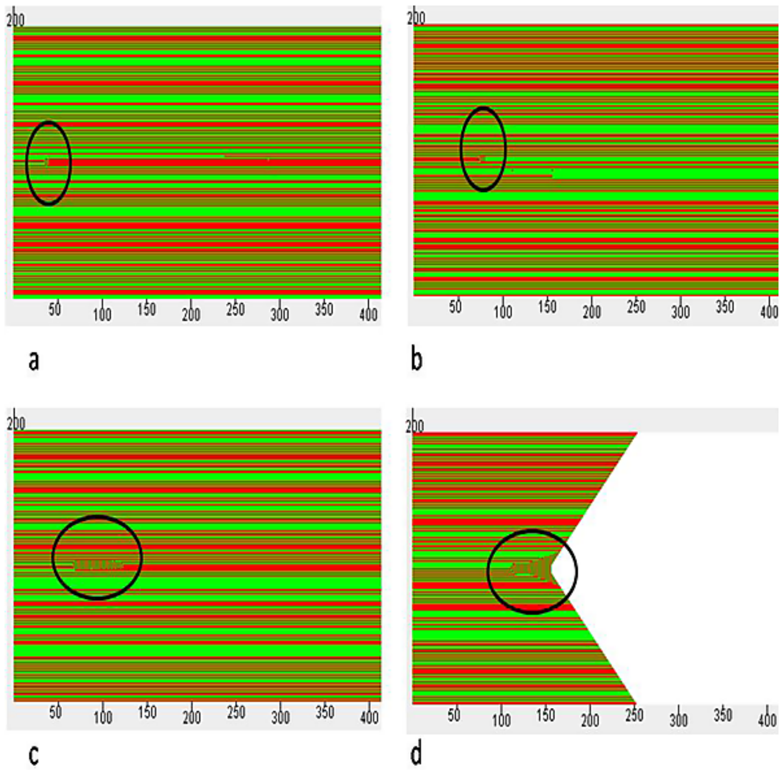

Fig. 6. Cluster pattern upon external shocks s hitting the 100^{th} market site given $b_c = 5.08$ and $g = 0.4$. (a) $s = -10$, after a short adjustment, a new cluster is formed. Adjustment is highlighted by circle. (b) $s = -12$, a new stable cluster is formed. Also, there is an adjustment in cluster below the impacted market. (c) $s = -14$, more markets adjust and the adjustment time is longer. (d) $s = -16$, coupling effect cannot stabilize the cluster pattern. Market-collapse spreads to the whole market system. A contagion effect is observed from the impacted market to the rest of the market system.

Based on the above disturbance analysis, when a shock is not large enough, it seems market members far away from the shock originating market are not affected as their signs of deviation do not change. To verify whether these market members are affected or not, investigation in terms of magnitude is necessary. We conduct the magnitude analysis with procedure: First, create a common set of initial random numbers for the 200-market system at condition $b_c = 5.08$. Second, after 50,000 rounds of evolvement, the last time step values are denoted as step 1 $x_{i,1}$ for analysis. A small shock $s = 0.001$ is introduced to the 100^{th} market site such that

$$x_{100,1} = x_{100,1} + s$$

Third, deviations of the respective original and disturbed market system are recorded down for the next 400 periods. Denote the original and perturbed deviations as $x^o_{i,t}$ and $x^p_{i,t}$. Next, Subtract $x^o_{i,t}$ from $x^p_{i,t}$ to get the difference pattern $d_{i,t}$ with filter $s = 0.001$. $d_{i,t}$ can be expressed as below:

$$d_{i,t} = \left\{ \begin{array}{cc} x^p_{i,t} - x^o_{i,t} \text{ if } \left| x^p_{i,t} - x^o_{i,t} \right| \geq s \\ \\ 0 \quad \text{ if } \left| x^p_{i,t} - x^o_{i,t} \right| < s \end{array} \right| \tag{3}$$

Lastly, plot $d_{i,t}$ in spatio-temporal diagrams, in which green , white and red colors represent positive, zero and negative difference. Fig. 7 reports the difference patterns for different coupling strength g. Based on Fig. 7, the shock propagation can be categorized into two modes: diffusion and localization. Fig. 7 row one shows the diffusive shock propagation when coupling strength g is small. The diffusive propagation speed increases with coupling strength since the corresponding time required for the disturbance to reach the whole system decreases. If coupling strength is increased, the other propagation mode – localization – emerges: the disturbance is confined in a zone and does not disappear with time (Fig. 7.b.1). Row two shows a mixture of the two modes. The localized zones can be observed visually. The difference pattern shows an irregular mixtures of propagation modes. Our simulation results are similar to the finding of Kaneko [10] except the irregular propagation patterns in Fig. 7.b.2.

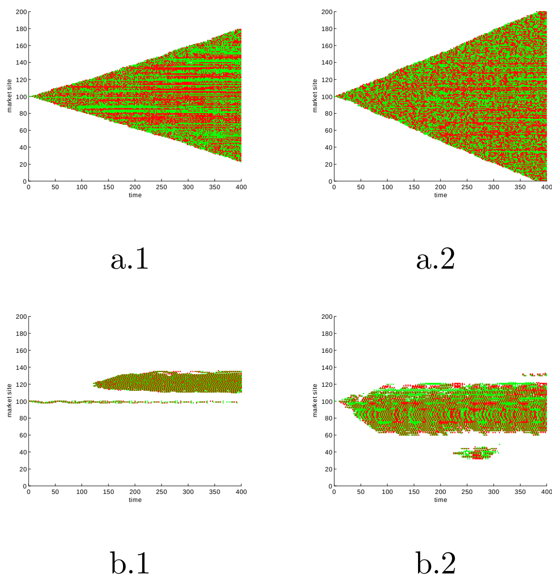

Fig. 7. Shock propogation upon a shock $s = 0.001$ hitting the 100^{th} market site given $b_c = 5.08$. When g is in very small range, disturbance diffuses with speed increasing with coupling strength g. (a.1) $g = 0.01$. (a.2) $g = 0.06$. Increasing g value, disturbance propagation has a localization mode and irregular pattern. (b.1) $g = 0.08$, localization mode. (b.2) $g = 0.47$, irregular pattern.

When a shock s has magnitude less than the avalanche level, the coupling market system can absorb the shock and disperses to other markets. In this sense, it can be conjectured that coupling has stabilizing effect. To verify this conjecture, set $b_c = 5.08$, a common set of initial random numbers for the 200-market is adopted to the market system with and without coupling effect, that is $g = 0$ and $g = 0.4$ respectively. At time step 40, a shock s hits 100th market such that its deviation $x_{100,40} = -2.29198$. Time series data of the adjacent markets are plotted. Without coupling, each market is isolated with price deviation switching randomly between positive and negative values. At time step 40, market 100 is hit by a shock and fluctuates dramatically. It takes around 20 time steps for market 100 to recover to its normal fluctuation path. As there is no market connection, other markets are not affected (Fig. 8.a). For the case of coupling, each market evolves in a way such that the price deviation is always positive or negative. At time step 40, market 100 is hit such that $x_{100,40} = -2.29198$. It takes market 100 less than 5 time steps to stabilize to normal fluctuation in the region of negative values. The adjacent markets 99 and 101 are impacted by the shock, especially market 101, which takes a similar time steps of market 100 to recover from the shock (Fig. 8.b).

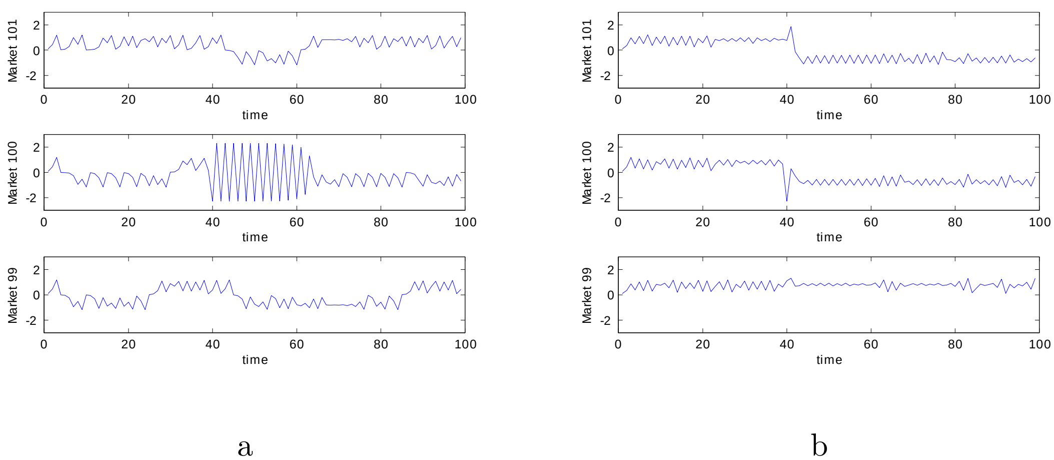

Fig. 8. Shock response comparison for coupling effect. (a) no coupling effect. (b) coupling effect $g = 0.4$.

4 Conclusion

This paper examines an asset market system consisting of multi-markets. Each market has a market maker and heterogeneous investors. An coupling market maker framework is proposed: market maker updates market price based on a weighted excess demand: including both domestic and abroad factors. The weight of excess demand is coupling parameter g. With the introduction of coupling effect g, duration of deviation remaining in either the "disjointed" positive or negative regions increases and persistent deviation appears. Market cluster sharing the same sign of deviation becomes apparent in the original chaotic interval characterized by erratic switching between positive and negative regions.

Check the isolated market phase diagram and distribution plots, it is found that coupling effect tends to segregate the distribution into quadrant I or III, that is, either the "disjointed" regions. This explains why the duration of deviation is enhanced. After the market cluster is established with coupling effect, enquiry about its stability arises. A series of disturbances are introduced to one of the markets. From the point of deviation persistence, when the disturbance is weak, only the adjacent markets are affected for adjustment and a new market cluster pattern is formed; if the disturbance is large enough, market system avalanche is generated from the initially impacted market. Next, we investigate the deviation magnitude difference created by the disturbance. Even if the disturbance is weak, disturbance can propagate to the market system with propagation modes of diffusion or localization, or the mixture of the two modes. Lastly, time series data of shock response shows ability of coupling effect to stabilize market member hit by shock.

The goal of this paper is to introduce coupling effect as a bridge for heterogeneous agents multi-market interactions. Numerical experiments have demonstrated the deviation persistence enhancement effect, which can also be found by using the agent composition function of He and Westerhoff [8]. More efforts of both numerical and theoretical works are still needed to further explore this area. Possible directions can be the application of coupling in financial markets.

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