Signal Extraction with Kalman Filter: A Study of the Hong Kong Property Price Bubbles

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Abstract

Since Flood and Garber (1980), the debate surrounding speculative bubbles has never subsided. A key obstacle to resolve this issue is the identification problem. A bubble is usually inferred from some assumed fundamental determinants of a price. These assumptions could be over-simplified. Furthermore, there might be data measurement errors. In this paper, we attempt to capture such errors with a latent state variable. This variable is extracted with Kalman filter. Based on our empirical comparisons, we find that it is possible to attribute the observed large price swings in the property market of Hong Kong during the 1980s and 1990s to a periodically collapsing rational speculative bubble.

JEL classification: G12, C12, C13, C52
Keywords: rational speculative bubble, misspecification or measurement error, Kalman filter

1. Introduction

As suggested by the title, this paper attempts to examine the experiences of speculative bubbles in the real-estate market of Hong Kong during the 1980s and 1990s. In the language of economists, a bubble exists in a price if the price is other than what is warranted by its fundamentals. The issue of speculative bubble arises because of uncertainties surrounding the “fundamentals”. In the real estate market, for instance, a buyer of a property is willing to pay a price which he perceives to be equal to the “fundamental” value. In assessing this “fundamental” value, he will make use of information on rental flows and future price changes. Unavoidably, his assessment must rely on expectations about some relevant future events. The expectations in turn are based on subjective, instead of the actual, probabilities of such events. Therefore, the assessment of market fundamentals is inherently subjective (Shiller 2001). As a result, the actual price will reflect the true “fundamental value” only by chance! That is a price will, in most cases, contain a bubble element.

Due to the difficulties associated with the assessment of the fundamental value, economists sometimes define bubbles differently. To them, a speculative bubble does not exist if there is no profit opportunity to be exploited. Skeptics of speculative bubbles argue that investors are rational and markets efficient, hence, speculative bubbles are,
theoretically, implausible. This is because that, when a market is efficient, the current price will fully and correctly reflect all relevant information. There exists no profit opportunity to be exploited, hence no incentive for speculation.

There is, however, no consensus on whether or not markets are efficient. An efficient market requires expectations to be formed rationally. If expectations are “backward looking” instead, it is possible to predict prices based on past trends, and excess profits could be earned. There are considerable evidences of backward looking expectations in the formation of asset prices, including real estate prices (Ito and Iwaisako 1995, Cho 1996; Malpezzi and Wachter 2002; Schnusenberg 1999; Lee and Ward 2000). But evidences of profit opportunity did not emerge until recently. Daniel and Titman (2000) show that a meta-strategy, that mechanically picks strategies based on the performance of stocks in the previous 10 years, has continued to earn consistent profits. Lee and Ward (2000) also found unexploited profit opportunities in the UK real estate market.

To verify the existence of a speculative bubble empirically is, nevertheless, a thorny issue. This is because that, in an empirical study, a bubble is typically inferred, rather than directly observed, from a fundamental model. Such a fundamental model itself is often a subject of dispute. Sometimes, the disputes arise about what the fundamental is. For instance, what determines the fundamental value of a house? Sometimes, the disputes center on the specification of a model. For example, most economists agree that the present-value model is a reasonable model for asset pricing, but they disagree about
which discount rate should be used in empirical studies (Flood, Hodrick and Kaplan 1986).

Even if economists can agree that a speculative bubble exists, they are often divided about the characteristics of the bubble. To Robert Shiller, “The idea that there has been a speculative bubble … is inherently a statement about some less-than-rational aspect of investor behavior” (2001). Many economists are, however, uncomfortable with the notion of irrationality. Perhaps because of that, there are far more theoretical and empirical studies devoted to the rational speculative bubbles than to the irrational ones (Flood and Garber, 1980; Brock 1982; Tirole 1982, 1985; Blanchard and Watson 1982; Obstfeld and Rogoff, 1983; Mussa 1984; Quah 1986; O’Connell and Zeldes 1988; Diba and Grossman 1988a).

A rational bubble arises from a market where all participants are rational. These rational participants make full use of all available information, asymmetric and incomplete though it may be. Mathematically speaking, the bubble component in a general solution to an economic model, supposing it is the correct one, is referred to as a rational, speculative bubble. We will follow this line of definition and call the bubble we investigate in this paper a “rational, speculative bubble.” Furthermore, this bubble is periodically collapsing. The bubble is inferred from a fundamental model, which is derived from a variant of the present-value asset pricing model. To mitigate the problem of misspecification, we will include in the fundamental equation a state variable. This
state variable is supposed to capture the specification or measurement errors (Hamilton 1994). For sakes of comparison, we will also estimate the model without this error term.

2. Is the Real Estate Market Prone to Speculative Bubbles?

Researchers in various countries point to speculation as a prime force behind cycles in real-estate markets (Malpezzi and Wachter 2002). We can suggest a number of reasons why a real estate market is more likely to be inefficient and more likely to see a speculative bubble arising. First, there is no organized short-selling in a real estate market. With no short sales, optimists will strongly influence real-estate prices. Second, there are high transaction costs involved. The high transaction costs will limit the ability of a pessimist to trade on his opinion. Third, there is a lengthy lag in supply. This lengthy lag will extend the period of excess demand in which excess returns can be earned. Such extended above normal returns will then superficially justify an exuberant property price (Hendershott 2000).

The lending processes of financial institutions may also contribute to the creation of a speculative bubble. Properties are often used as collateral for loans. An investor is likely to be able to borrow against a property when its price is rising, even though he earns substandard returns on his investment project. A manager of a financial institution who lends against real-estate collaterals also has an incentive to under-price the loan. This is because that, in good states, the under-pricing of a loan will increase the profits accruing to the manager. Evidences show that excess credit plays an important role in the creation

Myopic pricing behavior is another culprit (Herring and Wachter 1999, 2002). Myopic pricing is shortsighted pricing behavior that fails to take into account negative events which are likely to occur in the future. Specialists in cognitive psychology have found that decision makers have the tendency to underestimate shock probabilities (Tversky and Kahneman 1982). This tendency to underestimate the probability of a low-frequency shock promotes a speculative bubble in the real estate market.

Restrictions in supply are often cited as one of the key reasons for causing speculative bubbles in the real estate market. It is argued that, while speculation is usually thought of as a demand-side phenomenon, whether speculation behavior will be observed and a bubble will be formed depend on supply conditions. Malpezzi and Wachter (2002) in their simulation study show a very interesting result: speculation hardly matters at all in a market with elastic supply. Hence, they argue that markets with more responsive regulatory environments or less natural constraint will experience less speculation and price volatility.

3. Literature Review

The real-estate market has the longest and the most reliable history of boom and bust. That history stretches back to the early 1800s (Carrigan 2004). Because of the great
uncertainties surrounding its fundamental value, real estate has always been a major object of speculation.

Using data from 30 cities in the United States covering 14 years (420 observations), Abraham and Hendershott (1994) found that, as of the end of 1992, there was a 30% “above market” premium in prices in the Northeast, a 15%–20% premium in prices on the West Coast, and probably a significantly negative premium in Texas. In their model, Abraham and Hendershott incorporate a proxy for the tendency of a bubble to burst and a proxy for the tendency of a bubble to swell. They found that the proxy does indeed work and is especially useful in explaining the large cyclical swings in real house prices in the West. The lagged appreciation term that represents speculative pressures in the market performs admirably in soaking up volatility.


Bjorklund and Soderberg (1999) examine the 1985–1994 cycle in the Swedish property market and contend that the ratio of property value to rent increased too much, indicating that a bubble may have existed.
Among Asian experiences, Kim and Suh (1993) show that a bubble existed in the nominal and in the relative price of land in South Korea between 1974 and 89\textsuperscript{v}. Park et al (1998) suggests that the price bubble in South Korea in housing and land stood at 58\% and 40\% respectively in 1991 at its peak, disappeared almost completely by 1997. Lee (1997) conducted a test of bubble in the land price of Korea between 1964 and 1994. Using a structural model with GNP, interest rate and money supply as fundamentals, he found the hypothesis that only market fundamental drove the land price in Korea can be rejected.

Kim and Lee (2000) adapt the idea that the existence of an equilibrium relationship excludes the possibility of a price bubble. They conclude that, in the long run, nominal and real land prices are cointegrated with market fundamentals (approximated by nominal and real GDP respectively). However, in the short run, such cointegration relationship does not exist. This is consistent with the notion that speculative bubbles are periodically collapsing (Blanchard and Watson 1982). In the short run, speculative forces could drive prices away from market fundamentals. In the long run, fundamental forces will eventual reassert themselves.

Lim (2003) conducted two bubble tests based on the present value relation on the housing price of Korea, one is a modified volatility test (MRS test) suggested by Mankiw et al (1985), another combine unit-root test suggested by Diba and Grossman (1988b), and cointegration test by Campbell and Shiller (1987). Their MRS test show that the null hypothesis of market efficiency is rejected, indicating the existence of irrational bubble.
Their unit-root test and cointegration test however suggest that bubbles do not exist! This is in contrast with the findings of Xiao (2005) in her PhD thesis, which employ a Markov-switching ADF approach. However, the data series employed in her thesis are not identical to those by Lim (2003). This example show that the results of tests depend on the type of test employed. The authors also show that the test is sensitive to the choice of data.

The large swings of property prices in Japan in the late 1980s and early 1990s have intrigued many researchers. Ito and Iwaisako (1995) attempt to measure how much of the asset price variation observed in Japan in the late 1980s and the early 1990s can be attributed to changes in the “fundamentals”. The fundamental model they use is the standard present value model. Their conclusion is that “it seems impossible to offer a rational explanation of the asset price inflation in the second half of the 1980s by changes in fundamentals (page 10).” This point is reemphasized by the authors in a variance decomposition study (page 20). Basile and Joyce (2001) use the method by Fortune (1991) to measure the size of the asset price bubble, which is the difference between the ex post returns of an asset and the required return. They found that the land market bubble grows evenly through mid-1990s before declining.

4. The Experiences of Hong Kong

The fate of the Hong Kong economy seems intertwined with that of its property market. During most part of the 1990s, when the territory had high growth rate and low
unemployment rate, the property prices were also soaring high. When the property market tumbled after 1997, the economy suffered one of the worst and lasting recessions in its history (Gerlach and Peng 2004).

The residential property prices in Hong Kong were highly volatile over the 1990s. In 1991, the real price for the overall property market rose by 40%. It fell by 16.2% in 1995, which is followed by a remarkable increase of 18.9% and 20% in the next 2 years, and a rapid fall of 50% in 1998 (Chan, Lee and Woo, 2000). These drastic changes in property prices pose the question of “was there a speculative bubble?”

To the best knowledge of the authors, there have been relatively few papers devoted to studying speculative price bubbles in the property market of Hong Kong, even the experiences of this city suggest it to be one of the most interesting for this topic. Chan, Lee and Woo’s study is one of them. Their study uses a standard present value model with constant discount rate. The model assumes that property is a good investment which produces a stream of rental incomes over its lifetime. The current value of a property is therefore determined by the present value of the current rental income and the next period expected market price. There are two solutions to the price: a fundamental solution

\[ P_t = P_t^f \]

and a bubble solution.

\[ P_t = P_t^f + B_t \]

In addition, there could be a misspecification error, \( S_t \), which measures the deviations of the actual price data from the bubble solution. This could arise from the measurement
errors in the data, or wrong assumptions about the underlying parameters of the data generating process. Thus the total model noise is $B_i + S_i$.

Using signal extraction method of Durlauf and Hall (1989a,b), the authors conducted a flow and stock test to investigate the existences of $S_i$ and $B_i$. The data they use are monthly averaged rentals and quarterly averaged prices of the private domestic properties within the class A, which is defined as those apartments with sizes less than 39.9 m$^2$. The sample period runs from the first quarter of 1985 to the third quarter of 1997. Their results show that there exists misspecification error in the model noises as well as bubbles. The paths of the bubbles show that the bubbles exploded most sharply between 1990 and 1992, and between 1995 and 1997.

5. The Theory of Asset Pricing and the Kalman Filter

Similar to Chan, Lee and Woo (2000), the bubble we estimate in this paper is inferred based on the present value model. But Kalman filter method will be used instead. We will compare our result with that of Chan, Lee and Woo at the end of this paper.

a. The Present-value Model and Rational Speculative Bubbles

The major references for the model shown below are Campbell and Shiller (1988a, b). It will be used as a benchmark model in this paper for bubble quantification.
If economic agents are risk-neutral, the price of one equity share, $P_t$, will be equal to the expected discounted present value of the dividend accruing to ownership of the equity share during the ownership period, $D_t$, plus the price at which the share can be sold at the end of the ownership period, $P_{t+1}$. Mathematically,

$$P_t = \frac{E_t[P_{t+1} + D_t]}{1 + R_t}, \quad (1)$$

where

$P_t$ = the real price of the property asset at time $t$;

$D_t$ = the real total rents received during the period $t$;

$R_t$ = the time-varying real discount rate.

Define

$$r_t \equiv \log(1 + R_t); \quad (2)$$

hence,

$$r_t \equiv \log(E_t[P_{t+1} + D_t]) - \log(P_t). \quad (3)$$

In a static world, dividends grows at a constant rate, $g$, and the log of dividend-to-price ratio is also a constant. That is,

$$\log\left(\frac{D_t}{D_{t-1}}\right) = d_t - d_{t-1} = \Delta d_t = g, \quad (4)$$

and

$$\log\left(\frac{D_{t-1}}{P_t}\right) = d_{t-1} - p_t = \delta, \quad (5)$$

where
\( p = \text{the log of } P \); and

\( d = \text{the log of } D \).

Equations 4 and 5 imply that the asset price grows at the same rate as the dividend, and the ratio of the asset price to the sum of the asset price and dividend is also a constant. That is,

\[
\log \left( \frac{P_{t+1}}{P_t} \right) = \log \left( \frac{D_t}{\delta D_{t-1}} \right) = \log \left( \frac{D_t}{D_{t-1}} \right) = g ,
\]

and

\[
\frac{P_t}{P_t + D_{t-1}} = \frac{1}{1 + \frac{D_{t-1}}{P_t}} = \frac{1}{1 + \exp(\delta)} \equiv \rho .
\]

In such a world, the log of the gross discount rate, \( r_t \), would also be a constant. To see this, define

\[
\tilde{\xi}_t = \kappa + \rho p_{t+1} + (1 - \rho)d_t - p_t .
\]

Given the characteristics of the static world, \( \tilde{\xi}_t \equiv \kappa + g + (1 - \rho)\delta \equiv \bar{\xi} \), which is a constant. If we set

\[
\kappa = -\log(\rho) - (1 - \rho)\delta ,
\]

then

\[
\bar{\xi} = -\log(\rho) + g = \log \left( \frac{E_t[P_{t+1} + D_t]}{P} \right) = r .
\]

Thus, in a static world,

\[
r_t = \tilde{\xi}_t = \kappa + \rho p_{t+1} + (1 - \rho)d_t - p_t = \bar{\xi} .
\]
Bear in mind that in a world that is evolving over time, equation (11) would hold only approximately. Solve 11 for $p_t$ by forward iteration:

$$p_t = \kappa - \xi + \rho E_t[p_{t+1}] + (1 - \rho) d_t,$$

$$= \kappa - \xi + \rho(\kappa - \xi + \rho E_t[p_{t+2}] + (1 - \rho) E_t[d_{t+2}]) + (1 - \rho) d_t,$$

$$= \ldots$$

$$= \frac{\kappa - \xi}{1 - \rho} + \rho^i E_t[p_{t+i}] + (1 - \rho) \sum_{j=0}^{i-1} \rho^j E_t[d_{t+j}].$$

(12)

If the transversality condition, $\lim_{i \to \infty} \rho^i E_t[p_{t+i}] = 0$, is satisfied, we would have the fundamental solution for the price of property:

$$p_t = p_t^f = \frac{\kappa - \xi}{1 - \rho} + (1 - \rho) \sum_{j=0}^{\infty} \rho^j E_t[d_{t+j}].$$

(13)

This is the present-value model for asset pricing.

However, the transversality condition may fail to hold. In such a case, we would expect the price to contain a *rational speculative bubble*, $b$

$$p_t = p_t^f + b_t,$$

(14)

Where the bubble component

$$E_t[b_{t+i}] = \frac{1}{\rho^i} b_t.$$

(15)

This implies that

$$b_{t+1} = \frac{1}{\rho} b_t.$$  

(16)

If the log of the property prices and the log of the rents are $I(1)$ processes, the following model may be estimated instead:
\[
\Delta p_i^f = p_i^f - p_{i-1}^f = (1 - \rho) \sum_{j=0}^{\infty} \rho^j \left\{ E_i \left[ d_{i+j} \right] - E_{i-1} \left[ d_{i+j-1} \right] \right\}.
\] (17)

Suppose the growth of the dividends follows an AR(1) stochastic process,

\[
\Delta d_t = \phi \Delta d_{t-1} + \delta_t; \quad E[\delta_t] = 0, \quad Var[\delta_t] = \sigma_\delta^2.
\] (18)

Then

\[
\Delta p_i^f = \frac{1}{1 - \phi \rho} \Delta d_i - \frac{\phi \rho}{1 - \phi \rho} \Delta d_{i-1} \equiv \psi \Delta d_i + (1 - \psi) \Delta d_{i-1}.
\] (19)

If a bubble is present,

\[
\Delta p_i = \Delta p_i^f + \Delta b_t,
\] (20)

where

\[
\Delta b_{t+1} = \frac{1}{\rho} \Delta b_t.
\] (21)

b. Specification Error and Kalman Filter

In arriving at equation (19), we have made many simplified assumptions about the fundamental process, such as risk neutral and constant discount rate. Therefore \( b_t \) in equation (14) and (20) might, in fact, represent a misspecification or a measurement error instead of a bubble. To capture that error, we will make use of Kalman filter. First, let us express the present-value model in a time-invariant state space form.
Let $z_t$ be a 1-vector of state variables which represent mis-specification or measurement errors, $x_t$ a 2-vector of inputs, and $y_t$ a 2-vector of outputs. The state space model consists of two equations: the measurement equation,

$$y_t = Hz_t + Bx_t + \varepsilon_t; \quad E(\varepsilon_t) = 0, \quad \text{var}(\varepsilon_t) = R,$$

and the transition equation,

$$z_t = Fz_{t-1} + Ax_t + \eta_t; \quad E(\eta_t) = 0, \quad \text{var}(\eta_t) = V,$$

where the system matrices $H, B, F, A$, the measurement variance $R$, and the transition variance $V$ are all time-invariant.

Suppose,

$$y_t = \begin{pmatrix} \Delta p_t \\ \Delta d_t \end{pmatrix}, \quad z_t = \Delta s_t, \quad x_t = \begin{pmatrix} \Delta d_t \\ \Delta d_{t-1} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} b_t \\ \delta_t \end{pmatrix}, \quad \eta_t = \zeta_t,$$

$$H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad F = \beta = \frac{1}{\rho}, \quad B = \begin{pmatrix} \psi & 1 - \psi \\ 0 & \phi \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Where $b_t$ is the residual in the price which is unexplained by either the rental flow or the error term. $\zeta_t$ is a noise term in the transition process of the state variable. In restricting $H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, we have made the assumptions that the dividend process is not infected by this state variable. Therefore, the measurement equation is

$$\begin{pmatrix} \Delta p_t \\ \Delta d_t \end{pmatrix} = \begin{pmatrix} \Delta s_t + \psi \Delta d_t + (1 - \psi) \Delta d_{t-1} + b_t \\ \phi \Delta d_{t-1} + \delta_t \end{pmatrix}$$

(24)

And the transition equation
\[ \Delta s_{t+1} = \beta \Delta s_t + \zeta_t \]  \hspace{1cm} (25)

We assume that \( b_t, \delta_t \) and \( \zeta_t \) are uncorrelated, Hence,

\[ R = \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_\delta^2 \end{pmatrix}, \quad V = \sigma_\zeta^2. \]  \hspace{1cm} (26)

c. Signal Extraction with the Kalman Filter

The state variable, \( \Delta s_t \), is not observable but can be estimated using the Kalman filter, assuming the system parameters are known (Harvey 1989; Hamilton 1994). The Kalman filter consists of a set of recursive equations. Suppose we estimate the initial value of the state variable to be \( z_0 \), with estimation error \( P_0 \). The predicted value of the state variable and the prediction error at time \( t \), given the information set available at time \( t-1 \), \( \Xi_{t-1} = \{y_1,...,y_{t-1},x_1,...,x_{t-1}\} \), can be calculated using the prediction equations recursively forward:

\[ z_{t|t-1} = Fz_{t-1|t-1} + Ax_{t-1}, \text{ and} \]
\[ P_{t|t-1} = FP_{t-1|t-1}F^* + V. \]  \hspace{1cm} (27)

(28)

When time \( t \) information becomes available, we can update our estimation of the state variable and their estimation errors using the filtering equations recursively forward:

\[ z_{t|t} = z_{t|t-1} + \kappa_t e_{t|t-1}, \text{ and} \]
\[ P_{t|t} = P_{t|t-1} - \kappa_t H P_{t|t-1} \]  \hspace{1cm} (29)
\hspace{1cm} (30)

where

\[ \kappa_t = P_{t|t-1} H^* D_{t|t-1}^{-1}, \]  \hspace{1cm} (31)
\[ D_{t|t-1} = (HP_{t|t-1}H^* + R), \text{ and} \]  \hspace{1cm} (32)
\[ \varepsilon_{t|t-1} = y_t - Hz_{t|t-1} - Bx_t. \quad (33) \]

Once we have obtained the sequences \( \{z_{t|t-1}^T\}_{t=1}^T \), \( \{p_{t|t-1}^T\}_{t=1}^T \), \( \{e_{t|t}^T\}_{t=1}^T \), and \( \{P_{t|t}^T\}_{t=1}^T \), we can have a more efficient estimation of the state variable and its estimation errors, using the full set of information, \( \Xi_T = \{y_1, \ldots, y_T, x_1, \ldots, x_T\} \), and the following smoothing equations by backward recursion:

\[ z_{t|T} = z_{t|t} + J_t(z_{t+1|T} - z_{t+1|t}), \quad \text{and} \]
\[ P_{t|T} = P_{t|t} + J_t(P_{t+1|T} - P_{t+1|t})J_t, \quad (34) \]

where

\[ J_t = P_{t|t}^{-1}F^*P_{t+1|t}^{-1}. \quad (35) \]

The starting values for smoothing are \( z_{T|T} \) and \( P_{T|T} \) obtained from the filtering process.

d. Estimating Model Parameters

There are only five unknown parameters in the model. They are \( \beta \), \( \phi \), \( \sigma_\phi^2 \), \( \delta^2 \), and \( \sigma_\varepsilon^2 \).

These parameters are estimated by maximizing the log likelihood function of \( y_t \), \( t = 1, 2, \ldots, T \), which is

\[ \log L(y; x, \theta) = -\frac{nYT}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log |D_{t|t-1}| - \frac{1}{2} \sum_{t=1}^T e_{t|t-1}^T D_{t|t-1}^{-1} e_{t|t-1}, \quad (37) \]

where

\[ \theta = (\beta, \phi, \sigma_\phi^2, \sigma_\delta^2, \sigma_\varepsilon^2) \]
\[ y = (y_1, y_2, \ldots, y_T)^T \]
\[ x = (x_1, x_2, \ldots, x_T)^T. \quad (38) \]
(Harvey 1989; Hamilton 1994).

We obtained the estimates of $\theta$ using the EM algorithm (SAS/IML version 8; Dempster et al. 1977; Dellaert 2002). In applying the EM algorithm, we first obtain $\frac{\partial \text{Log}(y; x, \theta)}{\partial \theta_i}$, $i = 1,2,3,4,5$, then take the expectation of $\frac{\partial \text{Log}(y; x, \theta)}{\partial \theta_i}$ with respect to the information set $\Xi_T = \{y_1,...,y_T, x_1,...,x_T\}$ and set it to zero. By solving the set equations thereby obtained, we get the ML estimators:

$$\widetilde{R} = \frac{1}{T} \sum_{t=1}^{T} \left[ HP_{tT} H^+ \left( y_t - Hz_{t|t} - Bx_t \right) \left( y_t - Hz_{t|t} - Bx_t \right)^\top \right]$$

$$\widetilde{V} = \frac{1}{T} \left[ S_t(0) - S_t(1) (S_{t-1}(0))^{-1} S_t(1) \right]$$

$$\widetilde{F} = S_t(1) (S_{t-1}(0))^{-1}$$

$$\widetilde{D} = \left( \sum_{t=1}^{T} y_t x_t \right) - H \sum_{t=1}^{T} z_{t|t} x_t \left( \sum_{t=1}^{T} x_t x_t \right)^{-1}$$

where

$$S_t(1) = \sum_{t=1}^{T} P_{t,t-1|t} + z_{t|t} z_{t-1|t}$$

$$S_t(0) = \sum_{t=1}^{T} P_{t|t} + z_{t|t} z_{t|t}$$

$$P_{t,t-1|t} = \text{Estimated covariance between } z_t \text{ and } z_{t-1}.$$  \hspace{1cm} (40)

In order to compute 39, we need to estimate the values of the state variable and their associated estimation errors. Thus we have to provide a guess about the starting value of the system parameters. The estimation process consists of four steps:
1. Initiate the guessed system parameter values.

2. Run through equations 27 through 36, to obtain the sequences $\{z_{t|t-1}\}_{t=1}^{\tau}$, $\{P_{t|t-1}\}_{t=1}^{\tau}$, $\{z_{t|t}\}_{t=1}^{\tau}$, $\{P_{t|t}\}_{t=1}^{\tau}$, and $\{P_{t|T}\}_{t=1}^{\tau}$.


4. Repeat steps 2 and 3 until convergence occurs.

The initial system parameter values in our experiment are based on the preliminary OLS estimate of equation (19). The initial values of the state variable for the $(i+1)^{th}$ iteration is updated using estimates from the $i^{th}$ iteration by the set of equations

$$z_{0}^{i+1} = F^{i}z_{0|T}^{i}$$

$$P_{0}^{i+1} = F^{i}P_{0}^{i}F^{i'} + V^{i}$$

**e. Asymptotic Properties of the ML Estimators**

Suppose $\tilde{\theta}$ is the ML estimator of $\theta$ obtained by maximizing 37. Subject to certain regularity conditions (Caines 1988), then:

$$\sqrt{T}\varphi_{2D,T}^{1}(\tilde{\theta} - \theta_{0}) \xrightarrow{d} N(0, I),$$

that is,

$$\tilde{\theta} \xrightarrow{d} N(\theta_{0}, T^{-1}\varphi_{2D,T}^{-1}),$$

where $\varphi_{2D,T}$ is the information matrix from the sample of size $T$:

$$\varphi_{2D,T} = -\frac{1}{T}E\left(\sum_{t=1}^{\tau} \frac{\partial^{2}Log L_{t}}{\partial \theta \partial \theta|_{\theta = \theta_{0}}}\right),$$
with

\[ LogL_t = -\frac{ny}{2} \log 2\pi - \frac{1}{2} \log |D_{t|t-1}| - \frac{1}{2} \epsilon_{t|t-1}' D_{t|t-1}^{-1} \epsilon_{t|t-1}, \quad t = 1, 2, \ldots, T \]  

(45)

and

\[ \lim_{\tau \to \infty} \varphi_{rD,\tau} = \frac{\epsilon}{\tau} \rightarrow \varphi = -\frac{1}{T} \left( \sum_{i=1}^{T} \frac{\partial^2 LogL_i}{\partial \theta \partial \theta'} | \theta = \tilde{\theta} \right) \]  

(46)

The reported standard errors for \( \tilde{\theta} \) are the square roots of the diagonal elements of

\[ (T \varphi)^{-1} = \left( \sum_{i=1}^{T} \frac{\partial^2 LogL_i}{\partial \theta \partial \theta'} | \theta = \tilde{\theta} \right)^{-1}. \]

The Hessian is calculated numerically in our paper (Wheatley 2004, p. 267). The method is described below. First we collect the estimated parameters in an \( r \times 1 \) vector \( \tilde{\theta} \), run through the Kalman filter and calculate the log-likelihood of the data to give \( LogL(\tilde{\theta}) \). We then perturb one parameter at a time by \( \Delta = +0.01 \), run the Kalman filter and calculate the log-likelihood of the data again to give \( LogL(\tilde{\theta} + \Delta_i) \). We perturb one parameter at a time by \( \Delta = -0.01 \), run the Kalman filter again and recalculate the log-likelihood of the data to give \( LogL(\tilde{\theta} - \Delta_i) \). The Hessian is calculated using the formula

\[ \frac{\partial^2 LogL(y; \tilde{\theta})}{\partial \tilde{\theta}_i^2} \approx \frac{LogL(y; \tilde{\theta} + \Delta_i) - 2 \times LogL(\tilde{\theta}) + LogL(\tilde{\theta} - \Delta_i)}{\Delta_i^2}, \]  

(47)

where

\[ \Delta_i = A \ r \times 1 \ vector \ with \ all \ elements, \ except \ the \ i^{th} \ which \ is \ 0.01, \ being \ zero, \]  

and
\[ \theta_i = \text{The } i^{th} \text{ element of } \tilde{\theta}. \]

The standard error of \( \theta_i \) is approximated using the equation

\[
SE(\theta_i) = \sqrt{\frac{\partial^2 \log L(y; \tilde{\theta})}{\partial \theta_i^2}}^{-1}.
\]  \hspace{1cm} (48)

6. **Empirical Applications**

a. **Data**

The data we use in this paper are price and rent indices for building structures, as opposed to raw land, of Hong Kong. There is no convincing reason to believe that a bubble is more likely to exist in the price of a building than in that of a plot of land. Empirical studies on speculative bubbles use data on both. In fact, following the argument of Homer Hoyt (1933), if a bubble exists in the land price, it is likely to be transmitted to the housing price, and vice versa.

We do have a few reasons for choosing Hong Kong. During the 1980s and 1990s, the city experienced dramatic property-price swings (figure 1). Such swings were suspected by practitioners and academics to be the results of speculative bubbles. Although quality data on the property sector are available, there have been relatively few research papers devoted to the study of the speculative bubble in the property market of Hong Kong.
The price and rent indices of Hong Kong available for building structures are divided into four categories: domestic premises, office, flatted factories and retail premises. We are not aware of any data on the relative importance of each category. To avoid picking one which is not representative of the big picture, we will make use of all of these four categories. All data series come from the CEIC database, a comprehensive source of economic statistics for Asian economies.

The data series are monthly observations between December 1980 and January 2003 (a total of 266 observations). They are deflated by the CPI of Hong Kong. Each series makes use of two data sets of different frequencies: the first set is a quarterly data between December 1980 and September 2000, and the second a monthly data from January 1993 to January 2003. In order to combine them, we have converted the first set into monthly data by means of cubic spline. Thus, the first half of each series, running from December 1980 to December 1992, consists of the splined output from the first data set. The remaining half of the series is drawn from the second data set.

A plot of the price indices in figure 1 shows that they climbed persistently throughout the second half of the 1980s and the first half of the 1990s. This trend was reversed sharply in 1994. However, domestic and retail premises price indices shot up again in the late 1996, increasing on average by as much as 5.7% and 6.7% per month respectively (in real terms). This drama ended abruptly in the late 1997, following the Asian Financial Crisis. The office price index rose less dramatically in the early 1997. But its fall in the late 1997
was as dramatic. The flatted factories price was not caught in the 1997 drama. Its fall was slow but persistent after it peaked in March 1994.

The present value model suggests that the price and the rent indices should move more or less hand in hand. But this seems not the case in our data (figure 2). The price-rent ratio increased continuously during the 10-year long (1985-1994) expansion of the property prices, with that for the office sector the most volatile one. This ratio for the office and for the flatted factories sectors crashed all the way down to their respective historical low after the late 1997. The decline in this ratio since the late 1997 for the domestic premises or for the retail premises sector is much more moderate. Does this observation imply the failure of the present-value model and the existence of speculative bubble in Hong Kong? This is the issue the current paper is interested in.

In this paper we will employ in our estimation the log value of each data series. Wu (1997) argued that the use of the log, instead of the level, value has the advantage of avoiding negative prices, while allowing for negative bubbles. A Phillips-Perron unit root (abbreviated to pp) test show that the null of unit root can be accepted for the log values of each series considered (table 1). Therefore this paper estimates a state space model based on equation (17) to (21). That is we estimate equation (24) to (26). For reasons of comparison, we will also estimate equation (19), which excludes the state variable. We will refer to the later the “present value or pv model.”
We assume that both the growth of the rent and the growth of the state variable follow AR(1) processes. In doing so, we make use of the argument by Greene (1997, p. 584) that most time series can be reasonably represented by the AR(1) model, and that results of model selection exercises are highly data-sensitive\textsuperscript{viii}.

b. Empirical Results

The results of the estimations of the state space model are presented in Table 2. It is shown by the Breusch-Pagan-abnormality-robust LM test that, for each of the four series, at least two in three of the residuals from the estimated equations are heteroscedastic. Therefore, we have calculated the Newey-West autocorrelation consistency covariance, as well as the Hessian for $\rho$ and $\phi$. Their associated t-ratio are denoted as t(NW) and t(H) respectively.

The value of $\rho$ is insignificant at the conventional levels (1%, 5% and 10%) in all of the four categories. This implies that the current value of the state variable, recall that this variable represents misspecification or measurement error, is not systematically related to its past behavior. It also implies that $\psi$ is unity! That is, a 1% growth in the current rent will result in a 1% growth in the current fundamental price. But a 1% rent growth in the past will have a 0% impact on the current price behavior.

When t(H) is used, $\phi$ is significant at the conventional levels for all of the four categories considered. This means that the current rent growth depends on the growth of the rent in
the previous period. However, when hetereoscedasticity is taken into account, $\phi$ is significant at the conventional levels only for office and flatted factories. For these two series, the estimates show that a 1% rent growth in the previous period will result in a 0.78% and 0.57% rent growth in the current period for office and flatted factories respectively. If a less conventional level of significance, say 20%, is adopted, then $\phi$ is significantly different from zero for domestic premises as well, even when hetereoscedasticity is taken into account. The estimates show that the rent for retail premises has no obvious memory of its past behavior. A zero valued $\phi$ would reinforce our previous statement that $\psi$ is unity, and “a 1% growth in the current rent will result in a 1% growth in the current fundamental price. But a 1% rent growth in the past will have a 0% impact on current price behavior.”

We have adopted a Wald test to test the null hypothesis that the model parameters are jointly insignificant. This null is rejected at the conventional levels in all of the four cases considered, except for retail premises (table 2).

An estimation of equation (19) shows that the present value model is significant only for domestic premises and flatted factories. Again, the coefficient on the lagged rent is insignificant in all of the four cases. This implies that the past rent growth has no impact on the current price behavior. The coefficient on the current rent is only significant for domestic Premises and flatted factories.
The state space model, which incorporates a specification or measurement error, is obviously superior to the present value model. The former has a smaller sum of squared errors than the latter, either in a within sample fitting or in an out-of-sample forecasting (Table 4a). All of the test statistics, $R^2, \bar{R}^2, AIC, SBC$, indicate that the state space model is better than the present value model. The $\bar{R}^2$, the adjusted $R^2$, show that the state space model can explain more than 70% of the total variations in the office price index, more than 60% of that in the domestic premises price index. But the model can explain less than 40% of the variation in the price indices of the flatted factories and the retail premises. The present value model can, however, only explain 1% to 21% of the total variations in each price index. These conclusions are verified in figure 3 to 6.

However, the spate space model cannot explain the entire price variation. If we were to interpret the estimation residuals $\hat{b}$ as speculative bubble, then figure 7 tells us the following. Suppose the price of each category stands at its fundamental value in February 1981. A negative price bubble persisted in the office price index from March 1981 to February 1993. Between February 1993 and December 1997, this price index contained a positive speculative bubble most of the time. This positive bubble peaked in May 1994 and again in June 1997. This bubble dropped into the negative region after December 1997. The bubble in the domestic premises price index had a negative value between March 1981 and February 1991, but stayed positive for the remaining part of the time considered. This bubble culminated in June 1997. The bubble in the retail premises price index did not reach the positive region until mid-1991, but stayed there for most of the remaining part of the sample period. This bubble continued to rise for another two
months even after the eruption of the Asian Financial Crisis in July 1997. The bubble in the flatted factories was positive only between mid-92 and mid-95, given our assumption that the bubble has a zero value in February 1981. It peaked in January 1994.

Despite the slight differences in behavior, we can make some general statements. The property market of Hong Kong has experienced a bubble episode between 1993 and 1997, when property prices rose significantly above their respective fundamental values. The culmination of the bubble occurred right before the eruption of the Asian Financial Crisis. Furthermore, the bubble is periodically collapsing: it rises and falls, and rises again and falls again. Our results are compatible with those of Chan, Lee and Woo (2000).

7. Conclusions

Restrictions in supply are often cited as one of the key reasons for causing speculative bubbles in the real estate market. The total population in Hong Kong is about seven million, while there is only 50 km² of residential land in this territory. This makes Hong Kong one of the most densely populated cities in the world (Chan, Lee and Woo, 2000). Perhaps for that reason, the real property price of Hong Kong underwent extraordinary swings in the 1980s and 1990s. In size, the price swings have been as dramatic as those happened elsewhere in the world. In frequency, they have been more dramatic. This fact makes Hong Kong property market one of the most interesting for a study of speculative bubbles.
In this paper, we argue that a bubble is likely to exist in any asset whose fundamental value is difficult to assess. This uncertainty is the source of speculation, hence the source of speculative bubble. For a number of reasons, such as supply restrictions and institutional arrangements, property markets are more prone to speculative bubbles than other types of asset market.

We have attempted, in this paper, to find out if a variant of the present-value model can explain the movement of the property prices in Hong Kong. To deal with possible misspecification, we have added a state variable to the estimated equation. This state variable is meant to capture the misspecification or data measurement error. This error term is estimated using Kalman filter.

Our empirical study shows that even when an error term is included, the movement of the property price in Hong Kong cannot be entirely explained by the fundamentals. The fundamental model can explain more than 70% of the total variations in the office price index, more than 60% of that in the domestic premises price index. But the model can explain less than 40% of the variation in the price indices of the flatted factories and the retail premises.

If we were to interpret the estimation residuals from the price equation as rational speculative bubbles, then we can make the following statement. Speculative bubbles are periodically collapsing. The rise and fall of a speculative bubble is an important force behind the large swings in a property price of Hong Kong during the 1980s and 1990s.
The movements of the bubbles in the four price indices considered are quite similar. They were in the collapsing phase in the first half of the 1980s. They entered into an expansionary phase roughly after 1985. This expansion continued with little reversals until early 1994. Thereafter, the market quietened for two years. Then the bubble shot up again in all cases except for the flatted factories. They returned to a prolonged collapsing phase after the Asian financial crisis, with the flatted factories price index suffering the most significant drop in value. Unfortunately, we are unable to make an absolute statement about the proportion of bubble in each price index. To do so, we must make a bold assumption about which is the fundamental value of a price. If we boldly assume that the value of the bubble in each price index stood at zero value in February 1981 (the starting point of the usable sample), then we can say that each price index was below its fundamental value during the 1980s; and that for the most part of 1990s, the property market of Hong Kong was infected by a positive speculative bubble. Our conclusions are compatible with those of Chan, Lee and Woo (1980).
The property price of Hong Kong suffered a significant decline in the first half of the 1980s. In 1985, it embarked on a 10-year long expansion. But the path of the expansion is not smooth, especially that of the office price index. After crashed in early 1994, the domestic premises and the retail price indices experienced their sharpest increase of the sample period in the first half of 1997. That was followed by their steepest fall. The increase of the office price index in this brief period is not as significant. But its crash in the ensuing years was no less dramatic. The slipping of the flatted factories price index was unstopped since early 1994.

The price-rent ratio increased continuously during the 10-year long (1985-1994) expansion of the property prices. This ratio for the office sector is the most volatile one. This ratio for the office and for the flatted factories sectors crashed after in late 1997. The decline in this ratio for the domestic premises or for the retail premises sector is much more moderate.
Figure 3 How Well Does the State Space or the Present Value Model Explain the Price Movement? (Office)

(a) State Space Model (in log difference)

(b) Present value Model (in log differences)

(c) State Space Model (in log levels)
Hong Kong Office (KF)

In Log Level (Feb 81=1)

Price
Phat_kf

Feb-81
Feb-83
Feb-85
Feb-87
Feb-89
Feb-91
Feb-93
Feb-95
Feb-97
Feb-99
Feb-01
Figure 4 How Well Does the State Space or the Present Value Model Explain the Price Movement? (Domestic Premises)

(a) State Space Model (in log differences)

(b) Present value model (in log differences)

(c) State space model (in log levels)
Figure 5 How Well Does the State Space or the Present Value Model Explain the Price Movement? (Flatted Factories)

(a) State space model (in log differences)

(b) Present value model (in log differences)

(c) State space model (in log levels)
Figure 6 How Well Does the State Space or the Present Value Model Explain the Price Movement? (Retail Premises)

(a) State space model (in log differences)

(b) Present value model (in log differences)

(c) State space model (in log levels)

The fastest bubble growth in the domestic premises price index occurred between mid-1990 and mid-1992, and between the late 1996 and the mid 1997. It stood at its peak right before the eruption of the Asian Financial Crisis in July 1997. The collapsing of the bubble thereafter is less significant in both magnitude and speed compared to that of the office price index.
(c) The Flatted Factories Sector

Two of the sharpest growth of bubble in the flatted factories price index occurred between the mid 1986 and the late 1987, and between the late 1991 and the mid 1992. Unlike the others, the bubble in this index was not caught in the 96/7 explosion. Its decline since July 1997 was nevertheless like a waterfall.

(d) The Retail Premises Sector

The rise of the bubble in the retail premises price index had been gradual but persistent throughout the second half of the 1980s and the early 1990s. However, it shot up almost vertically between December 1996 and September 1997. Then it dropped as sharply between October 1997 and August 1998, but stabilized for the remaining part of the sample period.
## Unit Root Tests

### Table 1 Phillips-Perron Unit Root Tests

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<td>Lag=1</td>
</tr>
<tr>
<td>Office</td>
<td>Rho</td>
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</tr>
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<td>(Pr&lt;Rho)</td>
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<td>-1.3898</td>
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<tr>
<td></td>
<td>(0.8854)</td>
<td>(0.8468)</td>
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<td></td>
<td>τ</td>
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<tr>
<td>(Pr&lt;τ)</td>
<td>-0.63</td>
<td>-0.76</td>
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<tr>
<td></td>
<td>(0.8612)</td>
<td>(0.8278)</td>
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<td>H0: unit root</td>
<td>Accept H0 at 10%, 5% and 1% levels.</td>
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<tr>
<td>Domestic Premises</td>
<td>Rho</td>
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<td>(Pr&lt;Rho)</td>
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<td>-0.8384</td>
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<td>(0.9265)</td>
<td>(0.9000)</td>
</tr>
<tr>
<td></td>
<td>τ</td>
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<tr>
<td>(Pr&lt;τ)</td>
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<td>-0.61</td>
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<tr>
<td></td>
<td>(0.8963)</td>
<td>(0.8651)</td>
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<tr>
<td></td>
<td>H0: unit root</td>
<td>Accept H0 at 5% and 1% levels.</td>
</tr>
<tr>
<td>Flatted Factories</td>
<td>Rho</td>
<td></td>
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<td>(Pr&lt;Rho)</td>
<td>0.0042</td>
<td>-0.1415</td>
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<td></td>
<td>(0.9572)</td>
<td>(0.9495)</td>
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<tr>
<td></td>
<td>τ</td>
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<td>(Pr&lt;τ)</td>
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<td>(0.9573)</td>
<td>(0.9464)</td>
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<td>H0: unit root</td>
<td>Reject H0 at 10% and 5% levels; accept H0 at 1% level</td>
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<tr>
<td>Retail Premises</td>
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<td></td>
<td>τ</td>
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<td>(Pr&lt;τ)</td>
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<td>(0.8334)</td>
<td>(0.8105)</td>
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<td></td>
<td>H0: unit root</td>
<td>Accept H0 at 10%, 5% and 1% levels.</td>
</tr>
</tbody>
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## Parameter Estimates

### Table 2 The State Space Model

(a) Hong Kong Office

<table>
<thead>
<tr>
<th></th>
<th>( \rho )</th>
<th>( \phi )</th>
<th>( b )</th>
<th>( \delta )</th>
<th>( \zeta )</th>
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<tr>
<td>Parm</td>
<td>703.74517</td>
<td>0.78041</td>
<td>0.00095</td>
<td>0.00015</td>
<td>0.00081</td>
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<tr>
<td>t(H)</td>
<td>0.01065</td>
<td>21.34930*</td>
<td>1.37618</td>
<td>0.44427</td>
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<td>t(NW)</td>
<td>0.00627</td>
<td>2.22851</td>
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<tr>
<td>LM</td>
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<td></td>
<td>6.14363</td>
<td>26.95384</td>
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<tr>
<td>Wald</td>
<td>34.11522</td>
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(b) Hong Kong Domestic Premises

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<th>( \phi )</th>
<th>( b )</th>
<th>( \delta )</th>
<th>( \zeta )</th>
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<tr>
<td>Parm</td>
<td>1412.09758</td>
<td>0.51641</td>
<td>0.00037</td>
<td>0.00013</td>
<td>0.00027</td>
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<tr>
<td>SE</td>
<td>78415.88966</td>
<td>0.04891</td>
<td>0.00045</td>
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<td>0.00045</td>
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<td>t(H)</td>
<td>0.01801</td>
<td>10.55867*</td>
<td>0.82492</td>
<td>0.40133</td>
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<td>1.40871</td>
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<td>LM</td>
<td>13.57331</td>
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<td>28.75116</td>
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<td>Wald</td>
<td>24.55718</td>
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(c) Hong Kong Flatted Factories

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<th>( \phi )</th>
<th>( b )</th>
<th>( \delta )</th>
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<tr>
<td>parm</td>
<td>2263.80091</td>
<td>0.57090</td>
<td>0.00068</td>
<td>0.00023</td>
<td>0.00025</td>
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<tr>
<td>se</td>
<td>59437.90329</td>
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<td>0.00052</td>
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<td>t(H)</td>
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<td>11.98056*</td>
<td>1.32401</td>
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<td>t(NW)</td>
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<td>1.80586</td>
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<tr>
<td>LM</td>
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<td>20.68104</td>
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<td>Wald</td>
<td>29.55932</td>
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(d) Hong Kong Retail Premises

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<th>( \phi )</th>
<th>( b )</th>
<th>( \delta )</th>
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<tr>
<td>Parm</td>
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<td>0.00114</td>
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<td>SE</td>
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<td>0.00046</td>
<td>0.00072</td>
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<td>t(H)</td>
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<td>t(NW)</td>
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<tr>
<td>Wald</td>
<td>4.00195</td>
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</table>

**Note:** (i) Both LM and Wald test statistics are distributed \( \chi^2(4) \). The corresponding critical values are 13.28, 9.49 and 7.78 at 1%, 5% and 10% levels respectively; (ii) The critical value for \( t \) distribution is 2.576, 1.96 and 1.645 at 1%, 5% and 10% levels respectively; (iii) \( t(H) \) are \( t \)-ratios calculated using Hessian; \( t(NW) \) are that using Newey-West covariance. These notations are applicable for the rest of the tables in this paper.
Parameter Estimates

Table 3 The Present Value Model

|                | Hong Kong Office |                | Hong Kong Domestic Premises |                |
|----------------|------------------|----------------|-----------------------------|----------------
|                | Intercept rent Lagged rent | Intercept rent Lagged rent |                |                |
| Parameter      | -0.0013 1.0347 -0.056 | 0.0009 0.8228 0.1239 |                |                |
| t              | -0.54 5.17* -0.28 | 0.67 6.76* 1.02 |                |                |
| t(NW)          | -0.0735 0.0492 -0.0027 | 0.0586 0.0617 0.0093 |                |                |
| F (H0: model jointly insignificant) (Pr >F) | 21.67* (<0.0001) | 24.19* (<0.0001) |                |                |
| LM (H0: Homoscedasticity) | 12.5082* | 2.4758 |                |                |
| Wald (H0: model jointly insignificant) | 3.5316 | 0.3421 |                |                |

|                | Hong Kong Flatted Factories |                | Hong Kong Retail Premises |                |
|----------------|-----------------------------|----------------|-----------------------------|----------------
|                | Intercept rent Lagged rent | Intercept rent Lagged rent |                |                |
| Parameter      | -0.0028 0.4546 0.133 | -0.0005 -0.1045 0.1328 |                |                |
| t              | -1.62 4.02* 1.21 | -0.23 -1.22 1.59 |                |                |
| t(NW)          | -0.2148 0.0510 0.0153 | -0.1311 -0.0262 0.0349 |                |                |
| F (H0: model jointly insignificant) (Pr >F) | 13.76* (<0.0001) | 1.57 (0.1970) |                |                |
| LM (H0: Homoscedasticity) | 1.8779 | 27.4148* |                |                |
| Wald (H0: model jointly insignificant) | 6.1598 | 0.0376 |                |                |

Note: (i) The LM statistics is distributed \( \chi^2(3) \). The corresponding critical values are 11.34, 7.82 and 6.25 at 1%, 5% and 10% levels respectively; (ii) The Wald statistic is distributed \( \chi^2(2) \), with critical values 9.21, 5.99 and 4.61 at 1%, 5% and 10% levels respectively; (iii) If the null of homoscedasticity is rejected,wald and t(NW) are adopted; otherwise F and the conventional t are used.
## Model Comparison

### Table 4 Comparisons of Goodness of Fit

#### (a) Sum of Squared Errors

<table>
<thead>
<tr>
<th></th>
<th>Hong Kong Office</th>
<th></th>
<th>Hong Kong Domestic Premises</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>12-step Forecast</td>
<td>Estimate</td>
<td>12-step Forecast</td>
</tr>
<tr>
<td><strong>State space</strong></td>
<td>0.14509</td>
<td>0.00327</td>
<td>0.06119</td>
<td>0.00172</td>
</tr>
<tr>
<td><strong>Present value</strong></td>
<td>0.41849</td>
<td>0.00407</td>
<td>0.14069</td>
<td>0.00228</td>
</tr>
<tr>
<td><strong>State space</strong></td>
<td>0.13448</td>
<td>0.00439</td>
<td>0.20100</td>
<td>0.00327</td>
</tr>
<tr>
<td><strong>Present value</strong></td>
<td>0.23007</td>
<td>0.00632</td>
<td>0.64850</td>
<td>0.00526</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Hong Kong Flatted Factories</th>
<th></th>
<th>Hong Kong Retail Premises</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>12-step Forecast</td>
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</tr>
</tbody>
</table>

#### (b) Goodness of Fit

<table>
<thead>
<tr>
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<th>Hong Kong Domestic Premises</th>
<th></th>
<th>Hong Kong Flatted Factories</th>
<th></th>
<th>Hong Kong Retail Premises</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State space</td>
<td>Present value</td>
<td>State space</td>
<td>Present value</td>
<td>State space</td>
<td>Present value</td>
<td>State space</td>
<td>Present value</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.71625</td>
<td>0.19044</td>
<td>0.64314</td>
<td>0.21660</td>
<td>0.40172</td>
<td>0.11686</td>
<td>0.38598</td>
<td>0.01756</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.71187</td>
<td>0.18424</td>
<td>0.63763</td>
<td>0.21060</td>
<td>0.39248</td>
<td>0.11009</td>
<td>0.37649</td>
<td>0.01003</td>
</tr>
</tbody>
</table>

All test statistics show that the state space model is superior to the present value model. The adjusted r-square show that the state space model can explain more than 70% of the total price variations in the office sector, more than 60% of that in the domestic premises sector, and near 40% of that in the flatted factories and the retail premises sectors. The present value model can only explain 1% to 21% of the price variations in the four sector.
References


50. SAS/IML user’s guide, version 8.


1 One of the most prominent economists among the skeptics is Milton Friedman (1969).

2 Sornette and Johansen (1997) demonstrate that a speculative bubble has its observable features.

iii Refer to Blanchard and Watson (1982) and and Grossman’s (1988a) on arguments for and against this class of bubbles.

iv The proxy for a bubble to swell is the price appreciation lagged one period; that for a bubble to burst is the deviation of accrual price from the equilibrium price in the previous period (Abraham and Hendershott 1994, 2, 4).
Details of the paper are not available. This summary is obtained from Kim (2000).

The CEIC Economic Databases have been established since 1992. Its core economic database is the CEIC Asia Economic Database with over 190,000 data series. Its prime sources of data include over 150 major government statistical agencies, over 80 recognized non-government issuing agencies, and over 300 reference statistical publications. Please visit http://www.ceicdata.com/ for more information on the profile of CEIC Data Company Ltd ("CEIC")

Since there is no trending in the data series, and the mean is obviously not zero, we have adopted the pp test with a constant mean.

An AR(1) instead of a random-walk price process, in fact, indicate the presence of a bubble (Shiller 1990).

Greene (1997, p. 548) argue that, in the presence of heteroscedasticity, a Wald-type test is more appropriate than a F test.

A F test is used where LM test accept the null of homoscedasticity. Otherwise, a Wald test is adopted.

$t(H)$ is used if the LM test accept the null. Otherwise $t(NW)$ is used.