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Competition between **highway operators**: can we expect **toll** differentiation?

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Abstract

Where there are alternative roads to the same destination, competition between profit maximizing road operators is possible. Tolls on such roads could perform two welfare enhancing functions; discouraging excessive driving and allocating drivers between roads. The second of these functions will typically require some roads to be more expensive to drive on, and to be less congested, than others. Bertrand equilibrium will not always peform this second function. It may fail to allocate the most impatient drivers to less congested roads, as it does not always deliver toll differentiation. The performance of this second function is dependent on the first. That is, whether or not competing roads will be differentiated by tolls and congestion, will depend in part on the importance of discouraging marginal drivers. The equilibrium will not generally be fully efficient, but will often provide efficiency gains over other decentralized options.

Key Words: congestion, road pricing, networks, market structure

JEL: R41, C72, D62, L13

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1 Introduction

When motorists do not have to pay to use roads, they may not have incentives to drive efficiently. One reason is well understood. Too many people may choose to drive, and drive too much, because they don't take into account the increased congestion experienced by other drivers. However, there is also a second reason. Impatient drivers may be willing to pay more, to use 'Lexus lanes' or roads with minimal congestion. Toll differentiation can increase welfare, by encouraging patient and impatient drivers to select roads with high and low congestion respectively (Verhoef & Small 1999, Small & Yan 2001, Parry 2002).

One approach to pricing congestion is to allow private companies to operate roads and toll drivers. For example, a single private company might operate both roads. A private company might operate one road and the other road might not be tolled. Or competing firms might each operate one of the roads. Verhoef & Small (1999) examined the first two of these three options, with results that were discouraging. Both options *reduced* efficiency from a benchmark of no tolls.

The current paper examines the third option, competition. Unfortunately, no general result on the efficiency consequences is available. This is not surprising as even a single-road monopolist might charge an efficient toll (Edelson 1971). However, an example is presented, in which duopoly outperforms untolled roads as well as the other two options mentioned above.

Recall the two ways that tolls can raise efficiency; discouraging excessive driving and allocating drivers between roads. Competition will typically deliver positive tolls, and hence perform the first function. But the prospects for the second function are not so clear. **The following discussion ex-**

amines whether toll differentiation is required for efficiency, and whether it can be expected as a result of a duopoly equilibrium.

It is a standard result that Bertrand competition between providers of non-differentiated goods leads to a symmetric equilibrium (all firms charge the same price) and to efficiency. But this result is not applicable to congestible networks. The reason is that otherwise identical networks have different levels of congestion when they charge different prices (Häckner & Nyberg 1996). Engel, Fischer & Galetovic (1999) examine a roading oligopoly, finding that equilibria are not generally efficient. de Palma & Lindsey (2000) present an example in which a duopoly equilibrium is more efficient than one or both roads being untolled.

Previous research does not provide a compelling reason to expect **competing roads to deliver different levels of** congestion. de Palma & Lindsey (2000), Häckner & Nyberg (1996) and Lee & Mason (2001) all find symmetric equilibria. However none of these studies address the same question as the current paper. de Palma and Lindsey consider queue rather flow congestion. While queue congestion is a suitable assumption for some issues, it is not appropriate for examining differentiation between high and low congestion roads. de Palma and Lindsey's duopolists charge time varying tolls that completely remove congestion. Furthermore, the benefits from differentiation, identified by Verhoef and Small, are not relevant when drivers are homogeneous. Häckner & Nyberg (1996) find that equilibria must be symmetric if there are only two firms or if the disutility of congestion is only significant for high levels of utilitization. However, like de Palma & Lindsey, they assume homogeneous consumers.

Lee & Mason (2001) do allow consumers to have heterogeneous preferences, and do not assume that congestion takes the form of queues. They also find there will always be a symmetric equilibrium. Furthermore, they argue that competition in congested networks cannot deliver asymmetric equilibria. However, they assume that firms do not anticipate that changes in tolls will affect drivers' expectations about congestion.[2]

Previous literature has identified benefits from toll differentiation, but does not suggest that competition will deliver such benefits. Models of competition either do not incorporate all benefits, predict no differentiation in equilibrium, or both. In the following discussion, the possibility of toll differentiation is re-examined in a Bertrand model of a roading duopoly. It will be assumed that roads are subject to flow rather than queue congestion, that drivers (and sometimes roads) are heterogeneous and that drivers anticipate more expensive roads to be less congested.

Unfortunately, heterogeneous drivers and rational expectations are not easy to model. A firm's payoff need not be quasi-concave in its toll, and its best-response may be discontinuous (Verhoef & Small 1999, Lee & Mason 2001). Furthermore, games will not generally be supermodular. As a result, pure strategy equilibria need not exist. In addition, the analysis can be intractable. However, it is possible to obtain some results.

The assumptions of the model are outlined in the following section. Section 3 focuses on the second function of tolls - allocating drivers among roads - and abstracts from decisions about whether to drive at all. In this framework, some toll differentiation would be welfare improving, but unregulated duopoly may not deliver it. Even when competition does lead to toll dif-

[2] **They emphasize applications in which this assumption is reasonable, such as when firms cannot commit to prices in advance.**

ferentiation, **the wrong road may be more expensive**. However these pessimistic results depend on the absence of marginal consumers, who are introduced in section 4. An equilibrium will not generally deliver a first best outcome, but we can expect some toll differentiation when tolls actively reduce the amount of driving. An example is presented in section 5, illustrating the symmetric equilibria of section 3 and the asymmetric equilibria of section 4. Conclusions are drawn in section 6.

2 The model

Two roads connect the same two points. Each road is owned by a different firm. Firms 1 and 2 simultaneously choose tolls $p_1 \in \mathbb{R}_+$ and $p_2 \in \mathbb{R}_+$ respectively. Firms incur no extra costs from more drivers, and so firms maximize revenues. Consumers observe the announced tolls and then simultaneously choose which if any of the roads to drive on.

The two roads have utilization Q_1 and Q_2. Congestion is an increasing and differentiable function of utilization, i.e., $z_i'(Q_i) > 0$. Roads are heterogeneous when $z_1(\cdot)$ and $z_2(\cdot)$ are different functions.

Consumers are heterogeneous but anonymous. That is, they differ according to their disutility from congestion, but this disutility does not depend on the identity of other drivers on the road. A consumer **in group** i with taste parameter θ_i receives utility $u_i(z;\theta_i) - p$ from driving on a road with congestion z and paying a toll of p. She receives a reservation utility of u_o if she does not drive.

Utility from travel is twice differentiable in z and θ_i, strictly decreasing in z **and has a negative cross partial derivative. That is, motorists**

are willing to pay to avoid congestion, and motorists with higher values of θ_i are willing to pay more. In principle, a strong aversion to congestion could be either positively or negatively related to the value of travel. It could be the most patient or the most impatient that choose not to drive. We allow for both possibilities by dividing motorists into $i = 1, .., n$ subpopulations, where $u_i(z, \theta_i)$ is increasing in θ_i for some subpopulations i and decreasing for others. Subpopulation i has mass μ_i. The taste parameter, θ_i, is distributed according to F_i, which is continuously differentiable and strictly increasing on $[0, 1]$. Although homogeneity of drivers is ruled out by the assumption that F_i is increasing, it will be possible to examine some implications of homogeneity.

3 Results when everyone prefers to drive

If u_o is high enough, then some consumers will not drive in equilibrium. But in this section, we consider the case in which u_o is so low that everyone strictly prefers to drive. In this case, the only function that tolls need to perform is to allocate drivers between roads. The efficient allocation requires different tolls to be charged on the two roads.

Proposition 1 *Assume that $p_1 = p_2$ and every driver strictly prefers to drive. The outcome will not be a local total surplus maximum.*

The proof is **presented in Appendix C**. But the intuition is straightforward. If both roads charge the same toll, then they will also have the same level of congestion. Otherwise drivers would switch to the less congested road. In an efficient outcome, each driver would impose the same marginal external cost on other drivers, no matter which road she chose. The two roads will not generally have the same marginal externality when they are equally congested, if either roads or drivers are heterogeneous. So it will not be efficient for roads to be equally congested.

Consider the case in which drivers are homogeneous but roads are heterogeneous. All drivers would have the same willingness to pay to reduce congestion. Total surplus would **vary inversely with** aggregate congestion, $z_1 \cdot Q_1 + z_2 \cdot Q_2$. So long as the two roads have different elasticities of congestion with respect to utilization when $z_1 = z_2$, aggregate congestion can be reduced by moving drivers to the road with the lower elasticity. This reallocation of drivers can only be accomplished by differentiating the tolls.

Now consider the case in which roads are homogeneous, but drivers differ from each other. In this case, the findings of Verhoef & Small (1999) apply. Impatient drivers are willing to pay enough to compensate patient drivers for using the other road. Again there would be gains to price differentiation.

Although toll differentiation is required for efficiency, it need not result from competition. This is particularly clear when the roads are identical.

Proposition 2 *Assume that the two roads are identical (i.e. that $z_1(\cdot) \equiv z_2(\cdot)$). Assume further that there is a pure strategy Bertrand equilibrium in which every consumer strictly prefers driving to staying at home. Then both firms will charge the same toll, and have the same level of congestion.*

To see why this proposition holds, consider how the equilibrium is determined. Because of the assumption that higher θ_i implies stronger aversion to congestion, motorists' choices between the two roads can be characterized with thresholds. Let θ^i_M be the thresh-

old for subpopulation i, so that motorists with $\theta_i > \theta_M^i$ will prefer to drive on the less congested and more expensive road and those with $\theta_i < \theta_M^i$ will prefer the more congested one. Then θ_M^i is characterized as the value of θ_i for which a consumer is indifferent.[3]

$$u_i(z_2(Q_2), \theta_M^i) - u_i(z_1(Q_1), \theta_M^i) = p_2 - p_1 \tag{1}$$

Demands for the two roads are given by the following equations.

$$Q_1 = \sum_i \mu_i F_i(\theta_M^i), \qquad Q_2 = 1 - \sum_i \mu_i F_i(\theta_M^i) \tag{2}$$

The tolls only enter equation (1) in terms of the toll difference, and so $dQ_2/dp_1 = -dQ_2/dp_2$. Equation (2) implies that $dQ_1/dp_1 = -dQ_2/dp_1$. By transitivity, it follows that $dQ_1/dp_1 = dQ_2/dp_2$. This equality is substituted into the two firms' FOCs,

$$Q_1 + p_1 \frac{\partial Q_1}{\partial p_1} = 0, \qquad Q_2 + p_2 \frac{\partial Q_2}{\partial p_2} = 0 \tag{3}$$

to imply that $Q_1/p_1 = Q_2/p_2$. This means that if $p_1 < p_2$, then $Q_1 < Q_2$. But if the roads are identical, this is impossible. It would mean that some drivers were choosing road 2 even though it was more expensive and more congested. So there cannot be an equilibrium with $p_1 < p_2$, or by a similar argument, with $p_1 > p_2$.

The problem is more general than just with identical roads. If there is an asymmetric equilibrium, the more expensive road must have higher utilization. This is clearly impossible if the roads are identical, but it may be

[3] unless everyone in the subpopulation strictly prefers one of the two roads. Such subpopulations do not affect the result

suboptimal even when it is possible. Imagine that one road has greater capacity than the other. Efficiency may require the larger road to be more expensive, or it may require the smaller road to be more expensive. It depends on the elasticities of congestion and on the distributions, F_i. But only the former type of outcome could be a competitive equilibrium.

Proposition 2 supports the pessimism about asymmetric equilibria expressed by Häckner & Nyberg (1996), de Palma & Lindsey (2000), and Lee & Mason (2001). **However, it does rely on a strong assumption.** The prospect of staying at home is so unattractive, that everyone will choose to drive. This assumption is critical, and will be relaxed in the following section.

4 Results when not everyone drives

We now **allow some** potential drivers to stay at home. **Non-drivers are divided from drivers by the thresholds θ_L^i and θ_M^i. In subpopulations in which $u_i(z, \theta_i)$ is increasing in θ_i, high θ_i motorists always drive ($\theta_H^i = 1$), but low θ_i motorists may not. If they do all drive, then $\theta_L^i = 0$. Otherwise, the motorist with $\theta_i = \theta_L^i$ is indifferent whether or not to drive; $u_i(z_1, \theta_L^i) - p_1 = u_0$. Conversely, when utility is decreasing in θ_i, low θ_i motorists always drive ($\theta_L^i = 0$), but high θ_i motorists may not ($u_i(z_2, \theta_H^i) - p_2 = u_0$ or $\theta_H^i = 0$). As a result, demand is characterized in the following generalization of equation (2).**

$$Q_1 = \sum_i \mu_i \left(F_i(\theta_M^i) - F_i(\theta_L^i) \right), \; Q_2 = \sum_i \mu_i \left(F_i(\theta_H^i) - F_i(\theta_M^i) \right) \tag{4}$$

Toll differentiation is still required for efficiency.

Proposition 3 *If the two tolls are equal, the outcome is not efficient.*

The efficient tolls maximise total surplus.

$$S = u_0 \left(1 - Q_1 - Q_2\right) + \sum_i \mu_i \int_{\theta_L^i}^{\theta_M^i} u_i(z_1, \theta_i) dF_i + \sum_i \mu_i \int_{\theta_M^i}^{\theta_H^i} u_i(z_2, \theta_i) dF_i.$$

Incremental increases in p_2 have the following effect on surplus.

$$\frac{\partial S}{\partial p_2} = \sum_i \frac{\partial S}{\partial \theta_L^i} \frac{\partial \theta_L^i}{\partial p_2} + \sum_i \frac{\partial S}{\partial \theta_M^i} \frac{\partial \theta_M^i}{\partial p_2} + \sum_i \frac{\partial S}{\partial \theta_H^i} \frac{\partial \theta_H^i}{\partial p_2} \tag{5}$$

An increase in p_2 will sometimes increase total surplus, and sometimes decrease it. But, it is shown in Appendix C, that if we start from the highest total surplus that is attainable without toll differentiation, then surplus increases further when one of the tolls is increased incrementally.

Proposition 3 is analogous to proposition 1. A symmetric outcome continues to be inefficient, even when some potential drivers choose to stay at home. However, there is no analog for proposition 2. Competing duopolists may charge different tolls, even when roads are identical.

Proposition 4 *Assume that the two roads are identical. Any pure strategy Bertrand equilibrium, in which some people do not drive, is asymmetric.*

This proposition is demonstrated in Appendix D. The rationale is that there is an upward kink in the demand curve when $p_1 = p_2$, and hence an upward jump of marginal revenue. This means that there cannot be a profit maximum at this point, and so the two firms charge different tolls in equilibrium. **Recall that the rationale for proposition 2 was based on an identity between $\partial Q_1/\partial p_1$ and $\partial Q_2/\partial p_2$. This identity will not**

generally hold when higher tolls convince some potential drivers to stay at home.[4]

Although Bertrand duopoly can deliver toll differentiation, and hence some benefits, it does not generally deliver an efficient outcome. Efficient tolls reflect the disutility of congestion experienced by inframarginal drivers. In contrast, competing duopolists only care about marginal drivers - those at the thresholds. Although we do not have an analytic result about the efficiency consequences of duopoly, it is possible to calculate equilibrium tolls for simple examples. One such example is illustrated in the following section.

5 A simple example

Assume two identical roads and a single population, in which θ is distributed uniformly on $[0, 1]$**.** Utility from driving is $\theta \cdot (1-Q)-p$. As a consequence, θ_M is determined in the following counterpart to equation (1).

$$\theta_M \cdot (1 - \theta_M + \theta_L) - p_1 = {\theta_M}^2 - p_2 \qquad (6)$$

Equilibria fall into four categories. When u_o is low enough (eg. $u_o = -.6$), there is a symmetric equilibrium as described in Section 3. In such an equilibrium, $\theta_L = 0$ and θ_M is characterized by equation (6). When u_o is a little higher (eg. $u_o = -.3$), then there is an asymmetric equilibrium ($p_1 \neq p_2$) in which everyone drives ($Q_1 + Q_2 = 1$).[5] When u_o is higher again,

[4]**The sign of $\partial\theta^i_M/\partial p_1 + \partial\theta^i_M/\partial p_2$ depends on the sign of $\partial u/\partial\theta$. But whether this effect is overwhelmed by the effect on the other threshold also depends on this sign. Tabuchi & Thisse (1995) also find asymmetric equilibria in some spatial models, driven by kinked demand curves.**

[5]**This second type of equilibrium only arises because the example does not**

there is an asymmetric equilibrium in which some people stay at home, as described in Section 4. The two thresholds are determined by the intersection of (6) and the following condition showing indifference of the person with $\theta = \theta_L$, between driving ans staying at home.

$$\theta_L \cdot (1 - \theta_M + \theta_L) - p_1 = u_o \tag{7}$$

Finally, if u_o is too high, then no-one drives ($\theta_L = 1$). Bertrand equilibria can be found numerically. A range of equilibria, for various values of u_o, are presented in Table 1.

Duopoly equilibria are compared with three alternatives, (i) a monopoly owning both roads, (ii) a private firm operating one road with the other road untolled, and (iii) both roads untolled. As in Verhoef & Small (1999), the efficiency consequences of one tolled and one untolled road are very poor. However, in contrast to Verhoef and Small, the outcomes with a two-road monopoly are **typically more efficient than those with two untolled roads, except when everyone drives. Furthermore,** the outcomes in a duopoly equilibrium dominate the alternatives in terms of efficiency. Bertrand duopoly generally delivers greater total surplus than either monopoly or one untolled road, and (**usually**), a greater surplus than untolled roads. However, Bertrand equilibria are not first best efficient even when some drivers stay at home. Tolls tend to be too high and total utilization tends to be too low.

satisfy the assumption that $\partial u/\partial z < 0$ when $\theta = 0$. In such an equilibrium, there are marginal drivers (at $\theta = 0$) even though everyone actually drives. Consequently, proposition 2 does not apply. Furthermore, total surplus is not monotonic in u_o over this region

6 Conclusion

The main result of the paper is that vertical differentiation, a requirement for efficiency, can be expected in a Bertrand equilibrium in a wide range of cases. However, these cases do not include a totally inelastic total demand for commutes. While competition is unlikely to deliver full efficiency, it can lead to efficiency gains over other decentralized options.

Despite the encouraging results, some caution is in order. First, overall efficiency in a duopoly was only assessed in an example, and the example was chosen for tractability rather than plausibility. Even so, the encouraging results are consistent with de Palma & Lindsey (2000), and suggest that the competitive option may be worthy of consideration where possible.

A second reason for caution is the complexity of duopoly models. Best response functions are discontinuous when there are some potential drivers that stay at home. As a consequence, Bertrand equilibria may not exist. Furthermore, theoretical equilibria may seem more plausible as predictions when they are simple. Policymakers and private agents may find it difficult to predict outcomes in realistic settings. Perhaps, policymakers should proceed slowly, until more is known about how roading duopolies function in practice.

Appendix

A Demand functions at $p_1 = p_2$

When $p_1 = p_2$, θ_M is not well defined. The limiting allocations of drivers as p_1 approaches p_2 are different, depending on whether p_1 approaches p_2 from above or below. However, both these limiting allocations are possible outcomes when $p_1 = p_2$. Furthermore, every allocation with $p_1 = p_2$ has the same Q_1 and Q_2. Hence the demand functions are continuous at $p_1 = p_2$.

B Slopes of demand functions

Consider subpopulations with $u_\theta > 0$, and some non-drivers. $\theta_H^i \equiv 1$. θ_L^i and θ_M^i are determined by (1) and $u_i(z_1, \theta_L^i) - p_1 = u_0$. As p_2 approaches p_1 from above, we get the following.

$$\frac{\partial \theta_L^i}{\partial p_1} = -f_i(\theta_M^i)\frac{(z_2' + z_1')u_z^{iM} - u_z^{iL} z_1'}{\Delta_i}, \quad \frac{\partial \theta_L^i}{\partial p_2} = \frac{-f_i(\theta_M^i)u_z^{iL} z_1'}{\Delta_i}$$
$$\frac{\partial \theta_M^i}{\partial p_1} = -\frac{(u_z^{iM} - u_z^{iL})f_i(\theta_L^i)z_1' + u_\theta^{iL}}{\Delta_i}, \quad \frac{\partial \theta_M^i}{\partial p_2} = -\frac{u_z^{iL} z_1' f_i(\theta_L^i) - u_\theta^{iL}}{\Delta_i}$$

where $\Delta_i = f_i(\theta_M^i)u_z^{iM}(z_1' z_2' u_z^{iL} f_i(\theta_L^i) - (z_1' + z_2')u_\theta^{iL}) > 0$. In subpopulations with $u_\theta < 0$ and some nondrivers, $\theta_L^i \equiv 0$, and θ_M^i and θ_H^i are determined by (1) and $u_i(z_2, \theta_H^i) - p_2 = u_0$. As p_2 approaches p_1 from above, we get:

$$\frac{\partial \theta_M^i}{\partial p_1} = \frac{u_z^{iH} z_2' f_i(\theta_H^i) + u_\theta^{iH}}{\Delta_i}, \quad \frac{\partial \theta_M^i}{\partial p_2} = \frac{(u_z^{iM} - u_z^{iH})z_2' f_i(\theta_H^i) - u_\theta^{iH}}{\Delta_i}$$
$$\frac{\partial \theta_H^i}{\partial p_1} = \frac{f_i(\theta_M^i)u_z^{iH} z_2'}{\Delta_i}, \quad \frac{\partial \theta_H^i}{\partial p_2} = f_i(\theta_M^i)\frac{(z_1' + z_2')u_z^{iM} - u_z^{iH} z_2'}{\Delta_i}$$

where $\Delta_i = f_i(\theta_M^i)u_z^{iM}\left((z_1' + z_2')u_\theta^{iH} + z_1' z_2' u_z^{iH} f_i(\theta_H^i)\right) > 0$.

C Efficiency

To show that total surplus is not maximised when $p_1 = p_2$, we first find the best symmetric outcome, i.e., the highest total surplus attainable with a single price. Let $p_1 = p_2 = p$, and $z_1 = z_2 = z$, so:

$$S = u_0 (1 - Q_1 - Q_2) + \sum_i \mu_i \int_{\theta_L^i}^{\theta_H^i} u_i(z, \theta_i) dF_i$$

Then the best price satisfies the following FOC:

$$\frac{\partial S}{\partial p} = \sum_i \frac{\partial S}{\partial \theta_L^i} \frac{\partial \theta_L^i}{\partial p} + \sum_i \frac{\partial S}{\partial \theta_H^i} \frac{\partial \theta_H^i}{\partial p} = 0$$

The implication is that $p^\star = -\frac{1}{2} \sum_i \mu_i \int_{\theta_L^i}^{\theta_H^i} \frac{\partial u_i(Q,\theta_i)}{\partial Q} dF_i$. If we let $p_1 = p_2 = p^\star$, then equation (5) can be rephrased in the following way,

$$\frac{\partial S}{\partial p_2} = \nu \sum_i \mu_i \left(\frac{f_i(\theta_L^i)}{2} \frac{\partial \theta_L^i}{\partial p_2} - f_i(\theta_M^i) \frac{\partial \theta_M^i}{\partial p_2} + \frac{f_i(\theta_H^i)}{2} \frac{\partial \theta_H^i}{\partial p_2} \right)$$

$$\text{where } \nu = \sum_i \mu_i \int_{\theta_M^i}^{\theta_H^i} \frac{\partial u_i(z_2, \theta_i)}{\partial z} dF_i - \sum_i \mu_i \int_{\theta_L^i}^{\theta_M^i} \frac{\partial u_i(z_1, \theta_i)}{\partial z} dF_i < 0$$

Now plug in the comparative statics results from Appendix B. Subpopulations who all drive have $\partial \theta_L^i / \partial p_2 = \partial \theta_H^i / \partial p_2 = 0$, and $-\partial \theta_M^i / \partial p_2 < 0$. For subpopulations with $u_\theta > 0$ and some non-drivers, $\partial \theta_H^i / \partial p_2 = 0$ and

$$\frac{f_i(\theta_L^i)}{2} \frac{\partial \theta_L^i}{\partial p_2} - f_i(\theta_M^i) \frac{\partial \theta_M^i}{\partial p_2} = \frac{-u_\theta^{iL} f_i(\theta_M^i) + \frac{f_i(\theta_L^i) f_i(\theta_M^i) u_z^{iL} z_1'}{2}}{\Delta_i} < 0.$$

For subpopulations with $u_\theta < 0$ and some non-drivers, $\partial \theta_L^i / \partial p_2 = 0$ and

$$-f_i(\theta_M^i) \frac{\partial \theta_M^i}{\partial p_2} + \frac{f_i(\theta_H^i)}{2} \frac{\partial \theta_H^i}{\partial p_2} = \frac{\frac{f_i(\theta_H^i)}{2} \left(u_z^{iM} z_1' + (u_z^{iH} - u_z^{iM}) z_2' \right) + u_\theta^{iH}}{\Delta_i} < 0.$$

Recall from Appendix B that $\Delta_i > 0$. Therefore, as $\nu < 0$, total surplus increases if p_2 is raised incrementally from the highest level of total surplus attainable with a symmetric outcome.

D Equilibrium

Consider identical roads with $p_1 \leq p_2$. If there was a symmetric equilibrium, **then there should be no incentive to reduce** p_1 **or raise** p_2, i.e., $Q_1 + p_1 \partial Q_1 / \partial p_1 \geq 0$ and $Q_2 + p_2 \partial Q_2 / \partial p_2 \leq 0$ with $p_1 = p_2$. Therefore:

$$\sum_i \mu_i f_i(\theta^i_M) \left(\frac{\partial \theta^i_M}{\partial p_1} + \frac{\partial \theta^i_M}{\partial p_2} \right) \geq \sum_i \mu_i \left(f_i(\theta^i_L) \frac{\partial \theta^i_L}{\partial p_1} + f_i(\theta^i_H) \frac{\partial \theta^i_H}{\partial p_2} \right)$$

But the results of Appendix B show that this is not possible if some people do not drive. Subpopulations with no non-motorists have $\partial \theta^i_M / \partial p_1 = -\partial \theta^i_M / \partial p_2$, and $\partial \theta^i_L / \partial p_1 = \partial \theta^i_H / \partial p_2 = 0$. Subpopulations with $u_\theta > 0$, and some nondrivers, have $\partial \theta^i_L / \partial p_1 = 0$ and $f_i(\theta^i_M) \left(\partial \theta^i_M / \partial p_1 + \partial \theta^i_M / \partial p_2 \right) \leq f_i(\theta^i_H) \partial \theta^i_H / \partial p_2$, as $u^{iM}_z < u^{iL}_z$. Finally, subpopulations with $u_\theta < 0$, and some nondrivers, have $\partial \theta^i_H / \partial p_2 = 0$ and $f_i(\theta^i_M) \left(\partial \theta^i_M / \partial p_1 + \partial \theta^i_M / \partial p_2 \right) \leq f_i(\theta^i_L) \partial \theta^i_L / \partial p_1$, as $u^{iM}_z > u^{iH}_z$.

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	$u_o = -.6$	$u_o = -.3$	$u_o = -.25$	$u_o = 0$	$u_o = .25$	$u_o = .5$	$u_o = .75$	$u_o = .975$
duopoly	$p_1 = .5$	$p_1 = .3$	$p_1 = .3333$	$p_1 = .2919$	$p_1 = .2295$	$p_1 = .1584$	$p_1 = .0814$	$p_1 = .0083$
	$p_2 = .5$	$p_2 = .5492$	$p_2 = .4444$	$p_2 = .3271$	$p_2 = .2436$	$p_2 = .1635$	$p_2 = .0825$	$p_2 = .0083$
	$Q = 1$	$Q = 1$	$Q = .8334$	$Q = .5758$	$Q = .3570$	$Q = .2465$	$Q = .1166$	$Q = .0112$
	$S = .25$	$S = .2565$	$S = .2477$	$S = .2931$	$S = .4059$	$S = .5663$	$S = .7660$	$S = .9752$
monopoly	$p_1 = .6$	$p_1 = .3$	$p_1 = .25$	$p_1 = .4219$	$p_1 = .3402$	$p_1 = .2369$	$p_1 = .1221$	$p_1 = .0125$
	$p_2 = 1.055$	$p_2 = .7553$	$p_2 = .7053$	$p_2 = .4840$	$p_2 = .3662$	$p_2 = .2460$	$p_2 = .1241$	$p_2 = .0125$
	$Q = 1$	$Q = 1$	$Q = 1$	$Q = .4339$	$Q = .2952$	$Q = .1839$	$Q = .0872$	$Q = .0084$
	$S = .2148$	$S = .2148$	$S = .2148$	$S = .2655$	$S = .3906$	$S = .5597$	$S = .7644$	$S = .9751$
1 private	$p_2 = .4553$	$p_2 = .4553$	$p_2 = .4553$	$p_2 = .4553$	$p_2 = .2359$	$p_2 = .1408$	$p_2 = .0657$	$p_2 = .0063$
1 untolled	$Q = 1$	$Q = 1$	$Q = 1$	$Q = 1$	$Q = .5676$	$Q = .3369$	$Q = .1554$	$Q = .0147$
	$S = .2148$	$S = .2148$	$S = .2148$	$S = .2148$	$S = .3804$	$S = .5562$	$S = .7636$	$S = .9751$
both roads	$Q = 1$	$Q = 1$	$Q = 1$	$Q = 1$	$Q = .6340$	$Q = .3820$	$Q = .1771$	$Q = .0168$
untolled	$S = .25$	$S = .25$	$S = .25$	$S = .25$	$S = .3872$	$S = .5590$	$S = .7643$	$S = .9751$
total	$p_1 \leq .6$	$p_1 \leq .3$	$p_1 \leq .25$	$p_1 = .1953$	$p_1 = .1626$	$p_1 = .1154$	$p_1 = .0604$	$p_1 = .0062$
surplus	$p_2 = p_1 + .1308$	$p_2 = p_1 + .1308$	$p_2 = p_1 + .1308$	$p_2 = .2545$	$p_2 = .1886$	$p_2 = .1252$	$p_2 = .0625$	$p_2 = .0063$
maximum	$Q = 1$	$Q = 1$	$Q = 1$	$Q = .6831$	$Q = .4545$	$Q = .2798$	$Q = .1315$	$Q = .0126$
	$S = .2641$	$S = .2641$	$S = .2641$	$S = .2987$	$S = .4082$	$S = .5671$	$S = .7662$	$S = .9752$

Table 1: Outcomes in the example for various values of u_o

u_o is the reservation utility of drivers

p_1 is the price of the lower priced road

p_2 is the price of the higher priced road

Q is total utilization, $Q_1 + Q_2$

S is total surplus