QUANTITY PRECOMMITMENT AND BERTRAND COMPETITION
YIELD COURNOT OUTCOMES: A CASE WITH PRODUCT
DIFFERENTIATION: REPLY

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Since the publication of our paper (Yin and Ng, 1997), we have received some very helpful comments on the paper. In his recent comment, Schulz (2000) indicates that price reaction functions in the second-stage subgame of our model are discontinuous in certain circumstances so that Lemma 1 in the paper is incorrect. The editor, R. Damania, points out a mistake in the proof of Lemma 1 in his letter to us. We sincerely appreciate these criticisms, which are greatly helpful for us to refine our work. After a careful check, we find that not only Lemma 1 is invalid but the price reaction function given in Theorem 1 also has to be revised. Fortunately, these errors do not affect the main results of the paper. The erratum to the previous mistakes is given below.

Figure 1 below is borrowed from Schulz (2000), which can be obtained by applying demand functions (3)–(7) in our original paper. In the figure, line $C_i (i = 1, 2)$ plots the price pairs on which the demand for the firm $i$'s product just hits its capacity while the demand for the firm $j$'s product is not binding by either the upper or lower boundary; i.e.

$$C_i: p_1^i(p_j) = (1 - \gamma)\alpha + \gamma p_j - (1 - \gamma^2)K_i. \tag{1}$$

$Z_i$ in the figure $(i = 1, 2)$ is the line on which the demand for the firm $i$'s product just becomes zero while the demand for the firm $j$'s product hits neither the upper nor lower boundary; i.e.

$$Z_i: p_i = (1 - \gamma)\alpha + \gamma p_j. \text{ Or inversed, } p_j^i(p_i) = [p_i - (1 - \gamma)\alpha]/\gamma. \tag{2}$$

$A_{xy}$ gives the area, where the demand for firms 1 and 2's products has the characteristics of the first and second subscripts, respectively. $x$ and $y$ in the subscript can be $c$, $u$, and $z$, indicating that the demand is constrained by the production capacity, unconstrained demand and zero demand, respectively. The equations below $A_{xy}$ show the demand functions in that area and the demand in $A_{uu}$ is $q_i = \alpha/(1 + \gamma) - (p_i - \gamma p_j)/(1 - \gamma^2)$.

It should be noticed that Figure 1 presents a most comprehensive situation with nine areas, which requires small production capacities of both firms. When capacities are large, $C_1$ and $C_2$ move to left and down, respectively, so that some areas may disappear. An extreme is no capacity constraints and Figure 1 has only four areas: $A_{uu}$, $A_{uz}$, $A_{zu}$, and $A_{zz}$. With the help of Figure 1, we can revise the price reaction function as follows.

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Theorem 1'  

(I) If $2K_i + \gamma K_j \leq \alpha$, the price reaction function is

$$R_i(p_j) = \begin{cases} 
\alpha - K_i - \gamma K_j & p_j \in [0, \alpha - K_j - \gamma K_i] \\
\min\{p^2_j(p_j), \alpha/2\} & p_j \in [\alpha - \gamma K_i, \alpha]
\end{cases}$$

(II) If $2K_i + \gamma K_j > \alpha$, the price reaction function is

$$R_i(p_j) = \begin{cases} 
(\alpha - \gamma K_j)/2 & p_j \in [0, \alpha - K_j - \gamma K_i] \\
(\alpha - \gamma K_j)/2 & p_j \in (\alpha - K_j - \gamma K_i, \alpha - \gamma K_i) \cap \\
\min\{p^2_j(p_j), \alpha/2\} & p_j \in [\alpha - \gamma K_i, \alpha]
\end{cases}$$

Figure 1.

An interval $(x, y)$ below should be understood as an empty set if $x > y$ or both $x$ and $y$ are negative. If $x < 0$ but $y > 0$, the interval $(x, y)$ should be understood as $[0, y)$.

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Proof. (I) Consider $2K_2 + \gamma K_1 \leq a$.

(i) \[ p_1 \in [0, a - K_1 - \gamma K_2] \]

For a given $p_1$, firm 2 chooses the optimal price $p_2 = a - K_2 - \gamma K_1$ if $q_2 = K_2$ or $p_2 = (a - \gamma K_1)/2$ if $q_2 = a - p_2 + \gamma K_1$. The latter is the best price response iff it is in $A_{cu}$. Since $2K_2 + \gamma K_1 \leq a \iff a - K_2 - \gamma K_1 \geq (a - \gamma K_1)/2$, the price reaction function is $a - K_2 - \gamma K_1$.

(ii) \[ p_1 \in (a - K_1 - \gamma K_2, a - \gamma K_2) \]

There are two candidates of optimal price. One is to choose maximum price when the capacity is binding, which is the line $C_2$, i.e., $p_2^1(p_1)$. The other is

\[
p_2^3(p_1) = \frac{(1 - \gamma)a + \gamma p_1}{2}, \tag{3}\]

which maximises profits for demand $q_2 = a/(1 + \gamma) - (p_2 - \gamma p_1)/(1 - \gamma^2)$. Recalling (1), $p_1 > a - K_1 - \gamma K_2$ and $2K_2 + \gamma K_1 \leq a$, we have

\[
p_2^1(p_1) \leq [\gamma\alpha + \gamma p_1]/2 + [\gamma\alpha + \gamma p_1 - 2(1 - \gamma^2)K_2]/2 > p_2^3(p_1).
\]

Since $p_2^3(p_1)$ is the best price response iff it falls in $A_{uu}$, the above inequality implies that the price reaction function in this interval must be $p_2^1(p_1)$.

(iii) \[ p_1 \in [a - \gamma K_2, a] \]

(I)–(ii) shows that $p_2^3(p_1) < p_2^1(p_1)$ when $p_1 \in (a - K_1 - \gamma K_2, a - \gamma K_2)$. Thus, $p_2^3(p_1) < a - K_2$. Since $p_2^3(p_1)$ is flatter than $Z_1$, it is in $A_{ce}$ or $A_{au}$ on this interval and cannot be the best price response. Noting that $p_2 = \alpha/2$ maximises profit for demand $q_2 = a - p_2$, the best price response should be $Z_1$ if $p_2 = \alpha/2$ is above $Z_1$ or $p_2 = \alpha/2$ if it is below $Z_1$; i.e. $\min\{p_2^3(p_1), \alpha/2\}$.

(II) Assuming $2K_2 + \gamma K_1 > a$

(i) \[ p_1 \in [0, a - K_1 - \gamma K_2] \]

As (I)–(i) shows the price reaction function on this interval is $(\alpha - \gamma K_1)/2$.

(ii) \[ p_1 \in (a - K_1 - \gamma K_2, a - \gamma K_2) \]

In this interval, firm 2 has three possible price choices, i.e., $p_2 = (\alpha - \gamma K_1)/2$ or $p_2^1(p_1)$ or $p_2^3(p_1)$. Their corresponding profits are, respectively

\[
(\alpha - \gamma K_1)^2/4, \\
K_2p_2^1(p_1) = K_2[(1 - \gamma)\alpha + \gamma p_1 - (1 - \gamma^2)K_2], \\
\{\alpha/(1 + \gamma) - [p_2^3(p_1) - \gamma p_1]/(1 - \gamma^2)\}p_2^3(p_1) \\
= [(1 - \gamma)\alpha + \gamma p_1]^2/4(1 - \gamma^2)
\]

Let $\hat{p}_1$ be the point where the first profit figure is equal to the second, we have

\[
\hat{p}_1 = [(\alpha - \gamma K_1)^2/4K_2 + (1 - \gamma^2)K_2 - (1 - \gamma)\alpha]/\gamma. \tag{4}\]

Thus, $p_2 = (\alpha - \gamma K_1)/2$ leads to a larger profit than $p_2^1(p_1)$ iff $p_1 < \hat{p}_1$. Let $\bar{p}_1$ be the point where the first profit figure is equal to the third, we have

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\[
\hat{p}_1 = [(1 - \gamma^2)^{1/2}(\alpha - \gamma K_1) - (1 - \gamma)\alpha]/\gamma, 
\]  
which implies that \( p_2 = (\alpha - \gamma K_1)/2 \) brings more profit than \( p_3^*(p_1) \) iff \( p_1 < \hat{p}_1 \). Let \( p_1^* \) be the intersection of \( p_3^*(p_1) \) and \( p_3^*(p_1) \), i.e., \( p_1^* = [2(1 - \gamma^2)K_1 - (1 - \gamma)\alpha]/\gamma \). Since \( p_3^*(p_1) \) is steeper than \( p_2^*(p_1) \), we can see that \( p_3^*(p_1) > p_1^*(p_1) \) on the left of \( p_1^* \) and \( p_1^*(p_1) \) on the right. Moreover, for any \( p_1 \), if \( p_3^*(p_1) \) falls in \( A_{un} \), it results in more profit than \( p_3^*(p_1) \). It can also be shown that \( \hat{p}_1 > \alpha - K_1 - \gamma K_2 \) and \( p_3^*(p_1) \) is below the line \( C_1 \) (or its extension). Thus, the price reaction function shifts from \( p_2 = (\alpha - \gamma K_1)/2 \) to \( p_3^*(p_1) \) at \( p_1 = \hat{p}_1 \) if \( p_1 < \hat{p}_1 \) and \( p_1 \in (\alpha - K_1 - \gamma K_2, p_3^*/2) \) and then switches to \( p_3^*(p_1) \) at \( p_1 = p_1^* \) if \( p_1^* < \alpha - \gamma K_2 \). But if \( p_1 \leq \hat{p}_1 \) or \( p_1 \leq \alpha - \gamma K_2 \) and \( p_1 \notin (\alpha - K_1 - \gamma K_2, p_3^*/2) \), the price reaction function shifts from \( p_2 = (\alpha - \gamma K_1)/2 \) to \( p_3^*(p_1) \) at \( p_1 = \hat{p}_1 \) and remains on it until the end of the interval. If \( p_1 > \alpha - \gamma K_2 \) and \( \hat{p}_1 > \alpha - \gamma K_2 \) or \( p_1 > \alpha - \gamma K_2 \) and \( p_1 \notin (\alpha - K_1 - \gamma K_2, p_3^*/2) \), the reaction function remains on \( p_2 = (\alpha - \gamma K_1)/2 \) for the whole interval. Define

\[
\hat{p}_1 = \begin{cases} 
\hat{p}_1 & \text{if } p_1 \leq \min\{\hat{p}_1, \hat{p}_1\} \\
\infty & \text{otherwise}. 
\end{cases}
\]

The price reaction function in this interval is

\[
\begin{align*}
(\alpha - \gamma K_1)/2 & \quad \text{when } p_1 \leq \min\{\hat{p}_1, \hat{p}_1\} \\
\max\{p_2^*(p_1), p_3^*(p_1)\} & \quad \text{when } p_1 \geq \min\{\hat{p}_1, \hat{p}_1\},
\end{align*}
\]

(iii) \( p_1 \in [\alpha - \gamma K_2, \alpha] \)

Let \( p_1^{**} \) be determined by \( p_3^*(p_1) = \alpha/2 \) so that \( p_1^{**} = (1 - \gamma)/2 \alpha \). As (I)–(iii) shows, the price reaction function cannot be \( p_2 = \alpha/2 \) on the left of \( p_1^{**} \). Hence, on the left of \( p_1^{**} \), there are three possible price choices, i.e., \( p_2 = (\alpha - \gamma K_1)/2 \) or \( p_2^2(p_1) \) (i.e., line \( Z_1 \)) or \( p_3^*(p_1) \). The profit of adopting \( p_3^*(p_1) \) is equal to \( \{\alpha - [p_1 - (1 - \gamma)\alpha]/\gamma\} \{p_1 - (1 - \gamma)\alpha]/\gamma = (\alpha - p_1)(p_1 - (1 - \gamma)\alpha]/\gamma^2 \}. \) Let \( \overline{\gamma} \) be the point where this profit is equal to \( (\alpha - \gamma K_1)/2 \), i.e., \( \overline{\gamma} \) is the smaller solution of the equation

\[
p_1^2 - 2(\gamma)\alpha p_1 + (1 - \gamma)\alpha^2 + \gamma^2(\alpha - \gamma K_1)/4 = 0. 
\]

It can be shown that the intersection of \( p_2^*(p_1) \) and \( p_3^*(p_1) \) is on the left of \( p_1^{**} \) and \( p_2^*(p_1) \) is steeper than \( p_3^*(p_1) \). Then, if the price reaction function is \( p_2 = (\alpha - \gamma K_1)/2 \) at \( p_1 = \alpha - \gamma K_2 \), it shifts to \( p_3^*(p_1) \) at the \( p_1 = \hat{p}_1 \) when \( \hat{p}_1 < \overline{\gamma} \). Afterward, it switches to \( p_2^*(p_1) \) at the intersection of \( p_2^*(p_1) \) and \( p_3^*(p_1) \) to reach \( p_2 = \alpha/2 \). When \( \overline{\gamma} < \hat{p}_1 \) it shifts to \( p_3^*(p_1) \) at \( p_1 = \hat{p}_1 \) and moves toward \( \alpha/2 \). If the price reaction function at \( p_1 = \alpha - \gamma K_2 \) is \( p_2^*(p_1) \) or \( p_3^*(p_1) \), the development is similar, except that there is no shift. Thus, the price reaction function in this interval is

\[
\begin{align*}
(\alpha - \gamma K_1)/2 & \quad \text{when } p_1 \leq \min\{\overline{\gamma}, \hat{p}_1\} \\
\min\{\max[p_2^*(p_1), p_3^*(p_1)], \alpha/2\} & \quad \text{when } p_1 \geq \min\{\overline{\gamma}, \hat{p}_1\}.
\end{align*}
\]

With this new Theorem 1', Lemma 1 in our original paper should be changed to

2 The second condition is to ensure that \( p_3^*(p_1) \) falls in \( A_{un} \).

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Lemma 1’. Firm \(i\)’s price reaction function has one and only one discontinuity point at \(p_j^\ast\) or \(p_j^\ast\) or \(p_j^\ast\) when \(2\xi_i + \gamma K_j > \alpha\) and \(\gamma K_j < \alpha\). Otherwise, it is continuous.

**Proof.** Since max\{\cdot, \cdot\} and min\{\cdot, \cdot\} are continuous operators, the discontinuity occurs at the point where the price reaction function shifts from \(p_i = (\alpha - \gamma K_j)/2\) to \(p_i^3(p_j)\) or \(p_i^4(p_j)\). But this can happen only once when \(2\xi_i + \gamma K_j > \alpha\) and \(\gamma K_j < \alpha\) as Theorem 1’ illustrated.

The bold lines in Figure 2 illustrates a typical price reaction curve of firm 2 when the discontinuity conditions in Lemma 1’ are satisfied. Thus, there is a shift at \(p_i = \hat{p}_1\). If the line \(p_i^3(p_1)\) is lower, the shift may occur at \(p_i = \hat{p}_1 > p_1^\ast\). A rare situation is that \(\xi_i\) is very small but \(K_2\) is sufficiently large so that the line \(C_1\) is long but \(C_2\) is short. In turn, the shift occurs on the right of \(p_i = \alpha - \gamma K_2\).

Because of the discontinuity of the price reaction curves, the price subgame in the second stage does not guarantee a unique pure strategy equilibrium as indicated in Theorem 2 in the original paper. However, this theorem is not a necessity for Theorem 3. Hence, Theorem 3 holds for the discontinuous price reaction functions but the proof should be revised.

**Theorem 3.** In the equilibrium of the entire game, there are no idle capacities.

**Proof.** Suppose \((p_1^\ast, p_2^\ast, K_1, K_2)\) is the equilibrium of the entire game and at least one firm, say, firm 2, has certain idle production capacity. If the equilibrium leads to a pure price
strategy of firm 2, then \((p_1^*, p_2^*)\) must be on the line \(p_2 = (\alpha - \gamma K_1)/2\) or \(p_2^2(p_1)\) or \(p_2^3(p_1)\) or \(p_2 = \alpha/2\). Hence, \(p_2^e\) is independent of firm 2’s capacity and it has an incentive to reduce its idle capacity, which implies that \((p_1^*, p_2^*, K_1, K_2)\) cannot be the equilibrium of the entire game.

Consider now that firm 2 adopts a mixed price strategy in equilibrium. Since the mixed price strategy can only occur at the discontinuity point of the price reaction function, it implies that \(p_1^e = \hat{p}_1\) or \(\overline{p}_1\) or \(\overline{p}_1\). For the first two cases, \((p_1^e, p_2^e)\) is the intersection of the line \(C_1\) and \(p_1 = \hat{p}_1\) or \(C_1\) and \(p_1 = \overline{p}_1\). Because \(C_1\), \(\hat{p}_1\) and \(\overline{p}_1\) are independent of \(K_2\) (see (1), (5)–(7)), the price \(p_2^e\) is independent of firm 2’s capacity.

Turning to the case where \(p_1^e = \overline{p}_1\). Figure 3 illustrates the most complicated situation in the sense that both firms have discontinuous price reaction functions, resulting in multi-equilibria at \(E_1\) and \(E_2\) in the price subgame. \(^3\) Let us focus on \(E_1\). The shift of price reaction curve at \(\overline{p}_1\) from \(p_2 = (\alpha - \gamma K_1)/2\) to \(p_2^1(\hat{p}_1)\) (i.e., line \(C_2\)) implies that \(p_2^1(\overline{p}_1) > p_2^3(\hat{p}_1)\).\(^4\)

Recalling (1), (3) and (4), this inequality gives \((1 - \gamma)\alpha + \gamma \hat{p}_1 > 2(1 - \gamma^2)K_2\) and

\[
(\alpha - \gamma K_1)^2/4K_2 > (1 - \gamma^2)K_2. 
\]

Since \(E_1\) is on \(C_1\), we obtain from the reverse of \(p_1(p_2)\) that

\[
p_2^e = [\overline{p}_1 + (1 - \gamma^2)K_1 - (1 - \gamma)\alpha]/\gamma. 
\]

The demand for good 2 along line \(C_1\) is \(q_2^e = \alpha - p_2^e - \gamma K_1\). Thus, the firm 2’s equilibrium profit in the second-stage subgame is \(\pi = p_2^e q_2^e = p_2^e (\alpha - p_2 - \gamma K_1)\), which gives

\[
\frac{\partial \pi}{\partial K_2} = (\alpha - 2p_2 - \gamma K_1)\frac{\partial p_2}{\partial K_2} \\
= \gamma^{-2}(\alpha - 2p_2 - \gamma K_1)[(1 - \gamma^2) - (\alpha - \gamma K_1)^2/4K_2^2] < 0.
\]

\(^3\) In equilibrium \(E_2\), the firm 2’s capacity is binding. But actually, we can show that it is not the equilibrium of the entire game following the same arguments in the proof of \(E_1\). If firm 1’s price reaction function is continuous, then there exists only one equilibrium, \(E_1\). The intersection of \(\hat{p}_1\) and \(\overline{p}_2\) cannot be an equilibrium point because it falls in the area \(A_{uu}\).

\(^4\) Otherwise, it shifts to \(p_2^3(p_1)\) before \(\overline{p}_1\).
To get the second equality, (4) and (9) are used. From (8) and \( q_2^e > \alpha - 2p_2^e - \gamma K_1 \), the inequality is immediate, which implies that firm 2 has an incentive to reduce the capacity and \((p_1^e, p_2^e, K_1, K_2)\) is not an equilibrium.

When the capacity constraints of both firms are binding, the price reaction function is \( R_i(p_j) = \alpha - K_i - \gamma K_j \), which leads to the price subgame equilibrium (11) in our original paper. Therefore, the main conclusions, Theorem 4 and Proposition 1 there, hold.

REFERENCES
