Optimal Investment in Urban Drainage: 
A Framework for Cost-Benefit Analysis

Yew-Kwang Ng*
Department of Economics
Monash University

Abstract

This article addresses some basic issues (including distributional weights, discount rates, and the value of life) in the cost-benefit analysis of urban drainage and provides a framework for the estimation of the optimal amount of investment for flood mitigation. This involves: (i) estimating the expected total damage from flooding in present-value terms before flood mitigation; (ii) deriving the reduction in expected total damage as the average recurrent interval of flooding increases; (iii) estimating how this interval increases with the amount of investment in flood mitigation; and (iv) choosing the optimal investment by equating marginal benefit and marginal cost. The framework is also applicable to other accident or damage mitigation investments.

1. Introduction

In the past few years, heavy rain dramatically increased the flooding problem in Sydney. Two questions arise: how much additional investment in urban drainage should be undertaken to alleviate the problem? and, how should the investment be financed? This article provides a cost-benefit framework to estimate the optimal amount of investment for flood mitigation. The framework is fairly generally applicable to other cities and to other accident or damage mitigation investments.

2. What Costs and Benefits should be Included in the Analysis?

A ubiquitous problem in any cost-benefit analysis is to be exhaustive in including and measuring in dollar terms all benefits and costs and yet avoid double-counting.

In urban drainage, as in other urban development projects such as transportation, a likely neglect on the cost side is to include only the direct investment costs incurred by the relevant authority and to ignore the indirect costs imposed on the community in the form of inconvenience, time lost, etc. caused, for example, by blockage to traffic flow during construction. This problem is less important for a project which clearly needs to be undertaken and where the size of the project has a negligible effect on the (incremental) inconvenience costs. The problem is more relevant for projects whose overall desirability is in doubt. The inclusion of the inconvenience costs then may be decisive in leading to a negative decision. Also, if the size of the project significantly increases the inconvenience costs, an analysis which ignores inconvenience costs will recommend a larger than optimal investment.

The benefits of investment in drainage consist mainly of the avoidance or reduction in the various
types of costs associated with flooding. These costs include:

(i) costs of injury and loss of life;

(ii) costs of cleaning, repairing and replacing damaged buildings and other items;

(iii) costs incurred due to interruption of services such as traffic flow;

(iv) costs of anxiety, lost earnings, etc.; and

(v) costs of water pollution.

The benefits of investment in drainage consist of the expected reduction in these costs. Since both the costs and their expected reduction through drainage are only expected to occur in the future, probabilities and discount rates have to be used to convert them into present-value terms. Elsewhere (Ng 1984, 1987), the author has argued that the relevant costs and benefits should generally be treated on 'a dollar is a dollar' basis irrespective of the income levels of the affected persons, since the objective of equality is better pursued by the general tax/transfer system.

There may also be additional benefits from investment in drainage if the investment also improves the landscape and adds to the aesthetic value of the environment. Ignoring this possible minor benefit, the cost-benefit analysis of investment in drainage may be put in terms of the minimisation of the total costs (expressed in present-value terms) of investment (including inconvenience costs) plus the various costs of flooding. The amount of investment is chosen to minimise total costs. Though expressed as total-cost minimisation, full account is taken of all relevant benefits. If aesthetic values are included, they appear as negative costs.

While there are not many problems associated with expressing the costs of investment, repairs and replacement in monetary terms (we shall call these monetary costs), the expression of the cost of anxiety, inconvenience, pollution, and above all, loss of life in monetary terms is more difficult (we shall call these non-monetary costs). Some people (for example, The Institution of Engineers, Australia 1987) solve the problem of measuring non-monetary costs using a dual-objective approach.

Using the dual-objective approach, the various cost possibilities of a given investment are plotted on a graph in which the monetary costs are measured on one axis and reductions in injury and loss of life (that is, the non-monetary costs) on the other axis. The possibilities are shown as a U-shaped trade-off curve, T in Figure 1. For each level of drainage benefits, there is a trade-off curve T as in Figure 1. To concentrate on the choice between costs and lives lost alone, the figure is simplified by assuming that the level of drainage benefits is given. The upward-sloping section of the trade-off curve (that is, $K_1$) is the inefficient zone where the number of lives lost can be reduced without increasing costs. When such opportunities have been exhausted (point $K_1$), further reduction in lives lost requires higher costs, and typically at an increasing rate, explaining the shape of the trade-off curve. If no attention is paid to the number of lives lost, the point $K_1$, which minimises the expected annual monetary costs (of investment plus flooding) is chosen. However, since reducing the number of lives lost is one of the objectives, the relevant decision-makers may choose point $K_2$, involving a higher expected annual monetary cost but a lower expected annual number of lives lost.

In our view, the above dual-objective approach is really an opting-out approach. The analysts derive the trade-off curve $T$ and let the relevant decision-makers choose a point on $T$. Thus, the analysts opt out of the difficult decision of making a trade-off themselves. However, a choice has to be made, and any particular choice implies a specific (marginal) trade-off between expected annual number of lives lost and expected annual monetary costs (slope of $T$ at the chosen point $K_2$). Moreover, such a largely arbitrary choice may be based on less informed knowledge than the choice made by a more formal analysis. There is a body of economic literature and analysis which permits a more objective estimation of the appropriate trade-off the relevant group of people would be willing to undertake. Hence, in our

![Figure 1](image-url)
view, for any project involving a change to the probability of death/survival, a cost-benefit analysis is seriously incomplete without an analysis of how much the relevant group of people rationally would be willing to trade against the probability of saving life.

It might be thought that the value of life is infinite and a rational person would not trade his/her life for any amount of money. While a rational person may refuse to accept the certainty of death for any amount of money offered, this does not mean that he/she would also refuse to trade a marginal change in the probability of death/survival. In fact, every person makes many such trade-offs almost daily. For example, in crossing a road to buy a loaf of bread, a person subjects himself/herself to a slightly higher probability of death. The choice for a safer but more expensive airline also implies a certain trade-off. It is true that not all people make rational calculations in all their choices. However, the question as to how people would choose if they had the relevant information and acted rationally is still relevant.

Those believing that human life is sacrosanct may think that the appropriate trade-off is infinite or that the expected number of lives lost should be zero, or at least be minimised. However, this may not be possible, since the U-shaped T curve typically means that, even if we spent most of our GNP to prevent flooding, the expected number of lives lost would still be positive. It is clear that we should stop well before the point where we spend 90 per cent of our GNP on drainage. This implies that the optimal trade-off is not infinite. But what specific value should it be? An analysis of the amount of dollars a typical person in the affected group of people is willing to rationally trade for an increase in the probability of survival should be helpful in answering this question.

There are three methods that could be used to estimate the said trade-off. One is by using survey questionnaires; that is, asking people to reveal their trade-offs directly. This method is fraught with difficulties, especially with respect to the issue of life and death. A typical response to a questionnaire is usually careless, exaggerated or understated. Survey responses regarding the issue of life and death face an additional difficulty which affects the reliability of the results. Respondents may allow emotional attitudes to cloud their responses.

A second method to estimate people's trade-offs uses econometric analyses of data from actual choices made in different settings. The principal difficulty with this method is the lack of a sufficiently rich set of data necessary to derive the implicit trade-offs.

Due to the above mentioned difficulties, the present author is in favour of a third approach (see Ng 1992). This entails adopting a reasonable utility function supplemented by empirical data such as the income and wealth owned by the relevant group of people. A specific estimate of the trade-off on the probability of survival (useful for assessing part of the benefits of urban drainage) employing this method is feasible but beyond the scope of this article.

3. A Proposed Method to Estimate the Optimal Investment

3.1 General Considerations

Any given amount of investment may be used in a variety of ways with different results. The choice of the best way of using a given amount of investment to obtain the most effective result is largely a technical issue beyond the scope of this article. Rather, we concentrate on the choice of the optimal amount of investment.

An area may be flooded to different degrees. A flood that reaches or exceeds a given magnitude occurs with less frequency the higher the given magnitude. This is usually expressed as the average recurrent interval (ARI). The longer the ARI, the less frequent the expected occurrence. A zero degree of flooding is reached and/or exceeded all the time. Hence the ARI for this degree of flooding is zero, and the frequency of occurrence is infinite. The higher the degree of flooding, the longer the corresponding ARI.

The degree of flooding may be measured by the amount of water overflow, the depth of flooding at a particular defined position, the number of properties affected, some other objective measure, or even some semi-subjective measure such as the total damage caused by flooding. Irrespective of which measure is used, the degree of flooding does not go to infinity even if we let ARI go to infinity. Thus, the relation curve R relating the degree of flooding (denoted y) to the corresponding ARI (denoted x) is typically concave, as illustrated in Figure 2.

While the relevant ARI depends on the degree of flooding, it is convenient to speak about the ARI of flooding in general for a particular area. This concept can be made precise by defining a threshold degree of flooding below which the degree of flooding is regarded as too minor to be called a real flood. Ideally, the threshold degree of flooding should be defined such that the total damage from flooding below (above) this level is negligible (substantial). If the curve relating total damage to the degree of flooding...
is as shown in Figure 3, it may be sensible to define \( y \) as the threshold level of flooding (for flooding greater than total damage \( D \) increases sharply with a small increase in the degree of flooding \( y \)). If we plot \( y \) on Figure 2, the ARI for general flooding can then be determined as \( \bar{x} \). A flood that reaches or exceeds this threshold level occurs once every \( \bar{x} \) years on average. Since a 'flood' below this level is defined not to be a flood, one may then say that on average flooding occurs once every \( \bar{x} \) years for the area concerned.

![Figure 2](image)

The curve \( R \) in Figure 2 relates the degree of flooding to ARI. It may shift in the presence of:

(i) a change in climate, especially rainfall pattern in the relevant areas;

(ii) a change in the terrain and land usage in the relevant areas; and

(iii) investment in flood mitigation.

With respect to (i), the recurring heavy rain in Sydney in the past few years may just be a random aberration, but it also may reflect a changed climate. Given possible global warming associated with the greenhouse effect, a change in climate may not be unexpected. While the change may well be to a drier climate than to a wetter one, the recent recurring wet seasons should be taken as some indication that, other things being equal, a change to a wetter climate may be more likely.

Even without the greenhouse effect, the climatic data of a region should be continuously updated. In considering data more weight should probably be given to more recent observations. Moreover, even using equal weights for all data in the past, say, 100 years, the recent heavy rains in Sydney should still marginally increase the expected rainfall in the future. If the curve \( R \) had been derived for Sydney before the heavy rains in the past few years, the new curve incorporating the new information will lie somewhat above it. If there have been substantial changes in terrain and land usage (point (ii) above),
the effect on the curve also should be estimated and incorporated.

Our main concern is however with respect to shifting the curve rightward or downward through investment in drainage or flood mitigation. Given a particular curve $R$, how does investment in flood mitigation shift it down? In particular, does it shift rightward roughly proportionately over the relevant range to $R'$, or does it shift downward roughly proportionately over the relevant range to $R''$, as illustrated in Figure 4? If $R$ is approximately linear, $R'$ and $R''$ will coincide with each other.

The case of $R'$ is most interesting since it means that the ARI for each and every level of flooding is extended by roughly the same proportion. If this can be taken to be approximately true, it makes the analysis much simpler. The reason is that, since the ARI associated with each flood (and hence damage) level is increased by the same proportion, the total expected damage varies inversely with an ARI for any specifically defined flood level, the threshold level of flood $\bar{y}$ (the corresponding ARI being $\bar{x}$) in particular. For example, if the ARI for general flooding is doubled, the total expected damage is halved. The estimation of the optimal amount of investment then can be done in the following relatively simple steps.

Step 1: Estimate the expected total damage from flooding in present-value terms for all future years before flood mitigation.

Step 2: Derive the reduction in expected total damage as the ARI for general flooding increases (that is, the benefits of flood mitigation).

Step 3: Estimate how $\bar{x}$ is increased with increases in the amount of investment in flood mitigation (in other words, the costs of extending ARI), or how investment increases with $\bar{x}$.

Step 4: Choose the optimal ARI ($\bar{x}^*$) and the associated optimal amount of investment $I^*$ by equating marginal benefit and marginal cost.

These steps are further discussed below.

### 3.2 Estimating the Expected Total Damage

(Step 1)

To estimate the expected total damage for all future years before flood mitigation, classify the degree of flooding into a number of reasonably fine intervals. Then estimate the total damage (taking care to include all relevant costs without double-counting; see Section 2 above) if a flood within a given interval were to occur this year. This estimation could be assisted by past records of flood damage, insurance pay-outs, estimates of property values, and economic analysis (see Section 2 above, especially on the costs of expected lives lost). However, if past records are used, it is essential to convert the figures to make them up-to-date.

The above estimate of total damage has to be done for all intervals of the degree of flooding. Next, from the curve in Figure 2, derive the frequency of flooding for each interval. Multiply the estimate of total damage with the corresponding frequency and sum this product over all intervals to obtain the expected total damage from flooding for this year. A hypothetical example is provided below to illustrate the required calculation.

Here, the degree of flooding is taken to be classified into the following nine intervals: negligible, mild, medium, substantial, big, huge, critical, disastrous, and calamitous. In actual estimation, it may be desirable to classify into finer intervals and to use more objective yardsticks of classification; for example, depth of water at a defined spot, number of houses affected, etc. Note again that figures used in Table 1 below are purely hypothetical.

In Table 1, the last column is derived by simply taking the product of the previous two columns. For example, in the first row of figures, $0.24 = 0.2 \times 1.2$. The expected total damage for all possible degrees of flood is derived by aggregating all figures in the last column. The figure of $37.64$ million is then obtained as the expected total damage from flooding for the current year.

Assuming that the situation is to remain unchanged for all future years in the absence of flood

<table>
<thead>
<tr>
<th>Flood intervals</th>
<th>Total damage ($\text{million}$)</th>
<th>Frequency (times per year)</th>
<th>Expected total damage ($\text{million}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>0.2</td>
<td>1.2</td>
<td>0.24</td>
</tr>
<tr>
<td>Mild</td>
<td>0.4</td>
<td>0.5</td>
<td>0.20</td>
</tr>
<tr>
<td>Medium</td>
<td>2</td>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td>Substantial</td>
<td>10</td>
<td>0.11</td>
<td>1.10</td>
</tr>
<tr>
<td>Big</td>
<td>80</td>
<td>0.05</td>
<td>4.00</td>
</tr>
<tr>
<td>Huge</td>
<td>400</td>
<td>0.024</td>
<td>9.60</td>
</tr>
<tr>
<td>Critical</td>
<td>2000</td>
<td>0.008</td>
<td>16.00</td>
</tr>
<tr>
<td>Disastrous</td>
<td>10000</td>
<td>0.0005</td>
<td>5.00</td>
</tr>
<tr>
<td>Calamitous</td>
<td>50000</td>
<td>0.00002</td>
<td>1.00</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>37.64</td>
</tr>
</tbody>
</table>
mitigation, the present value of the expected total damage for all future years can be estimated by the following method.

Let the expected total damage from flooding in any given year grow at the rate \( g \) per year. This rate includes the rate of inflation as well as the expected growth in the real values of damage as population, real income and real property values increase over time. The present value of expected total damage in a future year \( t \) is then given by the formula

\[
D_t = D_0(1 + g)^t/(1 + r)^t
\]

(1)

where

\( D_t = \) expected total damage in year \( t \) (next year being counted as year 1) in dollars;

\( D_0 = \) expected total damage in the current year in dollars;

\( g = \) rate of growth in expected total damage in nominal terms (and \( g \) times 100 is the average annual percentage growth rate); and

\( r = \) nominal rate of discount (and \( r \) times 100 is the average annual percentage discount rate).

The expected total damage for all future years from and including the current year (denoted \( D^*_0 \)) can then be calculated as

\[
D^*_0 = D_0 + D_0(1 + g)/(1 + r) + D_0(1 + g)^2/(1 + r)^2 + D_0(1 + g)^3/(1 + r)^3 + \ldots
\]

(2)

Assuming \( r > g \) (the other case is discussed in Subsection 3.6), this infinite sum can be calculated as

\[
D^*_0 = D_0(1 + r)/(r - g)
\]

(3)

This formula can be easily applied. For example, if as in Table 1, \( D_0 \) is estimated to be $37.64 million and \( g \) estimated to be 10%, and \( r \) to be 12%, we have

\[
D^*_0 = $37.64m \cdot (1.12/0.08) = $210.784m
\]

as the estimated expected total cost of flood damage for all future years from the current year in present-value terms for our hypothetical case.

Since the expected damage runs into the distant future, the present value is much affected by the estimated growth rate \( g \) and the discount rate \( r \). Thus, care must be taken in the precise estimation of these figures. Some basic economic analysis would help in this respect.

Since flood mitigation is not expected to take effect before the completion of the investment project, it may be relevant to calculate the total damage for all future years not from year zero (that is, the current year) but from year \( c \) (when flood mitigation commences). The expected total damage from year \( c \) in present-value terms (at the current year) is

\[
D^*_c = D_0(1 + g)/((1 + r)^c) + D_0(1 + g)^c/(1 + r)^{c+1} + D_0(1 + g)^{c+1}/(1 + r)^{c+2} + \ldots
\]

(4)

Taking again \( r > g \), we may sum the above infinite series to

\[
D^*_c = D_0(1 + g)^c/((1 + r)^c - (r - g))
\]

(5)

Again taking the hypothetical case of \( D_0 = $37.64m \), \( g = 10\% \), \( r = 12\% \), but add the specification \( c = 3 \) (that is, assuming that flood mitigation will take effect three years from now), we may calculate the expected total damage for all future years from and including 1993 in the present value of 1990 as

\[
D^*_3 = $37.64m \cdot (1.12^3/0.02) = $1997m
\]

(6)

3.3 Estimating the Benefits of Flood Mitigation (Step 2)

Once the expected total damage for all years from the commencement year of flood mitigation have been estimated to sum to a certain figure in present-value terms ($1997 million in the hypothetical example above), the expected benefits from flood mitigation (assumed almost perpetual; with \( r \) significantly larger than \( g \), the sum to 100 years from now and the sum to infinity does not differ significantly) for the simple case where the R curve shifts rightward proportionately to \( R' \) (see Figure 4 and the corresponding discussion above) can be easily calculated. If the ARI for general flooding \( x \) is doubled, the expected total damage is halved and hence the total benefit from flood mitigation is half the estimated expected total damage ($993.5 million in the hypothetical example). More generally, we may express the expected total damage, as \( x \) increases from its unmitigated level \( x_{\text{u}} \), as

\[
D^*_x = \frac{x}{x_{\text{u}}}
\]

(7)

Hence the marginal benefit of extending \( x \) is

\[
\frac{d(D^*_x)}{d(x)} = \frac{D^*_x}{x^2} > 0
\]

(8)

Thus, the total benefit of flood mitigation increases with increases in the ARI for general flooding (\( x \)). However, the marginal benefit, though always positive, decreases with increases in \( x \), as
\[
\frac{d^2(D_x^3 / \bar{x})}{dx^2} = -\frac{2D_x^3 \bar{x}}{\bar{x}^3} \tag{9}
\]

is negative. Moreover, the marginal benefit decreases rather sharply with increases in \(\bar{x}\) as it does not just vary inversely with \(\bar{x}\) but varies inversely with the square of \(\bar{x}\).

The marginal benefit curve is plotted in Figure 5. While the height of this curve depends on the estimated \(D_x^3\), its general shape is as given in Figure 5, being determined by \(1/\bar{x}^2\).

3.4 Estimating the Costs of Extending ARI of Flooding (Step 3)

How total costs or the amount of required investment increases with the extension in the ARI of general flooding is largely a technical issue beyond the scope of this article. However, a few brief remarks are offered here.

There are economies of scale in some important aspects of investment in flood mitigation. For example, if a new drainage pipe is to be installed, the costs of disturbance to traffic and of digging do not increase substantially with the diameter of the pipe. On the other hand, the drainage capacity increases with the square of the diameter of the pipe. This suggests that the marginal costs of extending ARI may be downward sloping. The total costs of tripling the ARI is less than twice the costs of doubling it.

On the other hand, a doubling of drainage capacity need not necessarily double the ARI of flooding. Thus, the present author is not very certain about the precise relationship of how ARI varies with the amount of investment, even if assumed most efficiently utilised. However, in the Terms of Reference given by the Water Board (entitled ‘The Pricing Issues in Drainage’), it is stated that the relationship of investment to ARI is logarithmic. If this is so, then we have the total cost of extending ARI (\(\bar{x}\)) beyond its original level \(\bar{x}_u\) (which should be normalised to equal one) as \(TC = \alpha \ln \bar{x} + \beta\) where \(\alpha\) and \(\beta\) are constants. The marginal cost can then be readily derived as \(MC = \alpha / \bar{x}\) which is a rectangular hyperbola as shown in Figure 6.
3.5 Choosing the Optimal ARI and the Associated Optimal Investment (Step 4)

We now superimpose Figure 5 on Figure 6 to obtain Figure 7. The intersection of the marginal benefit curve with the marginal cost curve determines the optimal ARI \( x^* \). The required amount of investment to increase the ARI of general flooding to this level then constitutes the optimal investment.

The above analysis is put in marginal terms. It could as well be put in terms of total costs of flooding and total investment cost (including disturbance costs). The optimal point is chosen to minimise the sum of the two, as shown in Figure 8. The \( x^* \) in Figure 8 should be the same as that in Figure 7 if the curves are derived from the same set of estimates.

If it is doubtful whether some investment is better than no investment at all, the analysis in total terms is important. If the aggregate of all costs is minimised at the zero level of investment, no investment should be undertaken.

3.6 The Case when \( g \) Exceeds \( r \)

The infinite series in Equation (2) above sums to infinity if the annual rate of growth of expected total damage \( g \) exceeds the rate of discount \( r \). Since the universe is not expected to last for ever, a constant rate of growth in excess of the rate of discount is a misleadingly high rate. Thus, if \( g > r \) is the case, it is suggested that a closer look should be taken into the estimates for \( g \) and \( r \). We may have cases where \( g \) exceeds \( r \) for some years, but this is unlikely to be true for all future years.

If both \( g \) and \( r \) refer to nominal values and if the real rate of interest is negative (as was true for many years in the second half of 1970s), then \( g > r \) is quite possible for an \( r \) based on the market rate of interest. The case is that such a low \( r \) is not really sustainable in the long term. Taking a value of \( r \) sustainable in the long term, one will then not get an infinite sum. However, if the authority concerned can borrow long-term money at a negative real interest rate (as in the second half of 1970s), then it is a bargain to invest in such long-term projects as flood mitigation. Why didn't many public authorities and private concerns jump at such opportunities? Two explanations are possible. First, at times of low or even negative real interest rates, people are pessimistic about the future and see \( g \) as lower than \( r \), even if the latter is low or negative. Secondly, people may be irrational, ignorant or not provided with the right economic analysis. Perhaps one could add certain constraints as a third explanation. But then one has to explain the persistence of such non-optimising constraints.

3.7 The Case of a Non-Proportional Rightward Shift in the R Curve

If the shifting in the curve \( R \) (Figures 2 and 4 above) with respect to flood mitigation is not roughly proportional rightward as described for the case of \( R^* \) in Figure 4, the estimation of the benefits of investment in flood mitigation strictly should be undertaken using a more tedious method than the simple one used above. However, unless the departure from the proportional rightward shift is biased in a certain direction and the damage curve (Figure 3) is rather erratic, the simplified method usually will give good approximate results. Nevertheless, the complicated method is not difficult to apply, just somewhat more tedious. The method is outlined below.

Basically, the difference with the simplified method is that one has to derive a separate set of frequency figures (the second last column in Table I above) for each level of investment in flood mitigation. Given the knowledge of the relevant \( R \) curve, the frequency figures can be readily obtained, since frequency is the inverse of ARI. However, the ARI figures are usually in terms of a level of flood exceeding a defined degree. The frequency figures needed are frequencies within defined intervals. Nevertheless, the conversion of one into the other is straightforward. If the frequency of flooding in excess of 10' is 0.2 and the frequency of flooding in excess of 11' is 0.14, then the frequency of flooding between 10' and 11' is \( 0.2 - 0.14 = 0.06 \).
### Table 2  Frequencies of Flood within Different Intervals under Alternative Levels of Investment in Flood Mitigation (Hypothetical)

<table>
<thead>
<tr>
<th>Flood intervals</th>
<th>Zero</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>1.2</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Mild</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>Medium</td>
<td>0.25</td>
<td>0.2</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>Substantial</td>
<td>0.11</td>
<td>0.09</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Big</td>
<td>0.05</td>
<td>0.04</td>
<td>0.025</td>
<td>0.02</td>
</tr>
<tr>
<td>Huge</td>
<td>0.024</td>
<td>0.02</td>
<td>0.012</td>
<td>0.01</td>
</tr>
<tr>
<td>Critical</td>
<td>0.008</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Disastrous</td>
<td>0.0005</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Calamitous</td>
<td>0.00002</td>
<td>0.000015</td>
<td>0.000012</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

### Table 3  Expected Total Damage under Alternative Levels of Investment for Flood Mitigation (Hypothetical) ($million)

<table>
<thead>
<tr>
<th>Flood intervals</th>
<th>Zero</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>0.24</td>
<td>0.18</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>Mild</td>
<td>0.20</td>
<td>0.16</td>
<td>0.12</td>
<td>0.10</td>
</tr>
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<td>Medium</td>
<td>0.50</td>
<td>0.40</td>
<td>0.30</td>
<td>0.22</td>
</tr>
<tr>
<td>Substantial</td>
<td>1.10</td>
<td>0.90</td>
<td>0.60</td>
<td>0.40</td>
</tr>
<tr>
<td>Big</td>
<td>4.00</td>
<td>3.20</td>
<td>2.00</td>
<td>1.60</td>
</tr>
<tr>
<td>Huge</td>
<td>9.60</td>
<td>8.00</td>
<td>4.80</td>
<td>4.00</td>
</tr>
<tr>
<td>Critical</td>
<td>16.00</td>
<td>12.00</td>
<td>8.00</td>
<td>6.00</td>
</tr>
<tr>
<td>Disastrous</td>
<td>5.00</td>
<td>4.00</td>
<td>3.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Calamitous</td>
<td>1.00</td>
<td>0.75</td>
<td>0.62</td>
<td>0.50</td>
</tr>
<tr>
<td>Total</td>
<td>37.64</td>
<td>29.59</td>
<td>19.58</td>
<td>14.94</td>
</tr>
</tbody>
</table>

### Table 4  Aggregates of All Costs under Alternative Levels of Investment (Hypothetical) ($million)

<table>
<thead>
<tr>
<th>Type of costs</th>
<th>Zero</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood costs</td>
<td>1997</td>
<td>1570</td>
<td>1039</td>
<td>793</td>
</tr>
<tr>
<td>Investment costs</td>
<td>0</td>
<td>200</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>Disturbance costs</td>
<td>0</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>Aggregate costs</td>
<td>1997</td>
<td>1820</td>
<td>1599</td>
<td>1863</td>
</tr>
</tbody>
</table>
Once the sets of frequency figures are derived, the expected total damages from flooding for each level of investment then can be derived in the same way as was done in Table 1 above. Aggregating the expected total damages and the total costs of investment (including disturbance costs) and choosing the lowest aggregate level determines the optimal investment. This is illustrated in Table 2 for a hypothetical case. For the purpose of saving space, only four levels of investment are distinguished: zero, low, medium and high. In an actual analysis, it would be advisable to use many more levels.

Next, multiply the frequency figures (from Table 2) with the total damage figures (second column in Table 1) to obtain expected total damage under alternative levels of investment as shown in Table 3.

Next, convert the expected total damage for the current year (in the last row of Table 3) into ones for all future years from the commencement year of flood mitigation in present-value terms, using Equation (6). Enter these figures into Table 4, together with the figures for the total costs of investment and disturbance (also expressed in present-value terms).

The medium level of investment is chosen as it minimises the aggregate sum of all costs. A sensitivity analysis should also be undertaken to see how the optimal level of investment varies with changes in parameters (especially the growth rate in flood damage $g$ and the rate of discount $r$).

4. Concluding Remark

A framework for analysing the costs and benefits to determine the optimal amount of investment has been provided in this article. Since the relevant investment is typically substantial, an important problem is how it should be financed in order to be consistent with the objectives of efficiency and financial viability. This problem is tackled in a subsequent paper (Ng 1991).

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Endnote

1. A more complete depiction would show a third axis measuring the benefits of drainage (related to the level of investment).

References

Ng, Y.-K. 1992, 'The older the more valuable: Divergence between utility and dollar values of life as one ages', *Journal of Economics (Zeitschrift für Nationalökonomie)*, vol. 55, no. 1, pp. 1–16.