RELATIVE-INCOME EFFECTS AND THE APPROPRIATE LEVEL OF PUBLIC EXPENDITURE

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I. Introduction and general discussion

THE ISSUE of relative-income effects (that the welfare\(^1\) of an individual depends not only on his absolute income but also on his income relative to that of others) and the issue of the appropriate level of public expenditure have both attracted considerable attention.\(^2\) However, to my knowledge, the implication of the former on the latter has never been discussed. This paper shows that the existence of relative-income effects may mean that the optimal level of expenditure on public goods may be higher than indicated by using \(\sum \text{MRS} = \text{MRT}\). To avoid possible misunderstanding, it may be noted that the suboptimality of the traditional formula is due to the pre-existence of relative-income effects, not due to possible changes in relative incomes created by the public expenditure or the financing of it. That public finance and expenditure may entail a redistribution of income is well-known. Thus, to concentrate on the relationship between (pre-existent) relative-income effects and the optimal level of public expenditure, let us abstract away the problem of the financing of the public expenditure and the associated distributional effects, excess burden, etc.

When I first connected the two issues, it seemed to be straightforward that the existence of relative-income effects implies that the Pareto-optimal level of expenditure on public goods should be higher. This was based on simple reasoning. Pure public goods are available to all individuals and hence may be excluded from individual-specific income levels for the purpose of reckoning with relative-income effects. Resources used to produce private goods not only confer benefits on consumers but also...
impose external costs on others through the relative-income effects. This type of external costs does not apply to public expenditure, at least in our simple model where all public expenditure is on pure public goods and possible differences in preferences regarding different public goods are abstracted away (thus enabling us to treat public expenditure as involving a single dimension). The presence of relative-income effects will thus make the optimal level of private expenditure (which produces external costs) smaller.

As I analyse the problem more formally (next section), it becomes less simple than the above paragraph suggests. Private goods provide utility to a consumer as they have intrinsic consumption effects (food prevents hunger, provides energy, etc.) and, in the presence of relative-income effects, also increase his utility as they increase his relative income. Thus, while the presence of relative-income effects increases the external costs of private goods, it also increases their internal benefits. It does not necessarily follow that relative-income effects reduce the optimal level of private expenditure and increase that of public expenditure.

However, the usual method of estimating the optimal level of public expenditure (by \( \Sigma MRS' = MRT \), or \( \Sigma MV' = MC \)) may lead to a sub-optimal level. In estimating the marginal benefit of public expenditure relative to that of private expenditure, the latter (the marginal benefit of private expenditure) is likely to be taken to include the intrinsic consumption effects and the internal or direct relative-income effects (as these two things taken together constitute the worth of a private good as it appears to each individual) but not to include the external or indirect (through reduction in the relative incomes of others) relative-income effects. It is true that perfectly rational and informed individuals or a central authority that understand the argument of this paper will not make such a mistake, but I doubt that this is the case in real-life economies at the moment. If I am correct in assuming this, then the following interesting consideration suggests itself. As an economy grows in terms of per capita income, relative-income effects are likely to become more important relative to intrinsic consumption effects. The above-mentioned bias or mistake in estimating the optimal level of public expenditure may thus become more important. Recently, there has been a widespread concern about the huge share of public expenditure in GNP (which may be affected by factors other than the estimate of the optimal level of public expenditure). Recognizing the above bias, I wonder whether this concern has been well-founded. Since public expenditure is unlikely to be optimally distributed among different items and also unlikely to be provided in the most efficient way, one can

3 When per capita income is low, people are more concerned with getting fed and clothed, etc., i.e. more concerned with intrinsic consumption effects. As per capita income increases, most basic needs are met and more people attempt to derive satisfaction from having cars that are more modern than their neighbours' and jewelleries that are more expensive than their sister-in-law’s etc.
easily point to instances of wastes and over-supply even if the over-all level of public expenditure is not excessive. But when one comes to consider areas where public goods are inadequately provided, one may have a second thought in agreeing with those who believe that we have an excessive level of public expenditure. (This does not imply that certain areas under the public sector now could not be more efficiently transferred to the private sector.) Of course, many other factors (including the possible inefficiency of public enterprises, empire-building behaviour, burden of taxation, etc) have to be brought into consideration in assessing the appropriate level of public expenditure. What this paper does is to bring forth one factor that has not been adequately considered so far.

II. Analysis

While we loosely call the expenditure on public goods public expenditure, our argument applies strictly only to Samuelsonian public goods. Many governmental expenditures are transfer payments or public spending on private goods (some of which may have some aspects of public good or external effects). They do not qualify as public expenditure (defined here as expenditure on public goods). The latter includes things like defence, basic research, environmental protection, space exploration, etc. These items benefit the nation (perhaps in the future) as a whole. They do not constitute the private income of any specific individual and hence do not involve the relative-income effect. Government expenditures on things like education fall somewhere between purely public and private goods and thus will be abstracted away for simplicity. Also for simplicity, we take public expenditure to be unidimensional and lump all private goods into a composite good. The utility functions of individuals may thus be written as

\[ u^i = u^i(x^i, G, R') \quad (i = 1, \ldots, s) \]  

where \( x^i \) is the amount of the composite private good consumed by individual \( i \), \( G \) is the level of public expenditure, and \( R'^i = x'^i / X \) where \( X = \sum_{i=1}^{s} x^i \). Obviously, \( R'^i \) is the relative (private-good) income share of individual \( i \).

To derive the Pareto optimality conditions, we maximize, without loss of generality, the utility of individual 1 subject to the constancy of the utilities of all other individuals (\( u^i = \bar{u}^i, i = 2, \ldots, s \)) and to the production possibility constraint

\[ F(X, G) = 0 \]  

\[ \text{For the case of } n \text{ private goods, see the pre-publication version of this paper (Ng 1985b). The same result is obtained by the same analysis except that the apparent complication of the paradox of universal externality (Ng 1975) has to be overcome.} \]
We may write the relevant Lagrangean function as

\[ L = \sum_{i=1}^{n} \lambda^i (u^i(\cdot) - \tilde{u}^i) - \theta F(\cdot) \]  

(3)

where \( \theta \) and the \( \lambda^i \)’s are Lagrangean multipliers and \( \lambda^1 \equiv 1, \tilde{u}^1 \equiv 0 \). Assuming an interior solution, the first-order conditions are

\[ \lambda^i (u^i_x + u^i_R/X) - \sum_{j=1}^{s} \lambda^j u^j_{Rx}/X^2 = \theta F_X \quad (i = 1, \ldots, s) \]  

(4)

\[ \sum_{i=1}^{s} \lambda^i u^i_G = \theta F_G \]  

(5)

where \( u^i_x = \partial u^i/\partial x^i, \ u^i_R = \partial u^i/\partial R^i, \ F_X = \partial F/\partial X, \) etc.

An increase in \( x^i \) reduces the income shares of other individuals. This accounts for the summation term in (4). From (4) and (5), we have

\[ \sum_{i=1}^{s} \frac{u^i_G}{u^i_x + u^i_R/X} = \frac{\theta F_G}{\theta F_X + \sum_{j=1}^{n} \lambda^j u^j_{Rx}/X^2} \]  

(6)

In comparison with the standard optimality condition for public goods, (6) has an extra term added to the denominator on the L.H.S. and an extra one added on the R.H.S. The latter is a form of external diseconomy and serves to reduce the optimal amounts of private goods and hence to increase the optimal amount of public expenditure. The one on the L.H.S. is the extra internal effects of private good consumption as the latter (private good consumption) adds to the income of the individual concerned and hence does not only make him better off due to its intrinsic utility through \( u^i_x \) but also through the relative-income effect \( u^i_R \). We cannot thus conclude that the appearance of relative-income effects serves to increase the optimal amount of public expenditure.\(^5\)

To analyse the problem further, let us adopt an alternative formulation. We may write the utility function of an individual as

\[ u^i = u^i(x^i, G, X) \quad (i = 1, \ldots, s) \]  

(7)

where an increase in aggregate income \( X \), given \( G \) and \( x^i \), decreases his utility, i.e. \( u^i_X < 0 \).

Using (7) instead of (1), we have, instead of (6),

\[ \sum_{i=1}^{s} \frac{u^i_G}{u^i_x} = \frac{\theta F_G}{\theta F_X - \sum_{i=1}^{s} \lambda^i u^i_x} \]  

(8)

\(^5\) A referee suggests that, if each individual utility functions are Cobb–Douglas with equal exponents across all individuals (not necessarily across all goods), and if changes in the public output is financed/dispursed in such a way as to leave individuals just as well off in terms of direct consumption benefits, the optimal \( G \) stays exactly unchanged.
In comparison with the traditional conditions, an additional positive term \(-\sum_{i=1}^{s} \lambda u'_x\) is added to the denominator on the R.H.S. This seems to indicate that public expenditure should be higher than in the absence of the relative-income effects. However, a caution is in order. An increase in \(x^i\) increases the relative income of the individual concerned and may make him better off apart from the intrinsic consumption effect. This internal relative-income effect is captured by the term \(u'_R/Y\) in the denominator on the L.H.S. of (6) above but is included in \(u'_x\) in (8). Thus, it is incorrect to infer from (8) that public expenditure should be higher in the presence of relative-income effects.

What is valid to infer from (8) is that the optimal level of public expenditure should be more than what is indicated by \(\sum MRS^i = MRT\) if \(\sum MRS^i\) is evaluated according to \(\sum u'_G/u'_x\) and MRT by \(F_G/F_X\). That \(F_G/F_X\) represents MRT or (relative) MC (marginal cost) is straightforward. Whether \(\sum MRS^i\) or \(\sum MV^i\) (marginal valuation) is or is not evaluated according to \(\sum u'_G/u'_x\) depends on the case in question. To discuss this, it is convenient to derive a different form of (8):

\[
\sum_{i=1}^{s} \frac{\lambda' u'_G}{\lambda' u'_x + \sum_{i=1}^{s} \lambda' u'_x} = \frac{F_G}{F_X} \tag{8'}
\]

Now, \(\sum MV^i\) may be estimated centrally or by surveys of individual preferences (or perhaps by the Clark–Groves incentive compatible mechanism in the future). A perfectly rational and informed individual will not take account of just his marginal utility from public expenditure, \(u'_G\), and his marginal utility from his private expenditure, \(u'_x\), (which includes, in this formulation, both the intrinsic consumption effect and the internal or direct relative-income effect), but will also take account of the reduction in aggregate private income \(X\) as \(G\) increases and the implication of this on his utility through the external or indirect relative-income effects. If everyone is perfectly informed and rational in this way, the reported \(\sum MV^i\) will not just be confined to \(\sum u'_G/u'_x\) but will also reflect the term \(\sum \lambda' u'_X\). A non-optimal level of public expenditure need not therefore arise. However, it may be the case that a significant proportion of the individuals concerned fail to take adequate account of the indirect relative-income effects (i.e. the relative-income effects that work indirectly through the increase in \(G\) that reduces \(X\)), though they may take full or more adequate account of the direct relative-income effects (which operate through the reduction in each \(x^i\)).

Upon reflection, this is quite likely to be the case. Before I wrote this paper, if someone had asked me my valuation of a certain public project, I would no doubt have estimated the direct utility which I expected to derive from the project relative to the marginal utility of my private expenditure (which had been influenced to some extent by the relative-income effects)
Correct $\Sigma MV^i = \frac{\Sigma \lambda u_G^i}{(\lambda u_X^i) + \Sigma \lambda u_X^i}$

Conventional $\Sigma MV^i = \Sigma u_G^i / u_X^i$

$MC = \frac{F_G}{F_X}$

$G$ (expenditure on public goods)

Fig. 1

but I would not have included the indirect effect of the public project through its reduction of aggregate private expenditure and the corresponding relative-income effects on my utility. For one thing, the marginal utility of my private expenditure had *already* been shaped by relative-income effects while the indirect relative-income effects would operate only in the future. For another thing, one is direct and the other indirect.

It is thus likely that the reported $\Sigma MV^i$ on public expenditure will fall below the true $\Sigma MV^i$ due to the tendency for the negative term $\Sigma \lambda u_X^i$ in the denominator of $(8')$ to be ignored. The level of public expenditure thus determined will then fall short of the optimal level, as illustrated in Fig. 1.

The extent to which underestimation of the benefit of public expenditure will result depends also on the way the question is put. Usually, it may be put in such forms as, "What is the maximum amount you are willing to pay for project $X$?" or "What is your estimated benefit that you will obtain from the undertaking of project $X$?" In such forms, I believe that most people will not have included the full indirect effects and hence underestimation will result. This is so because one is then prone to compare the marginal utility of private expenditure (influenced by relative-income effects) with the direct utility of the public project only. On the other hand, if we ask, "Given that others (suitably defined) will pay a similar proportion of their income, what is the . . . ?", it is more likely that a higher proportion of
respondents will take the full indirect effects into account. Ideally, of course, if all respondents are made to read and understand this paper, no underestimation may be involved. In practice, a substantial degree of underestimation cannot be avoided. Our analysis suggests that at least a careful wording of the relevant question should be used to minimize the underestimation.

If the estimation of $\Sigma MV'$ is done centrally, a similar inadequate consideration of the indirect relative-income effects may also occur. It may be thought that, if the central estimator does not include the indirect relative-income effects $\Sigma \lambda' u_x$, he will also exclude the direct relative-income effect from $u'_x$, such that the estimated $\Sigma MV'$ is not necessarily biased downward. However, this need not be the case. The central estimator is likely to estimate the MV placed on public expenditure relative to that placed on private expenditure by individuals. In this estimate, he may include the direct relative-income effects in the MV of private expenditure since this is the amount on which individuals appear to place their valuation. But in his estimate of MV of public expenditure, he may just take account of $\Sigma u'_o$, without taking account of the indirect relative-income effects. If this is the case, underestimation of the relative value of public expenditure will result.

In the analysis above, (private) incomes are ‘full’ incomes inclusive of leisure times. What if individuals care only about relative incomes exclusive of leisure? Two differences (in comparison with the above) may be noted. First, if we use leisure as the numeraire in the estimation of the marginal valuation of public expenditure, no sub-optimal estimation will result from ignoring the indirect relative-income effects. (But the over-consumption of private goods remains; see below.) This is so since leisure does not have the external costs through the relative-income effects. However, since actual estimates of the values of public goods are in monetary and not in leisure terms this point does not affect the sub-optimal estimate of public expenditure argued above.

Secondly, not only does public expenditure tend to be sub-optimal, leisure consumption does too. I only say ‘tends to be’ because the problem is basically an over-consumption of private (non-leisure) goods due to the external costs. Public expenditure and private leisure together will be under-consumed. But we cannot be absolutely sure that each of the two items will be under-consumed without looking at the interrelationships between the three items. For example, one can imagine a case where leisure is very complementary to private goods but not to public goods. Then the over-consumption of private goods may indirectly cause the over-instead of under-consumption of leisure. One way or the other, the problem can be taken care of by imposing a tax on the item that produces external costs, an income tax in this case. Income taxes may thus be justified purely on efficiency grounds. (See Duesenberry 1949, Ch. 6 and Layard 1980, p. 738 on this same point. For a different ‘third-best’ efficiency ground of income
taxes due to the prevalence of specific external costs, see Ng. 1979, pp. 239–40. On yet another efficiency ground of taxation due to the defence burden of capital accumulation, see Thompson 1974.)

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REFERENCES


