The purpose of this paper is to comment on a recent paper by Frank (1970).

Figure 1 is based on Frank's figure 3, where the axes measure the face value of a loan $L$ and the rate of interest on the loan $r$. The opportunity cost of capital in the donor is constant at $p$. The opportunity cost of capital in the recipient country declines along $q(L)$ as more is lent because of declining returns to capital, but never falls below $p$.¹

The curves $C_1D_1$ and $C_2D_2$ represent iso-benefit curves to the recipient. $A_1B_1$ is an iso-benefit curve for the donor (negative grant value), and $A_2B_2$ is an iso-cost curve for the donor (positive grant value).² The curves $A_1B_1$ and $C_1D_1$ have a point of tangency at $P_2$. Similarly, $A_2B_2$ and $C_2D_2$ have a point of tangency at $P_1$. The set of all such points of tangency forms a contract curve $EF$. The curve $EF$ represents combinations of terms of lending and amounts of lending which are Pareto optimal. That is, both donor and recipient are better off at some point on the contract curve than at a point off the curve.” [Frank 1970, p. 1110]

The first mistake of the above argument is that the curve, $C_1D_1$ cannot bend backward (have a negative slope) as drawn in figure 1. The reason is simple. Below the line $q(L)$, the recipient would like to get more loan at given rate of interest. Compare the point $P_3$ with $P_2$. Since $P_3$ involves a larger loan at lower interest rate, the recipient must be better off at $P_3$ than at $P_2$. Hence $P_2$ and $P_3$ cannot be on the same indifference curve. To maintain indifference, a point of higher $L$ must involve higher $r$, hence the indifference curves must be positively inclined. And since the iso-benefit curves (such as $A_1B_1$) of the donor must be negatively inclined, the two sets of curves cannot have point of common tangency. For the segment

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¹ "If the recipient always has the option of reinvesting any grant from the donor in the donor country, he may obtain a rate of return equal at least to the opportunity cost of capital in the donor country" (Frank 1970, p. 1109).

² The grant value of a loan is the face value of the loan less the discounted stream of payments of interest and principal.
of the figure to the left of the point $p$, the iso-benefit curves for the recipient and the iso-cost curves for the donor are all positively inclined. Hence, common tangency is possible here. There is, however, a different consideration which suggests that no contract curve can be derived even in this segment.

The cost to the donor is measured by $D = (p - r) \cdot L/p$ (Frank 1970, p. 1112). Since $p$ is a constant, the donor's indifference curves are rectangular hyperbolas, such as $A_1B_1$, $A_2B_2$, $A'_1B'_1$, $A'_2B'_2$ in figure 2. With respect to the recipient's indifference curves, it is my argument that their slope (within the segment $OMNp$) is everywhere smaller than the slope of the donor's indifference curve at the same point. The reason is simple. For any point within this segment, the benefit to the recipient is higher than the cost to the donor, as $q$ is greater than $p$. The marginal rate of substitution (absolute value) between $L$ and $-r$ will be greater for the recipient than the donor. For any point within $OMNp$, there is a point on (and also many points above, as will be seen shortly) $MN$ where both parties are better off. This is only to be expected, as capital has a higher return in the recipient country up to $N$.

The indifference curves will, therefore, be as depicted in figure 2. Above
$MN$, the two sets of indifference curves coincide because both countries have the same rate of returns to capital, $p$.

If we accept Frank’s assumption (p. 1112) that the recipient can always borrow a smaller amount at the same rate of interest, so that the segment above $NS$ is not sustainable, the contract curve/surface is the line $RN$ plus the whole surface above $MN$: $R$ is the point where the line $NS$, which is the same as line $q(L)$ in figure 2, touches the highest iso-benefit curve $(A_2B_2)$ for the donor. If we assume, instead, that the recipient has to face a package (amount of loan and rate of interest) deal, the segment above $NS$ is then relevant. In this case, the contract surface is that above the line $MNQ$ but to the left of the recipient’s indifference curve $D_0S$ which represents zero benefit to the recipient.

From the above analysis, it may be concluded that Frank is quite incorrect in drawing his contract curve $EF$ below the point $N$. According to our analysis, as $q > p$ before $N$ is reached, points below the line $MN$ cannot be Pareto optimal.

Reference