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WHY DO PEOPLE BUY LOTTERY TICKETS? CHOICES INVOLVING RISK AND THE INDIVISIBILITY OF EXPENDITURE

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I. THE PROBLEM

Under the assumptions that, first, the pleasures of gambling may be neglected and, second, the marginal utility of income is diminishing, it can easily be shown that fair gambling is an economic blunder.\(^2\) Thus, the fact that people engage in unfair as well as fair gambles is clearly inconsistent either with utility maximization or with the assumption of diminishing marginal utility. Marshall resolved this contradiction by rejecting utility maximization as an explanation of choices involving risk. Milton Friedman and L. J. Savage pointed out that Marshall need not have done so, and suggested the hypothesis that marginal utility successively decreases, increases, and decreases; the total utility-curve being concave from above in the neighborhood of actual income and convex from above for higher and lower incomes.\(^4\) While not challenging the validity of their analysis, this note suggests an alternative resolution of the problem, which rests on the indivisibility of expenditure and its influence on the utility function of the consumer unit.

II. INDIVISIBILITY OF EXPENDITURE AND THE UTILITY FUNCTION

The assumption that the marginal utility of income is continually diminishing is derived from the assumption that the expenditure of the consumer unit is infinitely divisible. In fact, consumption expenditures are not infinitely divisible; for example, while a person may choose between having one motor car or two, or even between having a Volkswagon and having a Mercedes 220, he cannot have half a car or one and one-third cars.\(^4\) The indivisibility of expenditures is a recognized fact, demanding qualification of the equimarginal principle; and its presence implies a peculiar behavior of the utility function.

Consider a student just graduating from high school and contemplating a university education, which constitutes an indivisible expenditure. Let \(OX\) in Figure 1 represent his income and expenditure, and \(OY\) represent the marginal utility of income to him.

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1 I am grateful to Larry Sjaastad for much detailed assistance in clarifying the presentation of the argument of this note.


4 It may be suggested that expenditure on an automobile may be made divisible by resorting to renting. The possibility of renting the services of indivisible goods may reduce the influence of indivisibility but in practice does not effectively eliminate it. Moreover, renting and owning are not identical forms of consumption; and imperfections in the capital market frequently debar the consumer from the possibility of renting.
Excluding the possibility of university education, his marginal utility function is represented by the continuously declining curve \( MNPQ \). Let the fixed cost of a university education be \( OH \) and its total utility to the student be \( JOHK \). Then if his income is high enough (higher than \( OI_3 \), where \( I_1I_3 = OH \) and \( I_1N = HK \)) he will certainly choose the university education, and his marginal utility-curve will seem\(^5\) to assume the path \( MNEF \), where \( NE = OH \) and is parallel to the \( X\)-axis. On the other hand, if his income is less than \( OI_3 \), he will not choose added to the income \( OI_2 \), the marginal utility of income will be greater than \( I_3C \), for this increment will enable the individual to purchase items of consumption previously forgone to permit payment of his educational expenses. If the satisfaction derived from these other items is assumed for simplicity to be independent of whether or not he goes to university, \( D \) will be at the same height as \( A \), and \( DEF \) will be a rightward shift of \( ANP \). The individual's marginal utility function will be \( MNPDEF \) if he purchases the indivisible good, university training; the

![Diagram](attachment:diagram.png)

**Fig. 1**

the university education, since he will derive greater utility from spending his money on other items of consumption.

Somewhere between \( OI_1 \) and \( OI_3 \) there exists a level of income at which he will be on the margin of indifference about choosing the university education; let this income be \( OI_2 \). If he does not choose university education, his marginal utility-curve will be \( MNP \); if he does, it will be \( MABC \). By going to university he gains utility measured by the area \( NPC \) and loses utility measured by the area \( ABN \), and by the definition of \( I_2 \) these areas are just equal.

For a marginal increment of income

\(^5\) It is shown below that the actual path is \( MPDF \), upward step from \( P \) to \( D \) is attributable to the utility derived from the consumption of this good.\(^6\)

If the good "university education" were perfectly divisible, expenditure on it would have a marginal utility diminishing continuously in the usual way, represented in the diagram by the curve \( G'LF'' \) (the area \( G'JL \) being equal to the area \( LF'K \)). The individual would begin to consume the good

\(^6\) Considering that \( BC \) is spent on university education, the marginal utility curve could be thought of as \( MABCDEF \); but since the marginal utility of income is \( PI^2 \) in the absence and \( DI^2 \) in the presence of the university education, it is more appropriate to take \( MNPDEF \) as the marginal utility-of-income curve.
at income $OI_0$ (where $G'O = GI_0$) and would consume one unit of it at income $OI_4$ (where $FI_4 = F'H$). His marginal utility-curve in this case would follow the dotted curve $MGUF$, being higher than $MNPDEF$ in the segment $UF$ (the area $GPU$ being equal to the area $UDF$).

An alternative way of constructing the marginal utility-curve in the presence of indivisibility has been suggested to me in correspondence by Larry Sjaastad, to whom I am indebted for the following three paragraphs:

Referring to Figure 1, let $MGF$ be a segment of the marginal utility function, with income on the horizontal axis, when there are no restrictions on the manner in which the consumer allocates his income, that is, there is no indivisibility. Now let good $X$ be indivisible, and let $Y$ represent all other goods which are assumed to be perfectly divisible. Assume that there are at least two income levels, $I_0$ and $I_4$, such that indivisibilites do not matter—that at $I_0$ the consumer would demand exactly $N$ units ($N$ may be zero) of good $X$ if it were perfectly divisible, and that at $I_4$ he would demand exactly $N + 1$ units of $X$ if it were perfectly divisible. Thus, his marginal utility function must pass through points $G$ and $F$ even when indivisibilities are introduced. It will not follow the smooth curve from $G$ to $F$ as income is increased from $I_0$ to $I_4$, but rather the broken curve $GPDF$.

For simplicity, let us define our units such that the prices of all divisible goods are unity. In equilibrium then, the marginal utility of income will be exactly equal to the marginal utility of the goods which the consumer can vary. If the marginal utility of income is declining, the marginal utility of all goods must decline as they are increased in quantity relative to other goods. If the contrary were true—if even one good had increasing marginal utility—an increase in income would lead to increased consumption of that good and a reduction in the consumption of all others, and the marginal utility of each good would have increased, hence the marginal utility of income would have to be increased. Now in terms of Figure 1, as income is increased from the initial level of $I_0$, the consumer will initially spend all of that increase on good $Y$ (when good $X$ is indivisible), which causes the marginal utility of $Y$ to fall more rapidly than if he allocated part of his additional expenditure on $X$ as well. By our assumptions, the marginal utility of income is equal to the marginal utility of $Y$. Hence if $GF$ describes the marginal utility of income when both goods can be varied continuously, then marginal utility is described by something like $GP$ when $X$ can be varied only discretely. The area $GPU$ is the loss of utility caused by the indivisibility.

At some point between $I_0$ and $I_4$, income will be just large enough that the utility of consuming another unit of good $X$ is just equal to the utility associated with the units of $Y$ which must be given up to buy $X$. Just beyond that point, the individual will change his consumption pattern—he will buy another unit of $X$ and will consume less $Y$. As a result, the ratio of $Y$ to $X$ is sharply reduced, and consequently the marginal utility of $Y$ is sharply increased. Since the marginal utility of $Y$ is also the marginal utility of income, we have the vertical segment $PD$ in Figure 1. Moreover, point $D$ must lie above the curve $GF$, because at the income level just beyond $I_2$, the consumer is taking more $X$, hence less $Y$, than if $X$ were divisible. That is, his consumption of $X$ at income $I_2$ is the same as it would be at the larger income $I_4$ when $X$ is divisible. Thus his consumption of $Y$ must be less at the income level just beyond $I_2$ than is implied by the smooth curve $GF$, hence $Y$ has a larger marginal utility at this income level when $X$ is indivisible; therefore, the marginal utility of income just beyond income $I_2$ is actually greater in the indivisible case. Moreover, this remains true until income reaches $I_4$. The gain in utility is the area $UDF$, which is exactly equal to the area $GPU$ because total utility at income $I_4$ is unaffected by the indivisibility of $X$, as has...
been asserted above. The slope of the segment $DF$ must be greater than the corresponding segment of $GF$ for the same reason that the slope of $GP$ is greater than the corresponding part of $GF$—all increases in income are being spent on $Y$.

Thus the introduction of indivisibility of expenditure gives rise to peculiar behavior of the marginal utility of income function, the smooth curve $GF$ being converted into the broken curve $MNPDEF$.

III. GAMBING AND THE INDIVISIBILITY OF EXPENDITURE

Following Friedman and Savage, choices involving risk may be represented by the buying of fire insurance, that is, “the certain loss of a small sum (the insurance premium) in preference to the combination of a small chance of a much larger loss (the value of the house) and a larger chance of no loss,” and the purchase of a lottery ticket, that is, “a large chance of losing a small amount (the price of the lottery ticket) plus a small chance of winning a large amount (a prize) in preference to avoiding both risks.” In the case of insurance there is no contradiction between utility maximization and diminishing marginal utility, but in the case of lotteries and other forms of gambling there is such a contradiction if the pleasure of gambling is ignored, as is reasonable in the case of a lottery. The question is: Why do people buy lottery tickets? The analysis of the previous section provides a possible explanation.

Consider the example of the high-school graduate previously discussed, and assume that his ordinary income is insufficient to pay for a university education, but that he would go to university if he won a lottery prize. Let his situation be presented by Figure 2, where his ordinary income is $OX_1$, less than the critical level $I_2$; and let $X_1X_2$ be the price of a lottery ticket, and $X_2X_3$ be the (single) prize in the lottery.

If he is not going to enter university, then in terms of total utility he is gambling the area $A$ against a small chance of winning the area $B_1 + B_2$. Alternatively, if all goods including the university course are divisible, he is gambling the area $A + A'$ against a small chance of winning $B_1 + B_2 + B'_1 + B'_2$. In either case, since marginal utility is diminishing, he would not be behaving according to the rules of utility maximization if he bought the lottery ticket (assuming that the lottery is no better than fair). If,
however, the expenditure on university education is indivisible, he is gambling $A$ against a small chance of gaining $B_1 + B_2 + B'_2 + C$; and it may pay him to do so even if the lottery is less than fair.

This can be shown by a mathematical example. Suppose that the student's marginal utility function (the curve $GPQ$) excluding the possibility of university is $y = f(x) = 100/(x + 1)$, his existing income 25 units, the minimum cost of a university education 30 units, and the total utility of the education 150 units; the price of a lottery ticket 1 unit, and the single prize of the lottery 43.6 units. Under these assumptions, the areas $A, B_1, B_2, B'_2, C$ in the diagram can be calculated to be $^7$

$$A = \int_{25}^{35} \frac{100}{x + 1} \, dx = 100 \log 26 - \log 25 = 39.2,$$

$$B_1 = \int_{25}^{35.6} \frac{100}{x + 1} \, dx = 100 \log 38.6 - \log 26 = 39.5,$$

$$B_2 + B'_2 + C = \int_{7.6}^{35.6} \frac{100}{x + 1} \, dx = 100 \log 38.6 - \log 8.6 = 150.2,$$

$$\frac{A}{B_1 + B_2 + B'_2 + C} = \frac{3.92}{150.2 + 39.5} = \frac{3.92}{189.7} = \frac{1}{48.6}.$$

Hence if the chance of winning is better than $1/48.6$, it will pay the student to buy the lottery ticket.

Thus it has been shown that the introduction of indivisibility of expenditure permits the rationalization of gambling according to the principles of utility maximization, while maintaining the assumption of diminishing marginal utility of consumption.$^8$

Acceptance of the indivisibility of expenditure illuminates the analysis of behavior in the face of risk. In the first place, the fact that many people both buy insurance and gamble, that is, choose certainty and at the same time subject themselves to risk, can be explained without either destroying the assumption of utility maximization or introducing the supplementary assumption of increasing marginal utility of income over a certain range of income variation.

Second, indivisibility of expenditure explains why lotteries generally have several or many prizes. The Social and Welfare Services Lottery in Malaya, for example, offers the following prizes.$^9$

- First prize ............... $375,000
- Second prize ............. 125,000
- Third prize ............... 60,000
- Fourth prize .............. 30,000
- 5th prizes, each ........... 10,000
- 15th prizes, each .......... 5,000
- 30th prizes, each .......... 3,000
- 40th prizes, each .......... 2,000
- 75th prizes, each .......... 1,000
- Gift, each .................. 500
- Lucky, each ................ 250
- Consolation, each .......... 100

$^7$ The critical income level $I^2$ is obtained by solving the following equation:

$$\int_{I-30}^{I} \frac{100}{x + 1} \, dx = 5 \times 30$$

$$[\log (I + 1) - \log (I - 29)] = 1.5$$

$$I + 1 \over I - 29 = e^{1.5},$$

$$I = 37.6.$$

$^8$ The departure from behavior in accordance with this assumption is the consequence of the assumed indivisibility of expenditure and is not inherent in the nature of utility itself.

$^9$ A Malayan dollar is worth approximately one-third of an American dollar.
The explanation of the offer of multiple prizes suggested by indivisibility of expenditure is, first, that for most people the amount of expenditure involved is substantially smaller than the total sum to be paid out; second, that different potential purchasers of tickets have different indivisible expenditure totals in mind, and even the same individual may have different sums in mind for different uses.¹⁰

Third, expenditure indivisibilities help to explain the fact that lottery tickets are eagerly bought by the poor, and the "numbers game" and similar gambles flourish especially among the lower-income classes; for the effect of indivisibility of expenditure manifests itself most strongly in the case of low-income consumer units. The cost of purchasing a car, a house, a university education, or a business appears far beyond the means of a poor family, unless the money is obtained by gambling; but the same expenditure is only a small fraction of the income of the rich and raises little problem of indivisibility.

IV. COMPARISON WITH THE FRIEDMAN-SAVAGE HYPOTHESIS

The analysis presented here resembles that of Friedman and Savage in some respects, but differs from it in others. The relationship between the two may be clarified by reference to Figure 3, in which income is measured along the X-axis and total utility along the Y-axis. The Friedman-Savage utility function is represented by the curve ONQ, the interpretation of which is "to regard the two convex segments as corresponding to qualitatively different socioeconomic levels, and the concave segment to the transition between the two levels. On this interpretation, increases in income that raise the relative position of the consumer unit in its own class but do not shift the unit out of its class yield diminishing marginal utility, while increases that shift it into a new class, that give it a new social and economic status, yield increasing marginal utility."¹¹

![Figure 3](image-url)

The total utility function of the present analysis is represented by OKQ; the kink at K corresponds to the broken marginal utility-curve of Figures 1 and 2, and is caused by the existence of indivisibilities rather than inherent in the nature of utility. It differs in shape from the Friedman-Savage function in that the latter is a smooth curve with no kink. The curve OPQ represents the orthodox total utility function, which cannot explain gambling behavior.

¹⁰ In addition, of course, the offer of many prizes may lead the potential purchaser irrationally to believe that his chances of winning are higher than they really are; or the offer of prizes whose magnitudes match the amounts that different individuals would consider a fortune may exercise a psychological appeal.