Theoretical modelling and experimental identification of nonlinear torsional behaviour in harmonic drives

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Abstract

The demand for accurate and reliable positioning in industrial applications, especially in robotics and high-precision machines, has led to the increased use of harmonic drives. The unique performance features of harmonic drives, such as high reduction ratio and high torque capacity in a compact geometry, justify their widespread application. However, nonlinear torsional compliance and friction are the most fundamental problems in these components, manifesting themselves as a combination of stiffening spring together with hysteresis at reversal points. Accurate modelling of the static and dynamic behaviour is expected to improve the performance of the system.

This paper offers a model for torsional compliance of harmonic drives. A statistical measure of variation indicates the reliability of the proposed model. Two test setups have been developed and built, which are employed to evaluate experimentally the behaviour of the system. Each setup comprises a different type of harmonic drive, namely the high load torque and the low load torque harmonic drive. The results show an accurate match between the simulation torque obtained from the identified model and the measured torque from the experiment, which indicates the reliability of the proposed model.

1. Introduction

Invented by Walton Musser in 1955, primarily for aerospace applications, harmonic drives are high-ratio, compact torque transmission systems. As shown in Fig. 1, this nascent mechanical transmission, occasionally labelled 'strain-wave gearing', employs a continuous deflection wave along a non-rigid gear, the so-called 'flexspline', to allow gradual engagement of gear teeth. Besides a thin-walled flexible cap with small external gear of the flexspline, a harmonic drive also contains two other important components, namely a wave-generator, which is a ball-bearing assembly with a rigid elliptical inner-race, and a circular-spline, a rigid ring with internal teeth machined along a slightly larger pitch diameter than that of the flexspline. When properly assembled, the wave-generator is nested inside the flexspline, causing the flexible gear-toothed circumference on the flexspline to adopt the elliptical profile of the wave-generator. When the wave-generator is rotated, the engagement of the external teeth of the flexspline to the internal teeth of the circular spline will cause highly reduced rotation of the circular spline. Through this unconventional mechanism, gear ratios up to 500:1 can be achieved in a single transmission step. Fig. 2 illustrates the working principle of harmonic drives.

Under ideal assumptions, a harmonic drive transmission is treated as a perfectly rigid gear reduction. However, due to the relatively low torsional stiffness of harmonic drives, a more detailed understanding of the transmission flexibility is often required for accurate modelling. As described in a manufacturer’s catalogue [1], the typical shape of the stiffness curve consists in two characteristic properties: (i) increasing stiffness with displacement and (ii) hysteresis loss. To capture this nonlinear stiffness behaviour, the manufacturers suggest using piecewise linear approximations [1,2] whereas several independent researchers [3,4] prefer a cubic polynomial approximation. Even though in certain applications, a linear stiffness approximation can be adequate to model the behaviour of the stiffness [5].

The hysteresis loss in a harmonic drive is a phenomenon more difficult to model than the stiffness, yet sometimes it is ignored. Taghirad [6] proposed an advanced hysteresis model of torsional stiffness in the flexspline. He assumed that the hysteresis mainly arose from the structural damping of the flexspline. Seyffarth et al. [7] offered to model the hysteresis as a combination of Coulomb friction and a weighted friction function, represented by a hyperbolic function to capture the pre-sliding friction behaviour.
However, all researchers noted the inherent difficulties in finding an accurate model for the torsional stiffness of a harmonic drive. A unique behaviour of the hysteresis in the torsional stiffness of harmonic drive is investigated by Dhaouadi and Ghorbel [8], resulting in the conclusion that the hysteresis in the harmonic drive is indeed inherent (a hysteresis phenomenon such as that in magnetic and electric fields, see [9]) since angular displacement does depend on the previous deformations undergone by the drive. This conclusion leads us to acknowledge the existence of the unique property of non-local memory hysteresis in pre-sliding motion [10–12].

Recently, Preissner et al. [13] developed a more comprehensive, black-box model that focuses in particular on the hysteresis by using the Maxwell resistive-capacitor model. Their results confirm our findings [14,15] that nonlocal memory hysteresis is essential when developing a high-fidelity model for a harmonic drive.

The objective of this paper is to develop a grey-box model for the harmonic drive, perform parameter identification, and finally carry out experimental validation. Specifically, the proposed mechanical model allows for nonlinear torsional compliance, pre-sliding friction in the tooth engagement area giving rise to hysteresis effect, and macroscopic friction in the wave-generator and the motor. The focus is laid on simplified modelling, which however includes all the essential aspects of a harmonic drive. This model is therefore relevant for dynamic identification and control purposes.

In the following, Section 2 formulates the detailed mechanical model of the system and discusses the existing torsional stiffness model. Section 3 discusses the identification of the torsional stiffness in the low-torque class of harmonic drive, while Section 4 deals with the identification of a high-torque harmonic drive, which are then verified in the last part of the section. Finally, appropriate conclusions are drawn in Section 5.

2. Mechanical model

Following [15], the goal of modelling the harmonic drive system is to accurately describe the most essential phenomena with the simplest representation. This modelling starts from the assumption that under normal operation the effect of individual gear teeth can be ignored, in favour of stochastically averaged behaviour. This assumption is made based on the fact that anywhere from ten to more than fifty teeth can be in contact between the circular spline and flexspline. Consequently, in order to capture the cumulative effect of gear tooth meshing, a model, which describes sliding on a single continuous inclined plane [16], can be used. Based on this assumption, a harmonic drive model can be simplified by using translational motion model analogy (rather than rotational motion) as shown in Fig. 3.

The model consists of some planes, which represent the wave generator and gear tooth geometry of the actual transmission, and a linear spring representing torsional stiffness of flexspline in radial direction. Assuming that the surface of each component model is infinitely long, this model can be used for all ranges of transmission operation.

Comparing the actual harmonic drive to the model, two analogies can be drawn. First, horizontal motion in the model corresponds to tangential movement of the harmonic drive components, while vertical motion corresponds to the involute direction. Second, the slopes of the two inclined planes in the top and the bottom of the figure represent the transmission ratio of the harmonic drive.

Fig. 1. Harmonic drive components (reproduced from [1]).

Fig. 2. Operating principle of harmonic drives: the outer ring with internal teeth is the circular spline, the dark coloured flexible inner ring with external teeth is the flexspline, while the elliptical ball-bearing is the wave generator. The altering shape of the flexspline when the wave generator is rotated will cause reduced rotation of the circular spline. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 3. Schematic translation representation of the harmonic drive.
In commercial catalogues, every harmonic drive is assigned a transmission ratio, $N$. Specifically, given a known rotation of any two of the three main parts of the harmonic-drive as well as a value for $N$, the ideal rotation of the third part can be predicted by the equation:

$$\theta_{WC} = (N + 1)\theta_{CS} - N\theta_{FS}$$

where $\theta_{WC}$ is the rotation of the wave-generator, $\theta_{CS}$ is the rotation of the circular spline, and $\theta_{FS}$ is the rotation of the flexspline.

The torsional stiffness of the harmonic drives is represented by the nonlinear spring in combination with the frictional element between two inclined planes. The nonlinear spring has a hardening (i.e. increasing stiffness) property, while both of the two inclined planes cause hysteresis losses.

Specifically, Seyfferth et al. [7] assert that the hysteresis is attributed to the friction in the gear teeth meshing. In order to capture the behaviour of the hysteresis in the torsional stiffness of harmonic drives, they proposed to model that as the sum of hardening spring, $T_0(\Delta \theta)$, and hysteresis function of torsion angle, $T_h(\Delta \theta)$.

The nonlinear spring, $T_0(\Delta \theta)$, can be approximated by a third order polynomial function of the torsion angle:

$$T_0(\Delta \theta) = a_3(\Delta \theta)^3 + a_1 \cdot \Delta \theta$$

or a piecewise linear function of the torsion angle:

$$T_0(\Delta \theta) = \begin{cases} 0 & |\Delta \theta| \leq \theta_0 \\ k_1 \cdot \Delta \theta - k_0 \theta_0 & |\Delta \theta| > \theta_0 \end{cases}$$

where $\Delta \theta$ is the relative angular motion between circular-spline, $\theta_{CS}$, and wave generator, $\theta_{WG}$, or flexspline, $\theta_{FS}$, depending on which part is fixed, while $\theta_0$ is the piecewise linear threshold.

In particular, Seyfferth et al. estimated the hysteresis losses in the torsional stiffness as a combination of Coulomb friction and a weighted friction function (see Fig. 4), represented by a hyperbolic function as:

$$T_h(\Delta \theta) = T_h^* + [T_C - T_h^* \text{sgn}(\gamma(\Delta \theta - \Delta \theta^*)) \cdot \tanh(\gamma(\Delta \theta - \Delta \theta^*))]$$

where $T_C$ is the Coulomb torque, $\gamma$ determines the shape of the hyperbolic curve, while $T_h^*$ and $\Delta \theta^*$ represent the last reversal point of the hysteresis feature.

### 3. Torsional stiffness in Wave-Drive® component

In order to evaluate the torsional stiffness, we developed a test setup utilizing a low torque type of harmonic drive of Wave-Drive® from Oechsler AG, which has a transmission ratio of 50:1. The schematic of the setup is shown in Fig. 5. The circular spline is fixed to the ground with a locked-load mechanism, while a shaker applies a low torque to the output shaft, which is connected to the flexspline, through a small lever arm. A Bentley probe is utilized to sense the peripheral displacement (yielding the rotational angle), while a load cell measures the applied load from the shaker to the output shaft. The shaker command is prescribed by a low-frequency periodic function, thereby the hysteresis behaviour of the torsional stiffness can be observed comprehensively. The stiffness profile thus obtained is shown in Fig. 6.

Observing the hysteresis behaviour from the resulting torsional stiffness, we conclude that the shape of the hysteresis in the torsional stiffness, $T_h(\Delta \theta)$, is attributed to the (pre-sliding) friction behaviour during the relative motion between the teeth in the contact area, attributed to the non-local memory property of the hysteresis. This unique property of hysteresis characterizes the signature of pre-sliding friction behaviour [10].

In order to capture the torsional behaviour of harmonic drives, incorporating the nonlocal memory hysteresis behaviour, the Maxwell-slip model is used. It consists of parallel connection of stop type operators [17] to model the frictional behaviour in pre-sliding regime [18–20]. The stop-like operator is based upon the elastic-plastic behaviour in continuum mechanics. This operator shows the linear stress–strain relationship as a per Hooke’s law, when the stress is below a certain yield stress, which corresponds to a maximum force that the system can sustain, $W_{n}$, and mathematically is represented by:

$$T_{n} = \min(W_{n}, \max(-W_{n}, k_n(x_t - x_{t-1}) + T_{n-1}))$$

where $n$ represents the nth element in the model and the subscript $t$ indicates the time step.
When the elementary model is displaced from the equilibrium state, initially it will stick and it behaves like a linear spring with stiffness of $k_n$ until it reach the maximum force of $W_n$. Beyond this point the elementary model will slip, where the force in the element is equal to $W_n$. The parallel connection of Maxwell slip elements is illustrated in Fig. 7.

In identifying the torsional stiffness of Wave-Drive® component, the torque and corresponding torsion measurement are carried out in a straightforward way. Ten thousand points of random excitation signal with cut-off frequency of 5 Hz, at a 1000 Hz sampling rate, are used. The model described in Fig. 7 is utilized for capturing the torsional stiffness behaviour of this transmission component. Four Maxwell-slip elements are used in this modelling (see further below for justification). By means of least-square estimation, a set of parameters consisting of three parameters of the piecewise linear spring ($k_0$, $k_1$ and $h_0$; see Eq. (3)) and four pairs of Maxwell-slip parameters ($j_i$ and $W_i$) are optimized utilizing the MSE cost function described by:

$$MSE(\hat{T}) = \frac{100}{M \cdot \sigma_T^2} \sum_{i=1}^{M} (\hat{T}_i - T_i)^2$$

where $T$ is the actual torque, $\sigma_T^2$ is its variance and the caret denotes an estimated quantity, while $M$ indicates the number of data.

This identification gives satisfactory result with MSE of 0.52% and maximum error (normalized by the standard deviation of the real torque) of 0.0549 Nm. As a benchmark, the piecewise linear model in Eq. (3) as suggested by the manufacturer’s catalogue and Seyfferth’s model [7] are used to capture the torsional stiffness. The prior model results in the best fit of 8.7% MSE and maximum error of more than 0.1 Nm, while the latter exhibits MSE of 1.38% and maximum error of more than 0.0653 Nm. The main difference between Seyfferth’s model compared to the proposed one is attributed to the absence of the ability to capture the non-local memory effect as Seyfferth approximates the hysteresis behaviour in the torsional stiffness by using a symmetric hyperbolic equation.

The modelling results are illustrated in Figs. 8–10 for the case of the piecewise linear model, Seyfferth’s model and the proposed model, respectively. The upper panels of the figures show the actual friction torque together with the estimated one from the models, while the lower ones depict the error between the measured and estimated torques. Table 1 lists the optimized parameters of the three different models, i.e. piecewise linear, Seyfferth and the proposed one.

We can conclude that the proposed model exhibits better performance compared to the piecewise linear model, which lacks the hysteresis effect. In particular, Seyfferth’s model is evidently inferior at the instances of the motion (and torque) reversal points, which are associated with the non-local memory effect of the hysteresis. The Seyfferth model approximates the hysteresis with a symmetric hyperbolic function, where the memory effect is neglected. As a validation, the measured stiffness profile as shown in Fig. 6 that is obtained from periodic excitation will be used for validation test of the identification result. Fig. 11 illustrates the resulting estimated torsional stiffness of the test setup. The error in the objective function MSE lies within 0.57% (0.018 Nm of maximum error), assuring the ability of the model to capture the stiffness profile.

In order to demonstrate the significance of the proposed model in capturing the nonlocal memory effect on the hysteresis, performances of various modelling attempts are presented in Fig. 12, which depicts the MSE value as a function of the model used. Seyfferth’s model and the proposed model (HMS – Hysteresis Maxwell-slip) with different number of elementary blocks are compared in the figure. The numbers following the HMS terms on the horizontal axis of Fig. 12 represent the number of Maxwell-slip elements used in the model (1, 2, 4, 6 and 10 elements), while the values on top of the bars represent the maximum error.
in Nm and the vertical axis represents the performance of the model in MSE.

It can be concluded that the proposed model, which considers the nonlocal memory property, is able to capture the torsional behaviour of the harmonic drive satisfactorily compared to Seyfferth's model. In addition, as the proposed model can be seen as a discrete model, the more elementary models taken on the model, the smoother the modelling result will be. However, there exist an optimal number of elementary models, for which the result is not improving significantly when the number of element is increased. The figure shows that the performance does not differ substantially between 6 elements and 10 elements taken in the model.

4. Torsional stiffness in the pancake harmonic drive component

The second setup uses a pancake harmonic drive type HDF40 from Harmonic-Drive® Technologies. This harmonic drive has a transmission ratio of 80:1, the maximum output torque is 192 Nm and the approximated wave-generator inertia is 3.43 kg cm\(^3\). The harmonic drive is driven by a DC motor type M2AA from ABB. This setup is equipped with two incremental encoders to measure the position on the input side and the output side after the reduction, while the current applied to the DC motor is measured in the servo amplifier. The encoder on the output side is connected to the shaft through timing belt transmission in order to increase the encoder sensitivity. These signals are processed and recorded by a dSPACE® data acquisition board. The left panel of Fig. 13 shows the schematic of the setup.

For the purpose of torsional stiffness identification the output shaft, which in this setup is connected to the circular spline, is locked and mounted to the ground, while the DC motor applies torque to the input shaft. The current applied to the motor is assumed to be proportional to the motor torque and will be used to construct the torsional stiffness.

The input shaft connected to the wave-generator is driven by the motor, in which the torque balance can be written as:

\[ T_m = T_I + T_f + T_0 \]  

Table 1

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Fig. 9. Torsional stiffness identification result using Seyfferth's model. The upper panel shows the actual and the modelled applied torque, while the lower one depicts the error of the model. This model gives 1.38% MSE.

Fig. 10. Torsional stiffness identification result using combined piecewise linear and Maxwell-slip model. The upper panel shows the actual and modelled torque, while the lower one depicts the error of the model. This model gives 0.52% MSE.

Fig. 11. Estimated stiffness curve of the Wave-Drive®.

Fig. 12. Quantitative performances of various models.

Table 1

The identified torsional stiffness parameters of the Wave-Drive®.

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The input shaft connected to the wave-generator is driven by the motor, in which the torque balance can be written as:

\[ T_m = T_I + T_f + T_0 \]
where $T_m$ is the motor torque generated by the amplifier current, $T_f$ is the inertia torque from motor armature and shaft, $T_b$ is the friction torque in the motor and $T_0$ is the load torque driving the input shaft to the harmonic drive.

The equation of motion for the input shaft can then be readily written as:

$$J_1 \ddot{\theta}_{CS} + [T_b(\Delta \dot{\theta}) + T_f(\Delta \theta, \Delta \dot{\theta})]/(N + 1) = T_0$$  \hspace{1cm} (8)

where $J_1$ is the inertia of the wave-generator including the input shaft.

The equation of motion of the output shaft is then,

$$J_2 \ddot{\theta}_{WG} + T_b(\Delta \theta, \Delta \dot{\theta}) + T_f(\theta_{CS}, \dot{\theta}_{CS}) = 0$$  \hspace{1cm} (9)

where $J_2$ is the output shaft inertia and $T_f$ represents the output bearing friction in the system. However, this paper focuses on the identification of the torsional stiffness of harmonic drive, $T_b(\Delta \theta) + T_f(\Delta \theta, \Delta \dot{\theta})$, and the output bearing friction is not considered, while $\theta_{CS}$ is relatively small as the circular spline is locked to the ground.

The angular displacement input of $\theta_{WC}$ is converted to the output side through the constant gear ratio, which in this work equals $N + 1$, as the flexspline is fixed, the gear deformation being then:

$$\Delta \theta = \frac{\dot{\theta}_{WC}}{N + 1} - \dot{\theta}_{CS}$$  \hspace{1cm} (10)

In order to measure the torque applied to the harmonic drive, the modelling of the DC motor has to be done satisfactorily. Mechanical modelling of the corresponding DC motor under the same condition has been performed and gives promising result [19]. The motor torque was identified as a combination of inertia, load, viscous and friction torque, which was modelled by the advanced model of friction, namely the Generalized Maxwell-slip (GMS) [20]. As a measure of performance, the identification of the motor gives 0.38% mean square error (MSE), where this quantity value will also be used to measure the performance of the harmonic drive model in this paper.

For the purpose of identification of torsional stiffness in the pancake harmonic drive component, a filtered-random signal with 1 Hz cut-off frequency, in order to minimize the influence of inertia, and 1000 Hz sampling frequency is input to the system for this identification. Ten thousand points (10 s) of input and output are collected for training purpose. Eqs. (2)–(8) are utilized to identify the torsional stiffness behaviour of the pancake harmonic drive. For constrained motion, where the output shaft is locked and mounted to the ground, the angular displacement output, $\theta_{CS}$, is set to zero.

By subtracting the measured torque applied to the motor, $T_{m}$, from the estimated inertia, $T_i$, and friction torque, $T_f$, utilizing the GMS model as described in [20], the torque applied to the transmission unit, $T_{in}$, can be obtained. The identification result is tabulated in Table 2 for all parameters of $T_b(\Delta \theta)$ and $T_f(\Delta \theta, \Delta \dot{\theta})$.

In order to verify the quality of the model structure and its parameters, another filtered random signal, with a different seed from the training set, has been applied to the system. The measured stiffness profile can be seen in Fig. 14, while Fig. 15 shows the modelled stiffness profile of the system. Both figures qualitatively show good fit, with performances of 1.59% for the MSE and normalized maximum error of 0.62.

### 4.1. Model validation

In order to validate the modelling scheme, simulations of the system under unconstrained motion (unlocked load) are developed (see right panel of Fig. 13). All of the individual models obtained from previous identifications are combined and merged into one integrated model for the assembled system. The integrated system model uses the measured angular displacements of a typical
experiment as an input to the simulation. As a measure of the model performance, the applied motor torque of the simulation is compared to that of the experiment.

A certain low inertial load \( J_2 \) is attached to the output shaft, and, for verification purpose, a low frequency periodic signal is applied to the system. Figs. 16 and 17 show a comparison between the real torque and the estimated torque when periodic signals are commanded to the system, assuming that the frictional torque of the output bearing, \( T_F \), is very low. Pure sinusoidal signals at 0.2 Hz and 1 Hz are applied to the system, respectively, equipped with low gain proportional feedback to avoid drift in the system. Note that the experimental signals are filtered by a fourth order Butterworth filter (with cut-off frequency 10% of its sampling rate) to minimize the torque ripples in the experiments.

The upper panels of both figures show the displacement output measured by the output encoders, the middle panels show the actual torques and estimated torques, while the lower panels depict the error between the actual and estimated torques. The results in both figures show a good match between the simulation and experiment of the motor torque. This indicates the ability of the simulation to predict the dynamic behaviour of the system.

However, the estimated torque of the low velocity experiment is less accurate compared to that of the high velocity, because the smaller velocity signal gives smaller signal-to-noise ratio. One possible source of noise in estimating the derivation of the angular displacement into velocity and/or acceleration signal is the numerical differentiation of the position encoder signal. However, it should be mentioned at the end that to have a more accurate model of this system requires a complex gear meshing mechanism modelling.

5. Discussion and conclusion

A systematic way to capture the dynamic behaviour of harmonic drive component is conducted by a parsimonious representation. A simple but accurate model for the torsional compliance has been established, where the optimization of the model is achieved by means of heuristic nonlinear regression. As a measure
of the identification performance the MSE number is defined, by which the reliability and the accuracy of the model are quantified.

From the result of torsional compliance modelling, a piecewise linear model combined with non-local memory hysteresis, which is frequently used to capture pre-sliding friction behaviour, resolve the difficulties in determining the model of torsional stiffness in harmonic drive. Four elements of Maxwell-slip model have proven to be adequate to capture the hysteresis in the corresponding torsional stiffness. This model has an important advantage since it solves the pre-sliding friction behaviour without the need for memory stacks to keep track of motion reversal.

The modelling of the friction in the output side takes advantage of the high reduction ratio of the harmonic drive. Low velocity in the output shaft, which implies relatively constant Striebeck function, offers simplification of the friction model. Parallel connection of Maxwell-slip elements is shown to be adequate to mimic the behaviour of the friction in bearing output.

The model performance is assessed by a simulation verifying the experimental results for assembled system under the unconstrained motion cases. The simulated torque of the system is developed and compared to the experimental result. An accurate match in the result indicates the reliability of the model for wide operating conditions.

This proposed model has been utilized for control purposes of a system comprising harmonic drive element by deducing equivalent dynamic parameters and implemented to a gain scheduling system comprising harmonic drive element by deducing equivalent conditions.

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This proposed model has been utilized for control purposes of a system comprising harmonic drive element by deducing equivalent dynamic parameters and implemented to a gain scheduling controller [14]. However, it should be mentioned that knowledge of this behaviour from the model can be exploited for other control strategies.

**References**


