Structural response investigation of a triangular-based piezoelectric drive mechanism to hysteresis effect of the piezoelectric actuator

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Abstract

A new piezoelectric-based mechanism was proposed, modeled and tested. This mechanism was basically designed to be periodically excited while in contact with a driven surface. Two piezoelectric elements located by the angle of $90^\circ$ relative to each other are used to create elliptical motion on tip of the mechanism depending on the applied phase difference.

In order to ensure the designated elliptical motion is achieved, a dynamic analysis of the system is carried out utilizing the constitutive equations relating to the dynamic response of piezoelectric element with the voltage input under external loading. However, it is widely known that piezoelectric materials are subjected to hysteresis nonlinearity, which causes the constitutive equations cannot be implemented directly. On the other hand, the constitutive equations have been originally derived based on linear assumption that neglects the nonlinearity of hysteresis in piezoelectric materials.

In this paper, the hysteretic effect of the piezoelectric actuator is taken into consideration to characterize and model the dynamic response of the designated structure. The trajectory output of the structure is simulated using finite element approach while the excitation input to the model, incorporating the hysteresis properties, is predicted based on the proposed formulation. The simulation results exhibit good agreement to the tip trajectory of the mechanism at relatively low frequency ranging from 50 Hz to 200 Hz.

1. Introduction

In recent years, various types of piezoelectric-based mechanisms have been designed and developed to provide ultra-precision positioning systems for wide range of industries, e.g., semiconductor equipments, biomedical devices, surface scanning devices and storage media systems. The reason of special attention to such kinds of mechanisms is referred to specific characteristics such as fast response, micro/nano positioning resolution and less complexity compared to their electrical and magnetic counterparts. Many researchers attempted to study and characterize the dynamic behavior of this kind of mechanism including the piezoelectric effect, in particular for those in micro/nano positioning applications.

Sharp et al. [1] investigated the dynamic performance of an arch-shaped ultrasonic motor based on using the finite element method (FEM) approach. The comparison of measured displacement produced by the motor and the displacement...
from FEM modeling showed 20% discrepancy, which is attributed to three potential sources of compliance inline with the piezoelectric actuators in the motor assembly. In order to evaluate the dynamic behavior of a bimorph-type piezoelectric motor, a linear dynamical model was proposed. In this work, a friction model was utilized for modeling the contact parts to consider the effect of friction as a nonlinear element associating with the implemented linear dynamical model. Similarly, Merry et al. [3] studied the dynamic responses of a four-leg walking piezoelectric motor. The output of the linearized Lagrange-based dynamic equations of the system, which modeled the piezoelectric actuator using the linear constitutive equations, exhibits desirable agreement between the theoretical and experimental results when sinusoidal and asymmetric waveform inputs are applied to each tip of the legs. In an alternative design, the electromechanical transfer function is derived for ultrasonic wheel system to consider the dynamic characteristics of the entire system including its stability [4]. This transfer function is obtained based on the general concept of governing linear constitutive equations for driving piezoelectric actuators. In a different design, the flexural deformation of a piezoelectric rotary motor is predicted by utilization of the wave equation and linear assumptions for parameters of the system [5]. Ishikawalshikava et al. [6] proposed a new dynamical model for a spherical Multi-DOF ultrasonic motor based on nonholonomic dynamics. An alternative approach was used by Devos et al. [7] to estimate the dynamic response of the planar piezoelectric drive by using the frequency response function. A 3-DOF flexure-based parallel mechanism was dynamically modeled for micro/nano manipulation disregarding the constitutive equations used for modeling the piezoelectric actuators [8]. In control framework, using the neural network and inverse of hysteresis model enabled some of the researchers to design innovative strategy for both identification and control of the piezoelectric-based mechanism [9,10].

In this study, a piezoelectric-based drive mechanism is proposed to create a linear movement for a slider. The dynamic properties of the piezoelectric actuators used in this mechanism are measured at relatively low range between 50 Hz and 200 Hz, which basically aims to characterize the hysteresis effect of the actuator. In the following, the hysteresis of the actuator is taken into account with linear constitutive equations of piezoelectric actuator. According to this relationship, the amount of displacement provided by the actuator is evaluated at the contact point between the actuator and the structure of the mechanism. Using these values and FEM-based model of the structure, the directional trajectory at the end-effector of the mechanism is estimated. Finally, the values of deformations obtained in the simulation are compared with the values obtained by the experimental data for the end effector of the mechanism.

This article is organized into five sections. In the first place, the basic linear constitutive equations of a piezoelectric actuator are proposed under both static and dynamic operations. Then, the two-dimensional piezoelectric-based drive mechanism will be discussed comprehensively. In the next step, three steps of dynamic modeling will be considered subsequently, followed by Section ‘experimental results and discussion’. Finally, the conclusion of the paper will be discussed in the last section.

2. One-dimensional constitutive equation for piezoelectric actuator

Piezoelectric materials can be utilized in two different operating conditions, namely (i) the direct effect and (ii) the inverse effect. In the direct effect, electrical charge is produced by the material when a certain amount of force or pressure is applied. In the inverse effect, mechanical displacement is produced by applying electrical field to the piezoelectric actuator in an opposite manner with the direct effect. These effects can be written in mathematical expressions that determine the transformation of electrical energy to mechanical energy and vice versa. These constitutive expressions can be written for a linear piezoelectric actuator along its longitudinal axis (33-axis):

\[ S = s_{33}^E T + d_{33} E \]  
\[ D = d_{33} T + e E \]

where \( S \) (m/m) is the axial strain, \( T \) (N/m²) is the axial stress, \( E \) (V/m) is the electrical field, \( D \) (c/m²) is the electrical displacement, \( e \) is the dielectric permittivity, \( s_{33}^E \) (m²/N) is the compliance and \( d_{33} \) (m/V) is the piezoelectric induced-strain coefficient.

Generally, Eq. (1) represents the behavior of the actuator in the inverse mode and Eq. (2) characterizes the actuator response in the direct mode. For commercial piezoelectric actuators, some parameters such as \( s_{33}^E, d_{33} \) and \( e \) are provided by the manufacturer. In the following, internal stiffness, electrical field and free stroke of the stacked-type actuator can be subsequently estimated by the following relationships, which are utilized in later equations in this section:

\[ K_i = \frac{A}{h s_{33}^E} \]  
\[ E = \frac{V}{h} \]  
\[ U_{ISA} = d_{33} E l \]

where \( K_i \) is the internal stiffness of the piezoelectric actuator, \( A \) is the cross-sectional area of the actuator, \( l \) is the total length of the actuator, \( V \) is the maximum voltage applied to the piezoceramics, \( h \) is the thickness of each layer of the stacked-type piezoelectric actuator and \( U_{ISA} \) is the free stroke of the actuator.
However, Eqs. (1) to (5) basically describe the relationship of the mechanical and electrical gates only in static condition. When piezoelectric actuator is excited dynamically, the equations cannot be implemented straightforwardly. In addition, when the actuators are attached to a mechanical structure, in this case is the piezoelectric drive mechanism, the dynamic properties of the structure will affect the behavior of the actuators. In order to derive the constitutive equations under dynamic condition, Newton’s second law of motion is utilized to transform the linear algebraic equations which have been shown in Eq. (1) into differential equation of motion. From Newton’s second law of motion we have,

\[
\frac{dT}{dx}A = \rho A \frac{d^2u}{dx^2}
\]  

(6)

where, \( T \) is the axial stress, \( \rho \) is the density of the piezoelectric actuator and \( A \) is the cross-sectional area of the piezoelectric actuator. Using Eq. (6) and the relationship utilized for defining the axial strain, \( S=du/dx \), a second order differential equation is obtained by substituting into Eq. (1):

\[
\frac{d^2u}{dx^2} = \rho s_{33}^E \frac{d^2u}{dt^2} + d_{33} \frac{dE}{dx}
\]  

(7)

Assuming uniform electrical field along the actuator length allows us to eliminate the term \( dE/dx = 0 \) from Eq. (7). Finally, Eq. (7) is converted into the wave equation as

\[
\frac{d^2u}{dx^2} = \rho s_{33}^E \frac{d^2u}{dt^2}
\]  

(8)

The wave equation can be easily solved by

\[
u(x,t) = (c_1 \sin \gamma x + c_2 \cos \gamma x)e^{i\omega t}
\]  

(9)

where \( \gamma \) is the wave number which is given by,

\[
\gamma = \omega \sqrt{\rho s_{33}^E}
\]  

(10)

The coefficients of \( c_1 \) and \( c_2 \) can be evaluated by determining the boundary conditions. Assuming \( \nu(0,t) = 0 \), we find \( c_2 = 0 \) and the final equation is given as,

\[
u(x) = c_1 \sin \gamma x
\]  

(11)

To evaluate \( c_1 \), another boundary condition has to be applied in Eq. (11). This boundary belongs to the head of the actuator, which has contact with the structure of the piezoelectric-drive mechanism. At the point of contact, force will be generated in both actuator and the mechanism. This force is identical between two substances at the point of contact and expressed by

\[
T(l)A = -K_e(\omega)\nu(l)
\]  

(12)

where \( T(l) \) is the stress generated at the contact point, \( A \) is the cross-sectional area of actuator’s head, \( \nu(l) \) is the displacement of the actuator at contact point and \( K_e(\omega) \) is the dynamic stiffness of the mechanism which is function of frequency and stated by the following relationship:

\[
K_e(\omega) = -\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + i2\zeta \left( \frac{\omega}{\omega_n} \right) K_e \right]
\]  

(13)

where \( \omega_n = \sqrt{k_e/m_e} \) is the natural frequency, \( \zeta \) is the damping ratio and \( k_e \) is the static stiffness of the piezoelectric-drive mechanism that are all achieved by using the frequency response analysis of the structure at a certain operating frequency range of interest.

Using Eqs. (1) and (11), we have:

\[
c_1 \gamma \cos \gamma l = s_{33}^E - K_e(\omega) c_1 \sin \gamma l + d_{33} E
\]  

(14)

and

\[
c_1 = \frac{d_{33} E}{\gamma \cos \gamma l + (s_{33}^E/A) K_e(\omega) \sin \gamma l}
\]  

(15)

Since \( s_{33}^E/A \) is the inverse of \( K_l \) regarding to Eq. (3), if the numerator and denominator of Eq. (15) is multiplied by \( l \), the ratio of \( K_e(\omega)/K_l \), known as stiffness ratio, \( r(\omega) \) will be obtained as shown in the denominator of Eq. (16),

\[
c_1 = \frac{d_{33} El}{\gamma l \cos \gamma l + r(\omega) \sin \gamma l}
\]  

(16)
Finally, the displacement of head of the actuator in accordance to the structure is given by,

$$u_l(\omega) = \frac{d_{33}EI}{j[l \cot \gamma] + r(\omega) \sin \gamma} e^{j\omega t}$$  \hspace{1cm} (17)

where, referring to Lyshhevski [11], $d_{33}EI$ is defined as a free stroke of the actuator based on Eq. (3).

3. Two-dimensional piezoelectric-based mechanism

Fig. 1 illustrates the configuration of the proposed mechanism, which consists of two piezoelectric elements. If the actuators are excited in phase, the tip of the mechanism will displace only on the vertical direction and, on the contrary, if the actuators are excited at different phase angle, the head of the mechanism will deliver an elliptical motion in $x-y$ plane accordingly. This type of motion can be used, e.g., to actuate a slider on top of the mechanism.

As illustrated in Fig. 1, this mechanism consists of three main parts, i.e.,

1. Triangular-based body
2. Two piezoelectric actuators
3. Spherical head (end-effector)

The structure carries the load imposed by the actuators as well as maintaining the spherical head. Two piezoelectric actuators are located in their places by prestressing screws from the bottom part of the mechanism, where the mechanism is made from anodized-aluminum.

Fig. 2(a) shows the mechanism with a simple slider, while Fig. 2(b) depicts the mechanism with air-bearing slider.

4. Modeling of the mechanism

In this section, the analysis of the mechanism will be discussed. In the first place, the frequency response analysis is carried out to evaluate how far our model satisfies the behavioral matching with the real set-up. Subsequently, dynamic modeling of the mechanism is performed to characterize the response of the model owing to the different inputs applied at the point of contact between actuator and the mechanism.

4.1. Frequency response analysis

In this section, the frequency response of the model is evaluated along $x$- and $y$-directions in terms of acceleration (m/s$^2$) versus frequency (Hz) and then they are compared with the results obtained from the impact hammer testing of the real set-up in the frequency range of interest between 50 Hz and 250 Hz.

This analysis is carried out to study how far the model match to the real set-up dynamically at a certain frequency interval ranging from 50 Hz to 250 Hz and to have an idea on the stiffness of the mechanism in both $x$- and $y$-directions. Fig. 3 shows the comparison of frequency response between the model and the real mechanism for both directions.

As can be seen from the figure, despite an uncertainty on the measurement results, the frequency responses of the model tend to show similar behaviors with that of the real structure. In particular, the frequency response in the $x$-direction resembles better than that in the $y$-direction. This can be easily understood as the $y$-direction appears to be stiffer than the $x$-direction, which is due to the boundary condition of the structure.
Fig. 2. (a) Mechanism operation with simple slider and (b) mechanism operation with air-bearing slider.

Fig. 3. (a) Frequency response for x-acceleration and (b) frequency response for y-acceleration.
4.2. Hysteresis effect in piezoelectric actuator

In Section 2, Eqs. (1) to (17) were derived disregarding the nonlinear relationship between the voltage input and displacement output in the actuator, where in practice, piezoelectric actuators is subjected to severe hysteretic nonlinearity. This property consequently influences the response of the actuator and causes some deviations from linear system. The hysteresis nonlinearity is affected by some parameters such as input frequency and input magnitude applied to the piezoelectric actuator. As a result, due to this effect, the behavior of the actuator shall be determined. For this purpose, a dedicated set-up was built to investigate the nonlinearity on a piezoelectric actuator. This set-up consists of a piezoelectric actuator (P-887.90, from PHYSIK Instrument Co.) that is able to provide maximum displacement of 36 ± 10% μm at maximum applied voltage of 120 V. A power amplifier (E.663, PHYSIK Instrument Co.) is used to drive the piezoelectric actuator and a capacitive sensor (CS1, Micro-Epsilon Co. with resolution of 0.75 nm and operating temperature range of −50 °C to 200 °C) is utilized to measure the displacement of PZT actuator in terms of voltage (see Fig. 4).

One set of sinusoidal input with harmonic voltage signal (biased sine with amplitude of 50 V and 50 V bias) was prescribed to the piezoelectric actuator at varying frequencies ranging from 50 Hz to 200 Hz. The purpose of choosing on particular amplitude at varying frequencies is mainly to study the behavior of the actuator at different frequencies.

In this experiment, the maximum displacement achieved for the piezoelectric actuator in the presence of hysteresis will be incorporated with Eq.(17) obtained in the previous section in order to predict the equivalent displacement at head of the actuator where a contact is made between actuator and structure of the piezoelectric-drive mechanism. Fig. 5 shows the hysteresis curves from 50 Hz to 200 Hz and Table 1 displays the maximum displacement provided at each frequency.

4.3. Dynamic analysis

Section 2 discusses the implementation of the constitutive equations of the piezoelectric actuator under dynamic condition when the actuator makes contact with the structure of the piezoelectric-drive mechanism. Since these equations were derived regardless to the hysteresis nonlinearity in piezoelectric actuator, the displacement generated in the model of the mechanism will not give a good estimation as compared to those obtained in practice.

In the previous subsection, the hysteretic nonlinear properties of the piezoelectric actuator were investigated. It was realized that the free stroke of the actuator decreases when the input frequency is increasing, whereas the free stroke of the actuator is assumed to be invariant with frequency changes based on the linear constitutive equations.

This imperfection in constitutive equations motivates us to integrate the nonlinearity existing in the displacement with Eq. (17). In this equation, the value $d_{33}EI$ corresponds to the free stroke of the piezoelectric actuator. In order to make use of Eq. (17) in the presence of hysteresis, the $d_{33}EI\epsilon_{\text{tot}}$ term in Eq. (17) is substituted by the hysteresis data obtained from the experiments:

$$u_{1}(l,\omega) = \frac{U_{\text{hys}}}{\gamma \cot \gamma l + \gamma \omega \sin \gamma l}$$

$$u_{2}(l,\omega) = \frac{U_{\text{max}}}{2} \times \frac{1}{\gamma \cot \gamma l + \gamma \omega \sin \gamma l} + \frac{U_{\text{max}}}{2} \times \frac{1}{\gamma \cot \gamma l + \gamma \omega \sin \gamma l} \times e_{\text{tot}}$$

Fig. 4. Set-up configuration.
where, $U_{hys}$ in Eq. (18) is the hysteretic displacement obtained by the data from the experiment and $U_{max}$ in Eq. (19) is the maximum stroke of the actuator obtained in frequency range of interest based on the data in Table 1. The value $u_1(l, \omega)$ represents the hysteretic input displacement at the point of contact while $u_2(l, \omega)$ stands for pure sinusoidal input displacement at the same location. These values are compared in Fig. 6.

**Table 1**

<table>
<thead>
<tr>
<th>Input frequency (Hz)</th>
<th>Maximum stroke of the actuator (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>30.65</td>
</tr>
<tr>
<td>75</td>
<td>29.65</td>
</tr>
<tr>
<td>125</td>
<td>20.32</td>
</tr>
<tr>
<td>150</td>
<td>17.2</td>
</tr>
<tr>
<td>189</td>
<td>13.74</td>
</tr>
<tr>
<td>200</td>
<td>13</td>
</tr>
</tbody>
</table>

**Fig. 5.** Hysteresis curves with input magnitude of 100 V at different frequency: (a) 50 Hz; (b) 75 Hz; (c) 125 Hz; (d) 150 Hz; (e) 189 Hz and (f) 200 Hz.
Fig. 6. Pure sinusoidal and hysteretic signals at contact point between actuator and the mechanism at different frequency: (a) 50 Hz; (b) 75 Hz; (c) 125 Hz; (d) 150 Hz; (e) 189 Hz and (f) 200 Hz.
From the finite element analysis, the equivalent dynamic properties of the mechanism have been estimated for the equivalent damping ratio of the mechanism, $\zeta_e$, the equivalent damping factor of the mechanism, $C_e$, the equivalent mass of the mechanism, $m_e$ and the equivalent stiffness of the mechanism, $K_e$ as observed in Table 2. These values were obtained by performing harmonic analysis of the structure in the frequency range of interest. The maximum displacements at the point of contact for both types of signals are shown in Table 3, while Table 4 tabulates the material properties of the piezoelectric actuator (P-887.90).

In the last step, we simulated the deformation at the head of the mechanism owing to the displacement applied to the contact point including two input signals of pure sinusoidal and hysteretic. This simulation is performed using ANSYS as the finite element tool. The flexible dynamic module is chosen to solve the model and estimate the deformation of the head of the mechanism versus the dynamic displacement in place of contact between actuator’s head and the structure of

**Table 2**
Equivalent dynamic properties of the mechanism.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Value (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_e$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$C_e$</td>
<td>0.1 (N.s/m)</td>
</tr>
<tr>
<td>$m_e$</td>
<td>$1.544 \times 10^{-1}$ (Kg)</td>
</tr>
<tr>
<td>$K_e$</td>
<td>$5 \times 10^7$ (N/m)</td>
</tr>
</tbody>
</table>

**Table 3**
Maximum displacements of the contact point between actuator and the mechanism at different frequency.

<table>
<thead>
<tr>
<th>Input frequency (Hz)</th>
<th>Maximum displacement (μm) of the contact point</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>18.23</td>
</tr>
<tr>
<td>75</td>
<td>17.52</td>
</tr>
<tr>
<td>125</td>
<td>11.87</td>
</tr>
<tr>
<td>150</td>
<td>10.07</td>
</tr>
<tr>
<td>189</td>
<td>8.063</td>
</tr>
<tr>
<td>200</td>
<td>7.653</td>
</tr>
</tbody>
</table>

**Table 4**
Material properties of the piezoelectric actuator, P-887.90.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Value (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{33}^e$</td>
<td>$2.97 \times 10^{-11}$ (m²/N)</td>
</tr>
<tr>
<td>$d_{33}$</td>
<td>$3.94 \times 10^{-10}$ (m/V)</td>
</tr>
<tr>
<td>$E$</td>
<td>2 (KV/mm)</td>
</tr>
<tr>
<td>$\varepsilon_{33}$</td>
<td>1750 (F/m)</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>$8.85 \times 10^{-12}$ (F/m)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>7800 (Kg/m³)</td>
</tr>
</tbody>
</table>

Fig. 7. Position of the applied input displacements.
the mechanism. To define the boundary conditions of the model for this analysis, the base of the mechanism is assumed to be fixed with zero displacement and zero velocity.

\[
\begin{align*}
U_x\text{, base} & = 0 \\
U_y\text{, base} & = 0 \\
U_z\text{, base} & = 0 \\
\frac{\partial U_x}{\partial t} & = 0 \\
\frac{\partial U_y}{\partial t} & = 0 \\
\frac{\partial U_z}{\partial t} & = 0
\end{align*}
\] (20)

The same amounts of input displacements expressed in Eqs. (18) and (19) are applied to the position of contact with a certain phase difference for left and right actuators as illustrated in Fig. 7.

Fig. 8 shows the maximum deformation of the mechanism’s head disregarding the hysteresis at frequency of 50 Hz and 200 Hz subsequently, while Fig. 9 depicts the deformation with regard to the hysteretic input at frequency of 75 Hz and 189 Hz.

5. Experimental results and discussion

In order to compare and contrast the simulation to the experimental results, a dedicated experimental set-up is prepared. A capacitive sensor is used to measure the displacement in two locations, \( V_1 \) and \( V_2 \), separated by 55° angle as shown in Fig. 10.
A transformation matrix as presented in Eqs. (22) and (23) are used to approximate the displacements along the \( x \)- and \( y \)-directions after the measurement made along the \( V_1 \)- and \( V_2 \)-axes.

\[
\begin{bmatrix}
    V_1 \\
    V_2
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & \sin \theta \\
    -\cos \theta & \sin \theta
\end{bmatrix}
\begin{bmatrix}
    V_x \\
    V_y
\end{bmatrix}
\]  

(22)

\[
\begin{bmatrix}
    V_x \\
    V_y
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & \sin \theta \\
    -\cos \theta & \sin \theta
\end{bmatrix}^{-1}
\begin{bmatrix}
    V_1 \\
    V_2
\end{bmatrix}
\]  

(23)

In the matrix equations, \( V_1 \) and \( V_2 \) represent for the displacements in the measured directions, whereas \( V_x \) and \( V_y \) represent displacement of the mechanism's head in \( x \)- and \( y \)-directions, respectively. The resultant shapes for \( x \)- and

---

**Fig. 9.** Dynamic modeling and total deformation for hysteresis input at different frequency: (a) 75 Hz and (b) 189 Hz.

**Fig. 10.** Location of the sensor relative to head of the mechanism.
y-displacements in the head of the mechanism are shown in Fig. 11 together with the simulated responses for two different cases, i.e., with and without the presence of hysteresis.

In this experiment, the magnitude of the input applied to both actuators for the case disregarding the hysteresis is chosen (50 V input with 50 V bias) with phase difference of 90°, that basically is the angle between two actuators.

As shown in Fig. 11, different shapes of closed curves for different frequencies ranging from 50 Hz to 200 Hz are observed. This representation reveals that each operating frequency provides a certain type of motion in both head and entire the structure.

**Fig. 11.** Experimental and modeling results of the mechanism at exciting voltage of 100 V with phase difference of 90°: ---, measured signal, ---: model output to hysteretic input, ---: model output to pure sinusoidal input at different frequency: (a) 50 Hz, (b) 75 Hz, (c) 125 Hz, (d) 150 Hz, (e) 189 Hz and (f) 200 Hz.
Considering the directional deformation along x- and y-directions, different results have been obtained as can be seen in Figs. 11 and 12. Tables 5 and 6 provide comprehensive information about directional deformations along x- and y-axes, in which the measured signals from the experiment and simulation results from two different simulation cases are presented. These results shall be considered from two points of views. In one hand, as expected from Section 4.1, the maximum deformation in x-direction as presented in Table 5 exhibits less discrepancy between the simulation and measurement compared to that of the y-direction. This can be easily understood as the x-direction intuitively has lower stiffness than the y-direction. On the other hand, the directional deformation from the measurement in both directions

![Graph](image)

**Fig. 12.** (a) Error between measured signal and pure sinusoidal input, hysteretic input along x-direction, (b) Error between measured signal and pure sinusoidal input, hysteretic input along y-direction.

**Table 5**

<table>
<thead>
<tr>
<th>Measured maximum deformation of the mechanism’s head (µm)</th>
<th>Maximum deformation of the mechanism’s head due to pure sinusoidal input (µm)</th>
<th>Maximum deformation of the mechanism’s head due to hysteretic input (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Hz:13.4</td>
<td>50 Hz:13.5</td>
<td>50 Hz:13.4</td>
</tr>
<tr>
<td>75 Hz:12.3</td>
<td>75 Hz:12.6</td>
<td>75 Hz:12.2</td>
</tr>
<tr>
<td>125 Hz:8.6</td>
<td>125 Hz:7.9</td>
<td>125 Hz:8.1</td>
</tr>
<tr>
<td>150 Hz:7.1</td>
<td>150 Hz:6.7</td>
<td>150 Hz:7.1</td>
</tr>
<tr>
<td>189 Hz:5.6</td>
<td>189 Hz:5.4</td>
<td>189 Hz:5.5</td>
</tr>
<tr>
<td>200 Hz:5.3</td>
<td>200 Hz:5.1</td>
<td>200 Hz:5.6</td>
</tr>
</tbody>
</table>
shows closer values to the simulation of the model that considers the presence of the hysteresis in comparison to the modeling disregarding to the hysteresis effect.

As shown in Fig. 12, unlike the hysteretic signal, pure sinusoidal input yields less accurate tracking performance due to inherent linearity existing in this signal. Fig. 12(a) shows error between two signals (pure sinusoidal and hysteretic) with measured signal for $x$-direction owing to increasing in frequency value, while in Fig. 12(b), the error is shown for $y$-direction. Both Figs. 11 and 12 can be obviously the representative of the efficient modeling of the structural response when nonlinearity is associated with the structure, meanwhile, when the effect of nonlinearity is taken into consideration into the structure's response, the accuracy of the modeling will be improved.

6. Conclusion

In this study, the constitutive equations of the piezoelectric actuator are derived in conjunction with the structure of the piezoelectric-based mechanism under dynamic condition. Since the equations do not consider the presence of hysteresis that is evidently appears in piezoelectric materials, the hysteresis property obtained from direct measurements is integrated to these equations. After integrating hysteresis to the equation, the displacement generated by the piezoelectric actuator is simulated at the point of contact with the structure for different frequencies, ranging from 50 Hz to 200 Hz. In order to estimate the displacement on top of the mechanism, a finite element analysis (FEA) is incorporated using the ANSYS software. Afterward, the response is simulated and compared to the real response obtained from the experiment.

As a result, better agreement is observed between the simulation results and the experimental results owing to the nonlinear hysteretic input applied to the mechanism compared to the model without considering the nonlinear pure sinusoidal input. In short, introducing the hysteresis term on the constitutive equations of the piezoelectric actuator enables us to model the dynamic behavior of the system satisfactorily.

References