ON THE CHAOTIC RESPONSE IN A ROBOT JOINT MECHANISM DUE TO BACKLASH

Tegoeh Tjahjowidodo, Farid Al-Bender, Hendrik Van Brussel
Mechanical Engineering Department
Division PMA, Katholieke Universiteit Leuven
Celestijnenlaan 300B, B3001 Heverlee, BELGIUM
tegoeh.tjahjowidodo@mech.kuleuven.ac.be

Abstract
Estimation of modal parameters of mechanical structures is usually carried out by utilising the Frequency Response Function (FRF) for linear systems, or skeleton identification such as Hilbert transform for nonlinear systems. However, these techniques could be applied only when the output of a system is periodic for a periodic input. Under certain excitation conditions, chaotic behaviour might occur so that the response is aperiodic. In that case, chaos quantification techniques, such as Lyapunov exponent and correlation dimension are proposed in the literature. This paper attempts to quantify the chaotic behavior of the response of the system with backlash, and correlate the quantification parameters to the system’s parameters of the simulated system, in particular the backlash size.

For validation purposes, a robot arm mechanism comprising backlash element in the joint was set up. It is found that the corresponding mechanism shows aperiodic vibration response under certain periodic excitation input. However, it is not immediately clear if a linear stochastic process is not the cause of the aperiodic behavior. In order to verify the presence of chaotic behavior in the response, a surrogate data test is utilized. Once we are convinced that the chaotic process dominates the signal, quantification can be carried out after application of appropriate filtration techniques. By correlating the chaos quantification results to the modal parameters of the system, the estimation of the backlash size can be achieved.

Key words
chaos, Lyapunov exponent, correlation dimension, backlash

1 Introduction
The classical modal parameters estimation techniques used for linear dynamic mechanical structures such as the Frequency Response Function (FRF) with shock excitation or forced excitation are not suitable for a nonlinear system. For a number of nonlinear dynamic systems, several detection and identification procedures have been developed. The application of the Hilbert Transform (HT) can be effective for practical analysis of the modal parameter identification [Feldman, 1994a, 1994b; Tjahjowidodo et al., 2004]. Wavelet analysis was in particular shown to be another identification approach, which offers significant improvement in comparison with HT technique, though it suffers from memory hunger and processing time [Stazewski, 1998; Tjahjowidodo et al., 2005a]. Unfortunately, these techniques are applicable only for a ‘well-behaved’ system, i.e. where the output is periodic for a periodic input. Under certain conditions in dynamical systems (specifically nonlinear mechanical systems) chaotic behavior might occur, rendering the aforementioned techniques inapplicable.

[Lin, 1990] found that a simple mass-spring-damper system comprising backlash element might give chaotic response under certain excitation condition. He also demonstrated this behaviour in a supported beam with a mass at its midpoint. A backlash stiffness nonlinearity was introduced by providing motion constraints on both sides of the mass. However, he did not present the chaos quantifiers nor correlate the quantization to the model parameters of his system. Theodossiades et al. [Theodossiades et al., 2000] showed the chaotic responses in a more complex system, namely a gear pair system with backlash and periodic stiffness, under certain condition. Feng et al. [Feng et al., 1998] studied the chaotic response on a model of a rattling system and presented the bifurcation diagram of the chaotic behaviour as a function of excitation frequency and amplitude. Trendafilova et al. [Trendafilova et al., 2001] tried to exploit the chaos quantifiers for the purpose of fault detection in a real robot joint. She used the high-frequency component of the response besides the excitation frequency to quantify the chaoticity and correlated it with the backlash size in the robot joint. However, she did not explain the route of chaos in this high-frequency component signal.

This paper attempts to correlate the chaos quantifiers (e.g. Lyapunov exponent ($\lambda$) and Correlation Dimension), with the modal parameters of a chaotic
system, in particular, for our case, the backlash size. Such correlation methodology could be further developed so as to deal with other nonlinear systems such as defect qualification and quantification. Early damage detection in a mechanical system is another potential to exploit the model. Subsequently, this paper confirms experimentally the possible presence of chaotic response in a real mechanical system and characterises it.

For the experimental part of this investigation, a robot joint mechanism incorporating a backlash component developed at KULeuven/PMA was used. The experimental setup consists of a link mechanism, which is driven by a DC motor. Rotation input from the DC motor is reduced by a harmonic drive. In this setup, backlash was introduced in the connection of the harmonic drive to the shaft. Under certain operational conditions, the corresponding mechanism shows chaotic vibration response [Tjahjowidodo et al., 2005b]. However, it is not immediately clear if noise (stochastic process) is not the cause for the aperiodic behavior. In order to verify this, we first have to recover the embedding dimension of the space and the time lag of the signal for calculating the chaotic quantifications (Lyapunov exponent and Correlation Dimension). Afterwards, surrogate data test is used in order to check the hypotheses for the presence of chaotic behavior in the nonlinear system and of the linearly correlated noise [Theiler et al., 1992; Trendafilova et al., 2001]. Once we are convinced that the chaotic process dominates the signal, we have to separate the true response from the measurement noise. For this purpose a simple nonlinear noise reduction method (based on the 'simple nonlinear prediction' algorithm [Tsionis, 1992; Kantz et al., 1997]), which is more suitable for chaotic signals, is employed for the recorded signal.

In the following, section 2 discusses the theoretical treatment and simulation of the system under consideration. Section 3 describes the experimental system with a backlash element, presents the results and discusses the phase-space reconstruction of the resulting response. In this section, we also try to detect the presence of the nonlinearities (and chaotic behavior) behavior on statistical basis. Finally, some appropriate conclusions are drawn in section 4.

2 Theoretical Basis and Simulation

Lin [Lin, 1990] shows theoretically that under certain excitations, a simple nonlinear mechanical system with backlash might manifest chaotic vibration. He demonstrates that a simple system comprising a backlash spring, as shown in Figure 1, and having a dynamic equation:

$$m\ddot{x} + c\dot{x} + k_1x + F(x) = A\cos\omega t$$

(1)

where \(F(x)\) is the restoring force of the nonlinear backlash component given by:

$$F(x) = \begin{cases} k_0(x - x_0 \cdot \text{sign}(x)), & x \geq x_0 \\ 0, & x < x_0 \end{cases}$$

is found to behave chaotically under certain excitation conditions. Table 1 gives two sets of system’s parameters pertaining to chaotic behavior, for certain excitation force specifications, where \(m\) is the mass of the system, \(k_1\) and \(k_0\) are stiffnesses, \(c\) is a damping coefficient, and \(x_0\) the backlash (or play) size.

Figure 1. Schematic of a nonlinear mechanical system with backlash component.

Table 1. Parameter sets of vibration system

<table>
<thead>
<tr>
<th>CASE</th>
<th>(m) (kg)</th>
<th>(k_1) (N/m)</th>
<th>(k_0) (N/m)</th>
<th>(c) (Ns/m)</th>
<th>(x_0) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1</td>
<td>0</td>
<td>40000</td>
<td>8</td>
<td>0.005</td>
</tr>
<tr>
<td>#2</td>
<td>1</td>
<td>1000</td>
<td>31000</td>
<td>8</td>
<td>0.005</td>
</tr>
</tbody>
</table>

It can be shown that there exist certain sinusoidal excitation forces for CASE 1, with \(A = 100\) N and for CASE 2 with \(A = 240\) N, both at \(\omega = 40\) rad/s, which cause the response to behave chaotically as the phase plots show in Figure 2. In order to examine the influence of each parameter on the nature of resulting response, we use dimensional analysis to normalise the variables and reduce the number of parameters. By combining variables in dimensionless groups, one may gain more insight in the problem.

Figure 2. Phase plots of the chaotic response of two different mechanical systems with backlash component.

Introducing new variables of time and displacement \(\tau = \omega_0 t\) and \(p = x / x_0\), where \(\omega_0^2 = k_0 / m\), we may rewrite the dynamics equation for CASE #1 as follows:

$$m_0\ddot{x}_0 + p^2 + c_0 x_0 p + k_0 x_0 F(p) = A_0 \cos \omega_0 t$$

(2)

where the primes indicate differentiation with respect to \(\tau\) and \(F\) is the backlash spring function. This equation reduces to:

$$p^2 + 2\zeta \dot{p} + F(p) = \alpha \cos \frac{\omega_0}{\omega_0} \tau$$

(3)
where $2\zeta = c/\sqrt{k_0 m}$ and $\alpha = A/(k_0 x_0)$ and $\bar{F}(p)$ is backlash stiffness characteristic in normalized form:

$$
\bar{F}(p) = \begin{cases} 
p - \text{sign}(p), & |p| \geq 1 \\
0, & |p| < 1
\end{cases}
$$

That is to say that the problem is characterized by two parameters, $\alpha$ and $\zeta$.

In order to see how a chaotic motion evolves when the forcing amplitude increases (or equivalently the backlash size decreases), we generate the bifurcation diagram as a function of the parameter $1/\alpha$ for CASE 1; Figure 3 presents the results.

As for the chaos measure, we calculate the maximum Lyapunov exponent of the resulting response. This is based on a unique property of chaotic behavior that two trajectories starting very close together will rapidly diverge from each other. The divergence (or convergence) of two neighboring trajectories can be used as a chaos quantification measure, which is the Lyapunov Exponent ($\lambda$). Figure 4 shows the Lyapunov exponent as a function of dimensionless backlash parameter for CASE 1, for $\zeta = 0.02$, providing another representation for the bifurcation diagram in Figure 3.

Examining Figure 4 reveals however that the correspondence between $\lambda$ and $1/\alpha$ is not unique (one-to-one). This means that more than one value of the parameter $1/\alpha$ may have the same Lyapunov exponent. To overcome this problem, we need to excite the system by a certain set of sinusoidal inputs with varying excitation level at fixed frequency. By observing the evolution of the chaotic response in the amplitude, $A$, which is inversely proportional to $1/\alpha$, we can estimate the backlash size in the system. Different degree of the backlash will expand (for higher backlash) or contract (for lower backlash) the pattern of Lyapunov exponent function in Figure 4.

As an alternative or complementary approach, another chaos quantification parameter, namely the correlation dimension, can also be used. Figure 5 shows similar representation to Figure 4, except that the vertical axis represents the correlation dimension.

Effect of damping on the chaotic response

In order to see how a chaotic motion evolves with different damping ratios, we simulate the system of CASE#1 for different damping ratios ($\zeta$) starting from $\zeta = 0.02$ until $\zeta = 0.24$. Figure 6 shows the results. Note that the first panel of Figure 6 provides the same plot as in the Figure 3 for $\zeta = 0.02$. 

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Recalling equation (3), where the dynamic problem is characterized by two parameters, \( \alpha \) and \( \zeta \), we then need to correlate the effect of both parameters on the chaos quantification. Figure 7 shows the relation of Lyapunov exponent with parameters \( \alpha \) and \( \zeta \) in contour plot. One can conclude from the figure that the introduction of higher damping levels is an effective way to diminish and eventually eliminate the chaotic motion. At high damping, there is almost no chaotic response since the Lyapunov exponents are very low for any value of \( 1/\alpha \).

![Figure 7. Lyapunov exponent vs \( \zeta \) and \( 1/\alpha \).](image)

Once we have the Lyapunov exponent map as shown in Figure 7, conceptually, we can estimate the parameters, \( x_0 \) and \( \zeta \), by observing the evolution of the chaotic response as the system is excited by certain sets of sinusoidal signals with different excitation level at fixed frequency. Assuming that the damping value remains constant for any set of experiments, we can correlate the evolution of the chaotic behaviour in \( 1/\alpha \) to the parameters, \( x_0 \) and \( \zeta \), by observing the evolution of \( \lambda \) as a function of \( 1/\alpha \) similar to that of Figure 4. Based on Figure 7, different degree of backlash will expand (or contract) the profile of \( \lambda \) as a function of \( 1/\alpha \), while the changes of the damping value (\( \zeta \)) will alter it.

### 3 Experimental study

An experiment was carried out on the outer (second) link of a two-link mechanism as shown schematically in Figure 8. The aim of this experiment is to identify the backlash size of the second link joint. For this purpose, a certain degree of backlash (approximately 1.5°) was introduced in the joint of this link. The first link was fixed while the second one was made to oscillate over a certain range. This link was driven by a servomotor through a toothed belt and a harmonic drive. The vibration responses were measured with two rotary encoders. The first encoder measured the angular motion input to the harmonic drive, and the second one measured the relative oscillation between the first link and the second link.

![Figure 8. Schematic drawing of a two-link mechanism](image)

Under large amplitude periodic force excitation of a periodic force, it is found that the link system shows aperiodic behavior as can be observed from Figure 9. For this case, the system was excited by periodic motion with 1.54 Hz fundamental frequency and amplitude of 2.68°. However, we cannot ascertain whether this aperiodic response is dominated by the presence of linearly correlated noise.

With the aim of having better understanding on how the chaotic motion arises when the modal parameters of the system change, in particular the backlash size (as implied by \( \alpha \) in dimensional analysis), the experiment was carried out in several different excitation levels ranging from 1.34° to 5.73°, where it was observed that the aperiodic response persisted.

![Figure 9. Output response at 1.54 Hz and 2.68° input excitation. The solid line represents the output response while the dashed line is the input excitation. The left-scale is for the output and the right-scale is for the input.](image)

### 3.1 Quantifying chaos

The easiest way in obtaining the Lyapunov exponent, for instance, can be done by observing the separation of two close initial trajectories on the attractor and taking the logarithm of the separation. But this method cannot be applied directly to experimental data for the reason that we are not always dealing with two (or more) sets of experimental data that have close initial conditions. Experimental data typically consists of single observable discrete measurements. Reconstructing
the phase space from the time series with appropriate time delay and embedding dimension makes it possible to obtain an attractor whose Lyapunov spectrum is identical to that of the original attractor. Mathematically, a reconstructed phase space can be described as follows [Tsonis, 1992; Hilborn, 1994]:

\[ y(k) = [S(k), S(k+t), S(k+2t), \ldots, S(k+(d-1)t)] \] (4)

where \( S(k) \) is the time series from a single observation, \( t \) represents appropriate time delay for phase space reconstruction and \( d \) is a proper embedding dimension for phase space reconstruction. Now, if we choose two points in the reconstructed phase space whose temporal separation in the original time series is at least one ‘orbital period’, they may be considered as different trajectories on the attractor. Hence, the next step in determining the largest Lyapunov exponent for single observable time series is searching the nearest neighbor of certain points, in the sense of Euclidean distance, which can be considered as fiducial trajectories.

In a phase space reconstruction procedure, we must ensure that the points in each dimension (coordinate) are independent of each other. Therefore, time delay \( t \) must be chosen so as to result in points that are not correlated to previously generated points. The **Average Mutual Information** (AMI) technique can be used for determining appropriate time delay parameter for nonlinear time series. Abarbanel [Abarbanel, 1996] suggested that the value of \( t \) for which the first local minimum of the AMI occurs should be taken as time delay, and this is analogous to the time delay when the auto-correlation function attains zero value in linear case. Figure 10 shows the mutual information of the response shown in Figure 9. The first minimum value of the mutual information is approximately 0.174 sec.

The next step in reconstructing phase space is to recover the appropriate number of coordinates \( d \) of the phase space. The idea of a number of coordinates \( d \) is a dimension in which the geometrical structure of the phase space is completely unfolded. The basic method in determining the embedding dimension in phase-space reconstruction is the False Nearest Neighbor method. Suppose the vector \( y^{NN}(k) \) is a false neighbor of \( y(k) \), having arrived in its neighborhood by projection from a higher dimension, because the present dimension \( d \) does not unfold the attractor, then by going to the next dimension \( d+1 \), we may move this false neighbor out of the neighborhood of \( y(k) \). Thus, if the additional distance is large compared to the distance in dimension \( d \) between nearest neighbors, we have a false neighbor. Otherwise, we have a true neighbor.

In order to have a straightforward representation of the minimum embedding dimension, Cao [Cao et al., 1997] defined the mean value of \( E_1 \), which generally represents the relative Euclidean distance between \( y^{NN}(k) \) and \( y^{NN}(k) \) in two consecutive dimensions. Cao’s number \( E_1 \) consequently will stop changing when the dimension \( d \) is greater than the minimum embedding dimension \( d_0 \). Figure 11 depicts the Cao number as a function of embedding dimension. It can be observed that \( E_1 \) approaches a constant value for a dimension higher than four. Thus, we can conclude that the minimum dimension that will totally unfold the phase space is 5.

![Figure 10. Average mutual information as a function of the time lag for CASE 1 system excited by 2.68° input excitation.](image1)

![Figure 11. Minimum embedding dimension for CASE 1 system excited by 2.68° input excitation.](image2)

![Figure 12. Phase plots of output responses with certain discrete unit time delay of corresponding mechanical system with excitation frequency of 1.54 Hz and amplitude level respectively from left to right and top to bottom: 1.34°; 2.07°; 2.68°; 3.41°; 3.65°; 3.78°; 4.87°; 5.73°.](image3)
Nevertheless, from the results we have so far, it is not clear in any of the cases if noise (linearly stochastic process) is not the cause for the observed irregular behavior. Surrogate data test is utilized to identify whether the behavior of a signal is caused by the process. This method first specifies some linear process as a null hypothesis, then generates surrogate data sets, which are consistent with this null hypothesis, and finally computes a discriminating statistics for the original and for each of the surrogate data sets. In order to generate the surrogate data sets, the original data are transformed in such a way that all structures except for the assumed properties are destroyed. The generated surrogate data sets are assumed to mimic only the linear properties of the original data. [Theiler et al., 1992] state that a Fourier Transform algorithm is very consistent with the hypothesis of linearly correlated noise. This method is achieved by Fourier transforming the original data and substituting the phases with random numbers. After transforming back into the time domain, we get a new time series without affecting the power spectrum. If the discriminating statistic values (namely the maximum Lyapunov exponent, the average mutual information and/or the correlation dimension) computed for the original data is significantly different from the generated surrogate data, then the null hypothesis is rejected and we conclude that the data is not linearly stochastic noise and the nonlinearity is detected. Since we are motivated by the possibility that the underlying dynamics may be chaotic, our original choices for discriminating statistics are the chaotic quantifications. The correlation dimension, $D_2$, is the most frequently used as a discriminating statistic in surrogate data test. $D_2$ is computed as a limit of the correlation sum or the correlation integral [Abarbanel, 1996]:

$$D_2 = \lim_{r \to 0} \frac{\log |C(2, r)|}{2 \log |r|}$$

where $C(2, r)$ counts all the points within distance $r$ of each other.

Generally, if the irregularity in the data is chaotic, going to a higher embedding dimension will not change the result of $D_2$. On the contrary, if the data is in fact noise, the correlation dimension will not converge to a specific value in going to a higher embedding dimension.

Figure 13 shows the plots of the correlation dimension as the discriminating statistics against the embedding dimension, $m$, for four suspected chaotic responses in Figure 12 (excited at levels 2.68°, 3.41°, 3.65° and 3.78°). All of the plots in Figure 13 show that for all the cases, the values of the correlation dimension for the original data and the surrogates differ substantially. We can also conclude that the figures show the convergences of the correlation dimension for the original data, while the surrogates show no convergences. The estimated dimensions of the original data are about $d = 2.50, 2.10, 1.55$ and $2.10$ in ascending excitation level order, respectively, which shows that the underlying dynamics is in fact chaotic.

### 3.3 Noise reduction

A story of an experimental analysis is never complete without discussing the noise reduction step. The noise reduction step plays an important role in estimating the largest Lyapunov exponent to quantify chaotic behaviour. One of the problems in estimating the largest Lyapunov exponent of a ‘noisy’ signal concerns the minimum embedding dimension required to completely unfold the noisy attractor of the signal. The Simple Noise Reduction [Kantz et al., 1997] will be utilized in this work, since it offers superiority, in the calculation time, and simplicity. The Simple Noise Reduction techniques are closely related to the future prediction theory. For prediction we have no information about the quantity to be forecast other than the preceding measurement, while for noise reduction we have a noisy measurement to start with and we have the future values. Hence we aim to replace the noisy measurement with a set of ‘predicted values’ containing errors, which are on average less than the initial amplitude of the noise.
Figure 14 shows the result of simple noise reduction method of output response when the system was excited using $3.41^\circ$ excitation level, compared to the un-cleaned one. One may see that the trajectory appears smoother after noise reduction. Verification and quantification of the noise reduction performance can be done on the basis of the correlation integral. For our case, since the noise level is not significantly high, this verification will not be discussed in this paper.

![Figure 14. Cleaned signal compared to the noisy one.](image)

The left figure shows the phase plot of the response of the system under excitation frequency of $1.54$ Hz and amplitude level of $2.68^\circ$ before noise reduction, while the cleaned signal can be seen in the right figure.

In Figure 15, we can see the plot of the Cao’s number $E_1$ versus its embedding dimension $d$. The solid line represents $E_1$ for original noisy signal, while the dashed line represents $E_1$ for the signal when its noise has been reduced. From the figure, we can see that the noisy signal needs a higher dimension to unfold its attractor compared to its cleaned counterpart. The noisy one takes a minimum of 5 embedding dimensions to completely unfold its attractor, while the cleaned one needs only 4.

![Figure 15. Minimum embedding dimension using Cao’s method.](image)

### 3.4 Maximum Lyapunov exponents

Having reconstructed phase spaces of the cleaned signals, the final step is determining the Lyapunov exponents of the corresponding signals. Table II shows the chaos quantification of Lyapunov exponents for corresponding results in Figure 12, starting from the lowest excitation level to the highest respectively. The evolution of the Lyapunov exponents as shown in the table obviously marks the bifurcation phenomenon in the system. The response behaves periodically at low excitation levels until it reaches certain level between $2.07^\circ$ and $2.68^\circ$, then the chaotic response grows. Subsequently, after certain excitation level (between $3.78^\circ$ and $4.87^\circ$), the chaotic behavior diminishes, and the response, again, behaves periodically.

<table>
<thead>
<tr>
<th>Exc. Level</th>
<th>$1.34^\circ$</th>
<th>$2.07^\circ$</th>
<th>$2.68^\circ$</th>
<th>$3.41^\circ$</th>
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<tbody>
<tr>
<td>$\lambda$ (bit/time)</td>
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<td>-</td>
<td>1.423</td>
<td>1.322</td>
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</table>

<table>
<thead>
<tr>
<th>Exc. Level</th>
<th>$3.65^\circ$</th>
<th>$3.78^\circ$</th>
<th>$4.87^\circ$</th>
<th>$5.73^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (bit/time)</td>
<td>1.533</td>
<td>1.802</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 4 Conclusions

The following conclusions can be drawn from this investigation:

- Under certain excitation conditions in a nonlinear system, there may exist some separate regions for which chaotic vibrations could occur. The transition to and from those regions is marked by bifurcation points. We have shown that, for this case, it would be possible to quantify the Lyapunov exponent, for each amplitude of excitation. Correlating the Lyapunov exponent with $\alpha = A/k_0 x_0$ could, in principle, yield the backlash size. In some cases, several different backlash sizes could give the same value of Lyapunov exponent. Introducing another quantification concept, namely correlation dimension, different backlash sizes for the same value of Lyapunov exponent can be distinguished.
- Correlating the chaos quantification with both parameters of damping ($\zeta$) and $\alpha$, and observing the evolution of the chaotic response as the system is excited by certain sets of sinusoidal signals with different excitation level at fixed frequency, conceptually yields the damping value and the backlash size.
- The presence of the chaotic behavior is confirmed in the surrogate data testing by using correlation dimension as the discriminating statistic. The discrepancies of the correlation dimensions between the original data and their surrogates, in some certain excitation conditions, ascertain the
presence of the nonlinearities in the system. Specifically, the convergence value of the correlation dimension against the embedding dimension proves that the underlying dynamics are chaotic.

- For a simple chaotic system, as gauged by its attractor’s dimension, with relatively low noise level, the Simple Noise Reduction method gives very good result. This noise reduction method is therefore very suitable for application to simple systems since it offers superiority in calculation time.

In short, although quite difficult to perform in practice, chaos quantification could be used as a quantitative mechanical signature of a backlash component. Noise reduction plays an important role in this quantification.

Acknowledgement

The authors wish to acknowledge the partial financial support of this study by the Volkswagenstiftung under grant No. I/76938.

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