A Generalized Inertial-Dependent Prandtl-Ishlinskii Model for Wide-Band Frequency Piezoelectric Actuator

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Abstract. Smart materials such as piezoceramics used in industrial applications, are subjected to nonlinear phenomenon of hysteresis which degrades the tracking performance of the actuator in operation. Depending on the operating frequency and input magnitude applied to the piezoelectric actuators (PA), the symmetric and asymmetric hysteresis loops can be observed. A generalized inertial-dependent Prandtl-Ishlinskii (PI) model is proposed in term of the stop operator to compensate the effects of nonlinearity in asymmetric hysteresis loops for PA. A nonlinear envelope function is assigned for the threshold of the stop operator to minimize the error between the model response and the measured response obtained through the experiments at frequency range from 200 Hz to 500 Hz.

Introduction

Hysteresis is a nonlinear phenomenon that appears in wide varieties of physical systems. Studying this behavior in special types of materials such as shape memory alloys, magnetostrictive actuators and PA’s has attracted significant attention of researchers over the recent years.

Depending on the material used in a particular application, the shape of the hysteresis loops varies from symmetric to asymmetric shape. Regarding to this characteristic, several relevant works have been done. Al Janaideh et al. [1] proposed a generalized PI model using the play operator to characterize the hysteresis in a PA by assigning a linear envelope function utilized for the threshold value of the play operator. To compensate the hysteretic effects, an inverse generalized model was proposed by using the inverse model as feed forward controller [2]. Ang et al. [3] revealed that a linear relationship exists between the slope of the hysteresis loading curve and the rate of control input in a rate-dependent PI model. A dead-zone operator was introduced for capturing the asymmetric shape of the hysteresis and inverse of the model was constructed to compensate the nonlinearity arisen from the hysteresis. To model the asymmetric hysteresis, Jiang et al. [4] proposed the new play operator which was composed of two parts of left and right sides with different equations to cover the asymmetric shape of the hysteresis.

In this paper, a generalized inertial-dependent PI model is proposed using the stop operator, meanwhile, in addition to considering the inertial and damping effects in the stop operator, an envelope hyperbolic function is presented for the threshold of the stop operator to characterize the asymmetric hysteresis at frequencies ranging from 200 Hz to 500 Hz in a PA.

Generalized Inertial-Dependent Prandtl-Ishlinskii Model

Since the hysteresis loops are not always symmetric due to dynamic conditions existing for the PA’s, the classical stop operator or play operator cannot characterize the hysteresis in such cases efficiently.
As a result, a generalized PI model shall be proposed to meet the requirements of hysteresis characterization under asymmetric condition. In this model, an envelope function is defined to adapt the variations of the threshold value to the changes occurred in the hysteresis loop through shifting from symmetric shape to asymmetric shape. This envelope function can be chosen in term of any mathematical function appropriately fitted with the shape of the hysteresis loop. As illustrated in Fig. 1, the generalized model is a nonlinear operator which is defined by the two increasing and decreasing functions of $\gamma_i$ and $\gamma_r$, in which an increase in input $v$ causes the output $w$ increases along $\gamma_i$ and a decrease in input $v$ causes the output $w$ decreases along $\gamma_r$. In this configuration, the play operator for any input $v(t) \in C_m[0,t_e]$ is defined by

$$F_{\mu}^{\prime}[v](0) = f_{\mu}^{\prime}(v(0),0) = w(0);$$

$$F_{\mu}^{\prime}[v](t) = f_{\mu}^{\prime}(v(t),F_{\mu}^{\prime}[v](t_i));$$

for $t_i < t \leq t_{i+1}$ and $0 < i \leq N - 1$

$$f_{\mu}^{\prime}(v,w) = \max(\gamma_i(v) - r, \min(\gamma_r(v) + r, w))$$

where, $\gamma_i, \gamma_r : R \rightarrow R$ are the envelope functions operating on the play operator. Also, for any input $v(t) \in C_m[0,t_e]$ , the stop operator is defined by

$$E_{\mu}^{\prime}[v;w_{-1}](0) = e_{e}(v(0) - w_{-1});$$

$$E_{\mu}^{\prime}[v;w_{-1}](t) = e_r(v(t) - v(t_i) + E_{\mu}^{\prime}[v;w_{-1}](t_i));$$

for $t_i < t \leq t_{i+1}$ and $0 < i \leq N - 1$

$$e_r^{\prime}(v) = \min(\gamma_i(v) + r, \max(\gamma_r(v) - r, v))$$

where, $\gamma_i, \gamma_r : R \rightarrow R$ are the envelope functions operating on the stop operator.

Alternatively, proposing the generalized model is not sufficient to characterize the hysteresis loop particularly at higher frequencies and amplitudes where the inertial effect is incorporated with the PA response. Since the stop operator and the play operator cannot characterize the hysteresis characteristics in smart actuators appropriately at higher frequencies in particular, a new model is proposed for the PA in term of the mass-spring-damper system which is sensitive to any variations of the frequency through the PA excitation. Using the translational equation of motion for mass-spring-damper system results the following relationships in term of the play operator in Eq. 5 and inversely in term of the stop operator in Eq. 6.

$$X = \alpha V + \beta F_{\text{preloading}} - m_{eq} \frac{d^2 f_{\text{playoperator}}(V)}{dt^2} - c_{eq} \frac{df_{\text{playoperator}}(V)}{dt}$$

$$f_{\text{stopoperator}}(X) = \beta F_{\text{preloading}} - m_{eq} \frac{d^2 X}{dt^2} - c_{eq} \frac{dX}{dt} - k_{eq} X$$

where, $m_{eq}, c_{eq}$ and $k_{eq}$ are the equivalent mass, damping factor and stiffness respectively. In according to the external preloading force applied to the actuator, the value of $F_{\text{preloading}}$ stands for the preloading force when the PA is exposed to the preloading condition. In Eq. 5 and Eq. 6, $V$ represents the input voltage applied to the PA as the input of the play operator and $X$ represents the desired displacement of the actuator as the input of the stop operator. $\alpha$ and $\beta$ represent correction coefficients of the input voltage in the play operator and the preloading force $F_{\text{preloading}}$ subsequently.
The reason of choosing the inertial-dependent model is due to taking into account two important terms of inertial and damping which are shown by \( m \ddot{x} \) and \( c \dot{x} \). If two terms of \( \dot{x} \) and \( \ddot{x} \) are substituted by \( \omega x \) and \( \omega^2 x \) subsequently upon the periodical input applied to the actuator (\( \omega \) is the symbol of input frequency), these terms will be function of the frequency. As a result, the modeling of the hysteresis can be performed more efficiently at higher frequencies and amplitudes as compared to the classical PI model. Integrating the inertial-dependent model and the generalized PI model enables us to characterize the asymmetric hysteresis loops at high frequency, meanwhile, this model can have the two properties of either inertial-dependent model and the generalized PI model.

**Modeling Results**

In order to characterize the asymmetric hysteresis loops occurring in PA, a generalized inertial-dependent stop operator was utilized. Two input sets of low amplitude voltage with magnitude of 40V and high amplitude voltage with magnitude of 80V were applied to the PA at frequencies in which the asymmetric shape of the hysteresis takes place. E.g. for low amplitude voltage, the asymmetric shape of the hysteresis is initiated from frequency of 300 Hz and for high amplitude voltage, the asymmetric hysteresis shape is commenced from lower frequency of 200 Hz. As discussed in the previous sections, the generalized PI model is used to characterize the hysteresis loops that have asymmetric curves and the purpose of using the inertial-dependent model is to capture those hysteresis loops that cannot be characterized by the classical PI model properly.

The generalized inertial-dependent PI model was implemented to characterize the hysteresis in the PA by utilizing eight operators. These modeling approach is parameterized by threshold values of \( r \) and multipliers of \( w \), where \( r \) adjusts the width and height of the hysteresis and \( w \) adjusts the slope of the hysteresis curve correspondingly. To convert the model into the generalized model, an envelope hyperbolic function \( \gamma_r = a_0 \tanh(a_1v + a_2) + a_3 \) was assigned to capture the asymmetric shapes of the hysteresis loop.

As can be seen in Fig. 2 and Fig. 3 the generalized inertial-dependent PI model is capable of hysteresis characterization at high frequency satisfactorily.
Fig. 2. The results of modeling with generalized inertial-dependent Prandtl-Ishlinskii for a piezoelectric actuator at low excitation voltage of 40V; a) frequency: 300 Hz; b) frequency: 400 Hz; c) frequency: 500 Hz
Fig. 3. The results of modeling with generalized inertial-dependent Prandtl-Ishlinskii for a piezoelectric actuator at low excitation voltage of 80V; a) frequency: 200 Hz; b) frequency: 300 Hz; c) frequency: 400 Hz;

Conclusion

In this study, a generalized inertial-dependent PI model was proposed to achieve more efficient model as compared to both Classical and inertial-dependent PI models. In the new model, an envelope function was defined to adapt the threshold value of the stop operator or the play operator according to the shape of the hysteresis for characterizing the asymmetric hysteresis loops particularly at high frequencies above 200 Hz for a PA. Incorporating this model with inertial-dependent model reasonably enabled us to capture hysteresis loops at high frequency range up to 500 Hz where the hysteresis shape is subjected to both inertial effect and asymmetric loop, meanwhile, the inertial dependent model can characterize the width, height and turning points of the hysteresis properly whereas the envelop function improves the characterization of the asymmetric hysteresis at frequency range of interest respectively.

References