A survey on hysteresis modeling, identification and control

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ABSTRACT

The various mathematical models for hysteresis such as Preisach, Krasnosel’skii–Pokrovskii (KP), Prandtl–Ishlinskii (PI), Maxwell-Slip, Bouc–Wen and Duhem are surveyed in terms of their applications in modeling, control and identification of dynamical systems. In the first step, the classical formalisms of the models are presented to the reader, and more broadly, the utilization of the classical models is considered for development of more comprehensive models and appropriate controllers for corresponding systems. In addition, the authors attempt to encourage the reader to follow the existing mathematical models of hysteresis to resolve the open problems.

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1. Introduction

When speaking of hysteresis, one refers to the systems that have memory, where the effects of input to the system are experienced with a certain delay in time. This phenomenon is originated from magnetic, ferromagnetic and ferroelectric materials. It is like the elastic property of materials in which a lag occurs between the application and the removal of a force or field and its subsequent effect. The output of the system cannot be predicted without knowledge about the current state of the hysteretic system.

The importance of this study manifests itself in a mathematical modeling of some systems that involve hysteresis such as in smart materials, magnetic fields or micro-sliding friction where hysteresis is dramatically appeared as compared to other (geometric nonlinear) systems.

Amid the smart materials, piezoceramics, one of the most researched materials, are extensively used in widespread applications involving vibration control [1], adaptive structural shape control [2–4], structural health monitoring [5,6], structural acoustic systems [7–9], hybrid transducers and ultrasonic motors [10,11], nanopositioning stages [12,13], and more applications for the reason of high precision, high speed position control, high stiffness and fast response. They also are able to endure compressive forces up to several tons, while providing high resolutions and high bandwidth strains simultaneously.

In addition to piezoceramics, shape memory alloys and magnetostrictive actuators are also classified in the category of smart materials. The application of magnetostrictive materials is reported in transducer design [14], acoustic and industrial applications [10,15,16], and hybrid transducers [17–19].

In comparison with piezoceramics and magnetostrictive materials, shape memory alloys (SMA) are relatively new invention of large field of smart materials [20]. SMA is widely used in industrial applications such as vibration attenuation in civil structures [21] and SMA-based microactuator [22–24], etc.

In spite of large variety of applications assigned for smart materials, they are all subjected to the main source of nonlinearity, namely the hysteresis. This type of nonlinearity might lead to performance degradation specifically in positioning applications. If this phenomenon is neglected, it will give rise to inaccuracy in open loop control and degrades the tracking performance of the actuator. Also, it could cause undesirable oscillations in the system which could even lead to instability in the closed loop.

In contrast to a simple (but incomplete) representation of friction, i.e., the classical Coulomb friction model approximation that defines the force only at non-zero relative velocity ($v \neq 0$), in fact, micro-sliding displacements are actually observed [25]. When a contacting body is sliding and moving away from a reversal point, the friction force predominantly appears as a function of velocity, similar as presented by the Coulomb model. However, when the motion is reversed, the frictional effects of the mechanism are also determined by displacement function. Therefore, at a certain instance after the motion reversal, the friction behavior depends not only on the velocity, but also on the displacement, where the particular relation between friction and displacement involves a so-called non-local memory hysteresis [25–30]. This unique behavior attracts many researchers to thoroughly model the behavior of the contact between the two surfaces, which is realized using asperity junctions that can deform elastically or plastically depending on the load and on the displacement and/or the relative velocity of the surfaces [31].

In high precision positioning applications, the effects of friction present in a mechanical system can lead to significant positioning error. In order to compensate the error due to frictional forces, an effective control strategy is a prerequisite. As a consequence of the complex behavior of friction, linear control strategies are generally unsuitable for providing an optimal performance for controlling a motion of systems with friction. If an accurate model of the system is available, a compensation of the error in the system can be made by applying a feedforward command that is equal to and opposite to the instantaneous force.

In this paper, the various types of hysteresis models are investigated as well as their applications in modeling and control. In the remaining part of this study, different types of methods which are utilized for parameter estimation, system identification and system control will be briefly discussed. This paper will be wrapped up by conclusion.

2. Hysteresis in a nutshell

The presence of hysteresis in ferroelectric-based materials (such as piezoelectric materials) is an important property which creates constitutive nonlinearities in the relation between input fields $E$ (V/m) and stresses $\sigma$ (N/m²) and output polarization $P$ (C/m²) and strains $\varepsilon$ (m/m) as illustrated in Fig. 1. As detailed in [20], hysteresis is directly associated with the non-centro-symmetric structure of ferroelectric compounds and is observed to some degree at essentially all drive levels.

One of the main characteristics of ferroelectric materials is polarization reversal or switching by an electric field. Application of an electric field in ferroelectric materials will reduce domain walls in ceramics [20]. This phenomenon
(reducing and increasing in domain walls) is observed in terms of the hysteresis loop in ferroelectric materials. For further understanding, the switching phenomenon is discussed based on Fig. 1 as follows [20]:

**Point A:** In maximum amount of positive field, all dipoles are aligned with the field and material acts as a single domain. Increasing the field beyond this point results in the linear constitutive relations between the polarization and the field.

**Point B (positive remanence):** At point B, the applied field is zero and the dominant polarization in material is a positive remanence polarization $P_R$. Within this regime, the linear direct and converse constitutive equations of piezoelectric actuators are satisfied.

**Point C:** As the field is reduced down to negative coercive field $-E_C$, the polarization begins to switch and results in the hysteresis loop that has to be modeled by some mathematical expressions other than those applied for linear direct and converse constitutive equations.

**Point D:** At minimum amount of negative electrical field, similar to point A but opposite in polarization, the material again acts as a single domain.

**Point E (negative remanence):** Increasing the field $E$ to zero at point E causes dipoles to reorient to the negative remanence polarization $-P_R$. The behavior of polarization at this point is analogous to that at point B.

**Point F:** As the field increases up to positive coercive field $E_C$, switching of 180° domains produces the burst highly nonlinear region in the $E$–$P$ relation which causes the hysteresis loop like point C.

To investigate the hysteresis phenomenon in ferromagnetic materials, reader may refer to [32,33] in which the hysteretic behavior in MnZn ferrite, NiZn ferrite, NiFe Tape and CoCr film is considered.

The complex phenomenon of hysteresis in mechanical systems motivates some researchers to explore the dynamic behavior of the systems in a simpler way. Al-Bender et al. [34] analyzed the dynamic behavior of mechanical systems comprising rolling elements that exhibit the pre-sliding friction phenomena by performing the describing function technique. Tjahjowidodo [35] evaluated the geometric-nonlinear-equivalent of the rolling friction by utilizing the skeleton method and wavelet analysis. These attempts are proven to be able to simplify the dynamic analysis of systems with hysteresis nonlinearity that helps us to develop appropriate controllers; however, these do not capture the hysteresis properties in terms of modeling. The next section will discuss mathematical models to capture the properties in various mechanical systems.

### 3. Mathematical models for hysteresis

In order to simulate the hysteresis phenomenon discussed in previous section, some mathematical models have been developed. These models are classified into two types: (1) operator-based (the models which use operators to characterize hysteresis) and (2) differential-based (the models which use differential equation to characterize hysteresis). Our review is commenced by describing four well-known operator-based models, namely (1) Preisach, (2) Krasnosel’skii–Pokrovskii (KP), (3) Prandtl–Ishlinskii (PI) and (4) Maxwell-Slip.

#### 3.1. Preisach model

This model is one of the most popular operator-based models to capture the hysteresis behavior in nonlinear systems. In general, the Preisach model is expressed using a double integrator in continuous form as

$$x(t) = \int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta}[u(t)] d\alpha d\beta$$

(1)
where \( x(t) \) is the displacement of actuator with respect to initial length and \( \mu(\alpha, \beta) \) is a weight function that can be selected by both experiment and experience. The operator \( \gamma_{\alpha\beta} \) is valued 0 or 1 upon the polarized direction of the input \( u(t) \) as shown in Fig. 2.

The total displacement is the summation of several operators and weighting functions which are connected in parallel. To realize the concept of integrator in the Preisach model, Fig. 3 illustrates the function of integrator in this model.

The double integral presented for the Preisach model is relatively complicated to solve. To simplify the complexity, a simpler numerical model is presented by \( \alpha-\beta \) triangle and its corresponding \( x-u \) diagram to make a visual contribution for the user. To understand the numerical solution from \( \alpha-\beta \) triangle, let us consider an input history consisting a little increase or decrease in voltage as shown in Fig. 4.

The voltage increases from \( \beta_0 \) to \( \alpha_1 \) then decreases to \( \beta_1 \) and this cycle is repeated periodically, each of increasing and decreasing voltage will form a rectangle \( S_i \); \( i=1, \ldots, n \) in which the integration of area of each rectangle will give the total displacement of the actuator [36].

If this double integral is solved numerically, the following equation will be substituted:

\[
x(t) = \int_{\beta_0}^{\alpha_1} \mu(\alpha, \beta) \, d\beta + \int_{\beta_0}^{\alpha_1} \mu(\alpha, \beta) \, d\beta + \int_{\beta_0}^{\alpha_1} \mu(\alpha, \beta) \, d\beta + \ldots
\]

or in general, it can be rewritten as

\[
x(t) = \sum_{k=1}^{n-1} [x(\alpha_k, \beta_{k-1}) - x(\alpha_{k}, \beta_k)] + x(u(t), \beta)
\]

If the voltage stop is located vertically in the \( \beta \)-axis, the actuator displacement will be computed numerically by

\[
x(t) = [x(\alpha_1, \beta_0) - x(\alpha_1, \beta_1)] + [x(\alpha_2, \beta_1) - x(\alpha_2, \beta_2)] + [x(\alpha_3, \beta_2) - x(\alpha_3, \beta_3)] + \ldots
\]

or in the form of summation, we have

\[
x(t) = \sum_{k=1}^{n-1} [x(\alpha_k, \beta_{k-1}) - x(\alpha_{k}, \beta_k)] + x(\alpha_n, \beta_{n-1}) - x(\alpha_n, u(t))
\]

The \( x(\alpha_k, \beta_{k-1}) \) is representative of \( x(\alpha_k) - x(\beta_{k-1}) \) in Eq. (6).
The Preisach model is widely used in the modeling of hysteresis in smart materials. This model shows good performance to characterize hysteresis satisfactorily at narrow-band frequency as well as no-load condition. The accuracy of the Preisach model is gradually deteriorated as the pre-loading force and input frequency to the actuator are increased [37].

To increase the accuracy of the Preisach model, the modified Preisach is proposed for an integrated piezo-driven cantilever beam under quasi-static condition [38]. In this case, the Preisach model is modified using the finer mesh triangle and weighting functions in order to estimate the hysteretic behavior of the structure more accurately. An example of data reduction to build triangle of the Preisach model is shown in Table 1.

In some approaches, a hybrid model can improve the performance of the Preisach model, e.g., the Preisach model is fused into a 3-layered neural network to overcome inherent disadvantages in the Preisach model which mainly arise from the number of data points to train the model [40]. Similarly, the prediction accuracy of the Preisach model is effectively improved using a neural network in which the input vector is designed such that the network is applied in real-time application [41]. In another type of hybrid model, the tabulated Everette function is used in terms of the Preisach model to reduce the number of data points required in the classical Preisach model [42].

The classical Preisach model is rate-independent; meanwhile, the model is not sensitive to variation of rate of input applied to the system. On the other hand, hysteresis is a rate-dependent phenomenon [36]. As a result, to achieve the rate-dependent Preisach model, some modifications are needed. To get the gist of it, these modifications have to change the model such that it is used in dynamic applications. In this respect, a hysteresis operator first order differential equation (HOFODE) can be replaced by the Preisach model [43]. Since the Preisach model is similar to the modified diagonal recurrent neural network (MDRNN) regarding the structural point of view, the proposed HOFODE is implemented using the MDRNN.

As a substitute, a neural network can be used with the Preisach model such that the input vector of the network includes the same input vector as utilized in the Preisach model in addition to the velocity of the actuator in order to predict the displacement output of the actuator over a wide range of frequency [36].

Dissimilar to [36,43] in which the Preisach model is replaced by neural network, a novel modified rate-dependent Preisach model is proposed based on the approximation of density function using the fast Fourier transform (FFT) [44].

The Preisach model can also be used in the control framework. The inverse of the Preisach model is used as feedforward compensator in a control loop. This compensator is normally used as a tool to mitigate the effect of hysteresis nonlinearity in materials. If the linear model of the actuator is approximated, the inverse of the Preisach model can be employed parallel with any types of linear controllers such as PD/lead-lag compensator to improve the tracking performance of the control system [45].

Similarly, the inverse of the modified Preisach model is used as feedforward controller with PID controller to improve the performance of the linear PID controller [46] and also to control the joint angle of a manipulator driven by pneumatic artificial muscles [47].

The methodology proposed in [47] is pursued in [48] in which a linear–nonlinear system identification and control are carried out using the composite approach which combines the nonlinear properties of the piezoelectric mechanism which mainly arise from hysteresis in piezoelectric actuator and linear approximation of the mechanism regardless of nonlinearity in the system.

The dual integral in the Preisach mathematical model presents a big challenge to construct the inverse of the model easily. Using the inverse multiplicative technique enables us to split the Preisach model into non-memory and memory parts. Thanks to this methodology, it is only sufficient to derive the inverse of the non-memory part to obtain an expression for the control input signal [49].

Alternatively, the Preisach model is used along with some nonlinear controllers, e.g., a neural network sliding mode control is fused to the Preisach model to control a system with unknown hysteresis. In this work, neural network is utilized to predict the effect of hysteresis, while the sliding mode controller cancels the hysteresis effect on the system [50].

Table 1
Construction of $\alpha$–$\beta$ triangle [39].

<table>
<thead>
<tr>
<th>$\alpha$ (V)</th>
<th>$\beta$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30 19.98</td>
</tr>
<tr>
<td>30</td>
<td>19.40 19.25</td>
</tr>
<tr>
<td>60</td>
<td>16.22 14.32</td>
</tr>
<tr>
<td>90</td>
<td>11.81 8.90</td>
</tr>
<tr>
<td>120</td>
<td>8.50 6.66</td>
</tr>
<tr>
<td>150</td>
<td>5.96 4.11</td>
</tr>
<tr>
<td>180</td>
<td>3.78 1.78</td>
</tr>
<tr>
<td>210</td>
<td>1.22 0.58</td>
</tr>
<tr>
<td>240</td>
<td>12.86 12.09</td>
</tr>
<tr>
<td>270</td>
<td>10.97 9.13</td>
</tr>
<tr>
<td>300</td>
<td>8.25 6.29</td>
</tr>
<tr>
<td>330</td>
<td>5.46 3.33</td>
</tr>
<tr>
<td>360</td>
<td>4.34 2.46</td>
</tr>
<tr>
<td>390</td>
<td>1.22 0.57</td>
</tr>
<tr>
<td>420</td>
<td>0.57 0.57</td>
</tr>
</tbody>
</table>

Preisach function: $X(\alpha,\beta)$ (μm).

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Atomic force microscopy is a method to measure the surface roughness and topology within the manufacturing process. If the piezoelectric scanning system is built accurate enough, the results of the scanning process will be more reliable. To measure the surface roughness precisely, an indirect adaptive controller is used as feedback controller to control the motion of the piezoelectric scanning system. In order to increase the accuracy of the system, the inverse of the Preisach model is used as feedforward controller with the indirect adaptive controller [51].

In an innovative control system design, a Preisach model is substituted by plant model of a shape memory alloy actuator and cascaded with a PID controller. The error between the desired output and the output stemmed from the Preisach model is feedback to PID controller. The parameters of the PID controller are optimized using the genetic algorithm to minimize a cost function which is expressed in terms of sum of square of errors between the desired output and the output of the Preisach model [52]. Alternatively, the parameters of the PID controller are tuned by fuzzy logic rules in another design [53].

The combination of the LuGre friction model and the Preisach model enables us to characterize the hysteresis behavior in materials efficiently. Using the combined model, a back-stepping sliding mode control strategy is applied to seed the control algorithm and estimate the parameters of the controller by Lyapunov function, accordingly [54]. For detailed investigation of the control strategies designed for a system suffering from the hysteresis nonlinearity, readers may refer to Section 5.

Notwithstanding, the Preisach model is widely used for hysteresis characterization, it has some disadvantages that are listed below [40]:

1. Total amount of data collected through the experiment will influence the accuracy of the Preisach model.
2. Since this model uses double integrator, constructing the inverse of the model is not easy. In result, it is not convenient to use this model in real-time applications.
3. True physics of the system cannot be implemented through this model.

As a result, the Krasnosel’skii–Pokrovskii (KP) model and Prandtl–Ishlinskii (PI) model are derived from the Preisach model to overcome the disadvantages existing in the Preisach model.

3.2. Krasnosel’skii–Pokrovskii (KP) model

After proposing the Preisach model by a German physicist F. Preisach, the Russian mathematician, Krasnosel’skii, introduced the Preisach model into a pure formalized mathematical form in which hysteresis is modeled by linear combination of hysteresis operators [55].

Contrary to the operator used in the Preisach model that creates jump discontinuities in the model, the KP model allows a great combination of functions to be replaced by the Preisach operators. The KP kernel, the elementary operator of the KP model which is a special case of generalized play operator, is a continuous function on the Preisach plane including minor loops within its major loop, where the Preisach plane is defined as

\[ \mathcal{P} = \{ (p_1, p_2) \in \mathbb{R}^2 : U^- \leq p_1 \leq p_2 \leq U^+ - a \} \]

where \( U^+ \) and \( U^- \) represent the positive and negative saturation values of input \( u(t) \in C[0, T] \) respectively. The positive constant value, \( a \), is the rise constant of the kernel. Let \( C[0, T] \) denote the space of continuous piecewise monotone functions on the interval \( [0, T] \), then the elementary KP hysteresis operator is expressed as

\[ K_p(u, \xi_p) : C[0, T] \rightarrow y[0, T] \]

where \( \xi_p \), parameterized by \( P \), represents the initial condition of the kernel and the memory of the previous extreme output of the kernel, and \( y[0, T] \) is the output space. For a specific amount of \( u(t) \), the KP operator \( K_p(u, \xi_p) \) maps points \( (p_1, p_2) \) to the interval \([ -1, 1] \) and is given by

\[ K_p(u, \xi_p)(t) = \begin{cases} \max \{ \xi_p, r(u(t) - p_2) \} & \text{if } \dot{u}(t) > 0 \\ \min \{ \xi_p, r(u(t) - p_1) \} & \text{if } \dot{u}(t) < 0 \end{cases} \]

where \( r(\cdot) \) is the Lipschitz continuous ridge function. The ridge function can be expressed in any form but it is popular to be selected in terms of a continuous piecewise linear function defined as

\[ r(\chi) = \begin{cases} -1 & x < 0 \\ -1 + \frac{2\chi}{a} & 0 \leq x \leq a \\ +1 & x > 0 \end{cases} \]

Using the above equations, the KP operator is illustrated in Fig. 5. By comparing the Preisach operator, it is evidently observed that the KP operator consists of family of curves that are bounded by the curves \( r(u - p_2) \) and \( r(u - p_1) \) between the envelop ranging from \( p_1 \) to \( p_2 + a \).

The KP hysteresis output is expressed using the double integral as follows:

\[ y(t) = \int_0^t \int_0^t K_p(u(t), \xi_p) d(p_1, p_2) dp_1 dp_2 \]
where $\mu(p_1, p_2) \geq 0$ is the density of kernel KP which is utilized to weight the output of kernel KP. This formalism can also be interpreted as a parallel connection of an infinite number of weighted kernels which are shown in Fig. 6. This structure which is used in KP kernels provides more information of the nonlinearity than the Preisach operator, due to memory effect to record all previous extremes of hysteresis input–output behaviors [55].

Using the KP model enabled some researchers to characterize the hysteresis nonlinearity in magnetically shaped memory alloys which possess the asymmetric hysteresis loop [56]. In more comprehensive work to model the hysteresis nonlinearity in shape memory alloys (SMA), a KP-based model is developed to predict the hysteresis behavior of SMA both in minor loop as well as first order ascending curves attached to the major hysteresis loop, while the parameters of the KP model are identified only with some first order descending reversal curves attached to the major loop [55].

In terms of controller design, the inverse of the KP model is derived to linearize a system suffering from hysteresis, and then an adaptive control strategy is proposed to control a linearized system [57].

Similarly, a compensator is designed for a stack type piezoelectric actuator using the inverse of the KP model [58]. Although the KP model improves the performance of the Preisach model by utilization of the kernel operators, but it still exploits the double integral in its mathematical structure and causes some difficulties in constructing the inverse of the model as well as real-time application. To overcome this imperfection in the Preisach and the KP models, the Prandtl–Ishlinskii model is introduced as subset of the Preisach model which possesses the simpler mathematical structure in comparison with the Preisach and the KP models.

3.3. Prandtl–Ishlinskii (PI) model

PI is the subset of the Preisach model which is applicable for hysteresis modeling in materials. This model exploits two essential operators: (1) stop operator and (2) play operator.

3.3.1. Stop operator

The stop operator is implied by $\omega(t) = E_r[v](t)$ in which $v$ is the set of inputs, $w$ is the set of outputs, $r$ is the threshold value, and the function $E$ plays as the stop operator function.

Suppose that $C_m[0, t_F]$ is the space of a piece-wise monotone continuous function, a function $e_r: \mathbb{R} \to \mathbb{R}$ is defined as

$$e_r(v) = \min(r, \max(-r, v)) \text{ with } r \geq 0$$

(11)

For any input $v(t) \in C_m[0, t_F]$ and any initial value $w_{-1} \in \mathbb{R}$, the stop operator can be expressed by the following formulations:

$$E_r[v; w_{-1}](0) = e_r(v(0) - w_{-1})$$

(12)
\[ E_t[v; w_{-1}](t) = e_t(v(t) - v(t_i)) + E_t[v; w_{-1}](t_i) \]

\[ t_i < t \leq t_{i+1} \quad \text{and} \quad 0 \leq i \leq N - 1 \]  

(13)

It is also assumed that the function \( v \) is monotone on each of the sub-intervals \([t_i, t_{i+1}]\). This operator basically determines the height of the hysteresis region in the input–output or \((v, w)\) plane [59]. This operator can be seen in Fig. 7(a).

### 3.3.2. Play operator

Play operator is a continuous and rate-independent operator. In this operator, repeatedly, suppose \( C_m[0, t_E] \) represents the space of piecewise monotone continuous function. The function \( v \) is monotone on \([0, t_E]\) and each of the sub-intervals \([t_i, t_{i+1}]\).

The operator is defined by

\[
F_r[v](0) = f_r(v(0), 0) = w(0)
\]

(14)

\[
F_r[v](t) = f_r(v(t), F_r[v](t_i)); \quad \text{for} \quad t_i < t \leq t_{i+1} \quad \text{and} \quad 0 \leq i \leq N - 1
\]

(15)

where

\[
f_r(v, w) = \max(v - r, \min(v + r, w))
\]

(16)

The relation between the stop operator and the play operator is expressed as

\[
E_t[v; w_{-1}](t) + F_r[v; w_{-1}](t) = v(t)
\]

(17)

Basically, two main elements of the play operator are the input \( v \) and the threshold value \( r \) [59]. The play operator is shown in Fig. 7(b).

In a continuous format of the play operator, the hysteresis relationship between the output, \( y \), and the input, \( v \), can be represented by the following integral:

\[
y_p = qv(t) + \int_0^t p(r)F_r[v](t)dr
\]

(18)

where \( p(r) \geq 0 \) is a density function which is usually identified by experimental data and \( q \) is a constant. For the reason of convenience, the value, \( R \) is assumed \( R = \infty \) in the literatures. For further understanding of the operators applied in the PI model, reader may refer to the procedure of the simulation in MATLAB Simulink presented in [61]. In [62], a modified PI is proposed in terms of the right-hand and the left-hand play operators to model the multi-loop ascending and descending hysteresis loops.

In practice, depending on the input magnitude and the input frequency applied to the actuator, the hysteresis loop shows different behaviors. One of these behaviors appears as an asymmetric hysteresis loop. The generalized rate-independent PI is proposed to overcome the imperfections that we face in modeling the asymmetric hysteresis loops with the classical PI. The only change made in the classical model to transform it to the generalized one is attributed to the threshold value. Two envelop functions as \( \gamma_l \) and \( \gamma_r \) are defined instead of the threshold value, \( r \), in the classical model that are subsequently represented for left and right envelops of the hysteresis curve [63]. Regarding the shape of the hysteresis curve, different mathematical functions such as linear, tangent hyperbolic, etc. can be used as envelop function. The generalized PI model is
expressed for the play operator as
\[ F_1^t(v)(0) = f_1^t(v(0), 0) = w(0) \] (19)
\[ F_1^t(v)(t) = f_1^t(v(t), F_1^t(v)(t_i)) \] (20)
For \( t_i < t < t_{i+1} \) and \( 0 \leq i \leq N - 1 \)
\[ f_1^t(v, w) = \max(y_v(v) - r, \min(y_v(v) + r, w)) \] (21)
and for the stop operator, we have
\[ E_1^t(v; w_{-1})(0) = e_1^t(v(0) - w_{-1}) \] (22)
\[ E_1^t(v; w_{-1})(t) = e_1^t(v(t) - v(t_i) + F_1^t(v; w_{-1})(t_i)) \] (23)
For \( t_i < t < t_{i+1} \) and \( 0 \leq i \leq N - 1 \)
\[ e_1^t(v) = \min(y_v(v) + r, \max(y_v(v) - r, v)) \] (24)

Fig. 8 shows an example of asymmetric hysteresis loop and the respective envelope functions for ascending and descending hysteresis curves [63].

A different approach is used to model the asymmetric hysteresis loop occurring in piezoelectric actuator utilized for atomic force microscopy imaging. In this approach, as an alternative approach to [63] at which envelop functions are proposed for ascending and descending loops, different slope values describe the forward and backward hysteresis loops [64].

As described earlier in Section 3.1, the hysteresis phenomenon apparently appears as a function of frequency. Meanwhile, by increasing or decreasing the input rate or input frequency, the intensity of the hysteresis will increase or decrease consequently. Due to this property, different PI models are proposed in terms of rate-dependent model. In [60, 65, 66], a new mathematical model is proposed with a logarithmic threshold function in the play operator of the classical PI model which is the function of the input rate applied to the actuator. This function is written as
\[ \tau = \alpha \prod_{l=1}^{z} \ln \left( \beta_l + z_i \left| v(t) \right| \right) \] (25)
where \( \alpha \) and \( \beta_l \) are the positive constants, \( \beta_l \geq 1 \), \( e \geq 1 \) and \( z \) determines the number of operators in a particular problem, whereas \( v(t) \) stands for the rate of input applied to the material. Based on this definition, the output of the rate-dependent play operator is obtained by means of the following formulation:
\[ y_p(k) = w_k v_k + \sum_{k=1}^{\infty} g_k \beta_k F_{\tau_k} \] (26)
where \( w_k \) and \( g_k \) depend on the nature of hysteresis and the type of material used in the actuator. These values can be selected either as constant quantities or in terms of mathematical expression as
\[ w(v, \dot{v}) = a_1 e^{-m_1 \dot{v}} e^{m_2 v} \] (27)
\[ g(v, \dot{v}) = a_2 e^{-s_1 \dot{v}} e^{s_2 v} \] (28)
where \( a_1 > 0 \) and \( a_2 > 0 \), \( m_1 \), \( m_2 \), \( s_1 \) and \( s_2 \) are constants to be determined by experimental data. \( F_{\tau_k} \) is the play operator as given in Eqs. (15) and (16). On the contrary, the threshold value \( \tau_k \) is a function of input frequency. In the following model, a generalized rate-dependent PI is designed to capture the asymmetric hysteresis curve at relatively wide range of frequency between 0 and 200 Hz [65].
In a different approach to create rate-dependent PI, a linear function is proposed for a piezoelectric actuator at low frequency which corresponds to the slope of the hysteresis curve to input rate applied to the actuator [67]. In this methodology, the weight parameters are updated by the following relationship:

\[ W_{hi}(\dot{v}(t)) = W_{hi} + c_i \dot{v}(t), \quad i = 0...n \]  

(29)

where \( c_i \) is the slope of the best fit line, \( W_{hi} \) is the intercept of the best fit line with the vertical \( W_h \) axis or the slope at zero input rate and \( i \) is the number of \( W_h \) vectors. As an alternative approach to [67], a novel direct inverse rate-dependent model is derived directly from the experimental data which is applicable to all the PI-based hysteresis compensation problems [68].

The PI model is acclaimed for the reason of simplicity to identify the nonlinear response of smart materials such as piezoelectric actuators, shape memory alloys, etc. The nonlinear response is represented to a certain amount of input force or input voltage in terms of displacement or velocity. On the other hand, researchers have come to the conclusion that the PI model can be used in general control framework in addition to utilization in system identification. This fact is fulfilled when the inverse of the PI model is utilized in a feedforward control loop. The inverse of the PI model is initially formulated by Krejci and Kuhnen [69]. This research is pursued by Kuhnen and Janocha [70] to obtain the more comprehensive inverse model for a wide range of hysteresis loops. The inverse model of the play operator is utilized in an alternative formalism with another type of controller to improve the response characteristics of smart actuators. In order to identify the asymmetric hysteresis in shape memory alloys and magnetostrictive actuators, a generalized PI model is utilized [71]. Through this design, the inverse of the model is constructed to compensate the effect of hysteresis in a control system. Further familiarization with inverse of the PI model, if \( H[v] \) represents the output of the play operator as

\[ H[v] = qv(t) + \sum_{j=1}^{q} p_j F_{r_j}[v](t) \]  

(30)

The inverse of the play operator is defined as

\[ H^{-1}[v](t) = q^{-1}v(t) + \sum_{j=1}^{q} g(\hat{r}_j)F_{r_j}[v](t) \]  

(31)

where \( \hat{r} \) is the threshold value of the inverse model and \( g \) is the density function of the inverse model which are expressed as

\[ \hat{r}_j = qr_j + \sum_{i=1}^{j-1} p_i(r_j - r_i) \]  

(32)

\[ g_j = -\frac{p_j}{(q + \sum_{i=1}^{j-1} p_i)(q + \sum_{i=1}^{j-1} p_i)} \]  

(33)

\( q^{-1} \) is the constant which is equal to \( 1/q \) and \( j \) is the number of operators. As mentioned above, the inverse of the PI model can be used either as feedforward controller individually in open loop control systems or together with other types of controllers in feedback control systems, e.g., the inverse generalized PI model is used with a robust nonlinear adaptive controller to improve the robustness and the stability of the close-loop system [72]. Design of the adaptive sliding mode controller through fusing the PI model is also addressed for nano-positioning control of a piezoelectric actuator [73]. The PI model and neural network approximator are utilized in order to ensure that all the close-loop signals of a robust adaptive controller are bounded and the tracking performance guarantees the tracking error to converge to an adjustable neighborhood of zero [59].

As we know, the output of the stop operator is voltage input to the actuator which is known as forcing function. In order to control the linear model of a magnetostrictive actuator, the combination of linear transfer function and nonlinear forcing function results in development of nonlinear variable structure adaptive controller for positioning control at different frequency regimes [74]. Similarly, using the stop operator leads directly to design a compensator for a piezoceramic [75]. The inertia-dependent stop operator is also proposed in [76] at which the inertial and the damping effects of the stack type piezoelectric actuator are associated with the stop operator of the Prandtl–Ishlinskii model.

In different frames of controller design, a modified rate-dependent PI model is presented with a dead-zone operator for asymmetric hysteresis loops [77]. In this case, the tracking performance of a feedforward controller which is seeded by constructing the inverse of the model reveals good agreement with the real data at relatively low frequency range between 0 and 40 Hz.

Constructing the inverse of the PI model is not always done easily. If ill-conditioned hysteresis occurs in hysteresis loop, the inverse model will deal with singularity which makes it difficult to analyze. The remedy is carried out in terms of proposing the extended PI operator to map hysteresis to a domain where inversion is performed readily [78].

A new methodology is proposed by Al Janaideh et al. [79] for analytical inversion of the rate-dependent PI model. This model is used as hysteresis compensator for micropositioning control of a piezoelectric actuator.
3.4. Maxwell-Slip model

This model is basically a tool to express hysteresis nonlinearity in both mechanical and electrical systems. Originally, this model was proposed to show the behavior of friction in mechanical systems. The function of this model is similar to the stop operator in the PI model.

This model operates with an elasto-slide element comprising of a massless linear spring and a massless block which are susceptible to Coulomb friction, \( F \). The relationship for this element is described by

\[
F = \begin{cases} 
  k(x-x_b) & |k(x-x_b)| < f \\
  f\operatorname{sgn}(\dot{x}) & x_b = x - \frac{f}{c}\operatorname{sgn}(\dot{x}) 
\end{cases}
\]  

(34)

where \( f = \mu N \), \( k \) is the spring stiffness, \( x_b \) is the block position and \( x \) is the displacement (see Fig. 9).

The relationship of the applied force and the displacement output is depicted in Fig. 10. From a to b, the spring is elongated but no block motion occurs until the applied force reaches up to Coulomb force, \( f_i \). At point b, the sliding motion starts and it continues to point c where the reverse motion is started. From point c to e, the spring is compressed but no block motion occurs. Again, the sliding is started from point e to f when the applied force reaches force \(-f_i\) [81,82].

The overall force is represented by a parallel connection of number of Maxwell-Slip elements as shown in Fig. 11 [80]. The instantaneous stiffness of the system, \( S_j \), appears as summation of the elementary springs, \( \sum_{i=1}^{n} k_i \).

Combining the linear second-order dynamics of the piezoelectric actuator and the nonlinear elasto-slip model enables us to characterize the total dynamic behavior of a piezo-stack actuator for both modeling and control [83].

In order to express the Maxwell-Slip model in terms of voltage and charge, a mathematical model is proposed [84]. This model adds one nonlinear spring element to rest of the linear spring elements of Maxwell-Slip model to capture both symmetric and asymmetric hysteresis loops. Finally, the inverse of the model is used for positioning control of a piezoelectric actuator.

In terms of biomedical application, a Maxwell-Slip model is utilized as a lumped-parametric quasi-static model to capture the hysteresis existing in the pneumatic artificial muscle dynamics in order to handle control strategy in terms of the input force applied to control the length of the muscle [85].

Similarly, electromechanical model is developed for both modeling and control of piezo-based structure [80].

In spite of some applications mentioned above, one of the problems in Maxwell-Slip model corresponds to the hysteresis rising curve. At this instant of time, the smart actuators need to start from a relaxed state which is not easily acquired in practice [86]. This reason gives rise to restrict the application of Maxwell-Slip model in modeling the hysteresis nonlinearity in smart materials.

Unlike the previous models (Preisach, KP, PI and Maxwell-Slip) that fall in the class of operator-based models, other types of hysteresis models in the category of differential-based are introduced in Sections 3.5 and 3.6.

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Fig. 9. Concept of the Maxwell-Slip model [80].

Fig. 10. Hysteresis behavior of force–displacement in Maxwell-Slip model [80].
3.5. Bouc–Wen model

In general, the Bouc–Wen model is expressed by means of a nonlinear differential equation as follows:

\[ \dot{z} = A \dot{u} - \beta |u| z^{n-1} - \gamma |u|^n \]

where the parameters \( A, \beta \) and \( \gamma \) determine the shape of hysteresis and \( n \) is an integer number. \( Z \) in this equation is a state variable which is integrated by the state-space equations of the system and \( \dot{u} \) is the derivative of existing input \[87\].

In this context, one can find myriad of proposed models in modeling and control of hysteresis. To model the hysteresis nonlinearity in a large scale magnetorheological damper, an innovative identification model is used to estimate the parameters of a Bouc–Wen model \[88\].

To transform the Bouc–Wen model into a rate-dependent one, this model is formulated together with a linear Hammerstein model in which the former one models the static hysteresis in a piezoelectric actuator while the latter is used to model the rate-dependent hysteresis in a relatively low range frequency from 1 Hz to 100 Hz \[89\].

The Bouc–Wen model can also be applied with different types of controllers, e.g., in one design, three control laws are defined in terms of PID controller for micro-positioning control of a piezoelectric actuator modeled by the Bouc–Wen hysteresis model \[90\]. The PID controller with time-varying parameter shows the best tracking performance among the others.

To evaluate the combination of fuzzy T–S model and the genetic algorithm in control system design, the linearization of the Bouc–Wen model is carried out by T–S fuzzy rule and the parameters of the model are estimated by genetic algorithm \[91\]. In the last part of controller design, an LMI-based controller is designed to stabilize the system in an optimal manner.

Alternatively, a fuzzy-based PD controller is used for a piezoelectric actuator which is dynamically modeled by the Bouc–Wen hysteresis model \[92\]. Simulation results reveal that the controller is robust against external disturbances.

In order to improve the performance of nonlinear system, a back-stepping nonlinear control is proposed for a piezo-based structure \[93\]. In this design, first of all, the Bouc–Wen model is established for modeling the hysteresis in actuator, then the back-stepping controller is designed based on the parameters of the model. Finally, the performance of the back-stepping controller is compared with a linear PID controller. In the area of nonlinear control, repeatedly, a model reference adaptive nonlinear controller is designed to control a piezoelectric moving element \[94\].

Since the Bouc–Wen model is not invertible, a Bouc–Wen least square support vector machine (LSSVM) is designed either to identify or compensate hysteresis in feedforward path without the need to model hysteresis inverse \[95\]. In other research, a multiplicative-inverse structure approach is proposed to design a compensator for the hysteresis nonlinearity in the piezoelectric actuator which is expressed by the Bouc–Wen model \[96\]. This strategy allows us that no more computation is carried out for the compensator as well as adaptation with the Bouc–Wen model.

3.6. Duhem model

This model is another type of differential-based hysteresis model which is presented in a more complex way than the Bouc–Wen model. This model was proposed in 1897 to simulate an active hysteresis. The design was based on the approach that the output \( w \) can change its characteristics when the input \( v \) changes its direction \[97\]. To express it into a mathematical form, suppose we have

\[ \frac{dw}{dt} = a \left| \frac{dv}{dt} \right| f(v - w) + \frac{dv}{dt} g(v) \]

where \( a \) is constant and positive.
This model is fulfilled, if the following conditions are satisfied:

Condition 1: \( f(.) \) is piecewise smooth, monotone increasing, odd, with \( \lim_{v \to \infty} f'(v) \) finite.
Condition 2: \( g(.) \) is piecewise continuous, even, with \( \lim g(v) = \lim f'(v) \).
Condition 3: \( f(v) > g(v) > \alpha v \int_0^v |f'() - g()d\xi| e^{-\alpha \xi}d\xi \) for all \( v > 0 \).

The differential Duhem model is solved:

\[
w = f(v) + [w_0 - f(v_0)]e^{-\alpha v} + e^{-\alpha v} \int_{v_0}^v [g(\xi) - f'(\xi)]e^{\alpha \xi}d\xi
\]

and the above solution can be simplified by

\[
w = f(v) + \varphi(v)
\]

Ascending \( w_i \) belongs to a time \( \sgn(\nu) = 1 \) and the solution changes to

\[
w_i(v) = f(v) + [w_0 - f(v_0)]e^{-\alpha (v - v_0)} - e^{-\alpha v} \int_{v_0}^v [f'(\xi) - g(\xi)]e^{\alpha \xi}d\xi \quad v \geq v_0
\]

and the descending \( w_i \) is when the \( \sgn(\nu) = -1 \) and the new solution is

\[
w_d(v) = f(v) + [w_0 - f(v_0)]e^{-\alpha (v - v_0)} - e^{-\alpha v} \int_{v_0}^v [f'(\xi) - g(\xi)]e^{\alpha \xi}d\xi \quad v \leq v_0
\]

As can be seen from both the relationships for \( w_i \) and \( w_d \), when \( v \) is going to \( \infty \) or \(- \infty \) the condition 2 is satisfied \[97\] \[41\]

\[
\lim_{v \to -\infty}[w_i(v; v_0, w_0) - f(v)] = 0
\]

\[
\lim_{v \to \infty}[w_i(v; v_0, w_0) - f(v)] = 0
\]

The Duhem model is rate-independent. The big challenge in this model is to find out \( f(v) \) and \( g(v) \) which are functions of input voltage influencing on both the model performance and the hysteresis loop \[86\]. In order to approximate these functions, in one research, a polynomial is proposed based on the well-known Weierstrass theorem \[98\]. Finally, the coefficients of these polynomials are estimated using the recursive least squares method.

The Coleman–Hodgon model, special case of the Duhem model, is used to model unknown hysteresis nonlinearity in a system \[99\]. Consequently, in this work, a robust sliding mode control scheme is proposed to mitigate the effect of hysteresis using the Coleman–Hodgon model without any need of constructing the inverse of the model.

Similarly, the Duhem model can be fused to a robust control law in order to achieve the desired precision in tracking performance \[97\]. As said earlier, since the Duhem model is a differential-based model, construction of the inverse is a difficult task. This fact motivated Feng et al. \[97\] to present an adaptive nonlinear controller based on the Duhem model to ensure the stability of the system globally in spite of unknown hysteresis existing in the system.

Based on the system identification performed for a giant magnetostrictive actuator (GMA), a combination of linear–nonlinear model is obtained to model the dynamic characteristics of GMA. Using this model, a robust sliding mode controller is designed to improve the tracking performance as compared to linear controllers \[100\].

In addition to hysteresis models discussed in Sections 3.1–3.6, reader may refer to some works done with other mathematical models for hysteresis characterization in materials \[101–103\].

4. System identification strategies in hysteresis characterization

Most of the models considered in the previous section consist of many parameters to build the shape of the hysteresis curve. In the first place, a suitable model has to be assigned to describe a nonlinear behavior of the system properly, and then the parameters of the proposed model have to be estimated. This matter can be considered from two different points of views. In one hand, an identifier can be designed and substituted to the model of the system for imitating the behavior of the real system as nearly as possible with a minimum error. This kind of identification is known as non-parametric identification. On the other hand, parameters of the proposed model can be estimated through an optimization tools. This type of identification is known as parametric identification in which the parameters of the system are estimated using several methods such as least mean square, recursive least square, genetic algorithm, particle swarm optimization, etc.

4.1. Least mean square-based system identification

Within this discipline, the model is often presented in a regression form, afterward, the parameters of the regression model will be estimated through the least square method on the input and output data sets which are taken from an experiment or simulation. The mechanism of this methodology is shown in Fig. 12.
Some of the regression models are listed below:

**ARX:**

\[ A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + \epsilon(t) \]

\[ A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_mq^{-ma} \]

\[ B(q^{-1}) = b_0 + b_1q^{-1} + \ldots + b_nq^{-nb} \] \hspace{1cm} (43)

$q$ is the delay operator.

**ARMAX:**

\[ A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})\epsilon(t) \] \hspace{1cm} (44)

where $C$ is equal to $C(q^{-1}) = 1 + c_1q^{-1} + \ldots + c_nq^{-nc}$

**ARMA:**

\[ A(q^{-1})y(t) = C(q^{-1})\epsilon(t) \] \hspace{1cm} (45)

**DARMA:**

\[ A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) \] \hspace{1cm} (46)

where $y$ is the output vector, $u$ is the input vector, $\epsilon$ is the error between model output and actual output.

Using the aforementioned models, the following regression model can be defined as:

\[ y(t) = \varphi^T \hat{\theta} + \epsilon(t) \] \hspace{1cm} (47)

in which $\varphi^T(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t)]$ are known as regression variables. For example, in an ARX model, this vector is recognized by

\[ \varphi^T(t) = [-y(t-1), -y(t-2), \ldots, -y(t-n), u(t-d), u(t-d-1), \ldots, u(t-d-m)] \]

And also, $\hat{\theta}$ is defined as unknown parameters which must be identified by the least mean square method. For an ARX model, the parameter vector will be represented as

\[ \hat{\theta}^T = [a_1, a_2, \ldots, a_n, b_0, b_1, \ldots, b_n] \]

The following cost function is used in least mean square:

\[ J(\hat{\theta}) = \sum_{t=1}^{N} [y(t) - \varphi^T(t)\hat{\theta}]^2 \] \hspace{1cm} (48)

The $\varphi^T(t)\hat{\theta}$ is the predicted output and $y(t)$ is the real output. To minimize this cost function, let us set the derivative of $\partial J(\hat{\theta})/\partial \hat{\theta}$ to zero, it will give the following relation:

\[ \frac{\partial J(\hat{\theta})}{\partial \hat{\theta}} = -2 \sum_{t=1}^{N} \varphi(t)[y(t) - \varphi^T(t)\hat{\theta}] = 0 \Rightarrow \sum_{t=1}^{N} \varphi(t)y(t) = \left[ \sum_{t=1}^{N} \varphi^2(t) \right] \hat{\theta} \]

Finally, by making the inverse, the unknown parameters can be estimated as follows [104]:

\[ \hat{\theta}_N = \left[ \sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right]^{-1} \sum_{t=1}^{N} \varphi(t)y(t) \] \hspace{1cm} (49)
Regarding system identification by the least mean square method, a bivariate probability density function (pdf) is used in terms of the Preisach model to capture hysteresis in shape memory alloys and ferromagnetic materials [105].

An adaptive inverse model approach is also used for micropositioning control of a piezoelectric actuator in which the hysteresis is mathematically modeled and the parameters of the model are updated by least mean square algorithm [106].

4.2. Recursive least square-based system identification

This method is a tool to estimate unknown parameters of a system in a dynamic manner. This method is the extended type of least mean square algorithm.

At the beginning of this algorithm, let us assume,

\[
\sum_{l=1}^{N} \varphi(t)\varphi^T(t) = R(t)
\]

\[
\sum_{l=1}^{N} \varphi(t)y(t) = f(t)
\]

Then we have

\[
\hat{\theta}_t = R^{-1}(t)f(t)
\]

where \( R(t) \) and \( f(t) \) can be written in time-dependent terms as

\[
R(t) = \lambda(t)R(t-1) + \varphi(t)\varphi^T(t)
\]

\[
f(t) = \lambda(t)f(t-1) + \varphi(t)y(t)
\]

\( 0 < \lambda(t) < 1 \) is the forgetting factor that will exponentially give less weight to earlier samples. By substituting (54) into (52), we have

\[
\hat{\theta}(t) = \hat{\theta}(t-1) + R^{-1}(t)\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}(t-1)]
\]

Calculating the inverse of \( R(t) \) is the time consuming process. To overcome this problem, the matrix inverse lemma is used to eliminate \( R^{-1} \) from (56). By utilizing this lemma, the recursive least square rule changes into the following form:

\[
\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{p(t-1)}{\lambda(t) + \varphi^T(t)p(t-1)\varphi(t)}[y(t) - \varphi^T(t)\hat{\theta}(t-1)]
\]

\[
p(t) = \frac{1}{\lambda(t)}[p(t-1) - \frac{p(t-1)\varphi(t)\varphi^T(t)p(t-1)}{\lambda(t) + \varphi^T(t)p(t-1)\varphi(t)}]
\]

\( \varphi \) is the regression variable vector and \( \theta \) is the parameter vector. \( p(t) \) is initialized by a large number in a positive definite matrix. These relationships show the time-dependent version of the least mean square algorithm that makes it adaptive and usable in on-line parameter estimation [104].

The hysteresis in piezoelectric actuators can be characterized using NMAX and NARMAX models [107]. The parameters of these models are estimated by the recursive least square method.

In another example of application of this methodology in parameter estimation, the dynamic equations of the impact drive mechanism including two masses and one piezoelectric element are implemented together with the hysteresis model of the piezoelectric actuator using the Bouc–Wen model [87]. After identification of the parameters by the adaptive recursive least square method, the inverse of the model is used as feedforward controller to alleviate the effect of hysteresis in the impact drive mechanism.

4.3. Genetic algorithm-based system identification

This algorithm is used as a heuristic method of optimization in a category of random search methods. In this methodology, parameters underlying estimation process are considered as chromosomes that form a string called gene as shown in Fig. 13. A certain amount of chromosomes are forming the population [108].

At the beginning of the algorithm, a fitness function is defined. The values of the initial chromosomes are selected randomly to form a population. The fitness function is calculated for each set of the chromosome and the algorithm is preceded through iterations till the fitness function reaches out the best value. Upon the optimization process, i.e., minimization or maximization, some steps of this algorithm can be briefly summarized as follows:

1. initialize the first population;
2. evaluate fitness function;
(3) select chromosomes;
(4) perform crossover;
(5) perform mutation;
(6) report best chromosome as solution; and
(7) go to step 2 for the next generation or iteration.

**Fig. 14** shows the summary of the conventional genetic algorithm.

In one research, three fitness functions are defined for estimation of the parameters of the Bouc–Wen model [109–111]. Kwok et al. [112] proposed an asymmetric Bouc–Wen model for characterization of hysteresis in a magnetorheological fluid damper. A novel genetic algorithm with adaptive crossover and mutation stage is developed to optimize the parameters of the model.

A dual-stage X–Y positioned which is operated by permanent magnet for course displacement and two piezoelectric elements for fine displacement is dynamically modeled [113]. The hysteresis nonlinearity in piezoelectric actuator is modeled by the Bouc–Wen model and the parameters of the model are identified by real-coded genetic algorithm. In an innovative research, the hysteresis nonlinearity is approximated in the blood vessels using the Maxwell-Slip model [114]. In this work, a new optimization strategy is opted to estimate the parameters of the model in terms of new genetic algorithm approach which exploits locally crossover with pressure selection for variable length genotype.

### 4.4 System identification by particle swarm optimization (PSO)

Over the past decade, many types of optimization methods have been developed and used in many applications specifically in identification, control, etc. The particle swarm optimization has been recently considered as a high performance optimization tool that is very easy to understand and implement. It is known as an alternative method to
The procedure of PSO is described as follows:

1. Initialize the time to zero and set a number for initial position \( x_j(0) \) and initial velocity \( v_j(0) \).
2. Evaluate the fitness of each particle \( F(x_j(t)) \).
3. Set the \( P_{best}(t) \) to the better performance via
   \[
   P_{best}(t) = \begin{cases} 
   P_{best}(t-1) & F(x_j(t)) \geq F(P_{best}(t-1)) \\
   x_j(t) & F(x_j(t)) < F(P_{best}(t-1)) 
   \end{cases} 
   \]
4. Set the \( g_{best}(t) \) to the position of article with the best fitness within the swarm as
   \[
   g_{best}(t) \in \{P_{best}(t_1), P_{best}(t_2), ..., P_{best}(t_n)/F(g_{best}(t)) = \min \{F(P_{best}(t_1)), ..., F(P_{best}(t_n))\} \]
5. Update the velocity vector for each particle according to both equations and the following rule:
   \[
   v_j(t+1) = \begin{cases} 
   V_{\text{max}} & v_j(t+1) > V_{\text{max}} \\
   -V_{\text{max}} & v_j(t+1) < -V_{\text{max}} \\
   v_j(t+1) & \text{else}
   \end{cases} 
   \]
   \( V_{\text{max}} \) is usually selected to be half of the length of the search space.
6. Update the position of each particle according to its equation.
7. Update the inertial weight.
8. Let \( t = t + 1 \).
9. Compute the new \( F(x_j(t)) \) until the iteration to be terminated or the least value for \( F \) to be achieved.

Ye and Wang [116] utilized improved particle swarm optimization (IPSO) combined with chaotic map to identify the parameters of the Bouc–Wen model proposed as hysteresis model for a system. Alternatively, the modified particle swarm optimization (MPSO) is used to identify the parameters of Scott–Russell mechanism which is driven by a piezoelectric element [114,115].

### 4.5. System identification by neural network

Neural network is one of the most powerful methods in system identification. It plays a role as a block box to substitute as a model of a plant. As mentioned earlier, the system identification is accomplished in two forms of parametric and non-parametric regimes. Most of the modeling performed by neural network is based on non-parametric identification; meanwhile, neural network is applied as a block box instead of a physical model of the system.

The dynamic neural network usually appears in the form of recurrent network (the network in which the output in current time is dependent to the output and the input of previous time \( s \)) to identify the behavior of a plant.

To apply neural network in hysteresis characterization, Dang and Tan [43] proposed a modified recurrent neural network which is replaced by the Preisach model for a piezoelectric actuator. This network covers a certain range of frequency to create more adaptive model to variation of frequency. This network is trained by set of input and output at wide range of frequency. For further understanding, the summary of the training procedure is rewritten as follows: Output of the neural network is given by

\[
y(k) = \sum_{j=1}^{n} w^j_k H_j(k) 
\]

where \( w^3 \) is the output weight and \( H_j(k) \) is the hidden layer neurons, and \( n \) is the number of hidden layer. Hidden layer is computed as

\[
H_j(k) = f(s_j(k)) 
\]

where \( f \) is the activation function and \( s \) is calculated by the following relationship:

\[
s_j(k) = (1 - \alpha)w^{22}_k H(k - 1) + \alpha \sum_{j=1}^{n} w^j_k x(k) 
\]

where \( \alpha \) is the learning rate and \( x(k) \) is the input signal.
\( \alpha \) is an adjustable factor and its value varies between 0 and 1, \( w^{22} \) is the recurrent weight and \( x(k) \) is the input vector of the network.

This network is trained (the weights are updated) by using the following cost function:

\[
E = \frac{1}{2} (y_r(k) - y(k))^2 = \frac{1}{2} e^2(k)
\]  

(65)

\( y_r \) is the desired output and \( y(k) \) is the neural network output.

The training process is followed by tracking the following derivatives in terms of steepest descent gradient method which is used for updating the weights and biases of the network.

\[
\frac{\partial E}{\partial w^3} = -eH_j(k)
\]  

(66)

\[
\frac{\partial E}{\partial w^{22}} = -w^3 f'(s_j(k))P_j(k)
\]  

(67)

\[
\frac{\partial E}{\partial w^1} = -w^3 f'(s_j(k))Q_j(k)
\]  

(68)

\[
P_j(k) = \frac{\partial S_j(k)}{\partial w^3} = (1 - \alpha)H_j(k-1), \quad P_j(0) = 0
\]  

(69)

\[
Q_j(k) = \frac{\partial S_j(k)}{\partial w^1} = \alpha x(k)Q_j(0) = 0
\]  

(70)

If we assign three different learning rates of \( \eta_1, \eta_2, \eta_3 \), the weights of three layers are updated as follows:

\[
w^1(k+1) = w^1(k) - \eta_1 \frac{\partial E}{\partial w^1}
\]  

(71)

\[
w^{22}(k+1) = w^{22}(k) - \eta_2 \frac{\partial E}{\partial w^{22}}
\]  

(72)

\[
w^3(k+1) = w^3(k) - \eta_3 \frac{\partial E}{\partial w^3}
\]  

(73)

This network is able to predict the output vector one step ahead and also to estimate the shape of hysteresis at different frequencies, accordingly.

One type of neural network as a form of radial basis function (RBF) is applied to characterize the behavior of the Preisach model as a rate-dependent model [117]. The configuration of their proposed system is shown in Fig. 15 and the proposed network is displayed in Fig. 16.

The RBF neural network is a three-layered neural network which contains input layer, hidden layer which comprises a Gaussian activation function and an output layer. The activation function of hidden layer is expressed as

\[
\phi_i = \exp \left( -\frac{|x - c_i|^2}{2\sigma_i^2} \right)
\]  

(74)

\( x \) is the input vector, \( c_i \) is the center of each input vector and \( \sigma_i \) is the width of the Gaussian function.

Also, a neural network can be replaced by the Preisach model [40]. The input vector of the neural network consists of ascending and descending values for voltage input at a certain frequency and the output of the neural network represents the displacement of the actuator.

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Dang and Tan [43] proposed a modified diagonal recurrent neural network or MDRNN. This network is trained dynamically to imitate the behavior of the Preisach model. Furthermore, in some works, the hysteresis nonlinearity is directly identified using dynamic neural networks [118–122].

5. Control strategies in hysteretic systems

The compensation control for nonlinear hysteresis systems has been reported in literature for wide ranges of application from piezoelectric actuations, micro-sliding friction, magnetorheological and magnetic damper, nanompositioning systems, shape memory alloy wires to medical devices with tendon–sheath mechanisms. In general, we can classify two main approaches for such compensator, namely (i) open-loop control with no feedback from the output and (ii) close-loop control with availability of output feedback.

In this section, a review on control approaches for nonlinear hysteresis systems will be presented.

5.1. Feedforward control with open-loop schemes

In some nonlinear hysteresis systems, output feedback is usually unavailable due to size constraints and safety problems, which limits the application of feedback control structures. Therefore, in such applications, only open-loop control is feasible to be implemented.

For this purpose, the inverse models of the nonlinear hysteresis systems, most of the cases, have to be developed prior to the application of the feedforward control. Some methodologies for obtaining the inverse model have been introduced in the previous sections. The developed inverse mathematical model of the hysteresis is subsequently utilized to determine the control input for the nonlinear system.

A general structure of a feedforward compensation for hysteresis system is illustrated in Fig. 17. In a case of trajectory tracking of a mechanical system with hysteresis property, the hysteresis inverse model provides a control input $u_{FF}$, which is represented as a function of a desired trajectory $y_d$, to keep the output $y$ to follow the desired trajectory.

The feedforward compensation approach has been widely implemented in various mechanical systems. In some applications, an inverse hysteresis model is sufficient to deal with the error compensation. However, it is only efficient for low-frequency systems regardless to the creep and vibrations.

As previously discussed, the development of feedforward compensation for hysteresis systems requires two major steps. The first one is to determine the parameters of the hysteresis model, which is based on nonlinear identification methods as presented in Section 4, while the second step is to inverse the hysteresis model to determine the inverse function $H^{-1}$ for controller purpose.

A “conventional” feedforward controller scheme is performed based on the inverse model. Rosenbaum et al. [123] applied an inverse Preisach model-based feedforward for improvement of an accurate control of electromagnetic actuators. Gu et al. [124] used an inverse of a modified asymmetric Prandtl–Ishlinskii-based open-loop compensation for piezoceramic actuators, where nonlocal memory behavior was also taken into consideration. Al-Janaideh et al. [125] utilized a feedforward compensation control scheme for a piezo-micropositioning actuator (PA) based on an exact inversion of the rate-dependent model under the condition that the distances between the thresholds do not increase in time. Similar approaches have been reported in [126–128]. In terms of damping control system, Smith [129] developed a general framework of inverse compensation technique for a class of ferromagnetic transducer including magnetostrictive actuator.
in which the hysteresis model was developed based on the wall theory. In [130,131], an inverse model was proposed for magneto-rheological dampers to enhance force tracking control under the effect of nonlinear hysteresis.

The challenges for the above approaches are the complexity of the inversion problem and the parameter sensitivity that is required to be met. In order to minimize the complexity, a direct inverse model-based feedforward is introduced. In this approach, the compensator does not need any inverse model, which leads to more computational time and complicated modeling. The inverse is directly constructed from the nonlinear hysteresis model. Xu et al. [134,135] used a direct inverse Dahl model in open loop control to compensate the hysteresis in piezoelectric actuators. Rakotondrabe [96] developed a new inverse multiplicative structure to compensate for the hysteresis nonlinearity in a piezoelectric actuator without using any inverse model for the compensator. Creep and inadvertent vibrations were considered in feedforward approach [136–139]. One thing that should be highlighted with regard to the direct inverse approach is that the model is easy to implement and does not require an intermediate step to evaluate the (inverse) hysteresis parameters.

Although the feedforward compensation offers advantages of simplicity and easy implementation, the tracking error is not significantly reduced if external loads or disturbances occur in the hysteresis systems. The reason is that the accuracy of the feedforward control depends on the performance of hysteresis observers which estimate un-measured states, e.g., internal state in Bouc–Wen model of hysteresis. In addition, offline identification of hysteresis parameters results in inaccurate estimation if the dynamics of hysteresis system and unknown effects are considered. To alleviate these drawbacks, a close-loop control is desired and it will be discussed in the next sections.

5.2. Close-loop control with feedback information

In a feedback scheme, typically, hysteresis nonlinearities are treated as uncertainties and the proposed controller will force the output $y$ to follow desired trajectory $y_d$ based on the tracking error between the input and the output. With the feedback scheme as shown in Fig. 18, the tracking performance can be improved in the presence of unexpected disturbances and dynamics of mechanical systems.

At low frequencies, integral control (or PID) has been used to provide a high gain feedback and overcome creep and vibrations in the systems suffering from hysteresis. Lin et al. [140] used a gray relation analyses with tuning PID controller to compensate for the hysteresis in micro piezo-stage. Hsin-Jang et al. [141] utilized optimal PID for improving tracking performance in a piezoelectric micropositioner. Abramovitch et al. [142] tuned PID gains to obtain desired tracking error for atomic force microscopy. Although the conventional PID is able to reduce the error, disturbances and uncertainties are still major challenges in these control approaches. To deal with such drawbacks, nonlinear adaptive controls are potential substitutes. One of the simple approaches in nonlinear adaptive control is the sliding mode control strategy. Li et al. [143] and Yongmin et al. [144] used an adaptive sliding mode control for minimizing the tracking error of a piezo-driven micromanipulator. Nguyen et al. [145] and Lu et al. [146] utilized decentralized sliding mode control for a steel frame structure using multiple magnetorheological damper. Although sliding mode control is proven to be able to satisfactorily reduce the unexpected effects of uncertainties and disturbances, the discontinuous property of the controller causes chattering problems. In order to respond to such challenges, a smooth adaptive control for hysteresis systems is introduced. In [147,148], a smooth robust adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash hysteresis is introduced. Esbrook et al. [149] proposed a nonlinear adaptive control for a commercial nanopositioner using PI model for hysteresis with desirable robustness under loading conditions. Other advanced control approaches such as a state feedback for diamond turning machines [150], optimal control and adaptive neural network methods have been applied to piezoelectric actuators [151]. Although the feedback control scheme offers advantages over feedforward scheme, notable tracking error is still observed as the hysteresis phenomenon by itself is considered as a source of

![Fig. 18. Block diagram of feedback control scheme.](image)

![Fig. 19. Block diagram of the feedforward and feedback control.](image)
disturbance and uncertainty in this scheme. To further improve the tracking performances in hysteretic systems, a combination of feedforward and feedback control is introduced and will be presented in particular in the next section.

5.3. Close-loop control with a combination of feedback and feedforward

Feedforward control scheme is normally implemented when the output feedback is difficult to obtain and the stability problem related to feedback is also difficult to guarantee. To improve the tracking error in hysteretic systems related to modeling imperfection, dynamics, and uncertainties, a combination of feedforward and feedback is preferred. Fig. 19 illustrates a combined structure of a feedforward and feedback compensation scheme.

In order to deal with the tracking error caused by hysteresis modeling, feedback loop is highly recommended. Anti-windup strategy is also considered to deal with chattering in the actuator. In this class of controllers, Chen et al. [128] and Riccardi et al. [152] applied adaptive control algorithms to deal with uncertainties and disturbances occurring in the shape memory which allow applications based on an inverse PI-model with close-loop feedback. Ru et al. [153] eliminated nonlinear hysteresis and creep effects in a piezoelectric actuator using adaptive inverse controller with online estimation of hysteresis parameters. PID controller based on tracking error has been used to eliminate other effects. There are some approaches on the adaptive control compensations for hysteresis systems without presenting experimental validation. Sun et al. [154] overviewed approaches on the robust adaptive control of nonlinear hysteresis systems with unknown model dynamics, uncertainties, and unknown control directions. Rakotondrabe et al. [155] implemented a robust feedforward–feedback control of hysteresis piezo-cantilever under the thermal disturbances. Different approaches in the control problems from piezoactuator to magnetic damper with global stability in the controller designs can be found in the literature. Readers may refer to Tao et al. [156], Zheng et al. [157], and Yangqiu et al. [158] for more details.

6. Conclusion

In this study, the various types of mathematical models of hysteresis were surveyed. This paper was organized to illustrate two classes of hysteresis models, namely the operator-based model and the differential-based model which are introduced to the reader. In the second part of the paper, the author addressed the specific issue like the several methods utilized for parameter estimation of the proposed models. Implementations of the presented models are subsequently discussed in the last section, in terms of controlling mechanical systems that comprise hysteresis nonlinear elements. Two major approaches of controller, namely feedforward and feedback, are detailed in the section. Feedforward controllers are commonly offered in the absence of output feedback in the systems. This approach is proven to be effective; however, it requires a well-developed inverse mathematical model of the system. As an alternative solution, adaptive feedforward controllers are available to improve the controller performance to deal with creeping and aging problems. Combined feedback and feedforward controller schemes are also presented upon the availability of output feedback in the systems. The cascaded controllers are normally implemented when, for instance, stability of the system is difficult to guarantee.

Within this work, the authors encourage readers to get familiarized with several tools required for modeling and identification of nonlinear systems, in particular to those that involve hysteresis property.

References


