Dynamic Analysis of Presliding Friction via Skeleton Method

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Abstract

Friction, which evidently appears in any mechanical systems incorporating parts with relative motion, manifests itself as material nonlinearity. Depending on its application, friction can be a desirable thing or a drawback in the system. In order to optimally utilize or compensate for the frictional effect in a system, characterization of the frictional behaviour is very crucial. The classical Coulomb friction model has been widely used especially in the field of control engineering to compensate for the static force in a system. In spite of its simplicity, the effectiveness of the Coulomb model in capturing the frictional behaviour in the presliding regime is very low and it does not incorporate the hysteresis property that evidently appears. Hitherto, a question might arise whether the Coulomb model sufficient to characterize friction in mechanical systems.

This paper deals with the characterization of equivalent modal parameters, namely the stiffness and damping elements, to effectively represent the dynamic behaviour of mechanical systems subjected to hysteresis friction forces. The analysis is carried out by using the skeleton technique, which employs the instantaneous amplitude and frequency of the excitation input and the response output of the system under a certain forced excitation. Observing the response signal of a free vibration system and performing mathematical manipulation on its analytic signal form, the equivalent modal parameters can be constructed. Subsequently, the dynamic analysis is performed in the equivalent system and as a conclusion the results show good agreement with the dynamic properties of the original system for wide range of excitation.

1 Introduction

Being a complex nonlinear phenomenon, friction is perhaps the most difficult part to identify and compensate for in a mechanical system. It is the result of interaction between one body over or along another and is dependent on many parameters, such as contact geometry, topography, surface materials, presence and type of lubrication and relative motion. Depending on the type of application, friction can be a desirable thing or a drawback in a system. Brakes, clutches, clamps and friction damping are few examples of the first situation, while the friction in slides, bearings and joints is an example of the second. Detailed description of friction phenomena can be found in literature (see, e.g. [1] and [2]).

The classical representation of friction, i.e. the Coulomb model, defines the friction force only for non-zero relative velocity in the sliding regime ($v \neq 0$). The model is impractical for friction compensation at motion stop and reversal, where the effect of stick-slip motion arises. In fact, motion never starts or stops abruptly and micro-sliding displacements are actually observed [1]. Friction has been distinguished into two main regimes: the pre-sliding and sliding regimes. When a contacting body is sliding and moving away from a reversal point exceeding a pre-sliding limit, which is commonly referred to as the sliding regime, the friction force predominantly appears as a function of velocity. On the contrary, when the velocity of the contacting body is decreased and the motion is reversed, the system is entering a pre-sliding regime, and the frictional effects of the mechanism are predetermined also by a nonlocal memory hysteresis function of displacement ([1], [3], [4], [5], [6], [7]).

Regarding to the complexity of the hysteresis phenomena on frictional elements, there are hardly any instances in the literature of the dynamic behaviour analysis of mechanical systems subjected to friction
elements. Among the limited number of the literature, Altpeter [8] presented the dynamic analysis of the moving parts in a machine tool, which are subjected to frictional elements. However, the hysteretic behaviour of the friction is not detailed in the analysis. Seelecke [9] presented a relevant work on the dynamic analysis of shape memory alloys that are well-known to have hysteresis properties. Capecchi and Vestroni [10] studied the steady-state dynamic analysis of hysteresis systems in geo-engineering. A more relevant study is presented by Al-Bender et al. [11], in which they analyzed the dynamic behaviour of mechanical systems comprising rolling elements that exhibit pre-sliding friction phenomena. The analysis is taken by performing the describing function technique [12], where the nonlinear system is linearized by considering only the output (response) at the fundamental frequency component when the corresponding system is excited by sinusoidal input. Consequently, the linearized components (in this case are the equivalent stiffness and equivalent damping) appear as functions of frequency. The approach offers a potential tool to derive the equivalent frequency response function. However, as the equivalent parameters (stiffness and damping) are presented as functions of frequency, a simulation of the response for any arbitrary input/excitation signals is rather tedious to perform.

As an alternative tool to present the equivalent dynamic parameters of a nonlinear mechanical system, the application of the skeleton technique can be utilized by using the relationship between the (nonlinear) natural frequency (-ies) and instantaneous amplitudes of the system. Feldman and Braun [13] reported that every typical nonlinear modal parameter in a system has a unique relationship between the natural frequency and the instantaneous response amplitude when the system is freely oscillating. The relationship is referred to as the skeleton curve. Furthermore, Feldman [14] proposed the method of FreeVib, which observes free vibration response of a nonlinear system and extract its instantaneous amplitude and frequency to construct the modal parameters from the skeleton curve by using the Hilbert transform approach. Extensive studies and practical application of this technique can be found in some papers ([15], [16], [17]). Hitherto, it is obvious that this method is applicable to predict the modal parameters of mechanical systems with geometric nonlinearities\(^1\) not to those with material nonlinearities\(^2\).

This paper deals with the derivation of equivalent (geometric-nonlinear) dynamic parameters, namely the stiffness and the damping, of mechanical systems with (hysteresis) frictional elements. In particular, this paper presents the theoretical dynamic analysis of a mechanical system with rolling friction and application of the skeleton technique to derive the equivalent modal parameters. In the following, Section 2 and Section 3 discuss briefly the theoretical basis of the hysteresis and the skeleton method. Subsequently, Section 4 presents the mathematical model of the dynamic system used in this analysis. In Section 5, the dynamic simulation of the system and the skeleton analysis will be discussed. The equivalent modal parameters will be derived in this section. The resulting parameters are analyzed from the dynamic point of view in Section 6. Still in the same section, the dynamic behaviour of the equivalent system will be compared with that of the original system with hysteresis friction models. Finally, some appropriate conclusions are drawn in Section 7 that can be taken up for experimental validation.

## 2 Presliding Friction

The hysteretic relation between the rolling friction force and the displacement can be characterized by a hysteresis function exhibiting a non-local memory property. The key features of this behaviour, as also illustrated in Figure 1, are: (i) the friction always follows a certain profile in function of the displacement (referred to as the virgin curve), passing through all reversal points, which have to be memorized, but, (ii) every time a loop of the hysteresis curve is closed, the last reversal point can be forgotten. Al-Bender and Symens detailed the behaviour of the corresponding friction in [18].

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\(^1\) In a system with geometric nonlinearity, the change in geometry as the structure deforms causes a nonlinear change of the parameter in the system.

\(^2\) In a system with material nonlinearity, the material behavior depends on the current deformation. This type of nonlinearity is usually associated with the existence of hysteresis behavior in the dynamics of the system.
When a mass subjected to a friction force is displaced, initially the friction force follows an odd function of displacement, the so called virgin curve, \( y(t) \) (see Figure 1). If the motion is reversed at \( x_m \), the trajectory of the friction force follows the ‘flipped and double stretched’ of \( y(t) \) and the system memorizes the reversal point 1. Despite its complexity, the above described behaviour can be modelled relatively simple by using a parallel connection of Maxwell-slip elements also called Jenkin’s elements (see e.g. [19]). The behaviour of a Maxwell-slip element can be represented by two state equations. If the elementary model sticks, it behaves like a linear spring, thus the elementary friction force, \( F_i \), can be modelled mathematically as:

\[
\frac{dF_i}{dt} = \kappa_i v \quad \text{(stick)} \tag{2.1}
\]

with \( \kappa_i \) being the stiffness of each elementary model and \( v \) is the velocity. The elementary model will slip if the friction force of each element reaches the maximum value of the force, \( W_i \), that it can sustain. Beyond this point the elementary friction force equals the maximum force, \( W_i \), obtained by solving the following equation:

\[
\frac{dF_i}{dt} = 0 \quad \text{(slip)} \tag{2.2}
\]

When the motion is reversed, the elementary model will stick, and it will behave again like a linear spring until the friction force reaches the maximum force, \( W_i \). Using a parallel connection of \( N \) Maxwell-slip elements, the total friction force is equal to the sum of all elementary friction force components,

\[
F_f = \sum_{N} F_i \tag{2.3}
\]

### 3 Skeleton Technique

According to Feldman [14], a large number of signals, including vibration signals of systems with geometric nonlinearities, can be converted into analytic signals in complex time. Let us consider the following free vibration equation of a single-degree-of-freedom system:

\[
\ddot{y} + 2h_0(A)\dot{y} + \omega_0^2(A)y = 0 \tag{3.1}
\]

where \( y \) is the response signal, \( m \) is the mass of the system, \( h_0 \) and \( \omega_0 \) are the symmetrical viscous damping and stiffness characteristic of the system, respectively. Feldman shows that Eq. (3.1) can be converted by the Hilbert transform to an analytic signal form:

\[
\ddot{Y} + 2h_0(A)\dot{Y} + \omega_0^2(A)Y = 0 \tag{3.2}
\]
where $Y$ is an analytic signal of the response of the system and can be represented in the form of the combination of slow varying function called envelope, $A(t)$, and instantaneous phase, $\phi(t)$:

$$Y(t) = Y(t) + j\hat{Y}(t) = A(t) \cdot e^{j\phi(t)}$$  \hspace{1cm} (3.3)

Taking the derivatives of the analytic signal, $Y(t)$, and solving the equations of the real and imaginary parts from Eq. (3.3), the dynamic properties of $\omega_0$ and $h_0$ can be written in the expressions of the instantaneous parameters:

$$\omega_0^2(t) = \omega^2 - \frac{A}{A} \cdot \frac{2A^2}{A^2} + \frac{A}{A} \cdot \frac{A}{A}$$  \hspace{1cm} (3.4)

$$h_0(t) = -\frac{A}{A} \cdot \frac{\phi}{2\omega}$$  \hspace{1cm} (3.5)

where $\omega$ is the time derivative of the instantaneous phase $\phi$. The technique is referred to as the FreeVib method.

Ruzzene et al. [20] reported that the envelope and instantaneous frequency extraction of a signal in Eq. (3.3) using the Hilbert-transform technique will introduce errors when the system is highly damped. As an alternative, Staszewski [21] and Tjahjowidodo et al. [22] proposed to implement the wavelet technique to extract the instantaneous properties of a signal. Due to space limitation, interested readers on the wavelet based extraction technique are suggested to refer to [21] and [22].

4 A Mechanical Vibration System Subjected to a Frictional Surface

In this paper, the dynamic behaviour of a mechanical vibration system subjected to a frictional force will be evaluated by means of the skeleton technique. Let us consider a mass supported by a linear spring and resting on a frictional surface as illustrated in Figure 2.

![Figure 2. Schematic of a mechanical vibration system supported by a spring, $k$, and frictional force, $f_{fr}$.

Table 1. Discretized parameters of the Maxwell-slip model.

<table>
<thead>
<tr>
<th>$W_1$</th>
<th>$W_2$</th>
<th>$W_3$</th>
<th>$W_4$</th>
<th>$W_5$</th>
<th>$W_6$</th>
<th>$W_7$</th>
<th>$W_8$</th>
<th>$W_9$</th>
<th>$W_{10}$</th>
<th>$W_{11}$</th>
<th>$W_{12}$</th>
<th>$W_{13}$</th>
<th>$W_{14}$</th>
<th>$W_{15}$</th>
<th>$W_{16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1998</td>
<td>0.1612</td>
<td>0.1300</td>
<td>0.1049</td>
<td>0.0846</td>
<td>0.0682</td>
<td>0.0550</td>
<td>0.0444</td>
<td>0.1433</td>
<td>0.1572</td>
<td>0.1723</td>
<td>0.1890</td>
<td>0.2072</td>
<td>0.2272</td>
<td>0.2491</td>
<td>0.2731</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>$\kappa_2$</td>
<td>$\kappa_3$</td>
<td>$\kappa_4$</td>
<td>$\kappa_5$</td>
<td>$\kappa_6$</td>
<td>$\kappa_7$</td>
<td>$\kappa_8$</td>
<td>$\kappa_9$</td>
<td>$\kappa_{10}$</td>
<td>$\kappa_{11}$</td>
<td>$\kappa_{12}$</td>
<td>$\kappa_{13}$</td>
<td>$\kappa_{14}$</td>
<td>$\kappa_{15}$</td>
<td>$\kappa_{16}$</td>
</tr>
<tr>
<td>0.0358</td>
<td>0.0289</td>
<td>0.0233</td>
<td>0.0188</td>
<td>0.0152</td>
<td>0.0122</td>
<td>0.0099</td>
<td>0.0080</td>
<td>$\kappa_9$</td>
<td>$\kappa_{10}$</td>
<td>$\kappa_{11}$</td>
<td>$\kappa_{12}$</td>
<td>$\kappa_{13}$</td>
<td>$\kappa_{14}$</td>
<td>$\kappa_{15}$</td>
<td>$\kappa_{16}$</td>
</tr>
<tr>
<td>0.2995</td>
<td>0.3284</td>
<td>0.3601</td>
<td>0.3948</td>
<td>0.4329</td>
<td>0.4747</td>
<td>0.5205</td>
<td>0.5707</td>
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</table>

The equation of motion of the corresponding system can be written as:

$$m\ddot{x} + f_{fr}(x, \dot{x}) + kx = F(t)$$  \hspace{1cm} (4.1)

where $F(t)$ is the force input, $x$ is the displacement output of interest, thus $kx$ term is the restoring force, $m$ is the equivalent mass of the mechanical system and $f_{fr}$ is the rolling friction force from the interaction
between two contact surfaces. The reason to support the mass block $m$ with a linear spring, $k$, is to avoid the drift of the mechanism that might occur because of the external force, $F(t)$. Table 1 shows a set of the system parameters corresponding to Eq. (4.1), where the friction force is represented as in Eqs. (2.1-2.3) with 16 elementary models ($N = 16$), resulting in 32 ($= 2 \times 16$) parameters of $W$ and $\kappa$. It has to be underlined here that these parameters are chosen arbitrarily but based upon realistic values. Since the values are not based on physical measurements and only qualitative results are examined, no units are assigned to the numerical values of the different variables.

5 Simulation and Skeleton Analysis

To evaluate the equivalent modal parameters of the corresponding system, some simulations are performed on the Matlab/Simulink environment. In this simulation, the Maxwell-slip friction model is implemented in an S-function block written in C language to allow fast computation time. Regarding to the study of the FreeVib method, the simulated system is freely oscillating by prescribing a non-zero initial velocity, $v_0$. Setting different initial condition will result in a different section of equivalent (nonlinear) stiffness and damping function. Therefore, in order to have complete pictures of the equivalent dynamic parameters, the simulation has to be performed repetitively by using varying initial conditions.

As for the initial simulation, $v_0 = 20$ is assigned as the initial velocity value to the system and the displacement response obtained from the simulation is depicted in Figure 3 together with the envelope function obtained using an extraction technique. Due to the superiority of the wavelet analysis for the parameters extraction as previously mentioned, all of the envelope extraction in the rest of this paper will be performed using the wavelet analysis.

An evident drawback of implementing the wavelet analysis for instantaneous properties extraction is attributed to the edge effect as also pointed out by Ruzzene et al. [20]. At the beginning and at the end of the signal, the envelope and instantaneous frequency extraction are not exact. Therefore, according to Figure 3, the estimation is exact only within the instance of 7 – 30 second as illustrated in the lower panel.

Utilizing Eqs. (3.4) – (3.5), the equivalent modal parameters can be derived. Plots of the restoring force and the damping force are shown in Figure 4. Please note that the restoring force plotted in the left panel of the figure has been subtracted with the restoring force of the linear spring, $k$. Thus the plot represents the equivalent restoring force of the rolling friction solely.

From the figure, we can see that the restoring force implies the presence of a softening spring effect. This can be easily understood, since at a very small displacement range – when the hysteresis forms a relatively small loop (see Figure 1) –, the hysteresis predominantly behaves as a linear spring as the state is ‘sticking’. In other words, from the Maxwell-slip modelling point of view, most of the elementary models are also in the sticking state. On the other hand, at a small displacement range, assuming a moderate frequency content of the excitation signal, the system will oscillate at a relatively low velocity. Therefore, intuitively, no damping effect will also appear at the vicinity of zero velocity. However, the result of the analysis of the equivalent damping force does not cover the damping value at the low velocity range as seen in the right panel of Figure 4. The lack of this information is attributed to the edge effect in the wavelet analysis, where the damping values at the vicinity of zero velocity correspond to the last part of the signal.

In order to refine the results, the skeleton technique is re-implemented at varying initial velocity values, $v_0$. Seven additional skeleton analyses are performed with different initial velocity values, from $v_0 = 10$ until $v_0 = 100$, where the resulting equivalent modal parameters are presented in Figure 5. Different initial velocity value, or equivalently, different energy of the signal will manifest itself at different region in the equivalent dynamic parameters as shown in the figures. In turn, composing all of partial equivalent parameters, complete sets of those parameters are constructed.
Figure 3. Free vibration signal of the simulated system.

Figure 4. Estimation of the equivalent restoring force (left) and damping force (right) of the rolling friction in the system under study based on the wavelet analysis.

Figure 5. Equivalent restoring force (left) and damping force (right) for the simulated presliding friction.

The left panel of Figure 5 depicts the complete restoring force of the system under study. At low displacement, the equivalent spring exhibits a maximum stiffness ($\approx 5$) as implied by the local gradient of the restoring force. The approximated value comes close to the theoretical total stiffness at low
displacement (\(= \Sigma k\)). However, an accurate local stiffness at very low displacement is nearly impossible to evaluate (due to a reason that will be discussed in the next paragraph). The stiffness subsequently decreases until the restoring force reaches its maximum force at a point, where the virgin curve about to reach its sliding force (\(\pm 1.4\) displacement unit; as shown from the virgin curve plot later in Figure 6).

As for the damping force, from the right panel of Figure 5, we can see that the force is saturating at the force value, which is slightly higher than the nominal sliding force of the virgin curve (\(\pm 1.05\) force unit). One of the reasons to have an over-estimated Coulomb force is due to the absence of the hysteresis at the sliding regime that has to be compensated for. Interesting evidence appears at the vicinity of zero velocity, as can be seen in the inset plot of the same figure, where we can see a dead-zone region within \(\pm 0.05\) velocity range. At low velocity, that is to say, within a small distance after reversal points, the system is primarily characterized by the equivalent stiffness of the friction. Intuitively, one would say that the damping force at low velocity, indeed, will be very low. However, the reason for having the dead-band region in the equivalent damping force is due to the discretization in the Maxwell-slip model.

According to Table 1, the highest element stiffness is associated to the element with the smallest maximum allowable force, \(W\), namely the last element with \(W_{16} = 0.008\). Consequently, when the displacement response is damped and the amplitude is becoming less than or equal to the corresponding limit, \(W_{16}\), the system is switched to an undamped vibration system with a constant stiffness (\(= \Sigma k + k = 10\)) and no equivalent damping element acting on the system. Thus, the system is freely vibrating with certain (velocity) response amplitude, which corresponds to the dead-band limit of the equivalent damping parameter as shown in the right panel of Figure 5. This is also the reason, why the local stiffness on the equivalent stiffness parameter at very low displacement is also relatively difficult to estimate.

![Figure 6. The equivalent stiffness in comparison to the virgin curve.](image)

In particular, Figure 6 shows the positive part of the constructed equivalent stiffness together with the virgin curve of the rolling friction element. The figure confirms the previous proposition that immediately after its reversal point, the system is predominated by the total stiffness of the hysteresis elements. Subsequently, the (equivalent) damping is taking its part and the (equivalent) stiffness effect is becoming less compared to that of the virgin curve.

### 6 Dynamic Analysis

In order to analyze the dynamic behaviour of the system and its equivalent modal parameters, the frequency response function including the higher order forms will be evaluated. Prior to analyzing the system, the equivalent modal parameters are fitted into continuous functions. Considering the behaviour of the equivalent parameters, a combination of exponential function and gamma function is proposed:

\[
F_\gamma(x) = \sum_{i=1}^{N} a_i \cdot (1 - e^{-\gamma x}) + \frac{c}{\Gamma(d \cdot x^d)}
\]  

(6.1)
where $F_2$ represents the restoring force or damping force and $\chi$ is the input variable, i.e. displacement or velocity. The exponential term can be expanded to $M$ series to capture the shape/profile of the restoring force, while $\Gamma$ is the gamma function to refine the shape. The resulting fitted functions are shown also as bold curves in Figure 5, while the optimized parameters for both equivalent dynamic properties, using $M = 2$, are tabulated in Table 2.

Table 2. Parameters of fitted equivalent dynamic properties of the system with dry friction

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restoring force</td>
<td>0.9108</td>
<td>0.7944</td>
<td>-0.6822</td>
<td>0.1123</td>
<td>0.5321</td>
<td>1.2749</td>
<td>0.9575</td>
</tr>
<tr>
<td>Damping force</td>
<td>1.8756</td>
<td>0.4044</td>
<td>-0.5568</td>
<td>1.3146</td>
<td>0.6020</td>
<td>1.5213</td>
<td>0.3157</td>
</tr>
</tbody>
</table>

Subsequently, the frequency response functions are evaluated by performing sinusoidal force at different frequencies and amplitudes and analyze the responses. To have a direct comparison with the original system, the simulation will also be carried out on the original system with the Maxwell-slip model. In addition, a simplified model incorporating a Coulomb friction element represented by a signum function with 1.05 break-away force is also simulated as the third case. The simulations consider range of frequency from 0.1 Hz to 1 Hz, in which the equivalent natural frequency is covered.

The first column of Figure 7 shows the first three orders of the absolute frequency response function (AFRF) obtained from the original system. The higher order AFRF’s present the harmonic response of the system on the corresponding order. For example, the second order AFRF at frequency $f_a$ represents the output of the system at the frequency $2 \times f_a$ as a response of an excitation input at frequency $f_a$. Similarly,

3 The analyses are taken without using the linear spring as considered in the previous sections. The reason is to emphasize more the frictional effect on the dynamic results.

4 The excitation amplitudes for all cases in the simulation are 0.1, 0.2, 0.5, 0.8, 1, 1.5, 2, 5, 10 force unit.

5 An AFRF presents the frequency content of the respective response without dividing them by the amplitude of the excitation. The reason to use the AFRF, instead of the popular FRF, is to show direct pictures of the output level with respect to the amplitude excitation level.

![Figure 7. AFRF's for three different cases. The first column is for the original system, the second column is for the one with Coulomb friction, the third column is for the system with the equivalent modal parameters. Different row shows different order of AFRF.](image-url)
the second and the third column depict the AFRF’s for the Coulomb model and the proposed equivalent model, respectively. The plots demonstrate good agreement between the original case and the equivalent case, while the Coulomb case fails to capture the behaviour of the system under study; especially for the low amplitude excitation cases (please note the arrow in the figure that indicates the increasing amplitude of the excitation signal). Interestingly, the equivalent system is able to mimic the original system even for the low amplitude excitation level.

7 Conclusion

This paper has evaluated and presented the derivation of the equivalence of the mechanical system comprising hysteresis (rolling) friction element to geometric nonlinear elements regarding its dynamic behaviour. After an appropriate study of the hysteresis phenomenon and the skeleton technique, it is shown that the equivalent (nonlinear) modal parameters can be evaluated and constructed. As expected, the equivalent stiffness of the rolling element at very low displacement corresponds to the total stiffness elements of the hysteresis model. Subsequently, the stiffness value is decreasing after it reached its maximum restoring force, which corresponds to the point where the virgin curve reaches its sliding state. From this point onward, the stiffness of the equivalent spring continuously decreases until it loses its stiffness at high displacements.

On the other hand, the equivalent damping force is increasing from a relatively low value until it reaches its maximum force that corresponds to the force the friction element can sustain. One interesting thing occurs at the vicinity of zero velocity when we construct the equivalent damping force via the skeleton technique, where a dead-zone region appears. The effect manifests itself because of the discretization in the Maxwell-slip model that is utilized in the skeleton analysis. In order to overcome the problem, a continuous friction model [23] seems to be a good solution. However, the chirp excitation signal will cause non-closed loop hysteresis that might lead to a stack overflow problem.

Based on the dynamic analyses of the original system, the simplified system with Coulomb element and the presented equivalent system, it is shown that the proposed equivalent parameters offer a good correspondence with the original one. Slight differences occur when the system is excited by relatively low excitation signals, when the non-local memory hysteresis is exposed extensively. In particular, model simplification only by utilizing a piecewise linear function of a damping force as presented by the Coulomb friction function, sometimes is not sufficient to capture the dynamic behaviour of the system satisfactorily. Rearrangement of a (nonlinear) restoring force up to certain extent, sometimes, is necessary to better capture the frictional behaviour.

References


